Topographical surveying

Herbert Michael Wilson

[*Frontispiece.*]

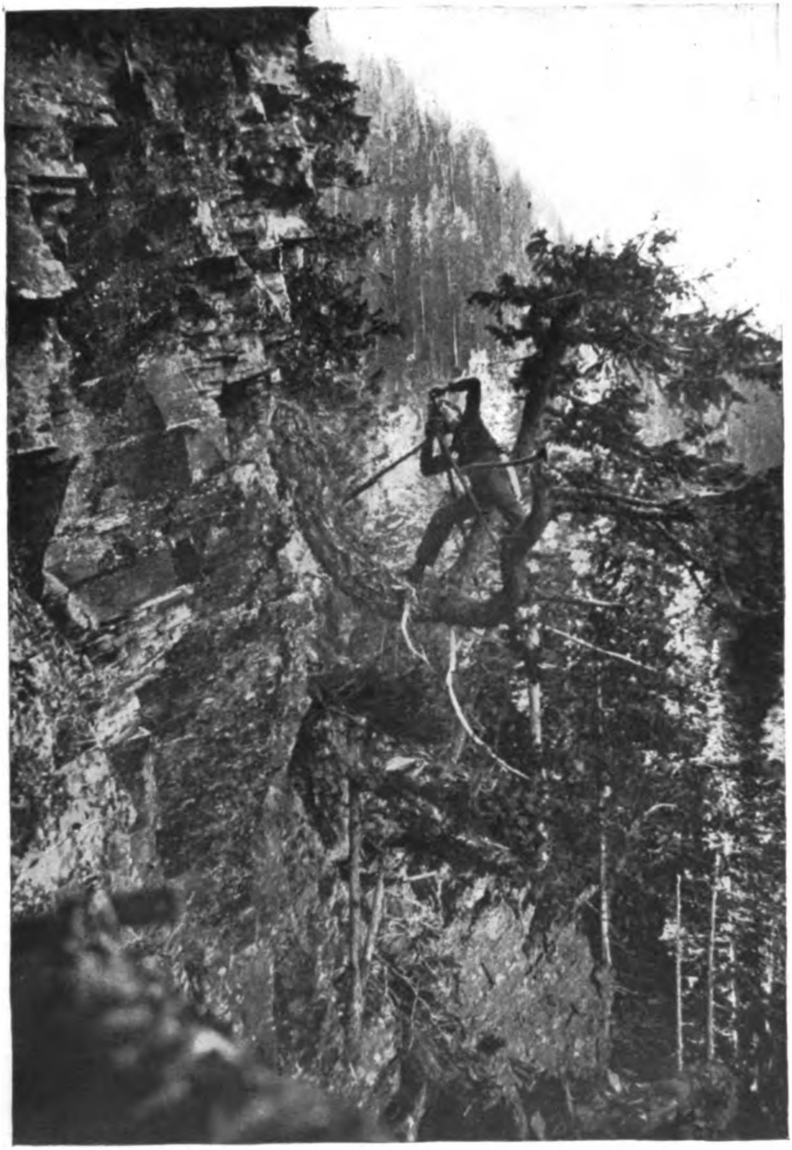

PLATE I.—SURVEYING UNDER DIFFICULTIES.

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TOPOGRAPHIC SURVEYING.

INCLUDING

GEOGRAPHIC, EXPLORATORY, AND MILITARY MAPPING,

WITH HINTS ON

CAMPING, EMERGENCY SURGERY, AND PHOTOGRAPHY.

BY
Nathaniel
HERBERT M. WILSON,
Geographer, United States Geological Survey; Member American Society of Civil Engineers; Author of a "Manual of Irrigation Engineering," etc.

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PREFACE.

THIS book has been prepared with a view of bringing together in one volume the data essential to a comprehensive knowledge of topographic surveying. It has been my aim to cover the varied phases of all classes of surveys which are made with a view to representing on maps information relative to the features of the earth's surface. The methods elaborated are chiefly those which have been developed in recent years by the great government surveying organizations and by such few private corporations as have kept in touch with the most modern practice; but I have endeavored to go beyond these, and, guided by personal experience, to adapt them to the most detailed topographic as well as to the crudest exploratory surveys. The hope is entertained, therefore, that the engineer who may be called upon to conduct an exploratory survey in an unknown region, or to make a detailed topographic map as a preliminary to construction, will find herein descriptions and examples of the methods he should employ, the essential tables for the computation of his results, and hints which will guide in the equipment of his party.

I have sought to avoid any detailed description of those instruments or methods which are elaborated in works on general surveying. The volume is devoted practically to higher surveying, and presupposes a knowledge of all the more elementary branches. At the same time, many of the subjects treated are essentially elementary, and these are briefly described, in order that all the facts which the topographer must know and all the formulas and tables which he must have at hand in the field may be brought together. An effort has been made to present the subject in the most practical form. Accordingly, care has been taken to avoid an elaboration of the mathematical processes by which the various formulas have been derived, as they are to be found in detail in several well-known treatises to which textual reference is made. To give more immediate aid to the working surveyor, examples of the various computations are presented, as are illustrations of the instruments, methods, and resulting maps from surveys actually executed.

The mode of presentation is not that usually followed in such works. Instead of describing the instruments or their uses independently, each is described in that portion of the text in which its employment in field surveying is most prominently mentioned. The tables are not brought together at the end of the volume, but each is placed in that portion of the text which relates to its use. The object is to produce a handy reference-book for use in the field, as well as a text-book for guidance in college instruction. It is believed that, by this arrangement, if a topographer in the midst of his field-work desires information on a special point, it can be found, with accompanying examples and tables, gathered together in one chapter or clearly indicated by cross-references. Again, the method of treatment usually followed in works of this class consists in, first, a description of the astronomic methods on which general map surveys must be based, and then a description of primary triangulation as a basis for the detailed topographic surveys which are finally described. I have reversed this order and have adopted the more natural method of commencing with the simplest operations and advancing gradually towards the most complex and refined. Each subject is treated in the same manner. It is believed that the work has thus been made especially useful to the inexpert topographer and the student.

The volume consists, in fact, of three separate books or treatises: (1) Topographic Surveying, (2) Geodetic Surveying, and (3) Practical Astronomy. The first has been subdivided into three parts: Plane Surveying, Hypsometric Surveying, and Map Construction; and these are preceded by a preliminary characterization of the relations existing between topographic, geographic, and exploratory surveys. This latter distinction is essentially arbitrary, as all are of a kind, and differ only in degree of detail and the consequent speed and generalization in procuring the field results. The general subject of Geodetic Surveying has been subdivided into Terrestrial Geodesy and Astronomic Geodesy, and the treatment of these differs but slightly in method of arrangement from that usually pursued. Part VII is devoted to such practical hints as it is believed will essentially aid those who have the organization and command of camping parties.

I am especially indebted to the courtesy of Professors Ira O. Baker, J. B. Johnson, and John F. Hayford for the use of numerous electrotypes and plates from their well-known works on surveying and geodesy; and to the Secretary of the American Society of Civil Engineers for electrotypes of illustrations in articles by me. I am also indebted to Messrs. W. & L. E. Gurley, Young & Sons, and G. N. Saegmüller for electrotypes of instruments illustrated in their catalogues. I have used freely the excellent Manual of Topographic Methods of the U. S. Geological Survey, written by Mr. Henry Gannett; in a few instances I have copied verbatim examples contained therein, and I desire to express appreciation of his courtesy, and of that of the Director of the U. S. Geological Survey in extending this privilege. To the latter I am also indebted for an opportunity to procure the colored illustrations published herewith, which were printed from the admirable copper-plates of the U. S. Geological Survey. Spe-

cifications and several illustrations of tents and other camp equipage were obtained through the courtesy of the Quartermaster-General of the U. S. Army. For much in the chapter on Photography I am indebted to Lieut. Samuel Reber's Manual of Photography and to E. Deville's Photographic Surveying.

Finally, I desire to express appreciation of the assistance I have received in editing manuscript and proof from many coworkers on the U. S. Geological Survey, more particularly from Messrs. W. J. Peters, S. S. Gannett, and E. M. Douglas on the subjects of geodesy and astronomy; E. C. Barnard and A. H. Thompson on topographic surveying; C. Willard Hayes and G. K. Gilbert on topographic forms and definitions; N. H. Darton on photography; and to Mr. W. Carvel Hall for assistance in reading proof. Two lists of works of reference are published, on pages 490 and 809, in which are cited the titles of all those works to which the reader is referred for further details. From nearly all of these some example or illustration has been obtained.

H. M. W.

WASHINGTON, D. C., Feb. 22, 1900.

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PART I.

TOPOGRAPHIC, GEOGRAPHIC, AND EXPLORATORY SURVEYING.

CHAPTER I.

KINDS OF MAP SURVEYS.

1. Classes of Surveys.—Surveys may be grouped under three general heads:

1. Those made for general purposes, or information surveys.
2. Those made for jurisdictional purposes, or cadastral surveys.
3. Those made for construction purposes, or engineering surveys.

Information surveys may be exploratory, geodetic, geographic, topographic, geologic, military, agricultural, magnetic, or hydrographic. Geodetic surveys are executed for the purpose of determining the form and size of the earth. They do not necessarily cover the entire surface of the country, but only connect points distant from each other 20 to 100 miles. Topographic and geographic surveys are made for military, industrial, and scientific purposes. To be of value they must be based upon trigonometric or triangulation surveys, but not necessarily of geodetic accuracy.

The mother map, or that from which all others are derived, is the topographic map. This is made from nature in the field by measures and sketches on the ground. It is the original or base map from which can be constructed any variety of maps for the serving of separate purposes. The historian may desire to make a map which will indicate the places upon which were fought great battles, or on which are located the ancestral estates of historic families. The geologist may desire to indicate the location of certain rock formations. The promoter of railways or other engineering works may desire to represent the route of his projected road or the location of city water-supplies or real-estate subdivisions. For these several purposes the topographic or base map furnishes the original data, or foundation, on which can be indicated, in colors or otherwise, any special class of information.

Cadastral surveys define political and private property boundaries and determine the enclosed areas. Such surveys are executed for fiscal and for proprietary purposes, and their value depends upon the degree of accuracy with which they are made. A cadastral survey is not necessarily based upon triangulation and may be only crudely executed with compass and chain. To thoroughly serve its purpose, however, it should be based on geodetic work of the greatest refinement. It does not necessarily cover the entire area enclosed, but only points and lines which mark the boundaries.

Engineering surveys are executed in greater detail than any of the above. They may preferably follow some of them and are preliminary to the construction of engineering works. They are conducted with such detail as to permit the computing of quantities of materials to be moved and the exact location of the various elements of the works which are to be constructed. Engineering surveys may be made for the construction and improvement of military works, as forts, navy yards, etc.; for constructing routes of communication, as roads, electric lines, canals; for reclamation of land, as irrigation and swamp surveys; for the improvement of natural waterways, as river and harbor surveys; or for the improvement of cities, as city water-supply and sewage disposal.

2. Information Surveys.—All surveys have a twofold purpose:

1. To acquire certain information relative to the earth; and

2. To spread this among the people.

The acquirement of the information is the field survey. The dissemination may be in the form of manuscript, illustrations, or sketch maps, as in the case of exploratory surveys; of a map only, as in the case of topographic surveys when the map embodies the whole result; or it may be a combination of the two, as in the case of geographic surveys.

In addition to the above primary classes of information surveys are the numerous minor differences in the method of field-work, including the instruments used, the degree of care in obtaining the information, and the mode of recording the results in notes or maps. The instrumental work of *exploratory surveys* is usually of the crudest and most haphazard kind, the observations having to be taken and the notes recorded incidentally and by such means and at such time as the primary necessities of the expedition, those of moving forward over the route traversed, will permit. Moreover, from the necessity of the circumstances such surveys are rarely homogeneous, never covering completely any given area; else they would cease to be exploratory. Being disconnected, they are fixed from time to time with relation to the earth by such astronomic observations as will frequently check the interrupted route surveys in relation one to the other.

Topographic and geographic surveys differ essentially from exploratory surveys, but from each other only in minor details of scale, degree of representation of relief, and the note taken of the sphericity of the earth. *Topographic surveys* are generally executed on so large a scale and with such care and detail that account need rarely be taken of the sphericity of the earth in plotting the resulting map, and they are therefore based on geodetic data only as they merge into geographic surveys. Moreover, all important natural and artificial features may be represented on the resulting map because of its large scale.

Geographic surveys merge imperceptibly, on the one hand, into topographic surveys, as the scale of the latter becomes so small and the area depicted on a given map sheet so large that the shape of the earth must be considered. On the other hand, they may be plotted on so small a scale and the relief be depicted by such approximate methods that they merge imperceptibly into exploratory surveys, being practically of the same nature as the latter excepting that they cover a given area in its entirety.

3. **Topographic Surveys.** — A topographic map is one which shows with practical accuracy all the drainage, culture, and relief features which the scale of representation will permit. Such scale may be so large and the area represented on a given map sheet be so small that the control for the field surveys will be procured by means of plane and not of geodetic surveying. On the other hand, the scale may be so small and the area represented on the given map sheet so large as to require control by geodetic methods.

The mistake is often made of assuming that a topographic map is special and not general. It is general, as it is not made for the purpose of constructing roads and highways, though it becomes a very valuable aid in their projection; nor is it made for the purposes of reclaiming swamp-land or irrigating arid land, but it furnishes general information essential to a preliminary study and plan for their improvement. The outcome of a topographic survey being a topographic map, it should be judged by the map, and the map should be judged by the manner in which it serves the general purpose. Above all, of two maps or works of any kind made for the same purpose and serving that purpose equally well, that the cheaper one is the better is a well-recognized principle of engineering.

In the prosecution of a general topographic survey only such primary points should be determined geodetically as are essential to the making of the map. About one such point per one hundred square miles is a fair average for a one-mile to one-inch map. Such spirit-level bench-marks should be set and recorded as are obtained in carrying bases for levels over the area under survey. On the above scale about one bench to five square miles is a fair average.

4. Features Shown on Topographic Maps.—The features exhibited on topographic maps may be conveniently grouped under the three following heads:

1. The hydrography, or water features, as ponds, streams, lakes.

2. The hypsography, or relief of surface forms, as hills, valleys, plains.

3. The culture, or features constructed by man, as cities, roads, villages, and the names printed upon the map.

In order that these various features may be readily distinguishable and thus give legibility to the map, it is usual to represent the hydrography in blue, the relief in brown, and the culture in black. In addition to this, wooded areas may be indicated in a green tint.

The object of a topographic survey is the production of a topographic map. Hence the aim of the survey should be to produce only the map; neither time nor money should be wastefully expended in the erection or refined location of monuments; the demarkation of public or private boundary lines; or the establishment of bench-marks beyond such as are incidental to the work of obtaining field data from which to make the map. The erection, location, and description of boundary marks is the special work of a property or cadastral survey. The erection, description, and determination of monuments and bench-marks as primary reference points is the work of a geodetic survey. The determination of many unmarked stations for map-making purposes is the work of a topographic survey.

5. Public Uses of Topographic Maps.—For the purposes of the *Government or State* good topographic maps are invaluable. They furnish the data from which the congressman or the legislator can intelligently discover most of the information bearing directly upon the problem in hand, and they give committees great assistance in their decisions as to the need of legislation. If a River and Harbor bill is before Congress, or a bill relating to State Canals before the Legislature, by an inspection of such maps the slopes of the country through which the canal is to pass or in which the improvements are to be made may be readily ascertained. The sources of water-supply for a canal or river may be accurately measured on such a map and their relation to the work in hand intelligently ascertained.

If the *War Department* of the Government or the Adjutant-General's Office of the State desires to locate an arsenal, encampment ground, or other military work, or, above all, if it is to conduct active military operations in the field, such maps serve all the preliminary purposes of the best military maps. With the addition of a very little field-work during war times, such as the indication of ditches, fence lines, outbuildings, etc., on the mother or topographic map, a perfect military map may be obtained.

For the *Post-office Department* or private stage, express, or telephone companies, such maps furnish the basis on which an accurate understanding can be had of contracts submitted for star or other routes for carrying the mails or packages. As these maps show the undulations of the surfaces over which roads pass, their bends and the relative differences in length, the difficulties in travel on competing roads can be readily ascertained from them.

The *Land Departments* of the Government and State can discover on such maps not only the outlines of the property under their jurisdiction, but its surface formation. *Forestry Boards* can see indicated upon these maps the outlines of the various wooded áreas, besides the slopes of the lands on which these woods are situated, their relation to highways of transportation, railways, or streams, and the slopes to be encountered in passing through the woods on these highways.

The *Legal department* of the Government or State finds these maps of service in discussing political or property boundary lines, in ascertaining within what political division crimes are committed, or individuals reside with whom the officers of the law desire to communicate. It is difficult to see how any systematic or economical plan of road improvement can be advantageously made without the knowledge of existing grades, the physiography of the district through which the roads pass, and the location of quarries, which such maps present.

The whole system of making successive *special surveys* or maps for every new need is one of the most wasteful in our present public practice, nor can it be otherwise until one survey shall be made that answers all important official uses. The amount of money that has been expended in making small maps of numerous cities and villages would have mapped, on a general scale, many times the area of the country. Even when we have these special maps they do not fully answer the purpose for which they were intended, as they only show the small area included within the immediate plan of operations. The value of a stream for economic purposes cannot be fully ascertained by an examination of the stream at the point from which it is to be used, but the drainage basin from which it derives its supply should be surveyed, and its area and slopes be known. A good topographic map not only shows the relations between the natural and artificial features in the immediate neighborhood under consideration, but it shows the relations of these to the surrounding country.

6. Degree of Accuracy Desirable in Topographic Surveys.—It is difficult to set any standard for the amount of detail which the topographer must sketch on his map, or the amount of control which must be obtained for the checking of this detail. A topographic map may be so made as to serve many useful purposes and yet be almost wholly a sketch, scarcely controlled by mathematical locations. The same territory may be mapped on the same scale with little improvement in the quality of representation of topographic form and yet the work be done with such detail and accuracy and such amount of control as to make it useful for all practical purposes to which its scale adapts it.

With these facts clearly in view, it is evident that explicit *instructions to the topographer* are a practical necessity. Unlike any other surveyor the topographer must use his own judgment or be guided by instructions regarding the amount of time and money to be spent in obtaining detail and control, since the latitude permissible in mapping the same territory on the same scale varies greatly according to the uses to which the map is to be put. Such instructions should interpret the significance of scale and contour interval, and should cover the technical details of operations as found applicable to conditions and locality (Art. 7). They should also fix the method of making and preserving field-notes. There are a variety of methods of survey, of instruments, and of records which are generally applicable to any case, yet to the expert topographer there is practically only one best way for each, and this can be decided only after he has inspected the country or has otherwise acquired knowledge of its characteristics.

The scale and mode of expressing relief (Art. 191) must be fixed as well as the contour interval, if contours are employed, in order that all the data necessary for the construction of the map on this scale may be obtained. The methods and instruments should be stated in order that those best suited to the conditions may be selected in the beginning. The mode of record should be fixed in order that there may be uniformity in the results brought into the office, provided there are various topographers working on the same area. Such instructions are to the topographer what specifications are to the contractor, yet they cannot quite carry the force of law because of the unforeseen exigencies which may arise and which require departure from fixed instructions in accordance with the best judgment of the topographer.

In topographic mapping it is sometimes desirable to make hasty *preliminary or reconnaissance maps* of a region in order that some information of the area may be immediately obtained. Such maps are practically sketches covering an extensive area and without adequate framework of control, yet they contain most of the information required in the early development of the region. The error has too frequently been made of giving such maps the ear-marks of accuracy by representing the relief by numbered contours. In this they are misleading. Contours indicate precision and should justly be taken as accurate within the limits of the map scale. As has been aptly stated by Mr. J. L. Van Ornum, "accuracy is expected where exactitude is shown, and the conclusion is just that inaccuracy in representation is inexcusable." Where for any reason the desired accuracy cannot be attained for lack of the proper control, the resulting map is merely a sketch-map, and relief should be indicated not by contours but by hachures or by sketched contours; that is, lines in contour form, but disconnected and unnumbered. Such sketch-maps are useful as representations of topographic form, but are valueless as base-maps on which to plan great public improvements, the inception of which is so closely connected with topographic surveys.

A topographic map well executed is, to quote Captain George M. Wheeler, "the indispensable, all-important survey, being general and not special in character, which underlies every other, including also the graphic basis of the economic and scientific examination of the country. This has been the main or principal general survey in all civilized countries. The results of such a survey become the mother source or map whence all other fiscal examinations may draw their graphic sustenance." Such a characterization of a topographic survey can apply only to one accurately made and on which every feature represented is as accurately shown as the scale of map warrants.

In *planning a topographic survey* the controlling factor of the scale must always be kept clearly in mind, as this is the ultimate criterion which decides the method of survey and the amount of time and money to be expended in its execution. The underlying law of topographic mapping is that applied to other engineering works, namely, no part of the construction, nor any part of the survey, should be executed with greater detail or at greater expense than will permit it to safely perform the duties for which it is intended. Thus, in mapping an extended area, traverse methods alone for *horizontal control* are insufficient unless performed with the greatest exactitude. The primary triangulation on which such a survey is based should be no more accurate than will permit of plotting the points with such precision that they shall not be in error by a hair's breadth at the extreme limit to which the triangulation is extended. The secondary triangulation should be executed with only such care as will permit of plotting without perceptible error on the scale selected and within the limits controlled by the nearest primary triangulation points. Simpler methods of securing horizontal control may be adopted for the minor points within the secondary triangulation, and these methods, be they by plane-table triangulation (Chap. IX) or by traverse (Chap. X), need be nothing better than will assure the plotting of the result without perceptible error and within distances controlled by the nearest secondary triangulation points. Finally, minor details may be obtained by the crudest methods of traverse, range-finding, pacing, or sketch-board (Arts. 81, 116, 95, and 61), providing that the distances on the map over which such methods are propagated shall be so small as to warrant their not being perceptibly in error within the limits of the controlling points of the next higher order.

As with the horizontal control so with the *vertical control*, no more time should be expended or precision attempted in determining elevations than are necessary to obtain the data essential to the mapping of the relief accurately to the scale limit. Where relief is to be represented by contours of a small interval and on a large scale, or where the slopes of the country are gently undulating or comparatively level, the leveling must be of a high order that the contours may be accurately placed in plan. In country having slopes as gentle as 5 to 10 feet to the mile, a difference of a few feet in elevation may mean that distance in error in the horizontal location of the contour if the elevations are not determined with accuracy. On the other hand, in precipitous mountain country much less care is necessary in the quality of the leveling, since a large error in vertical elevation may be represented in plotting by the merest fraction in horizontal plan. For a large contour interval in country of moderate slopes less accuracy is essential in the determination of the elevation. For contours of 20 feet interval errors of elevation varying from 5 to 20 feet or more may be made, depending upon the steepness of the slope and the consequent nearness in horizontal plan of one contour to the next. The same ratio applies to greater contour intervals. Therefore the methods pursued in determining the elevations should begin with a careful framework of spirit-leveling (Art. 129), and the amount of this should be only so great as to insure that the dependent levels of less accuracy shall not be so far in error as to be appreciable for the scale and contour interval selected and for a given slope of country. Based on these spirit-levels rougher elevations by vertical angulation with stadia (Art. 102) or by trigonometric methods (Art. 159) may be employed, and tied in between these may be elevations by aneroid (Art. 174), the latter being checked at intervals sufficiently frequent to assure that the resulting elevations shall not introduce appreciable errors in the location of contours.

The same rules should apply to the frequency with which vertical control points are determined. These should be so close together for the scale of the map and for the contour interval selected that in connecting them by eye in the course of the sketching no error appreciable on the scale shall be introduced. Any map, the best obtainable, is but a sketch controlled by locations. No one would undertake to determine the elevation and horizontal plan of every point on a contour line. Control positions on contours are only determined with sufficient frequency to insure comparative accuracy in connecting them. Bearing on this same point is the fact that such connection by sketching can undoubtedly be done with greater accuracy on the plane-table board with the terrane in view than from notes platted up in office or from photographs or profile drawings.

Where relief is to be represented by hachures or broken sketch contours, precision in absolutely fixing the vertical element is of the least moment. It is generally desirable in making such maps to write approximate altitudes at prominent points, as stream junctions, villages, or mountain summits, but the chief desideratum is relative differences in elevation in order that the number of the sketched contours and their frequency, or the degree of density of the hachuring, may give an index to the amount of relief.

7. Instructions Relative to Topographic Field-work.—The following instructions are those issued by the Director of the United States Geological Survey for the guidance of topographers in the field:

1. At least two primary triangulation points or a primary control line

should be platted on each atlas sheet previous to commencing field-work.

2. On each atlas sheet, in addition to primary levels, such other elevations should be obtained instrumentally that aneroids need never be left without a check elevation for distances exceeding five inches. These control elevations may come from profiles of railroads, spirit-levels, or from vertical angulation.

3. Plane-table triangulation must be conducted on the large sheets to a scale of 1:45000 or 1:90000, and it is desirable that as fast as intersections are obtained by the topographer the vertical heights of stations and intersected points should be computed.

4. In conducting plane-table triangulation, as many hilltops, churches, houses, and other notable features should be intersected and located as is possible, in order to furnish the basis of connection with the traverse work, while gaps or passes and salients on ridges should also have their positions and elevations determined as far as possible from the plane-table stations.

5. Secondary topographic control must precede topographic sketching and the filling in of minor details of the map. To this end, on the inauguration of field-work, the topographer in charge should so arrange spirit-leveling as to control a given portion of the area under survey. He should execute plane-table triangulation or run main controlling traverse lines prior to commencing detailed topographic sketching. His principal assistant may meantime be engaged in running additional control traverses, accompanied by vertical angulations or levels, and his traversemen may be engaged on minor traverses.

6. Field sheets must be as few in number and as large as the character of the topography will permit, and all main control must be adjusted thereon; this to be done before filling in of minor detailed sketching is commenced. These minor details may be obtained by traverse on separate sheets, but must be transferred to and adjusted on main field sheets at once, so that no uncompleted spaces shall be left on them in the field.

7. For fifteen-minute sheets sketching will be done only by the chief of party or competent assistant and on completely adjusted control; or from sketch stations, only when traverse control and elevations have been previously obtained, that all may be adjusted as sketching progresses.

8. All prominent objects within a reasonable distance of the traverse should be sighted with the alidade, and lines drawn to them from the various traverse stations. Sights should also be taken to such objects as may be located by the topographer. If necessary, the traversemen should occasionally ascend low hills to check aneroid elevations with those obtained by triangulation and to sight to other hills, churches, etc., for purposes of orientation.

9. All permanent buildings, other than barns or sheds grouped about a dwelling-house, must be indicated by the traverseman on his plane-table sheets and transferred by the topographer to his field sheet before sketching. The outlines of wooded areas must be shown on the sketch sheets.

10. Elevations must be adjusted between check points previous to sketching. When this is done sketching may be commenced, and must be in continuous contours. Occasional breaks may be permitted, but simple "sketch" work must not be done.

11. The topographer in charge will be responsible not only for the quality of the topographic work, but also for the quality and management of the spirit-leveling done under his direction and for the location and marking of the bench-marks, each of which he should endeavor to examine personally. Permanent bench-marks must all be located on the resulting topographic map and the elevations written thereon.

12. Only so much of the field sheets must be inked in the field as can be done with sufficient care to permit of their being accepted as final drawings, and of their being directly photographed or photolithographed (excepting where land-survey plats are used as field sheets). Accordingly, only such inks should be used as will photograph readily, namely, mixed burnt sienna or Higgins' orange for contours, Higgins' black for culture, and only mixed Prussian blue with about one-tenth burnt sienna or orange for drainage.

13. A full record must be made on the title-page of each note-book, stating character of work, locality, atlas sheet, and date of record; also, name of topographer and maker of notes. A similar record is to be made on field and traverse sheets, with the addition of scale and contour interval.

14. Plats, on a large scale, should be made or obtained at all villages and cities, showing the streets and houses in detail.

15. The determination of names of streams, mountain peaks, villages, and other places of note should receive particular attention. They should be obtained and recorded with authorities, so as to ascertain local usage and spelling.

8. Elements of a Topographic Survey.—From a constructive point of view a map is a sketch corrected by locations. The making of locations is geometric, that of sketching is artistic. However numerous may be the *locations* they form no part of the map itself, serving merely to correct the sketch which supplies the material of the map. Every map, whatever its scale, is a reduction from nature and consequently must be more or less generalized. It is therefore impossible that any map can be an accurate, faithful picture of the country it represents. The smaller the scale the greater the degree of generalization and the farther must the map depart from the original. The larger the scale the smaller the area brought together on a given map, and the less it appeals to the eye which grasps so extended a view of nature. There is, however, for the purposes of making information maps, a scale which is best suited to every class of topography, and the best result only will be obtained by selecting the relation of horizontal scale and contour interval which fits the particular topography mapped.

By far the most important work of topographic mapping is the *sketching* (Arts. 13, 15, 17, and 193), and this should be done by the most competent man in the party—presumably its chief. He should not only sketch the topography because of his superior qualifications for that work, but also because the party chief is responsible for the quality of all the work, and only in the sketching, which is the last act in map-making, has he full opportunity for examining the quality of the control and of the other elements of the work executed by his subordinates. The map-sketcher is therefore the topographer, and it is in the matter of generalization or in the selection of scale and the amount of detail which should be shown for the scale selected that the judgment of the topographer is most severely tested. This is the work in which the greatest degree of proficiency can only be attained after years of experience. The topographer must be able to take a broad as well as a detailed view of the country, and to understand the meaning of its broadest features that he may be able best to interpret details in the light of those features (Chap. VI). It is only thus that he can make correct generalizations, and thus that he is enabled to decide which detail should be omitted and which preserved in order to bring out the predominant topographic features of the region mapped.

The *correctness of the map* depends upon:

(1) The accuracy of the locations;

(2) Their number per square inch of map;

(3) Their distribution;

(4) The quality of the sketching.

The first three of these elements defines the accuracy of the map, and the greatest accuracy is not always desirable because it is not always economical. The highest economy is in the proper subordination of means to ends, therefore the quality of the work should be only such as to insure against errors of sufficient magnitude to appear upon the scale of publication (Art. 6). The above being recognized, it is evidently poor economy to execute triangulation of geodetic refinement for the control of small-scale maps, and, providing the errors of triangulation are not such as are cumulative, the maximum allowable error of location of a point on which no further work depends may be set at .01 of an inch on the scale of publication.

The second condition, the *number of locations* for the proper control of the sketching, is not easily defined. It depends largely upon the character of the country and the scale and uses of the map. Any estimate of it must be based on unit of mapped surface and not of land area. For rolling or mountainous country of uniform slopes or large features (Fig. 4), from $1\frac{1}{2}$ to 3 locations and 2 to 5 inches of traverse per square inch of map should, with accompanying elevations, be sufficient. On the other hand, in highly eroded or densely wooded country (Fig. 34) as many as 3 to 6 locations and 5 to 10 inches of traverse, per inch of map may, with accompanying elevations, be necessary to properly control the sketching. Again, in very level plains country (Fig. 6) less than one location and but 2 to 5 inches of traverse, with accurate elevations, will suffice to furnish adequate control.

The same is true of the third element of accuracy, the *distribution of locations*. In rolling, hilly country of uniform slope the control should be obtained chiefly at tops and bottoms and changes of slope. The same is true of heavy mountains, excepting that all summits and gaps on ridges must be fixed, as well as all changes in side slopes and a few positions distributed about the valley bottoms. In flat plains the positions determined should be locations on the contours themselves and at changes in their direction. In highly eroded regions locations of all kinds should be distributed with considerable uniformity, so as to control every change of feature or slope.

The fourth element, the *quality of the sketching*, depends wholly upon the artistic and practical skill of the topographer —in other words, upon his possession of the topographic sense, which may be described as his ability to see things in their proper relations and his facility in transmitting his impressions to paper. This is by far the most important and difficult requirement to meet, and one which takes a longer apprenticeship on the part of the topographer than all the others combined.

CHAPTER II.

SURVEYING FOR SMALL-SCALE OR GENERAL MAPS.

9. **Methods of Topographic Surveying.**—Three general methods of making topographic surveys have usually been employed in the past:

First, traversing or running out of contours by means of transit, chain or stadia, and level;

Second, cross-sectioning the area under survey with the same instruments; and

Third, triangulation of the territory under survey with such minuteness as to get a sufficient number of vertical and horizontal locations to permit of connecting these in office by contour lines.

All three methods are slow and expensive, while the first two are unfitted to the survey of large areas, because of the inaccuracies introduced in linear or traverse surveys.

A *fourth method, and that which this book is designed to expound*, is that always employed by the United States Geological Survey as well as to a lesser degree by several other American and European surveys. It is fitted to make topographic maps for any purpose, on any scales, and of any area. This consists of a combination of trigonometric, traverse, and hypsometric surveying to supply the controlling skeleton, supplemented by the "sketching in" of contour lines and details by a trained topographer. In this method the contour lines are never actually run out nor is the country actually cross-sectioned. Only sufficient trigonometric control is obtained to tie the whole together, the minor control between this being filled in: first, in the most favorable triangulation country almost wholly by trigonometric methods; second, in less favorable triangulation country by traverses connecting the trigonometric points.

There are two general methods of making a contour topographic map which have been aptly styled the "regular" and the "irregular." These might be respectively called the old and the new. The old or *regular method* includes the surveying and leveling of a skeleton work of controlling traverse or triangulation and the cross-sectioning of the terrane into rectangular areas, the outlines of which are traversed and leveled. In addition the leveled profiles and traverses are continued between this gridironing at places where important changes of slope occur, and finally the survey and leveling of flying lines or partial sections is extended from each station. By this method the base of each level section or the contour line or line of equal elevation is determined by setting the instrument in position where this level line intersects the profile, and using the telescope as a leveling instrument with its cross-hairs fixed on a staff at the height of the optical axis, a line is then located by tracing successive positions of a stadia rod or by locating by intersection successive points on the level line, and a line drawn through these points locates the contour curve. In addition, parts of several level sections are plotted from one station by intersection on, or location of a staff, and by the determination of its height above or below the instrument by vertical angulation. In this mode of topographic surveying pegs are usually driven at regular intervals and their heights determined by spirit-level and vertical angulation.

The new or *irregular method* of topographic surveying consists in determining by trigonometric methods the position and height of a number of critical points of the terrane and connecting these by traverses and levels, not run on a cross-section or rectangular system, but irregularly, so as to give plans and profiles of the higher and lower levels of the country, as ridge summits or divides and valley bottoms or drainage lines, such lines being run over the most easily traversed routes, as trails or roads. With the numerous positions and heights determined by the triangulation, and on these traverses as controlling elements, contour lines are sketched in by eye and by the aid of the hand-level on a plane-table with the country in constant view. This is the method now generally employed by expert topographers, and the work is so conducted that the development of the map proceeds with the survey of the skeleton and rarely necessitates the return to a station when once occupied. Moreover, it calls for the location of less points and the running of fewer traverses and profiles, and these over more easily traveled routes, than the former method. It is therefore more expeditious, cheaper, and the resulting map is a better representation of the surface, as it possesses not only the mathematical elements of instrumental location, which in the old method are arbitrarily connected in office, but also the artistic element produced by connecting the lines of equal elevation in the field, with the country at all times immediately before the eye.

10. Geological Survey Method of Topographic Surveying.—In average country, favorable for triangulation, comparatively clear of timber and well opened with roads, a skeleton trigonometric survey (Chap. IX) is made, by which the positions and elevations of all summits are obtained, as well as the horizontal positions of a few points in villages or at road crossings, junctions, etc. This constitutes the upper system of *control* (Fig. 1). Below and between this is a network of road traverses (Chap. X) supplemented by vertical-angulation (Chap. XVII) or spirit levels (Chap. XV) for elevations, and these follow the most easy routes of travel, not cross-sectioning the country in the true sense, but following all the lower lines or stream bottoms, as well as the gradients pursued by roads (Fig. 2). Between these two upper and lower sets of control points there are therefore many intermediate ones obtained by road traverses, and the topographer, by observation from the various positions which he assumes and with the knowledge he possesses of topographic forms, sketches the direction of the contour lines. These are tied accurately to their positions by the large amount of mathematical control already obtained, supplemented by additional traverses or vertical angles where such are found wanting. (Art. 162.)

The *instruments* used are as various as are the methods of survey employed; the essential instruments being the plane-table and the telescopic alidade (Chap. VII), which invariably replace the transit (Art. 85) or compass (Art. 91), so that all surveying is accompanied by mapping at the same time, and there is no tedious and confusing plotting from field-notes to be done later in office. Nor are any of the salient features of the topography of the region lost through neglect to run traverses or obtain positions or elevations, all omissions of this kind being evident from an inspection of the map while in process of construction. The distances are obtained by triangulation with the plane-table (Art. 73) and by odometer measurements (Art. 98), supplemented off the roads by stadia measures (Chap. XII) or in very heavily wooded country by chaining (Art. 99) and pacing (Art. 95).

The *underlying principles* of this method of topography are, first, a knowledge of and experience in various methods of surveying, and a topographic instinct or ability to appreciate topographic forms, which is acquired only after long practice; and, second, a constant realization of the relation of scale to the amount of control required and methods of survey pursued; no more instrumental work being done than is actually required to properly control the sketching, and no more accurate method being employed than is necessary to plotting within reasonable limits of error. Thus, where trigonometric locations (Chap. IX) are sufficiently close together, crude odometer traverses (Art. 98) or even paced traverses (Art. 95) can be run with sufficient accuracy to tie between these with inappreciable errors. Where trigonometric locations are more distantly situated, the spaces between them must be cut up by more accurate traverses, as those with stadia (Chap. XII) or chain (Art. 99), these again being gridironed by less accurate odometer or paced traverses. Again, a primary system of spirit-leveling (Chap. XV) or accurate vertical triangulation (Chap. XVII) is employed only for the larger skeleton, these elevations being connected by less accurate vertical-angle lines or flying spirit-levels, and these again by aneroid (Art. 176), each method being employed in turn so that the least elements of control obtained may still be plotted well within a reasonable limit of error in horizontal location of contour line.

Finally, *speed and economy* are obtained by traveling the roads and trails in wheeled vehicles or on horseback, at a rapid gait from instrument station to instrument station; the slower process of walking being only resorted to where roads and trails are insufficient in number to give adequate control and view of every feature mapped.

11. Organization of Field Survey.—The party organization and the method of distributing the various functions of topographic surveying among the members of the party must necessarily differ with the scale of the map and the character of the region under survey. The work involved in making a topographic or geographic map may comprise four operations:

First. The location of the map upon the surface of the earth by means of astronomic observations.

Second. The horizontal location of points, which is usually of three grades of accuracy: primary triangulation or traverse; secondary triangulation or traverse; and tertiary traverse and meander for the location of details.

Third. The measurement of heights, which usually accompanies the horizontal location and may be similarly divided into three classes, dependent upon their degree of accuracy.

Fourth. The sketching of the map.

If the area under examination is small or the scale be of topographic magnitude, the first of the foregoing operations may be omitted, when the topographic party will have (1) To determine the horizontal positions of points; (2) To measure the heights of these points; and (3) To sketch in the map details as controlled by the horizontal and vertical locations so procured.

Where map-making is executed for geographic or exploratory purposes and on a small scale in open triangulation country, as that in the arid regions of the West, the skilled force may consist of only the topographer in charge. Where the map scale is increased up to topographic dimensions or the country is hidden from view by timber or because of its lack of relief, the topographer may be assisted by one or more aides whose functions will be variously performed according to the conditions of the country.

12. Surveying Open Country.—In making a geographic map on scales varying, say, from one-half mile to four miles to the inch in open, rolling, or mountainous country suited to triangulation, all sketching and the execution of the plane-table triangulation (Chap. IX) or other control should be done by the topographer in charge. He may be aided by one to three assistants according to the speed with which he is able to work and the difficulties encountered by the assistants in leveling (Chap. XV). It is assumed that the topographer has a fixed area to map, and that within this area he is in possession of the geodetic positions (Chap. XXIX) of two or more prominent points and the altitude of at least one.

With the positions of these points platted on his plane-table sheet (Art. 188) he proceeds, as outlined in Article 54, to make a reconnaissance of the area for the erection of signals and to locate prominent points on summits and in the lower or drainage lines of the country by plane-table triangulation (Art. 73). Meantime, one assistant may be running lines of spirit-levels (Chap. XV) for the control of the vertical element, while one or two assistants are making odometer (Art. 98) or stadia traverses (Chap. XII) of roads or trails for the control of the sketching and the mapping in plan of the roads and streams. This preliminary control executed, the topographer adjusts to his triangulation locations the traverses run by the assistants (Art. 81), and writes upon them in their proper places the elevations determined by leveling or or vertical angulation (Chap. XVII).

In Fig. 1 is shown a typical *triangulation control sheet*, the directions of the sight lines being indicated so as to show the mode of derivation of the various locations. The stations and located intersection points are numbered in order to show the sequence in which they were procured. The *traversing* executed for the same region is illustrated in Fig. 2, from which it will be seen that merely the plans of the roads with their various bends, stream crossings, and the houses along them were mapped. Hill summits and other prominent objects to one side or other of the traversed route were intersected (Art. 84) in order to give additional locations and to facilitate the adjustment of the traverse to the triangulation. The closure errors of the various traversed circuits are shown, and an inspection of these makes it clear that in every case the errors in traverse work are so small as not to affect the quality of the control, because the adjustment of the traverses by means of points on them which are located by the plane-table triangulation will distribute the errors in the various road tangents in such manner as to make them imperceptibly small on the resulting map. The product of such adjustment is shown on Fig. 3, which is the base on which the topographer begins his sketching. On this *sketch sheet* are the locations obtained by him in the execution of his plane-table triangulation, the traverses as adjusted to this control, and

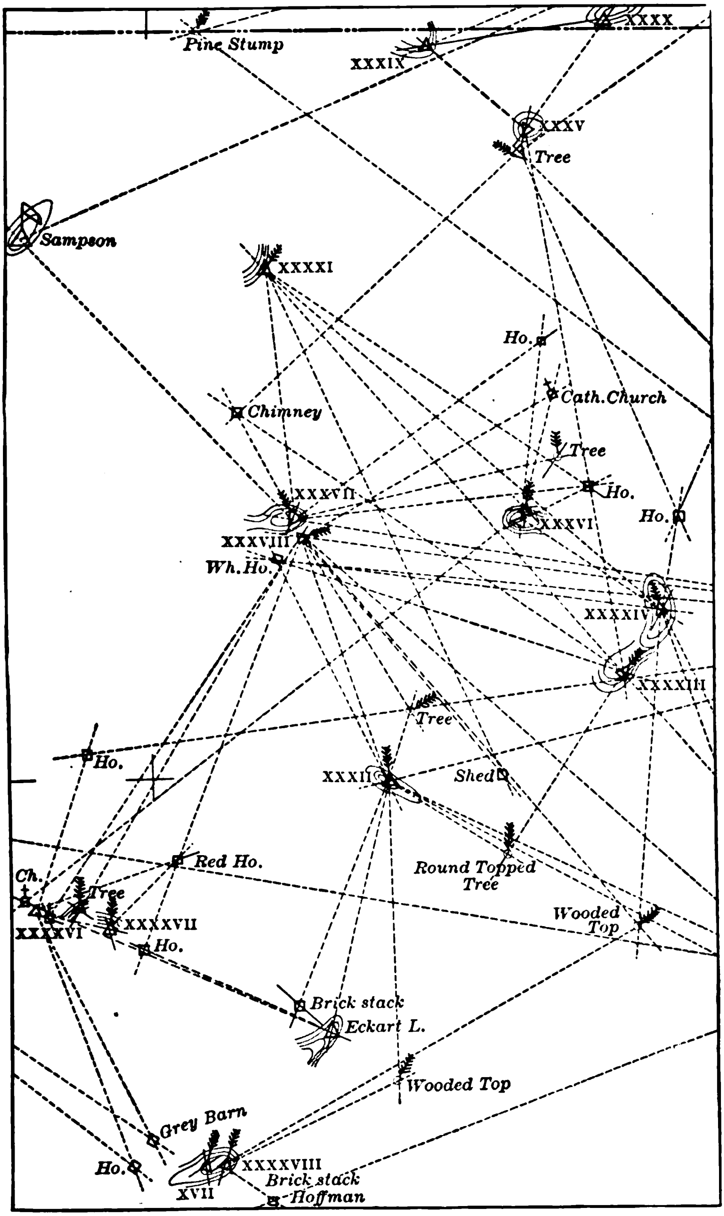


FIG. 1.—DIAGRAM OF PLANE-TABLE TRIANGULATION. FROSTBURG, MD.
Scale $\frac{1}{11800}$.

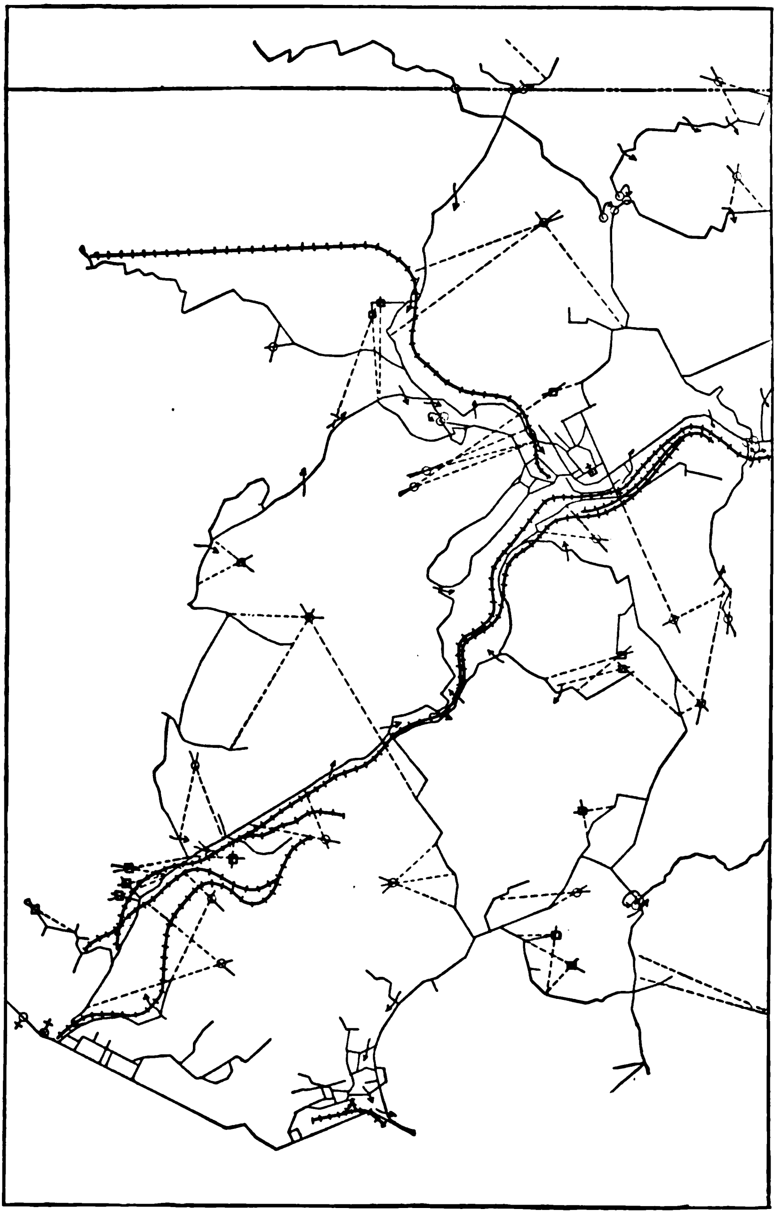


FIG. 2.—ROADS, HOUSES, AND LOCATIONS RESULTING FROM TRAVERSE.
FROSTBURG, MD.
Scale $\frac{1}{62500}$.

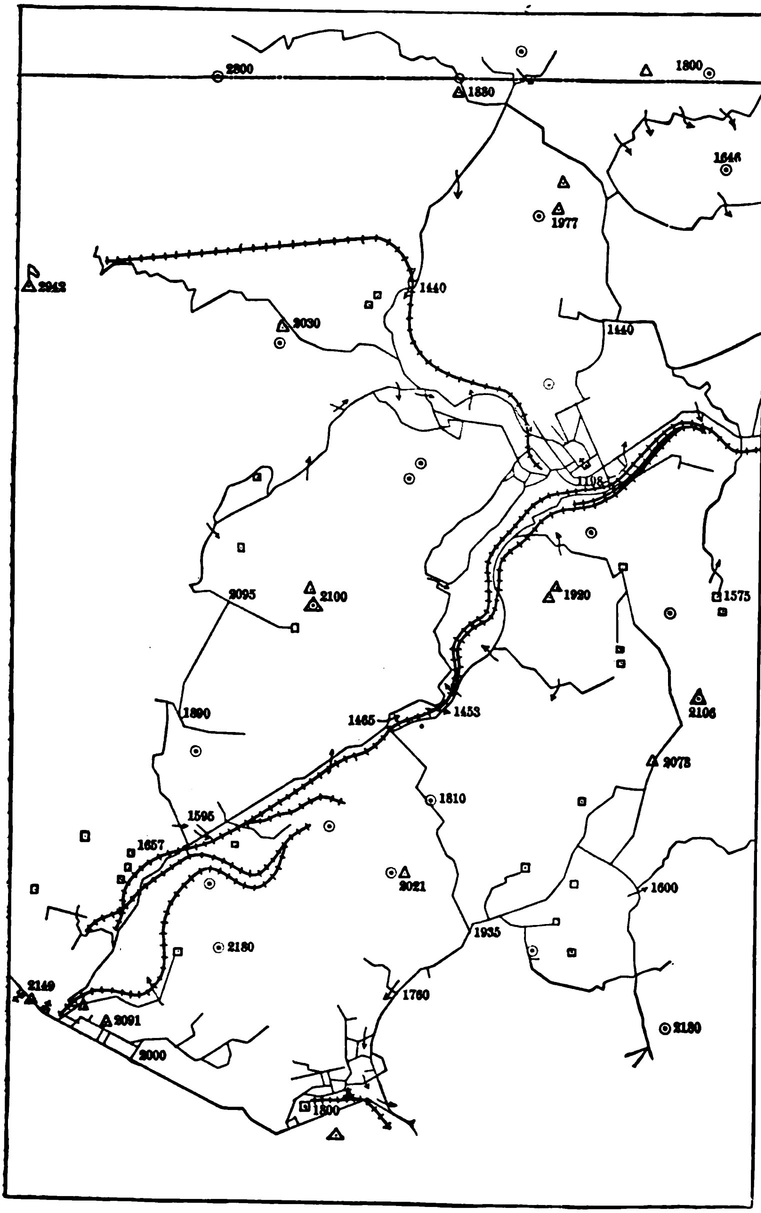


FIG. 3.—ADJUSTED SKETCH SHEET. FROSTBURG, MD.
Scale $\frac{1}{62500}$.

elevations from vertical angulation or spirit-leveling written in their appropriate places.

If the work be the making of a topographic map on scales larger than those above described, and the country be still of the same topographic character—namely, open, with salient summits,—a system of control similar to the above must in like manner first be executed by the development of plane-table triangulation and the running of control, traverse, and level lines. But the after-work of sketching the map will be conducted in a different manner than for the smaller scales, because of the greater detail required, the shorter distances to be traveled by the topographer in performing the work, and his consequent nearness to the various features which he is to map.

13. **Sketching Open Country.** — Having the control platted on the sketch sheet as shown in Fig. 3, and where roads are sufficiently abundant to cut up the map with traverses so near one to the other that the topographer may not have to sketch more than one-half to one inch to either side of his position, the sketching of the topography proceeds as follows:

Taking the sketch sheet on a board in his lap, the topographer for cheapness and convenience, because of the speed, drives over every road. Where these are not sufficiently near one to the other he walks in between them, pacing distance (Art. 95), and getting direction by sighting fixed objects, while he sketches the plan of the contour lines (Art. 193) as far as he can safely see them to either side of his path. This operation is performed by setting out from such fixed points as a road junction, a located house, or a stream crossing, the position of which is platted on his map and the elevation of which is known. Adjusting the index of his aneroid at the known elevation (Art. 176), he drives along, keeping the platted direction of the road parallel to its position on the ground and marking on the map the positions at which the various contours are crossed by his route. Thus, if his contour interval be twenty feet, at every change of twenty feet as indicated by the barometer he stops, and, knowing his position on the map either by reference to bends in the roads, houses, or by having counted the revolutions of his wheel from a known point, he glances along the trend of the slopes to one side or the other, following by eye the level line of his contour, and this he sketches in horizontal plan upon the map. At first he may be aided in this by a hand-level (Art. 156), but as he acquires skill with practice he is able to estimate the position and direction of the contour line to either side with great accuracy, and finally to interpolate other contours above and below that on which he is placed with such precision as not to affect the quality of the resulting map by a contour interval.

The aneroid being an unreliable instrument, he must not drive more than two or three miles without checking it at a well-determined elevation. This he is usually able to do at houses, or hill-summits, or other points the positions of which have been determined by his prior control. If he is not able so to check his aneroid, he hastily sets up his plane-table, reads with the telescopic alidade a few vertical angles (Art. 162) to hilltops or houses in sight and the elevations of which are known, and, with these angles and the distances which he can measure from his position to the points sighted as shown on the adjusted control, he is at once able to compute the elevation of his position (Art. 161) within two or three feet and thus check his aneroid. At the same time he is in similar manner able frequently to throw out other elevations by sighting from the position thus determined to houses or summits near by which may have been located by the traverse (Art. 84), and the heights of which he determines now from his angulation. The topographer thus sketches the whole area assigned him, not only mapping the contours, drainage, political boundaries, and other topographic features, but also checking the positions of houses and summits and the directions and bends of roads and streams as located by the traverseman (Fig. 4).

Where the hills are more prominent and the slopes bolder and steeper, the topographer sketches these from his various view points by *interpolating contours* between the located control points. With the sketch-board in his lap or on the tripod and approximately oriented, looking about in various directions at hill-summits, houses on slopes, spurs, etc., which may with their elevations be platted on his map, he first sketches in plan the streams and drainage lines as well as the directions of slopes. Then he sketches the position of contour lines about such control points as summits, salients, and his own position. With these as guides he is then unable to go astray in the interpolation of the intermediate contours which complete the map of the area immediately about him.

The sketching of the topography for *large-scale maps* differs rather in degree than in kind from the above. The large-scale map covering as it does a relatively small area, the topographer is not under the necessity of traveling with such speed as to necessitate his using wheeled conveyance. At the same time the largeness of the scale places the roads at much greater distances apart on the map and necessitates his traveling between these to greater extent. It will thus be seen that the scale and the ability to travel over the country work harmoniously one with the other. For the smaller geographic scales the roads are so close together on the map as to afford sufficient control and sufficient number of viewing points for sketching the topography of the average open country, whereas on large-scale topographic maps these roads are in plan much farther apart, but the time consumed in walking between them is a comparatively small item because of the decrease in the distances to be covered.

In sketching a large-scale map the topographer will have about the same relative amount of primary control as above

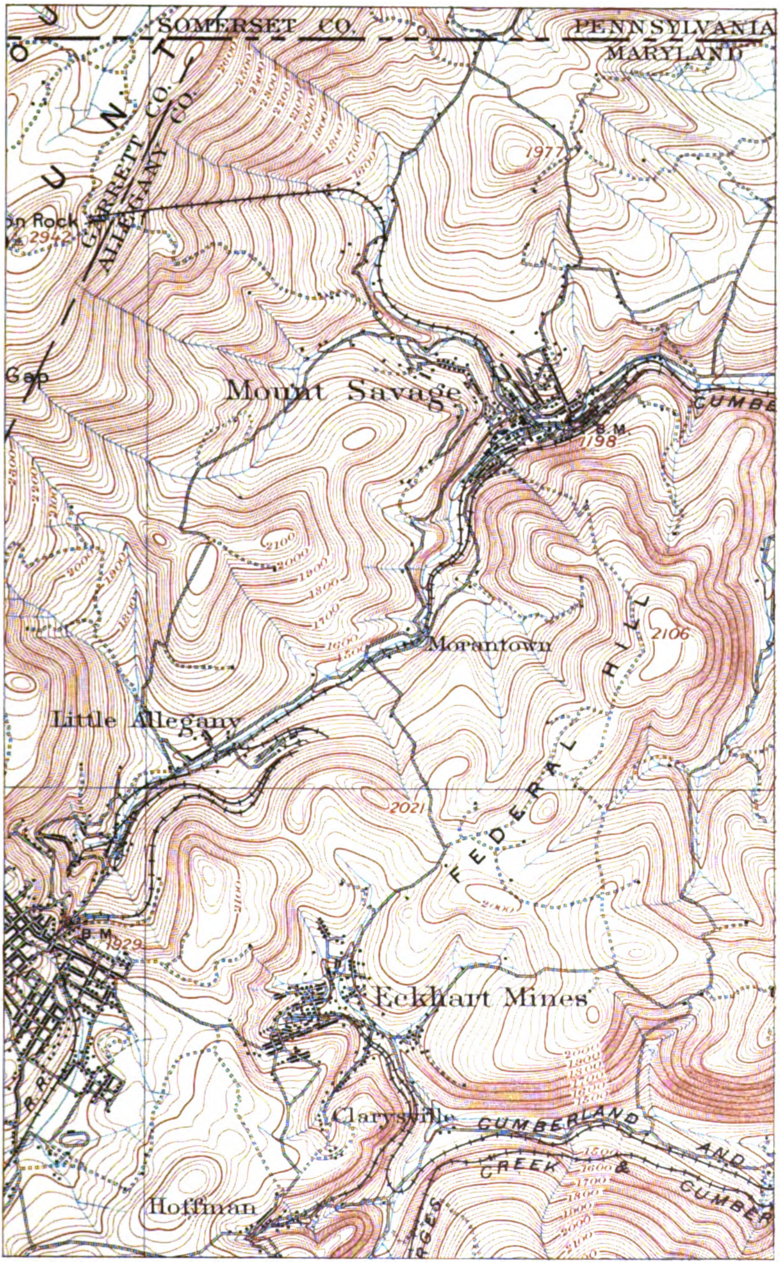


FIG. 4.—COMPLETED TOPOGRAPHIC MAP, FROSTBURG, MD.
Scale 1 to 62,500. Contour interval 20 ft.

indicated. Starting out with some known point and on foot, accompanied by one or more stadiamen, he sets up and orients his plane-table, and, having considerable areas to fill in on his map between his present position and his next recognizable natural feature, he posts the stadiamen at convenient changes in the slope of the country or at houses or trees or bends in the streams, and drawing direction lines and reading distances by stadia to these positions he obtains additional locations to control the sketching (Art. 101), which is executed as above described. In the progress of this work he not only determines horizontal positions by sighting to the rods held by his stadiamen, but also the vertical positions of the same points (Art. 102). For very large-scale maps and under some conditions the work may be expedited by permitting the assistants to sketch the contours immediately adjacent to their stadia stations, and these sketch notes must be given the topographer at frequent intervals to be transferred to his map. In this manner one topographer may handle from one to three stadiamen, providing he uses judgment in the selection of his and their positions. For smaller-scale topographic mapping the work may be expedited by the stadiamen riding on horseback from one position to another, or even by the topographer himself using this means to get about.

14. Surveying Woodland or Plains.—The securing of control in densely wooded country, as that of the Adirondack region or the woods of Minnesota, Michigan, and of Washington; or the securing of control for very flat plains country, as that of the Dakotas and Nebraska, is accomplished by different means than must be adopted in open triangulation country. Be the scale of the resulting map large or small, the primary control may be obtained most economically either by triangulation or by traverse methods. If the country is *wooded and rolling*, it may be more economical to clear the higher summits or to erect high viewing scaffolds upon them, from which to conduct a skeleton plane-table triangulation. Intermediate positions may be obtained by placing signal-flags in tall trees and locating these by intersection or using them to obtain other positions by resection. With practice the topographer will thus triangulate the most forbidding woods country more expeditiously than it could otherwise be controlled, by taking advantage of every outlook, as a rock on a hillside, a lake, a small clearing for a farm, or by clearing or signaling the commanding summits. He will thus occupy only such points as those just described, locating by intersection (Art. 73) from them the flags on the more wooded and forbidding ones which may be the more commanding positions, and using the latter again for carrying on his work by resection (Art. 74).

In *level plains* or in wooded plateau land the control may of necessity be executed only by traverse methods. In such case where the scale is of geographic dimensions one or two astronomic stations should be determined (Part VI), or for larger scales it may suffice to assume the initial position. From this primary traverse lines should be run (Art. 226) at considerable distance one from the other, depending upon the scale. For the one-mile scale a nearness of fifteen to twenty miles will suffice. For the two-mile scale these primary traverse lines may be double the distance apart; for a large topographic scale a relatively smaller distance, depending upon the map scale; for all scales a distance corresponding to fifteen to twenty-five inches on the map according to the topography.

Between these primary traverse lines others of less accuracy should be run as a secondary control. On these distances should be measured by wheel (Art. 98) when the vehicle can be driven in straight tangents, by stadia (Art. 101) in open irregular country, or by chain (Art. 99) or tape (Art. 97) through underbrush or dense wood. Elevations will be secured in the woods by vertical angulation to stadia (Art. 102) or by spirit-leveling (Chap. XV); in the open or plains by vertical angulation to fixed objects, as the eaves or chimneys or window-sills of houses, the platforms of windmills, etc. (Art. 160), or to the stadia-rod, as well as by spirit-leveling. The secondary traverse is usually executed by the party chief while his assistants are engaged in tertiary traverse for the filling in of topographic details or the procuring of vertical control.

The primary and secondary control having been procured as above, this should be platted on sketch sheets of the customary large plane-table size for open country (Art. 68), and preferably in small detached pieces placed on small boards of about six inches square, where the latter have to be carried through woods and underbrush. These control sheets will be not dissimilar to those described in Article 13, excepting that they will lack the location of points procured by angulation and will consist almost wholly of platted traverse lines. In order that the topographer when sketching may identify these lines on the ground, trees must be frequently blazed in woods when the traverses are being run and station numbers or elevations be written on the blazings.

15. Sketching Woodland or Plains.—With the control platted on the sketch sheet as just described, the topographer in *plains* work starts out and drives over the country much as described in Article 13, traveling over all the traversed roads and checking his aneroid by setting in at known elevations or by angulation to and from buildings and similar objects. As the country is relatively flat, the contour lines are at considerable distances apart in plan, and consequently a very small difference in vertical elevation makes a considerable change in the horizontal location of a contour. Therefore the determination of the vertical element should be of greater relative accuracy, that the resulting map may be correct.

In the *woods* the sketching is executed in an entirely different manner. Little skill is required in the depiction of the topography, as it is impossible to see the country and therefore to sketch it in the ordinary sense. The topographer is limited to sketching that which is directly under foot—in other words, to mere contour crossings—and in order that these may be connected the traverses must be much nearer together, and not only the topographer but his more skillful assistants are all engaged in sketching and traversing at the same time. Starting out with the primary and secondary control as obtained in the last article, the topographer travels over those traversed routes which have been blazed and sketches the contours upon these while his assistants run additional traverses over controlling routes, as along stream beds and ridge crests, and so close together as to completely command all the country under foot. These traverses will be of crude quality, directions being obtained by sight alidade (Art. 62) and traverse-table (Art. 61), and distances by pacing (Art. 95) or by dragging a light linen tape (Art. 97). Each day the topographer must adjust to his control sheet the traverses with accompanying sketching as executed by his assistants. With such a skeleton of topography on highest and lowest lines, i.e., contour crossings of streams and ridges, he can readily interpolate contours for most of the intermediate spaces and, following after his assistants, fill in those places which are not fully mapped.

In the execution of a survey under such conditions the topographer's work is largely supervisory and consists chiefly in the management of the work of his assistants, the adjustment of their sketching, and its inspection as he fills in the details omitted by them. There is little room for them to go astray, because they only sketch that which they walk over. The topographer should invariably reserve for himself the higher ridges, the ponds, and the more open places in order that quality and speed may be obtained by the utilization of his skill in that work which gives some opportunity for sketching at a distance from the traveled route.

16. Control from Public Land Lines.—In the western

United States where the public land surveys have been executed in recent years and with sufficient accuracy to furnish horizontal control, this may come almost wholly from the township and section plats filed in the United States Land Office. The topographer takes into the field paper on which sections and quarter sections are ruled and numbered. On these he writes at the proper section corners the elevations as determined from the primary spirit-levels (Chap. XV). He also indicates on the northern and western margins of each township the offsets and fractional sections as shown on the published land plats (Fig. 5). At some period during the progress of field-work the topographer adjusts the land-line work to positions determined either by primary triangulation (Chap. XXV) or traverse (Chap. XXIII), supplementing this by additional control where necessary.

17. Sketching over Public Land Lines.—With the control sheet prepared as described in the last article, the topographer proceeds to drive over the section lines on which roads have been opened. The control sheet is attached to a plane-table board. Starting from a known section corner, he drives in a straight line down one of the section lines to other section corners, determining his position by counting revolutions of the wheel (Art. 98) and sketching contour crossings as he progresses.

Starting out with a known elevation from spirit-levels (Chap. XV), he determines other elevations as he proceeds by setting up his plane-table at a section corner or opposite a house which he can locate by odometer distance, and reads vertical angles from the point of known elevation to houses, windmills, or other objects in sight (Art. 162), drawing direction lines to them as an aid in their identification (Art. 84). Driving on until he comes to one of these objects and being thus able to locate it on his plane-table, he measures the distance from it to the point from which the angle was taken and at once computes his elevation (Art. 161). Or, setting up his

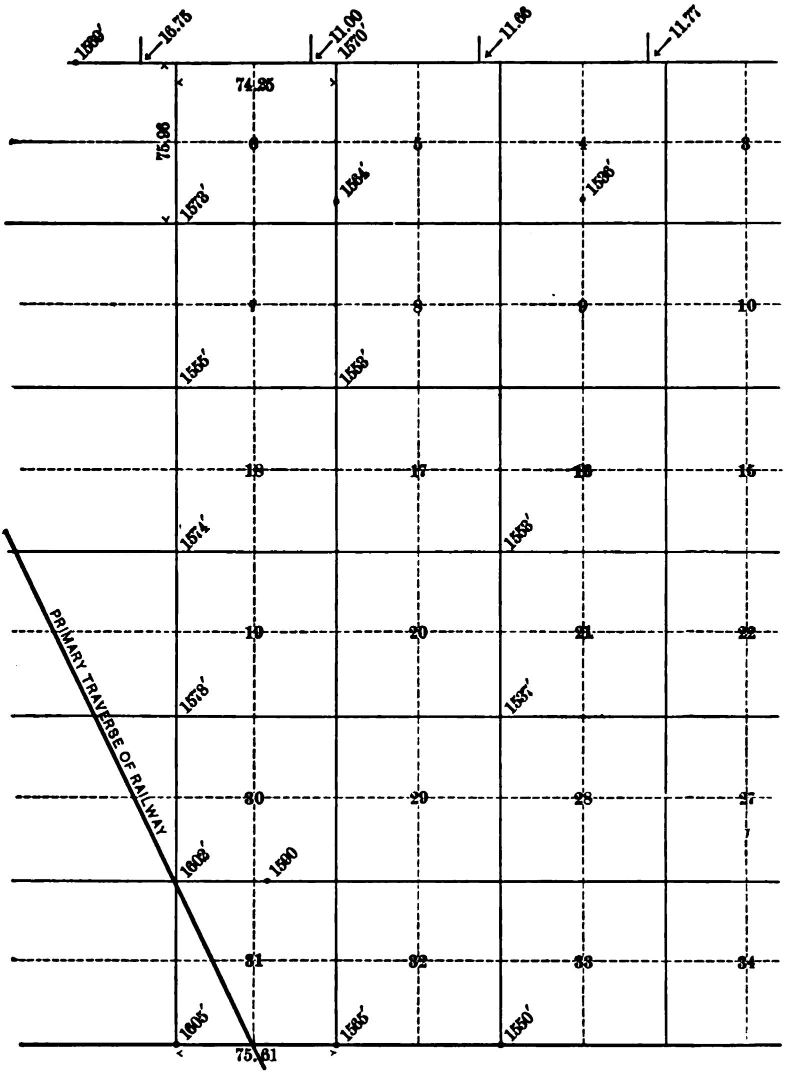


FIG. 5.—LAND SURVEY CONTROL FOR TOPOGRAPHIC SKETCHING.
NORTH DAKOTA.
Original scale 2 inches to 1 mile.

plane-table board from some known position, as determined from his section lines and odometer, he reads vertical angles to houses or windmills, the heights of which have already been determined by vertical angulation, and thus brings down to his present position an elevation by means of the angle read and distance measured on his board. In conducting vertical angulation in this manner care must be taken to sight at some well-defined point, as a platform or top of a windmill, the gable or top of a house or top of door-sill, etc.

As the sketching is a comparatively simple process under these conditions because of the flatness of the terrane, his work may be expedited by permitting his more skillful *assistants to aid in sketching.* In order that he may control their work he drives and sketches over those roads which parallel the roads of his assistants on either side, and in such manner obtains a clear insight into the work performed by them. The assistants may determine elevations either by vertical angulation, as does the party chief, or by aneroid frequently checked, say at distances not exceeding two miles between the better elevations obtained by the topographer. On such a sketch sheet as it comes from the plane-table board (Fig. 6) the roads have been clearly marked over the section lines and additional diagonal roads have been traversed or sketched directly on the plane-table board, controlled by section corners, the outlines of lakes having been obtained by stadia (Art. 101).

Where the topographic map is made at the same time as the subdivision of the public lands, as was the case in the Indian Territory surveys of the United States Geological Survey, the cost of executing the topographic survey scarcely exceeds the cost necessarily involved in making the land subdivision or cadastral survey. The only additional cost in the execution of the topographic survey is that for leveling. Fig. 33 is an example of the cadastral map resulting from such a survey of the public lands. The topographic map of the same region corresponds in appearance almost identically with that shown in Fig. 6, being shorn of the various symbols used on the Land Survey Maps.

18. Cost of Topographic Surveys.—As indicated in Tables I, II, and III, the cost of topographic surveying varies widely with the character of the country, the scale of the map, and the contour interval. Such topographic surveys as are executed by the United States Geological Survey range in cost for maps of a scale of one mile to the inch and 20-foot contour interval, similar to those described for *open country* in Articles 12 and 13, from $5.00 to $8.00 per square mile. Such as are described in Articles 14 and 15, for *plains or woodland*, range in price from $3.00 to $4.00 per square mile for the former to between $15.00 and $30.00 for the latter. The highest-priced work of this kind which can be executed being the woodland survey, and the cheapest country to map topographically being the open plain.

Land-survey country, as that instanced in Article 16, which is a survey of a portion of North Dakota, ranges in cost from $1.25 to $2.00 per square mile, where the topographic map is made on a scale of two miles to one inch and in 20-foot contours. For the same scale and in mountainous country, as that of the South and West, the cost is from $3.00 to $8.00 per square mile.

If any endeavor is made to do work for other purposes than the procurement of a topographic map, as for the determination of land lines or the staking out of canals or railroads, the cost of the survey is at once greatly enhanced. It is this which has added so greatly to the relative cost as shown in the tables cited of some private topographic surveys as well as of the cadastral surveys.

19. Art of Topographic Sketching.—Mr. A. M. Wellington aptly said of topographic surveying that "the sketching of the form of the terrane by eye is truly an art as distinguished from a science, which latter, however difficult it

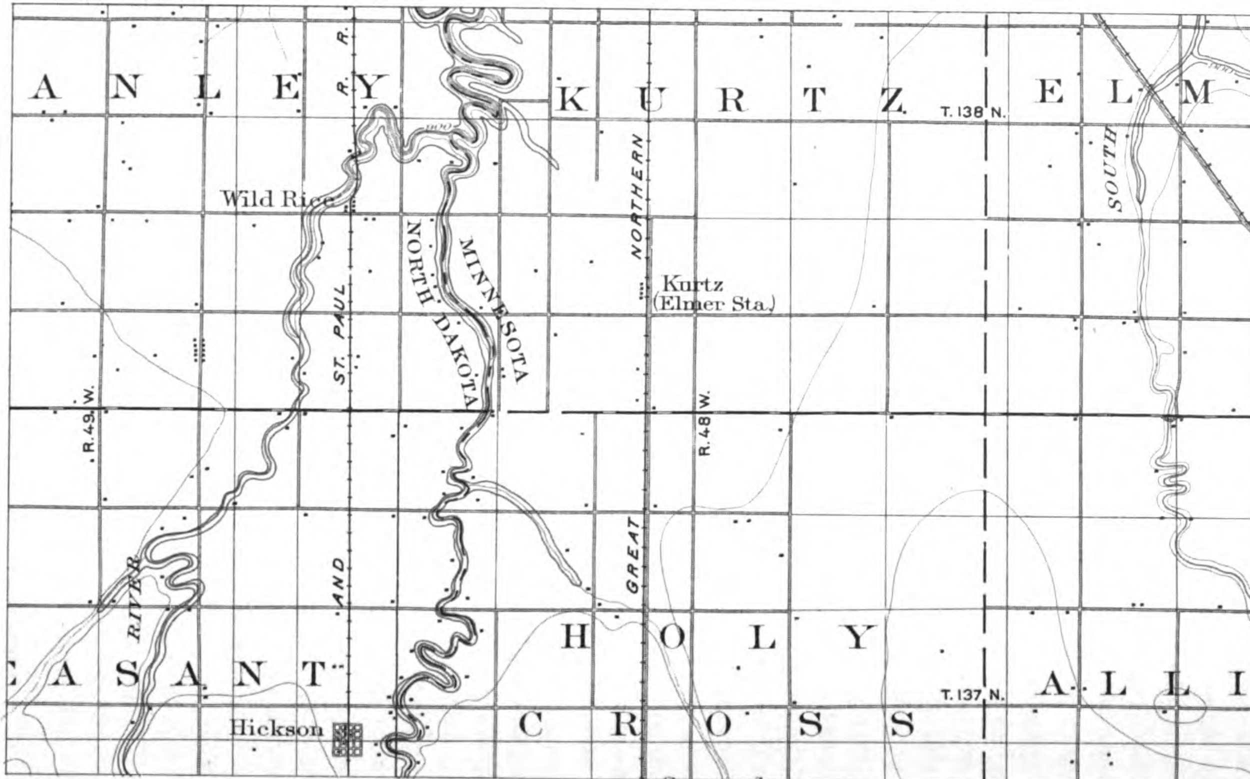


FIG. 6.—TOPOGRAPHIC MAP ON LAND SURVEY CONTROL, NEAR FARGO, N. D.
Scale 1 to 125,000. Contour interval 20 ft.

may be, is always susceptible to rigorous and exact analysis. An art, on the other hand, is something which cannot be taught by definite, fixed rules which must be exactly followed, though instruction may be given in its general principles."

In representing the heights and slopes of a given piece of country by contour lines, every case presents some peculiarities which must be met, as they are presented, by the topographer's own resources. No hard-and fast limit of minuteness of detail can be previously fixed. The scale chosen for the topographic map limits this to a certain extent, but its exact limits must be set by the topographer's own experience and good judgment, that he may *discriminate between important and trifling features;* those which are usual and common to the region being mapped, and those which are accidental or uncommon, and which should therefore be accentuated. Above all, the topographer must exhibit an alertness to distinguish as to what amount of detail should be omitted and that which should be included. Hesitancy in this is the chief source of slow progress. Valuable time may be wasted in the representation of features which may be lost on the scale of the work and which are common in all localities to the topographic forms being sketched; while features characteristic of such special topographic forms as those produced by eruption, erosion, or abrasion, or those indicative of the structure of the region and which give distinctive character to its topography, may be lost sight of or be covered up in the map by too careful attention to minute details.

The *characteristic features* of a terrane are best observed from a point nearly on the same level; and as between sketching features from above or below for a reasonable range, sketching from below is the better, as features viewed from any considerable height above are apt to appear dwarfed and much detail of undulation of the surface lost sight of. Yet, as a precise representation of the land requires sketching its forms from numerous positions at intervals not far apart, the necessity will rarely arise of observing surface forms from points of observation much above or below the surface represented, excepting in case of very small scale geographic or exploratory surveys.

20. Optical Illusions in Sketching Topography.—In sketching topographic forms by eye there are a number of optical illusions to which it is well to call attention, though the effect of these can be entirely overlooked in the sketching of detailed topography such as would be mapped on scales less than one mile to the inch. But for the sketching of topographic maps on smaller scales, where the eye has to be more depended upon, these illusions become more important. Most of these have been well classified by Mr. A. M. Wellington in his admirable work on railway location, and they are here summarized, with variations, from that work. Among the more serious of such illusions are the following:

1. The *eye foreshortens* the distance in an air line and materially exaggerates the comparative length of a lateral offset so as to greatly exaggerate the loss of distance from any deflection.

2. The *eye* exaggerates the sharpness of projecting points and spurs, and accordingly *exaggerates the angles.*

3. In looking, however, at smooth or gentle slopes from a distance, the tendency of the eye is to decrease the angle so that in such country as the rolling plains of the West *slopes look much gentler*, the inclinations much less, than they are in fact.

4. In this connection the eye is liable to make *slopes* looked at from a distance *appear steeper and higher* than they are in fact, when they are compared with known slopes and elevations of lesser dimensions near by.

5. Again, the unaccustomed eye, which mentally measures all dimensions by referring them to those with which it is acquainted, is apt to make a *divide or pass appear lower*

than a nearer divide or pass to which it is referred in one sweep of the vision, whereas it may be higher (Fig. 7).

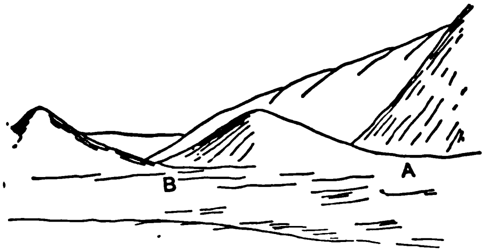


FIG. 7.—OPTICAL ILLUSION AS TO RELATIVE HEIGHTS OF DIVIDES. *A* is nearer and lower than *B*.

6. The *eye* invariably *exaggerates the steepness of the slopes* of mountains, these appearing to have inclinations of from 60 degrees to almost vertical, whereas in fact the steepest slopes are rarely as great as 45 degrees.

7. The eye trained to estimate slopes and distances in regions of large topographic features—that is, regions of extreme relief or differences of elevation—will be at a disadvantage in making similar estimates in a country in which the differences of elevation are small. The tendency of one accustomed to estimating the topographic forms in the Rocky Mountains, where differences of elevation and distances visible to one sweep of the eye are great, will be to overestimate heights and distances in the less rugged country of the Eastern States, where great detail in topography exists, and thus deceives the eye into an *exaggerated notion of the amount of the relief*.

8. In viewing the terrane with an idea of estimating its roughness as affording a possible route for railways, canals, or similar works, a rugged mountain gorge with occasional precipitous narrows, separated by river flats, may appear much more difficult and much rougher than it is in fact. This is especially so as compared with a gently undulating or rolling

country, which, when viewed from a distance, appears to be comparatively level, while a nearer view will show it to be full of elevations or depressions which will render construction most expensive, because of the rapid and numerous succession of large cuts and fills.

The effect on the eye and the mind is to *exaggerate the ruggedness of a country* which is difficult to travel because of such impediments as broken stone, fallen timber, creeks, and swamps, whereas a region where travel is easy and free, as in open rolling plains country or where good roads abound, is often estimated to be much simpler and more level topographically than is the other region.

CHAPTER III.

SURVEYING FOR DETAILED OR SPECIAL MAPS.

21. Topography for Railway Location.—Some of the worst errors in engineering location originate in reconnaissance, for the reason that the average reconnaissance surveys are not of areas, but of routes or lines, and there is great danger of serious error in the selection of the line to be surveyed. It may, accordingly, be stated that a *railway reconnaissance* should not be of a line, but of an area sufficiently wide on each side of an air line between the fixed termini to include the most circuitous routes connecting these. The results of such a survey should be embodied in a topographic map of greater or less detail, according to the nature and extent of the country. If the reconnaissance be of a great railroad, such as some of the Pacific roads, built through hundreds of miles of unknown country the resulting map should be on a small scale, perhaps 2 to 4 miles to the inch, and with contour intervals varying from 20 to 100 or 200 feet, according to the differences of elevation encountered and the probable positions of several locations. With such a map the number of possible routes may be reduced to two or three, and a more detailed topographic survey should then be made of these on which to plan the final location.

As ordinarily practiced, topographic surveys for railways are made by the older methods, with transit and chain or stadia and with spirit-level; notes of the surveys are kept with accompanying sketches in note-books, and these are reduced to map form in the office. The same results can be much more satisfactorily and more rapidly procured by using the plane-table in place of the transit, and the resulting map, being plotted in the field, is a more accurate and available representation of the terrane than can possibly be made from plotting notes in an office.

The *Germans*, who are very thorough in *taking topography for railroads*, divide the work into three separate surveys of different degrees of accuracy: first, recourse is had to the government topographic maps on a scale of approximately 1:100,000, and on this a preliminary route or routes are laid down: second, a more detailed topographic survey is made in the field on a scale of 1:2500 as a maximum or 1:10,000 as a minimum, with contour lines of 15 feet interval. This map is limited in area from a few yards to a few hundred yards in width, according to the nature of the country. Where no previous small-scale topographic survey exists, the base of this more detailed or second survey is a transit (Art. 87) or plane-table (Art. 83) and level (Art. 129) traverse, following as nearly as possible the approximate route of the proposed railway. Bench-marks (Art. 135) are established along this at distances of from 500 to 1000 feet, by which the aneroid may be checked. With this transit line completed on the proper scale, the topographer goes over the ground and, by means of distances from pacing (Art. 95) or odometer (Art. 98), and elevations by aneroid (Art. 176), constructs a hasty contour map on which are indicated all roads, water-courses, structures, high-water marks of bridges, width and height of existing bridges and culverts; and all other necessary topographic details as to the position of rock masses, strike and dip of strata, swamps, springs, quarries, etc.

On such a map as this, hastily and cheaply made, it is possible to plan the detailed topographic map, limited from a few yards to 100 or 200 yards in width and covering what will practically be the final route of the located line as obtained from the second survey. This *final detailed survey*, from which the paper location is to be taken, should be on a scale of from 1:500 up to 1:1000 and with contours of about 5 feet interval, more or less, according to the nature of the land. There is plotted on the plane-table sheet the transit and level base line previously run for the second survey, and the instruments now used by the topographer are of a more accurate nature, consisting of a plane-table (Arts. 58 and 83) for direction and mapping, two or more stadia rodmen for distances (Art. 102), while elevations are had by vertical angles with the alidade (Art. 59). On this final map are shown much the same topographic details as on the second, but all are more accurately located and the elevations are of a more refined nature. The data furnished by this final map will serve all the purposes of making a last *paper location* of the line, from which the engineer will in the field possibly deviate according to the appearance of the route traveled as presented to his eye when the location is laid down.

Mr. Wellington's location of the Jalapa branch of the Mexican Central Railway (Fig. 8) is an excellent example of a detailed contour topographic map for railway location. This was platted in the field on the scale of 1:1000, or about 83⅓ feet to 1 inch. The contour interval was 2 meters, or 6.56 feet.

22. Detailed Topographic Surveys for Railway Location.—Prior to making the location, which may be made in part from the notes of preliminary surveys, a narrow belt of topography should be mapped in detail, its width being restricted as far as possible, providing the preliminaries have been skillfully conducted or have been preceded by a small-scale topographic map executed with especial care along the possible routes of the location (Art. 21). On the detailed topographic map a *paper location* may be made, from which full notes of the alignment can be derived, the points of curve and tangent taken off, and a profile of the paper location pre-

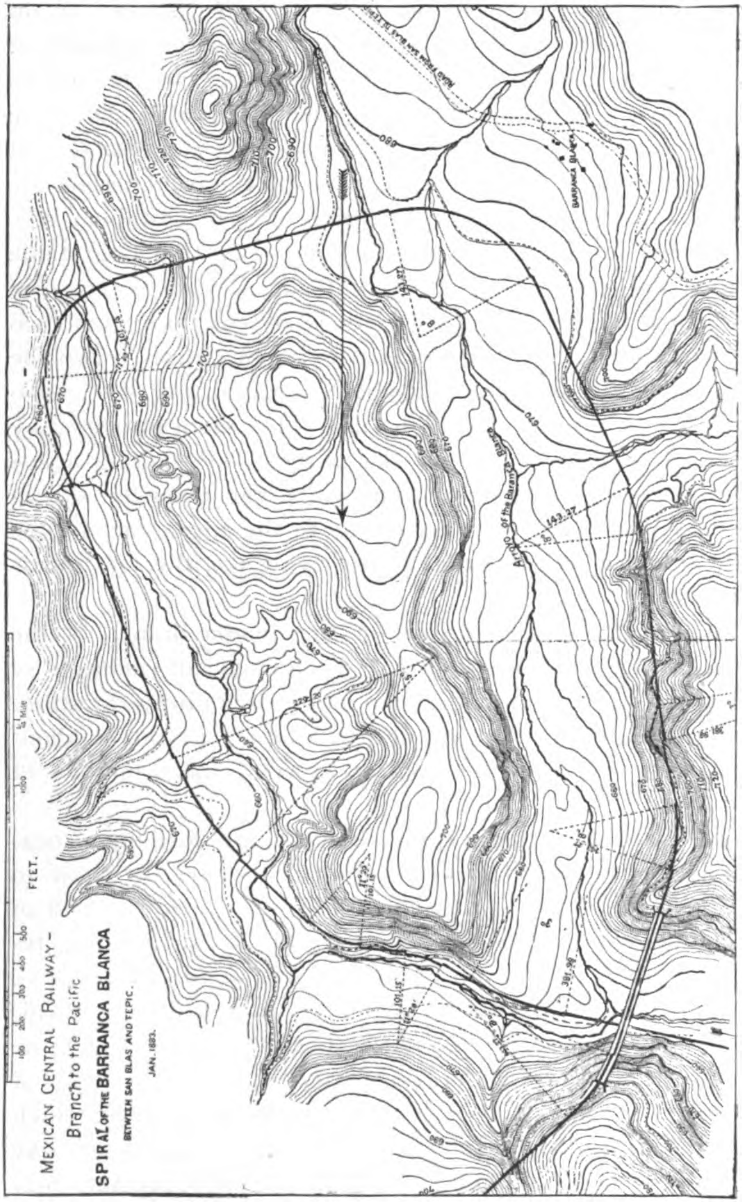


FIG. 8.—CONTOUR TOPOGRAPHIC SURVEY FOR LOCATION OF MEXICAN CENTRAL RAILWAY.
Scale of original 83⅓ ft. to 1 inch. Contour interval 6.5 ft.

pared. As has been stated of topographic maps for general purposes, the topographer should not trust too much to eye in sketching in his contour curves. For the making of the paper location the topography should be as exact and the contour lines should be as accurately placed as the scale of the map will permit, in order that a line may be located upon the map and a profile called off from it which shall agree as closely as possible with the subsequent transit location and spirit-level profile.

In making such a map it is neither necessary nor possible to locate every point on each contour, the horizontal and vertical locations of the contours being at such distances apart that their projections on the map will be so close together that in connecting them by eye in the field the topographer cannot go astray by an appreciable distance. With a detailed contour map made as described for the location of canals (Art. 23), a *grade contour* or location line may be drawn which will show where the plane of the roadbed will cut the natural surface and from which it will at once be seen whether or not the location is the most favorable the topography will permit.

The error into which many have fallen is in assuming too much or too little for the topography as a guide to location. The topographic map fails to show many essentials requisite in making a location, as it gives no evidence of the materials to be encountered, nor does it convey an adequate idea of the magnitude of the excavations and fills. The topographic map must be supplemented by a careful *visual reconnaissance* of the line which it covers. Such topography should therefore be restricted in its width and amount, and no attempt should be made to make a final location from such a map. On the other hand, where a topographic map is not made, and too much reliance is placed on the visual reconnaissance of the country, the greatest errors are at once introduced in encountering a bad system of gradients, in overlooking important towns, or in otherwise selecting inappropriate routes.

In planning the location on a detailed topographic map, the engineer should begin at a summit or similar fixed point, assuming or taking from a guide-map an initial elevation. Then with a pair of dividers he should step off such distances that these will correspond to the grade chosen and their termini end on the map above or below such contours as will give the proper differences in elevation to produce such grades. By this means a *grade contour* can be sketched in on the map and then connected by tangent lines. The latter must, in turn, be connected by throwing in curves the radii of which shall be as large as possible, care being taken that the grades on these shall be properly compensated. With such a paper location it is then possible, by means of scale and protractor, to take off the directions and distances in a note-book, when, with these as a guide, the located line may be run on the ground and changed or modified in the field as the visual observation of the engineer may suggest.

Speed in mapping railway topography varies greatly with the scale selected and the character of the land mapped. One party working in flat, desert country in Utah ran 20 linear miles in a day of 9 hours, including running of spirit-levels. The same party working later in mountainous country in Washington averaged during a long period of time less than ½ mile a day, in one instance working six weeks on a location through 1½ miles of canyon. A party working on railway location and mapping topography on the plains of Kansas made an average speed of 2.1 miles a day at an average cost, including all expenses, of $11.03 per linear mile. The Utah work averaged about $2.50 per mile, and the cost of much of the Washington work exceeded $100.00 per linear mile.

23. Topographic Survey for Canal Location.—Surveys for canal lines or lines of conduits, etc., are best made by having the leveling (Chap. XV) precede the plane-table or transit work. The level will then run out a grade contour having the requisite fall per mile, and the transit (Art. 87) or plane-table

(Art. 83) with chain measurements (Art. 99) will follow the level, locating this grade contour. Topography may be taken on either side by stadia (Art. 101) and plane-table so that in the final location of the canal the preliminary grade contour may be shifted to suit the sketched topography, much as the line of a railway location would be shifted from similar data (Art. 22).

An interesting example of a detailed topographic survey for the final *location of* an *irrigation canal* is that made by Mr. J. B. Lippincott of the Santa Ana Canal, through a rocky canyon. This location was made upon a carefully prepared topographic map drawn on a scale of 50 feet to 1 inch, with contour interval of 5 feet. The maps were plotted from cross-section notes based on two connected and approximately parallel preliminary lines, the contour curves being sketched in the field to indicate intervening irregularities of surface. The preliminary controlling lines were carefully run with transit and chain, were frequently connected, and had a vertical interval of 70 feet. The space between and for thirty feet above the upper line, or for a total of 100 feet vertically, was carefully contoured. From the map thus prepared a more accurate cross-sectioning was made, and from these notes a new contour map of the ground was prepared on a scale of 30 feet to an inch over the more difficult portions of the line, after a preliminary location had been selected on the first contour map. Fig. 9 gives a plat of one of the roughest portions of this line, and on it are shown in small circles the various points located on each contour. The plane-table was used and was set up generally as shown by the station numbers and triangles on the preliminary and plotted traverses, and directions were measured to stadia-rods held at various points on the 10-foot contour lines (Art. 101). The positions of the contour lines at these points were therefore plotted, and the corresponding elevations were immediately connected as contour lines on the plane-table sheet. In this

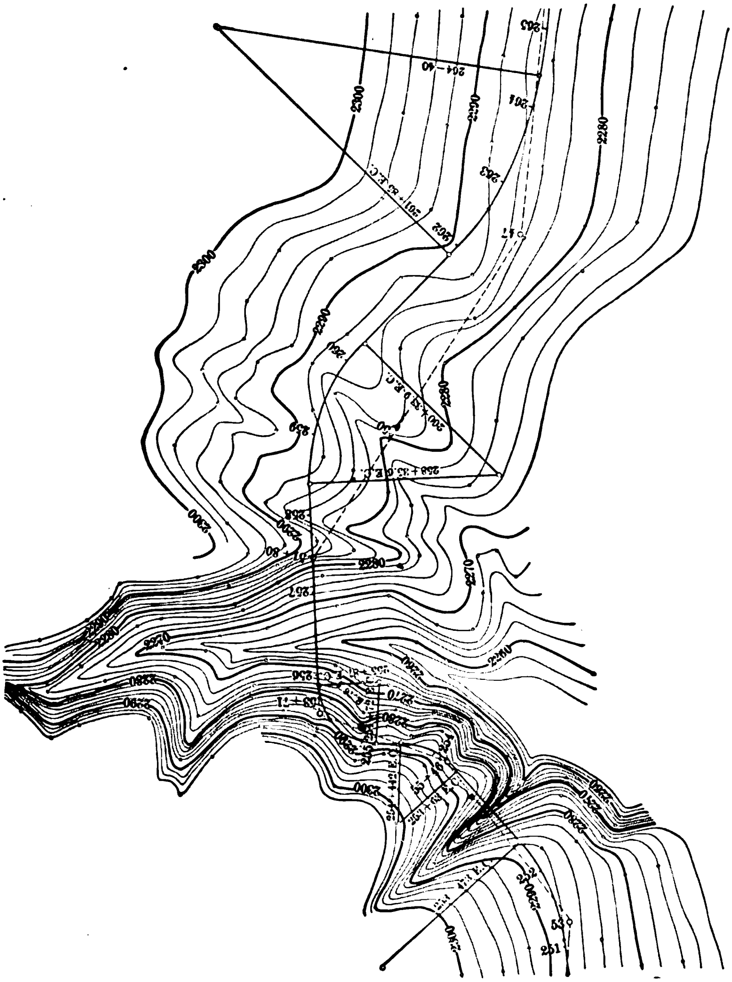


FIG. 9.—DETAILED CONTOUR SURVEY FOR CANAL LOCATION.
Original scale 100 ft. to 1 inch. Contour interval 2 ft.

way enough points were located on each contour to sufficiently control it, and the immediate 2-foot contour lines were interpolated by eye estimation in the field.

In doing this work *three* various *methods* were *tried:* (1), by locating the contour lines with slope-board and rod; (2), by locating the contours at right angles to the stations occupied by a levelman using a hand-level (Art. 156); and (3), by means of the plane-table and stadia (Art. 101). Mr. Lippincott says that as a result of these tests there is no question between the quality of the three classes of work; that without plane-table the work had to be plotted up in office and located points connected by estimation or from rough sketches; with the plane-table the same points were plotted immediately, in the field, and the connections between these made with the terrane in view, and that the resulting map by plane-table much more accurately expressed the slopes of the land than did the maps made by the other methods. The *speed* by the various methods was about the same. The party consisted generally of five persons, including the topographer, levelman, and rodman, and the speed was from 2500 to 4000 linear feet per day, actually locating four 10-foot contours and sketching in five or six more, a total of 100 feet vertical interval, and interpolating the 2-foot contours. Where side canyons and ravines were passed the slope-board was found to be entirely inadequate and helpless, while by the use of levelman and hand-level without the plane-table, and with taped traverse lines, the conditions were improved, but the work was of the crudest character so far as its topographic expression was concerned.

An example of a *preliminary topographic survey of a canal line*, made under the author with plane-table and on a small scale to determine the possibility of bringing the water from a stream or reservoir to certain lands for purposes of irrigation, is illustrated in Fig. 10. The scale of this illustration is denoted by the land section lines, each section

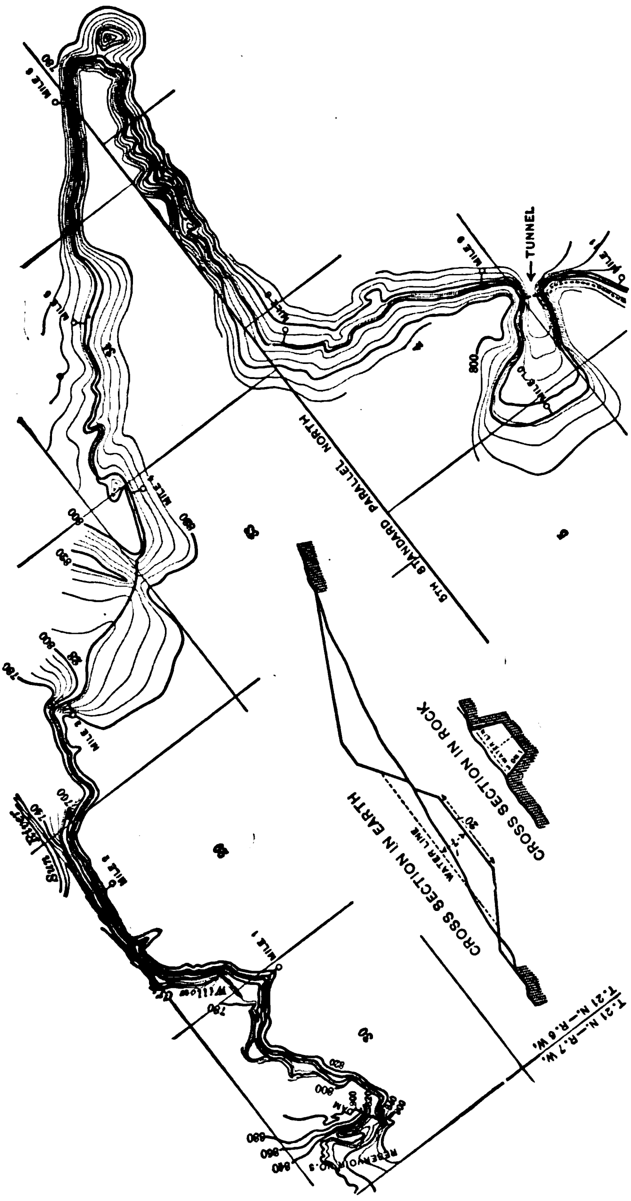


FIG. 10.—PRELIMINARY MAP OF CANAL. MONTANA.
Scale 4000 ft. to 1 inch. Contour interval 4 ft.

being a mile on a side. The original survey was made on a scale of 3000 feet to the inch, with a contour interval of 4 feet. The plane-table was accompanied by a spirit-level to determine grade, in order that the canal line might be given the required fall per mile.

24. Surveys for Reservoirs.—In making surveys of reservoirs for storage of water for city water-supply or for irrigation and similar purposes, the scale and contour interval depend necessarily on the dimensions of the reservoir. The former should be from 400 to 1000 feet to the inch, and the latter from 2 to 5 feet vertical interval. Special surveys should be made of possible sites for dams and waste-weirs on larger scales and with a contour interval of 1 or 2 feet, and several cross-sections of the dam site should be run and the topography taken in detail for a sufficient distance above and below the center line. If sufficient borings or trial-pits are sunk, a contour map of the foundation material may be constructed.

Perhaps the most satisfactory manner of making surveys of reservoir sites is instanced in the following practical example of one made by the author. A standard or base transit (Art. 87) and level (Art. 129) line is first run across the dam site, carrying the same a little above the highest possible flow line of the reservoir. From this should start a main transit and level line which should follow up the lowest or drainage line of the reservoir basin (Fig. 11, *A*, *D*, *G*), and this should be extended until it reaches an elevation corresponding to that of the highest probable flow line of the dam. Bench-marks (Art. 135) should be left as this line progresses, and stadia distances measured (Art. 102), and level elevations taken to points within the range of the level-telescope, as at *A*, *B*, etc. Based on this main transit and level line, a plane-table and stadia line (Art. 101), accompanied by spirit-leveling, should be run from the highest flow line of the dam cross-section around the corresponding contour line on one side of the reservoir, *H*, *I*, *J*, etc., and if the land be clear, stadia and level sights may be taken to the other contour lines within the range of the instrument, including sights on lines of equal elevation on the opposite side of the reservoir if the latter be small. If large, however, a number of flags may be located on the opposite side by triangulation (Art. 73) or by stadia observations, and cross-section lines be run to these, from which the data for constructing a contour topographic map can be obtained as at *I* and *L*.

Another example of a reservoir survey is illustrated in Fig. 12, which is a portion of the map of the Jerome Park reservoir site in the city of New York, and was platted on a scale of 400 feet to the inch with a contour interval of 10 feet. From such a map it is possible to compute the contents of a reservoir for each additional five feet of elevation, and on it land lines and property lines are shown in such manner as to indicate the damage which will be done by submergence.

25. **Survey of Dam Site.**—A typical illustration of the topographic map resulting from the survey of a site for a *dam for* closing a *storage reservoir* is shown in Fig. 13. This survey was executed with a plane-table (Art. 73), chain (Art. 99), and spirit-level (Art. 129) on a field scale of 400 feet to 1 inch, with a contour interval of 2 feet. The result of such a topographic survey is to indicate clearly the best alignment for the dam, providing the borings which must necessarily follow the selection of such alignment prove its feasibility.

An example of a topographic survey executed for selection of a site for a weir or *diversion dam in a river* is that illustrated in Pl. III. This shows the topography of the flood-bed of the Snake River between its high bluff banks, as well as the contouring of the bed of the river as shown by soundings. On this is indicated the best alignment for the diversion weir as well as for the canal head and headworks. The field work of the survey was executed with transit,

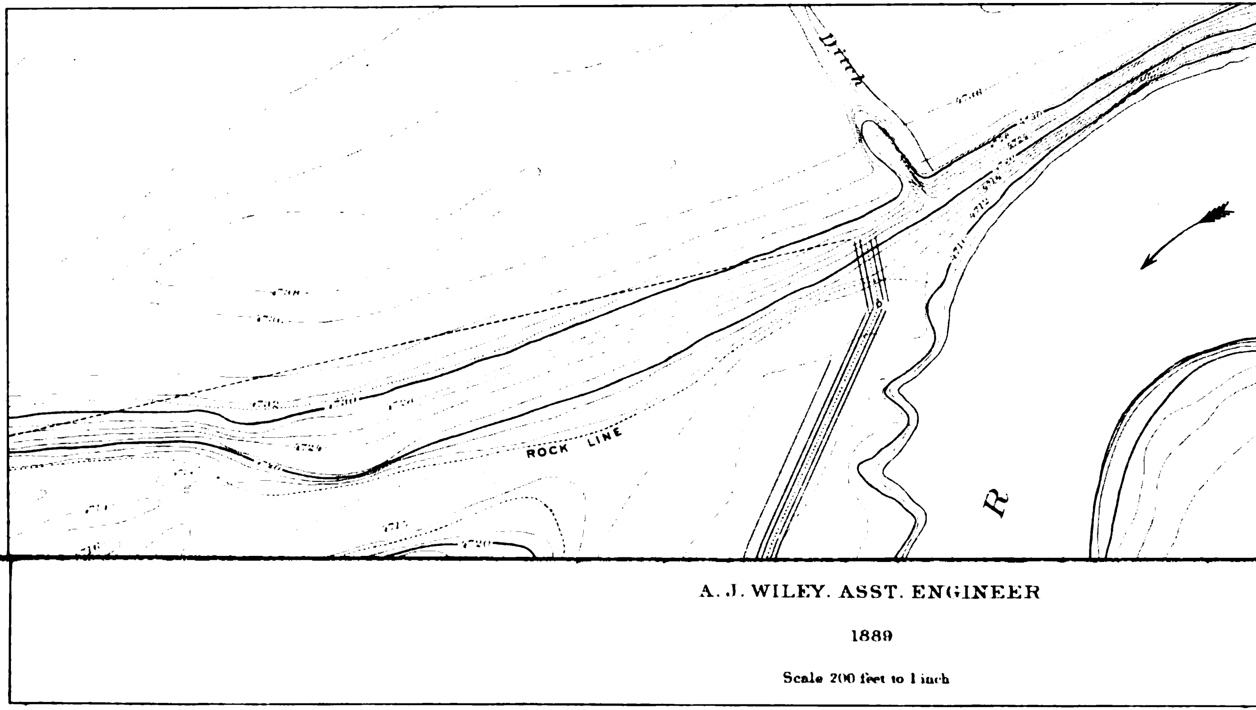


PLATE II.—CONTOUR TOPOGRAPHIC SURVEY OF SITE FOR DIVERSION DAM, SNAKE RIVER, IDAHO.

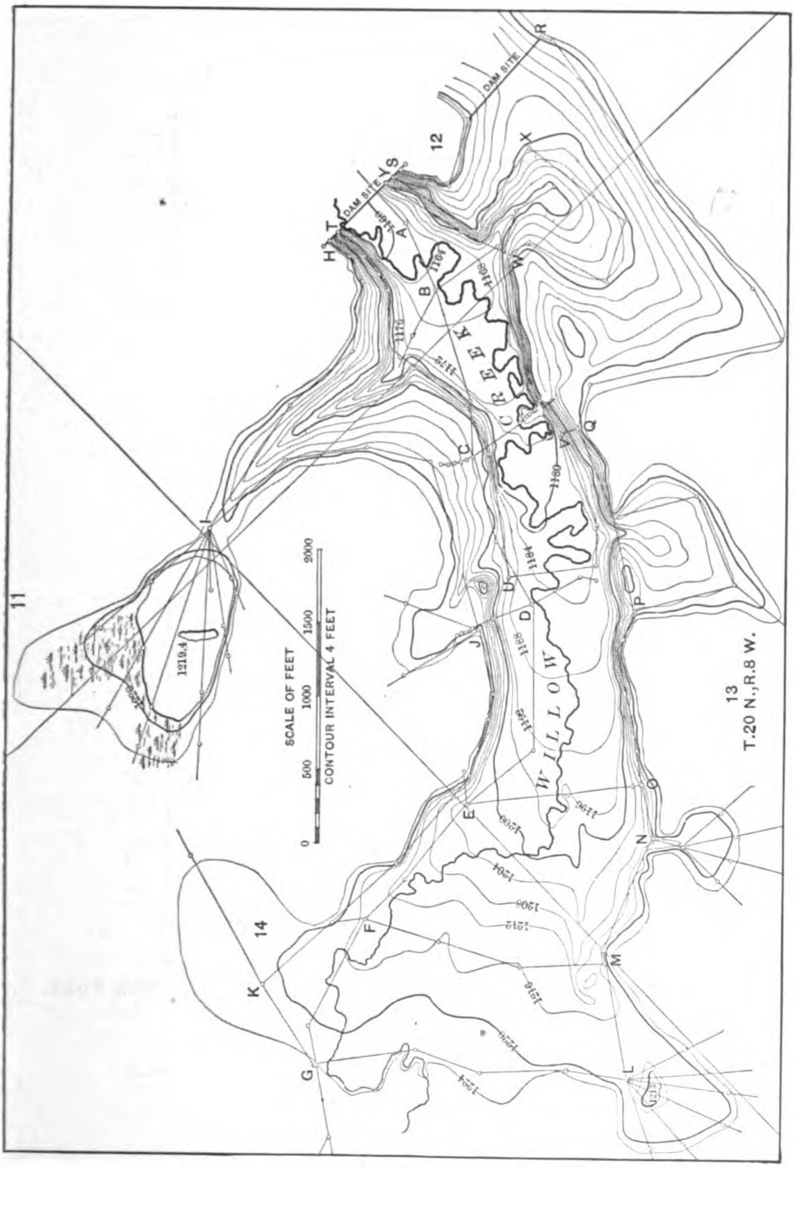


FIG. 11.—CONTOUR SURVEY OF A RESERVOIR SITE. MONTANA.

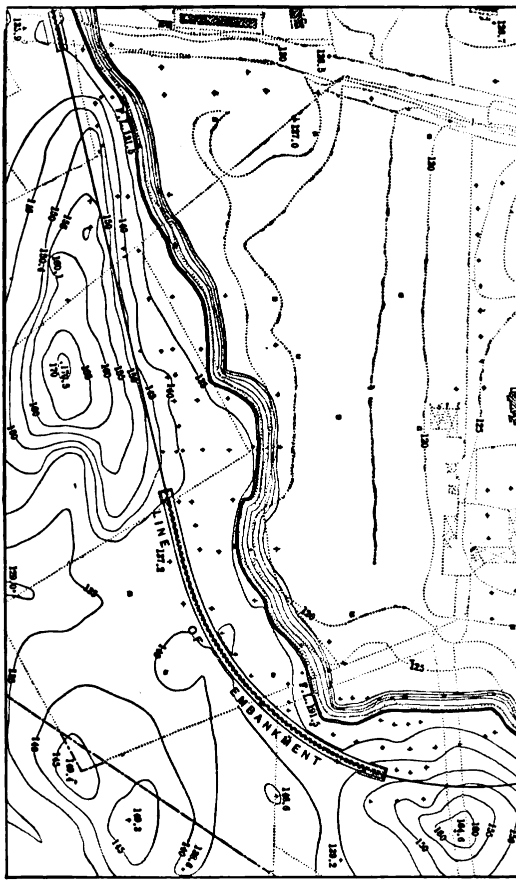


FIG. 12.—PORTION OF THE JEROME PARK RESERVOIR SURVEY. NEW YORK. Scale 400 ft. to 1 inch. Contour interval 10 ft.

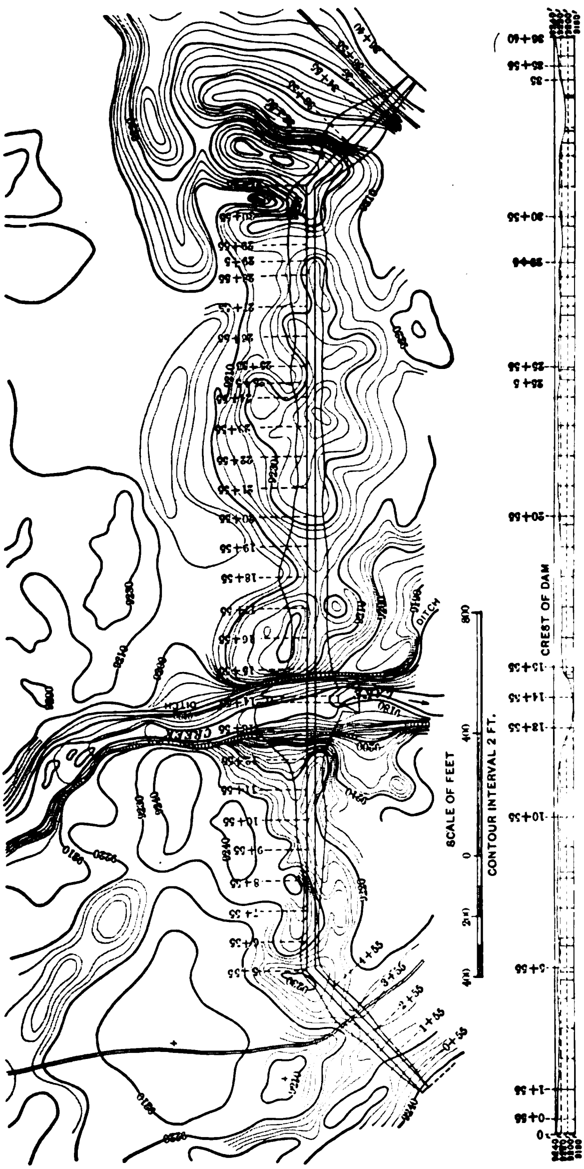


FIG. 13.—PLAN AND PROFILE OF TWIN LAKES DAM SITE. COLORADO.

plane-table, chain, stadia, and spirit-level on a scale of 200 feet to 1 inch, with a contour interval of 2 feet.

26. City Surveys.—Topographic surveys of cities are almost invariably made in conjunction with complete cadastral surveys of the same, and usually under three conditions:

(1) In laying out a plan for city extension or in making a plan for a projected city where little or no construction of streets, etc., exists;

(2) In making a complete survey of a city on which to plan future public works of all kinds; and

(3) A topographic survey of a city may be made merely for the sake of obtaining the resulting map.

An example of the first class of city topographic survey is that for the *survey* of the *town site* of the city of Allessandro, in southern California. Surveys of a town site on a comparatively level tract were made by Mr. J. B. Lippincott with such detail that irregular streets, parks, and other city improvements were planned on the resulting topographic map. The eastern portion of the tract mapped had a general rise of about one foot in one hundred, the roughness increasing toward the west until broken country was reached. The contour interval was one foot, and the scale 100 feet to the inch. A base line (Chap. XXI) was projected through the centre of the tract, measured with care, and stakes were set at every 500 feet. At each 2000 feet a right angle was turned off and lines run north and south to the boundaries, a large stake being set every 500 feet on these lines. After these were located, two transits were used, one on the base line and the other on the 2000-foot line, and the remaining stakes were located by intersection at the corners of the 500-foot blocks. Flags were placed on each 1000-foot stake for witness-stakes and to orient the plane-table, and levels were run from the center base line and around the outside of the tract, readings being taken carefully on all hubs.

In *making* the *topographic map* a plane-table was used

(Art. 53) and a spirit-level (Art. 130) was set up near it, the levelman placing two rodmen along distant contours, though it was sometimes found that four rodmen could be used on various contours. The plane-table was set over a stake and oriented by one of the flags, and on it were plotted the positions of the rodmen by stadia distances (Art. 102), and thus the contours were sketched. Sights were taken in the more level portions of the country, not closer than 50 or farther than 100 feet apart, and the rodmen were placed on a contour and kept upon it until they reached a distance of 500 or 600 feet from the plane-table. By this means about one square mile was mapped in a week of fair weather. Where the slope increased toward the west, 2, 4, or 5 feet contours only were located, and the others interpolated by sketching. Forty-seven working days were employed in mapping 3.25 square miles, during which 25,400 points were located, or 7800 to the mile, the cost being about $300 per square mile.

Of the second class of city surveys the two most prominent examples are those of the cities of St. Louis and Baltimore (Art. 27). The topographic survey of the city of Washington, made by the U. S. Coast and Geodetic Survey, is an example of the third and unusual class of city survey. This was made on a scale of 1 : 4800, and covers an area of 48 square miles. The result is a *topographic map pure and simple*, unaccompanied by the placing of permanent monuments or the obtaining and recording of accurate measures, as is necessary in making a cadastral survey of a city. This survey was based on a minute triangulation (Chap. XXV) while all the details of the topography were obtained by means of the plane-table (Art. 53), stadia (Art. 101), and Y level (Art. 129). The contour interval was 5 feet, and these contours were based on lines of Y-leveling run along all roads, avenues, and railroads. The plane-table stations were placed so close together as to encompass within the distance from station to station all the area within the range of the stadia. No system of precise bench-marks was left in the course of the leveling, but the Y level was freely used in tracing successive contours upon the ground, the points upon each contour being located by stadia. The degree of refinement of this survey seems excessive in view of the scale of the map, as the errors of actual location of the contours upon the map would greatly exceed the actual errors of leveling; moreover, as no provision was made for continual revision of the maps by leaving easily recognizable monuments, the value of such a survey is limited by the many changes due to rapid suburban development, which would render such maps out of date within a very short period of time.

27. **Cadastral and Topographic City Survey.**—The topographic surveys of the cities of Baltimore and St. Louis were made in conjunction with *complete cadastral surveys*, and show all property lines, widths from building line to building line in all streets, dimensions, and other incidental data relative to buildings, both public and private. The most complete example of such a survey is that furnished by the city of Baltimore. This was based on a system of triangulation executed with precision (Chap. XXV) and connected with a base line measured with much care with a 300-foot steel tape (Chap. XXI). This triangulation covers 54.7 square miles, and for its execution required several high observation-towers in addition to existing structures. Between the located triangulation points was an adjusted system of steel-tape traverse lines (Art. 87), executed in such number that no closed circuit of traverse exceeded 7500 feet in length. By these traverse lines there were located 3740 stations. Precise levels (Art. 140) were run over an area of 29.51 square miles. These levels included 141 miles of duplicate line, in which were established 606 permanent bench-marks, while elevations were taken at every street intersection by ordinary Y levels (Art. 129). The primary control averaged three triangulation stations and forty-two traverse stations per

square mile in the unbuilt sections of the city. On this control there was constructed a detailed topographic map on a scale of 1 : 2400 and with contours of 5 feet vertical interval (Fig. 14). In the execution of this work there were run many miles of stadia traverse and Y levels and of taped measurements of street widths and building-line dimensions, etc.

Three methods of obtaining topography were adopted: (1) that by transit and stadia, accompanied by notes worked up and plotted in the office; (2) that by means of plane-table and stadia, with complete map made in the field on the plane-table; and (3) that by transit and stadia, with notes worked up and plotted on a crude drawing-board in the field. The first two methods were employed in surveying only small areas, and were abandoned successively as not satisfactory. The third was that which was ultimately employed in mapping the larger portion of the city. In the prosecution of this latter method of work high-grade transits with fixed stadia wires and vertical and horizontal circles reading with verniers to 30 seconds were employed. All notes, as rapidly as obtained by measurement and by angulation, were plotted with an 8 inch protractor (Art. 89) and boxwood scale on the field drawing-board. Previously there had been plotted on the field sheets the primary triangulation and primary traverse locations (Chaps. XXV and XXIII) with lengths and azimuths of lines joining stations, and elevations of precise-level bench-marks. The party organization consisted of a topographer, a recorder, a draftsman, a levelman, and two stadiamen. As rapidly as the topographer read azimuths, distances, and vertical angles, the draftsman plotted the same, and the recorder worked out elevations furnished by the topographer and the levelman. After all observations had been taken and the horizontal locations and elevations plotted, the contours were drawn in on the field board by the topographer, and the party moved to the next station. The total

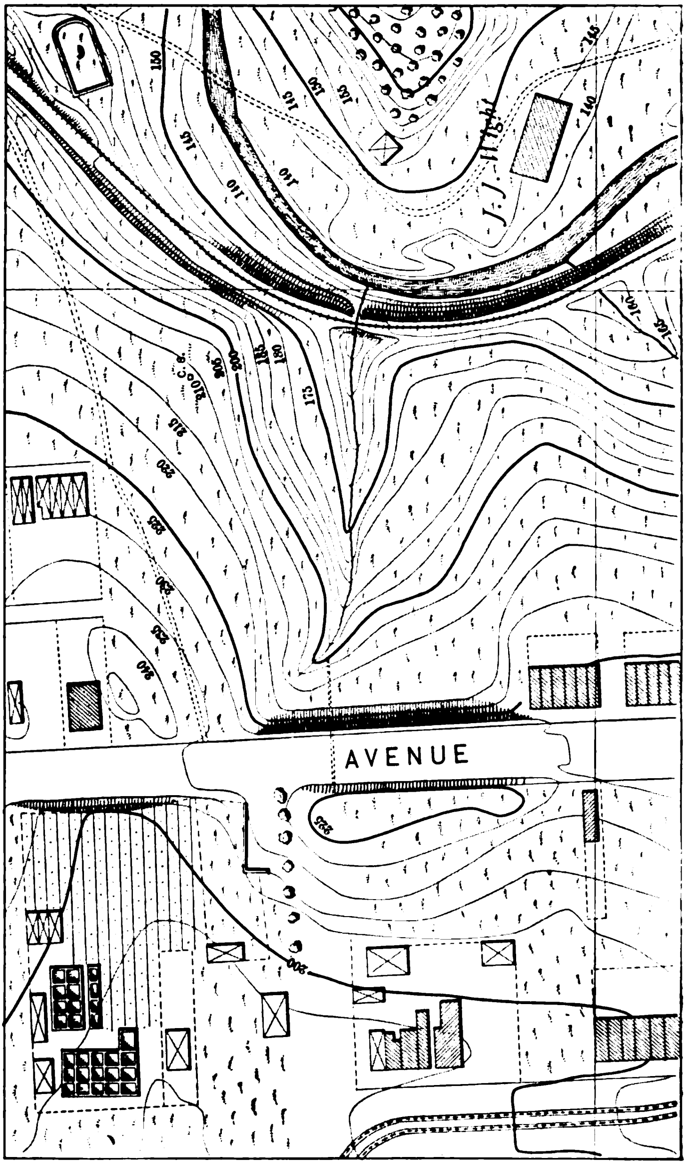


FIG. 14.—TOPOGRAPHIC AND CADASTRAL MAP OF BALTIMORE, MD.
Scale 200 ft. to 1 inch. Contour Interval 5 ft.

area of topographic survey was 32.2 square miles, in which there were located 213 miles of streets and alleys, 1147 precise points were occupied, 2320 stadia stations occupied, and 134,209 sights were taken, the average being 10 per acre.

28. Cost of Large-scale Topographic Surveys.— Special topographic surveys are usually prosecuted with a view to showing all the topographic details of a limited area, and are executed with such minuteness that the resulting map may be plotted on a large scale. Not uncommonly such surveys are of cadastral thoroughness, and the results may then be plotted on such a scale as will permit of showing in plan the minutest detail of houses and other structures.

Such a survey is the British Ordnance survey, plotted on a scale of 1 : 2500, and the topographic and cadastral surveys of the cities of St. Louis and Baltimore (Art. 27). Also topographic surveys for railroads, reservoirs, etc., plotted usually on scales of 400 to 1000 feet to one inch and with 2-foot to 10-foot contours (Arts. 22 to 24), on which property lines are also shown.

TABLE I.

SCALE AND COST OF DETAILED TOPOGRAPHIC MAPS.

Country.	Scale.	Relief.	Cost per sq. mi.
India	4 in. to 1 mi.	Hachures	$ 26.50
Baden	1 : 5000	....	80.00
U. S. Coast Survey	1 : 10,000	Contours	212.00
U. S. Lake Survey	1 : 10,000	"	120.00
U. S. Miss. and Mo. River Com	1 : 10,000	"	51.00
U. S. Coast and Geodetic Sur. of Dist. of Col	1 : 4800	5-ft. contours	3,000.00
Butte (Mont.) Special U. S. G. S	1 : 15,000	20-ft. "	83.00
Perkiomen Watershed, Penn	1 : 4800	10-ft. "	145.00
Croton Watershed, N. Y.	3 in. to 1 mi.	20 ft. "	17.50
Connellsville Coke Region, Penn	1 : 19,200	10-ft.	116.36

CHAPTER IV.

GEOGRAPHIC AND EXPLORATORY SURVEYS.

29. Geographic Surveys.—The *object of a geographic survey* is to fix the relative positions of points on the surface of the earth so that they can be referred accurately to a tangent plane and be therefore independent of the sphericity of the earth. The geographic survey of an extended area consists of three parts:

1. A geodetic survey, which permits of the projection of a primary system of controlling points on such a tangent plane.

2. Of a plane survey, for the projection of a system of intermediate controlling points upon the same plane and adjusted to the primary system.

3. Of a hypsometric survey, for the determination of the distances of the points established by the other two surveys above or below an assumed datum or basal plane of elevation.

The *results of a geographic survey* are presented—

1. In a geographic map, which is intended to give as complete an image of the area surveyed as the scale of representation will permit; and

2. In a report on the physical and statistical characteristics of the region surveyed.

The *methods employed* in the field execution of the geographic survey are described hereafter under various titles. An essential preliminary to the geographic survey is a geodetic survey based on astronomic positions, and the mode of obtaining this fundamental information is explained in Parts V and VI. With such primary control as is furnished by the geodetic survey the details of the geographic survey are executed by some of the various methods explained in Chapter II and Part II. They are essentially similar to those employed in the making of topographic surveys, differing therefrom chiefly in the employment of cruder and more rapid methods. Moreover, the amount of information to be gathered is more scattered and less detailed than that procured by topographic surveys, because the scale of the resulting map is smaller and therefore will not permit of the representation of minor details.

30. Instrumental Methods Employed in Geographic Surveys.—For the making of a geographic map the primary control must be executed by geodetic methods, but this need not be of the highest degree of accuracy, but only of such quality that the resulting errors will not be appreciable upon the scale of the map. For filling in the intermediate details the most useful instrument is the plane-table (Chap. VII), which may be employed for the execution of secondary and tertiary triangulation, for road traverses (Chaps. IX and X), and as a sketch-board on which to fill in the details of topography (Arts. 13, 15, and 17).

In the course of such work the methods employed will be of a crude nature. Signals will rarely be erected, natural objects being sighted both in the triangulation and in the traverse, and the number of stations and locations will be relatively few and far apart one from the other. They must, however, be fixed with such accuracy upon the scale of map that there will be at least two or three located points to each square inch of map surface. Thus on a map scale of two miles to one inch there may be an average of less than one location per square mile. On a scale of four miles to one inch there may be but one location to every two square miles. Again, on the latter scale, plane-table stations would, under favorable circumstances, be placed at an average distance of ten miles apart, and from each there would be sketched an approximate area of one hundred square miles. For it will be realized that on the scale of the map this implies sketching from each station to a distance from one to one and one-quarter inches in each direction and over territory controlled by intermediate locations averaging one-half inch apart.

The *intermediate details* of the geographic survey executed for small-scale maps should be filled in by the ordinary traverse methods, performed, however, with instruments fitted only for the execution of approximate work. Thus a very light traverse table (Arts. 61 and 63) or a prismatic compass (Art. 91) should be used for directions, while distance may be obtained by wheel (Art. 98) or pacing (Art. 95). Elevations will be determined in the prosecution of a geographic survey of this character by several methods, the amount of basal spirit-leveling (Chap. XV) being of the very least, perhaps only a few fundamental elevations per map sheet. The more important elevations will be obtained by trigonometric levels of primary or secondary quality (Chap. XVII), and the larger proportion of the intermediate elevations will be obtained by mercurial barometer with aneroid for elevations of less moment (Arts. 170 and 174).

31. Geographic Maps.—A geographic map is generally plotted on a small scale, corresponding with the least scales of general governmental surveys (Table II), and the limits of such scales are roughly between about one mile to one inch and six miles to one inch. For larger scales the resulting map might be classed as topographic, and for smaller as exploratory. On geographic maps various conventional signs (Art. 195) are employed to represent hydrography or drainage, culture or works of man, and relief or surface undulations. Such drainage features as streams, lakes, and ocean margins as may be of sufficient size to permit representation on the scales selected are shown. Such cultural features as are of a strictly public nature, as railways, the more important highways, cities, and political boundaries, should be shown. Surface undulations should be clearly represented by one or other of the various conventions. According to the scale of the map and the

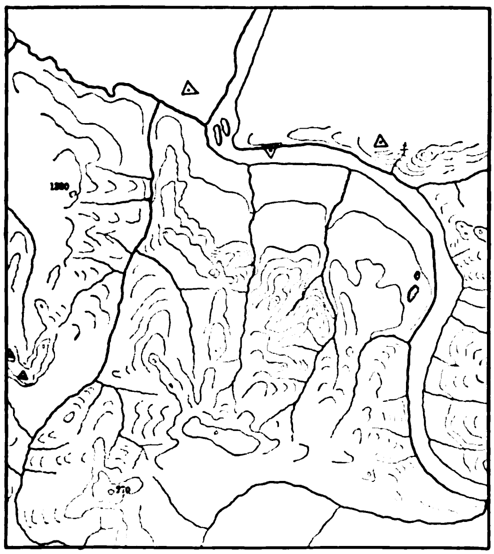


FIG. 15.—FIELD SKETCH MAP MADE ON PLANE-TABLE IN ALASKA. E. C. Barnard, Topographer. Scale 4 miles to 1 inch.

quality of the field survey, such representation may be by contour lines of considerable vertical interval, by sketch or broken contours representing relative differences of relief, or by means of hachures which represent in a conventional way degrees of relief, absolute relief being shown only by written figures of elevation.

In Fig. 15 is shown a portion of the sketch contour map made in Alaska by E. C. Barnard of the U. S. Geological Survey. In Fig. 16 the same map is shown after it has been drawn up in office. The contour interval of this is 200 feet, and the scale 4 miles to one inch. Yet these are not true contours, and should preferably not have been represented as such, since the amount of vertical con-

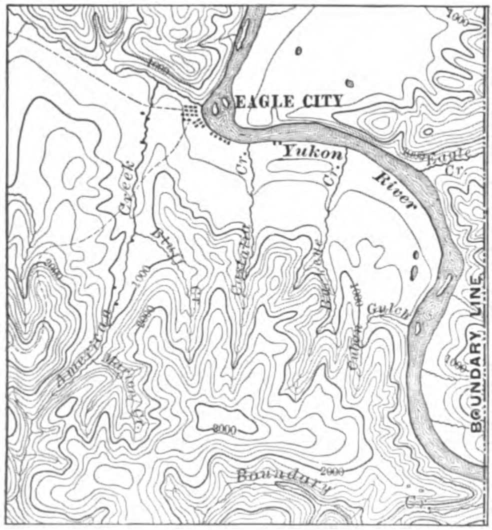


FIG. 16.—GEOGRAPHIC CONTOUR MAP MADE FROM FIG. 15.
Scale 4 miles to 1 inch. Contour interval 200 feet.

trol was too small to give that exactness implied by contour lines.

32. Features Shown on Geographic Maps.—A geographic map should show with sufficient completeness all the important topographic features of the area surveyed. It should depict especially physiographic peculiarities, which are the key to the origin of topographic forms. It will thus be realized that in their execution the geographer should have a clear knowledge of the relations of geology to topography (Art. 45).

Accordingly the amount of the *instrumental control* required will be the minimum which will permit accurate representation of the essential and predominant features. Between this control the shape and positions of the various streams may be sketched in such a manner as not only to show their direction, but their changes of direction as determined by accidents of broken or displaced stratification or the slope of the surface over which they flow. Moreover the map will distinguish between the rounded slopes of a synclinal and the abrupt sides and angular sections of an anticlinal gorge. It will show at a glance the position of a fault in the stratification by precipitous slope and exposed strata on one side, and on the other the gentle declivity of tilted surface rock. It is thus evident that the geographer must be largely guided in his depiction of the terrane by his knowledge of the geologic structure, so that the resulting map, while well controlled in relative place by instrumental locations, will, because of the necessity of generalizing topographic forms imposed by the small scale employed (Chap. VI), bring out the essentials or keynotes of such form rather than permit their burial under a mass of detail which is not essential to the purpose of the map.

33. **Geographic Reports.**—The smaller-scale geographic maps executed by governments, and in a few instances by railway enterprises in connection with surveys made for the gathering of general information relative to unexplored regions, should show not only the topography of the region surveyed, but the outlines of its forested areas. Above all it should be accompanied by reports on all those scientific and economic facts which will aid in developing the region under examination. Examples of such surveys are those executed by the Hayden survey in Colorado and Wyoming, and by the Wheeler survey in various portions of the United States west of the 100th meridian.

In both the resulting maps are based on geodetic control, and are published on various scales according to the objects of the survey. In the case of the Hayden survey a general scale of four miles to one inch was adopted (Fig. 20), and differences of elevation were shown approximately by contours having an interval of 200 feet. In the case of the Wheeler survey two general scales of four (Fig. 19) and eight miles to the inch were used in various localities, and the surface relief was depicted by hachures accompanied by occasional figures of elevation, actual elevations not being shown, as in the case of the Hayden survey, by contour lines. In the case of both surveys, in addition to showing the culture, drainage, and relief, the general topographic maps are accompanied by special maps showing the distribution of timbered, pasture, and barren land, and by other maps showing the surface geology. Finally, the whole was accompanied by extensive printed reports detailing the important scientific and economic features of the regions examined.

34. **Scale and Cost of Government Geographic Surveys.**—All civilized nations appreciate the value and necessity of good topographic maps of their territory. The principal nations of Europe have completed surveys that will generally subserve the purposes of geographic maps, or are now engaged upon such work. These European surveys are all based upon a computed triangulation and are usually made upon a scale not far from one mile to one inch or 1 : 63,360. They are sometimes larger and sometimes smaller. Their scales range between one mile for Great Britain up through Austria, France, Norway, Germany, and Russia, to two miles in the latter country. And from Great Britain they range down with larger scales through Sweden, Italy, Spain, Denmark, and Switzerland, the scale for the latter being a little larger than two inches to one mile.

A study of these maps is of value in determining the scale which should be adopted for a general geographic map of the United States, and as a result the scales chosen for the latter are from one to two miles to one inch. It is believed that the larger scale offers the best opportunity for the expression of such features of the country as the engineer, legislator, or investor desires to see expressed with some detail on a general map. If a still larger scale map is desired, it is usually for a small area, and for this purpose the indi-

TABLE II.

SCALE, COST, AND RELIEF OF GOVERNMENT GEOGRAPHIC MAPS.

Country.	Scale.	Relief.	Cost per sq. mi.
India	1 mile to 1 inch	Hachures	$ 11.00
Austria	1:75,000	Hachures and contours	400.00
Baden	1:25,000	Contour	22.20
Belgium	1:20,000	One meter	167.00
	1:40,000	Contours	
France	1:10,000	Hachures	52.00
	1:20,000		
Great Britain	1 mile to 1 inch	Hachures and contours	184.00
Italy	1:100,000	Contours, 5 and 10 meters	30.00 to 45.00
Prussia	1:100,000	Contours, 5 meters	71.00
United States Geological Survey: Middle Atlantic and Eastern States	1:62,500	Contours, 20 ft.	10.00
Geological Survey: Southern and Western States	1:125,000	Contours, 100 ft.	4.00
Geological Survey, Western States	1:250,000	Contours, 200 ft.	1.75
Hayden Survey	4 miles to 1 inch	Contours, 200 ft.	2.10
Wheeler Survey	4 miles to 1 inch	Hachures	2.25
" "	All; 2 to 8 miles to 1 inch	"	1.50

vidual desiring the map should, and probably would, make his own special surveys, as they would be conducted with a view to the inauguration of active engineering operations. The contour interval chosen for the geographic map of the United States varies according to the topography and horizontal scale from five to one hundred feet vertically. The smaller contour intervals are employed especially on very level costal plains, while the larger intervals must be used for the expression on the same scale of steep mountain slopes and valley walls. It has been found that this range of contour interval gives the best mean value for the expression of all characters of topographic form, permitting the proper depiction on the scale chosen of the steepest mountains and yet giving a fair idea of the value of the slopes on the more level surfaces.

35. **Exploratory Surveys.**—One of the essentials of an exploration is some form of survey which shall record the appearance of the country traversed. The primary requisite in such a survey is some means of measuring directions and distances along routes of travel. A well-equipped expedition should be provided with several varieties of instruments for this purpose lest some be lost or injured, and in order that those best suited to the exigencies of the case may be employed. Sometimes no effort is made to fix the geographic position of such surveys, but ordinarily and where the work is conducted under scientific auspices means are provided for the determination of latitude, longitude, and azimuth by simple instruments and with approximate accuracy.

Azimuths may be measured in route surveys with prismatic compass, or by means of a light plane-table, or with a light transit (Arts. 91, 63, and 85).

Distances may be measured by stadia, by pacing, by timing the gait of animals or of a boat rowed or drifting, and in extreme cases by mere eye estimation (Arts. 102, 95, and 96).

Where the exploration is of a compact area rather than of a route the survey may be best executed by *trigonometric methods* (Chap. IX), with light plane-table, with transit, or by photo-surveying methods (Chap. XIV). In such a case elevations may be conveniently determined by vertical angulation (Art. 160), when the resulting map will be rather geographic than exploratory in quality.

Astronomic position is determined in such a survey by latitudes observed with sextant, and longitudes obtained by chronometer (Arts. 336 and 328), or by lunar photographs (Chap. XXXVII). Azimuths are readily obtainable by observations on polaris with theodolite (Art. 312). In a compact trigonometric survey several careful determinations of latitude, longitude, and azimuth made at one point only are necessary. In running a route survey latitudes should be observed at distances not exceeding fifty miles, longitudes as frequently as convenient, according to the method, and azimuths on nearly every clear night.

The *sources of error* in such a crude route traverse are in the measurement of directions and distances. The former will be but slightly in error for the small scale of map selected if frequent azimuths are observed. Errors in distance will be fairly well compensated by the observations made for latitude. The most satisfactory way of determining longitude under such conditions is by means of ships' chronometers read at the point at which the expedition starts out, provided that be on the seacoast (Art. 328). The plotting of the final map will aid in keeping the longitude fairly well in check.

Elevations should be recorded by means of aneroid barometer (Art. 174), and the eccentricities of this may be kept in check by carrying a cistern mercurial barometer, which should be read hourly at each camp (Art. 170). Where the circumstances permit, a base barometric station should be established, at which the moving barometer should be compared with the stationary standard, and the latter should be read hourly throughout the duration of the expedition in order to permit of a reduction of the synchronous observations of the moving barometer (Art. 169).

36. Exploratory and Geographic Surveys Compared. The following is an interesting comparative group of maps illustrating the result of surveys of various degrees of accuracy. Fig. 17 is a small portion of the sketch map accompanying the report of Captain Zebulon M. Pike, and made in 1807. This includes the headwaters of the Platte and Arkansas rivers in Colorado, the point marked "Highest peak" being the summit now known as Pike's Peak, and "Blockhouse" being presumably the present location of Canyon City. This sketch map was made without the aid of instruments, and is entirely uncontrolled in distance or direction other than by estimates.

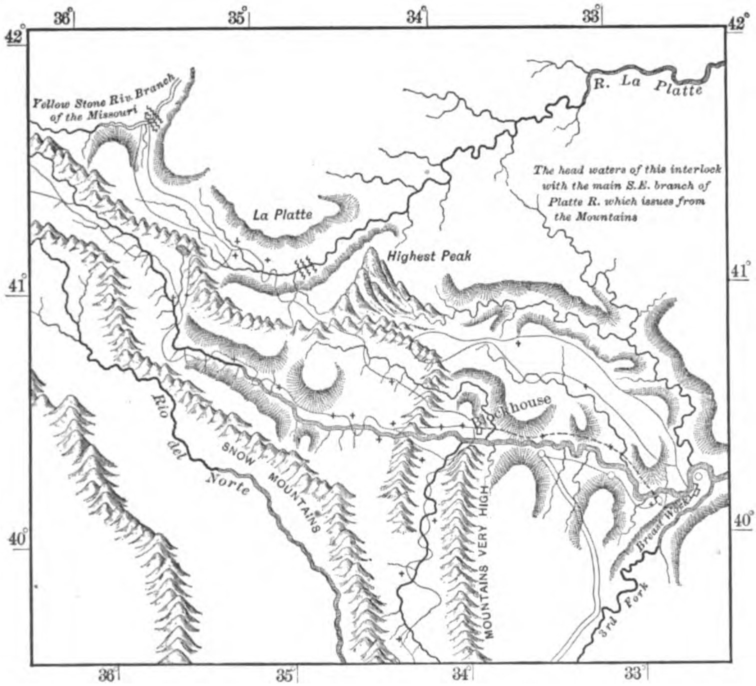


FIG. 17.—CAPT. ZEBULON PIKE'S MAP ABOUT PIKE'S PEAK, COLO. 1807.

Fig. 18 is a small portion of a map published in the report of Captain J. C. Fremont of an exploration across the Rocky Mountains in 1845. Geographic position is approximately fixed by means of sextant observations for latitude, chronometer, and lunar observations for longitude, and barometric observations for height. Between these sparsely scattered astronomic positions directions and distances are by estimate only, the route, however, being sketched at the time of travel.

Figs. 19 and 20 cover small portions of the same area on the west slope of Pike's Peak. The first is a portion of the U. S. Engineers' geographic map, scale of four miles to one inch, and made between 1873 and 1876. This map shows relative relief by means of hachures, actual relief being shown only by figures of elevation, the result of barometric or trigonometric observations. The surveying was executed by means of secondary triangulation with transit, expanded from

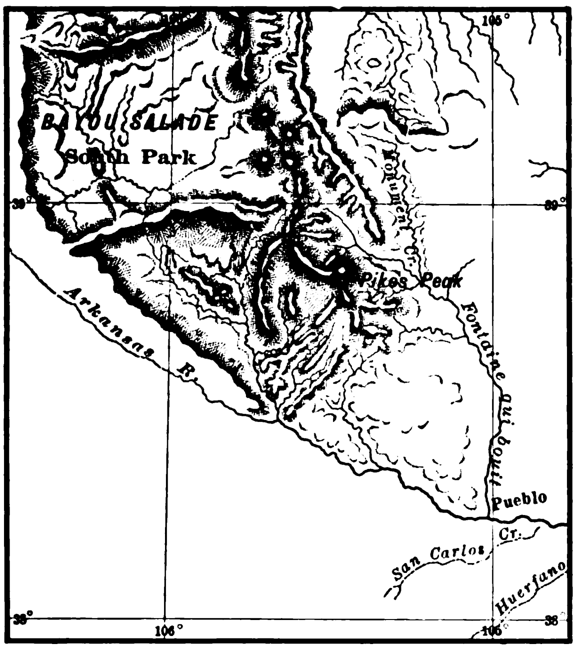


FIG. 18.—CAPT. J. C. FREMONT'S MAP ABOUT PIKE'S PEAK, COLO. 1845.

a primary triangulation executed with theodolite and based on an astronomic station and carefully measured base line. Intermediate details were sketched in from the secondary triangulation stations and by odometer traverses of roads. Fig. 20 is a small portion of the Hayden map covering the same area. This is a geographic map also on a scale of four miles to the one and executed at about the same time as the U. S. Engineers' map. The method of survey was practically the

FIG. 19.—WHEELER MAP ABOUT PIKE'S PEAK, COLO. 1876.
Scale 4 miles to 1 inch.

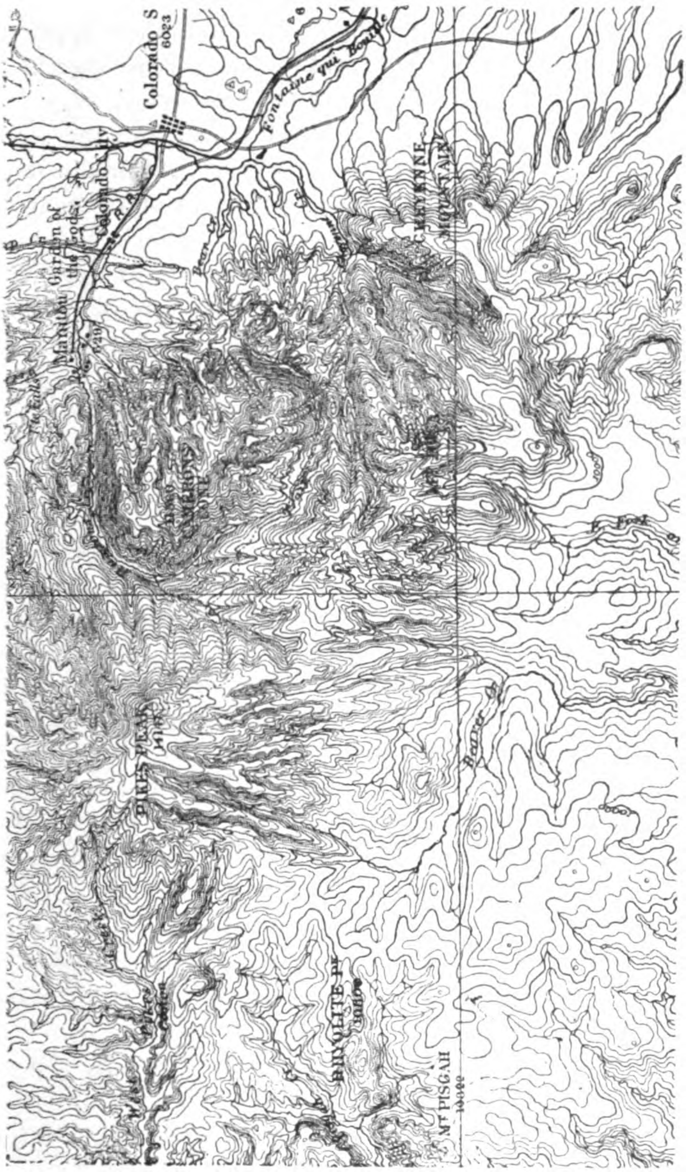


FIG. 20.—HAYDEN MAP ABOUT PIKE'S PEAK, COLO. 1875.
Scale 4 miles to 1 inch. Contour interval 200 feet.

same, but many more elevations were determined both by barometric and trigonometric methods, and from these approximate contours of two hundred feet interval were sketched, thus giving the relief with greater relative accuracy.

In Figs. 21 and 22 are shown small portions of the same area as mapped by the U. S. Geological Survey, the first in 1892 and the second in 1894. Fig. 21 is a fragment of an accurate geographic map on a scale of two miles to one inch, and with differences of elevation represented by contours of one hundred feet interval. Field-work was based on a careful primary triangulation and was executed by means of plane-table with telescopic and sight alidade for direction and vertical angulation, odometer traverses of roads, and sufficient spirit-leveling and stadia work to fill in the details. Fig. 22 is a large-scale topographic map of the same area executed on a scale of 1 : 25,000, approximately two and one-half inches to one mile, and with a contour interval of fifty feet. This was based on a plane-table triangulation and spirit-leveling accompanied by stadia traverses and intersections for both vertical and horizontal detail.

37. Methods and Examples of Exploratory Surveys.—The field-work of exploratory surveying includes the making of some form of record of the geography of the country passed over, which may be either kept in note-books and worked up in office or may be drafted in the field upon a sketch plane-table. Such surveys may be of a route only, especially where the course traveled is the bed of a narrowly confined stream, or through woods when little can be seen of the surrounding country, or it may be of an area when the explorers are traversing open country or high ridges which permit of an extended outlook over the region surrounding them.

The *personnel of an exploring party* should consist, if possible, of one individual qualified to conduct any form of survey, be it by transit, plane-table, compass, stadia, or estimate, as the circumstances may demand, and also competent to

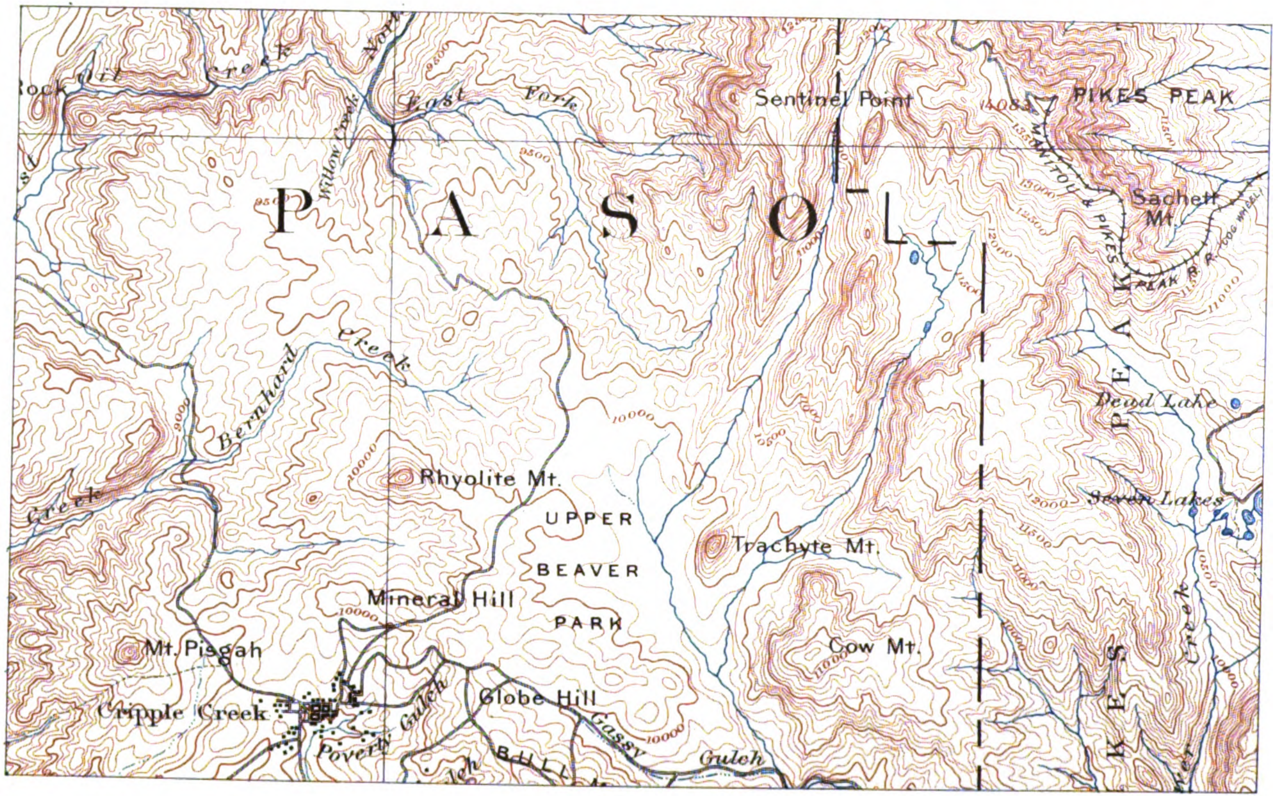


FIG. 21.—PORTION OF U. S. GEOLOGICAL SURVEY MAP ABOUT PIKE'S PEAK, COLO. 1892.
Scale 1 to 125,000. Contour interval 100 ft.

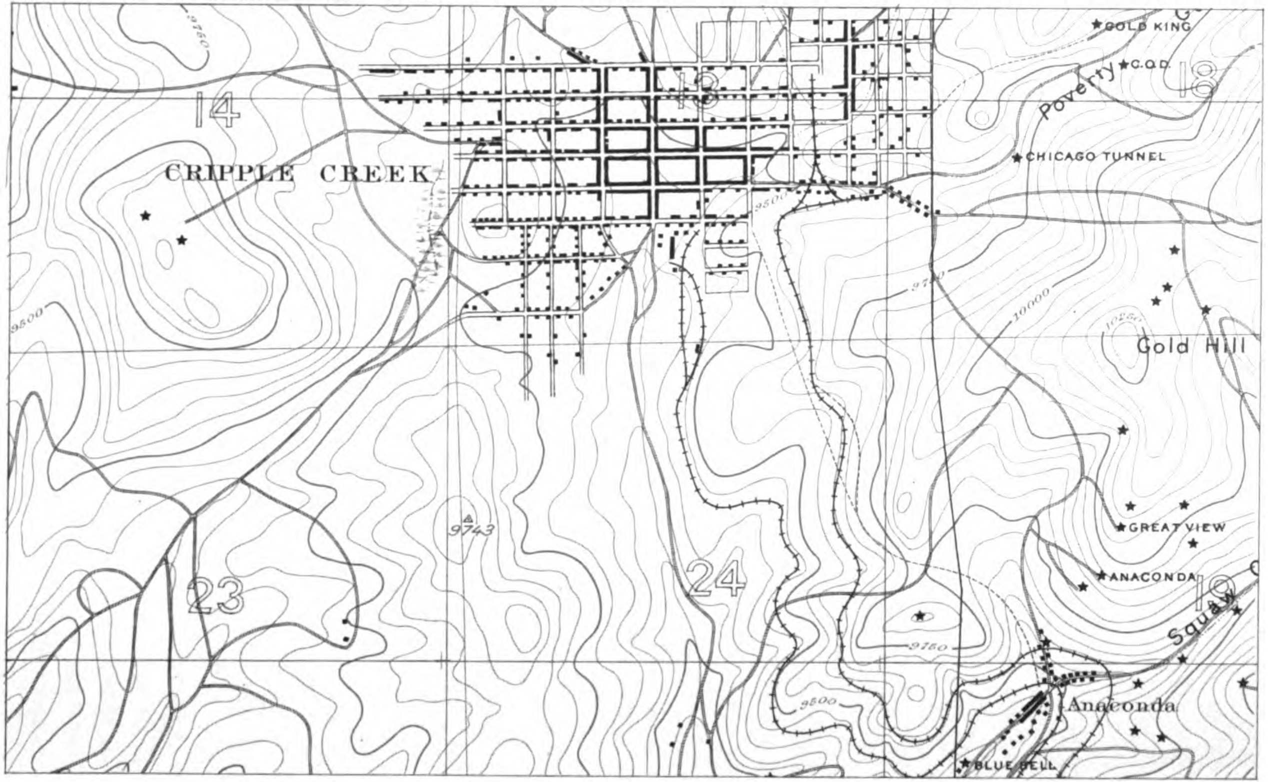


FIG. 22.—PORTION OF U. S. GEOLOGICAL SURVEY MAP ABOUT CRIPPLE CREEK, NEAR PIKE'S PEAK, COLO. 1894. Scale 1 to 25,000. Contour interval 50 ft.

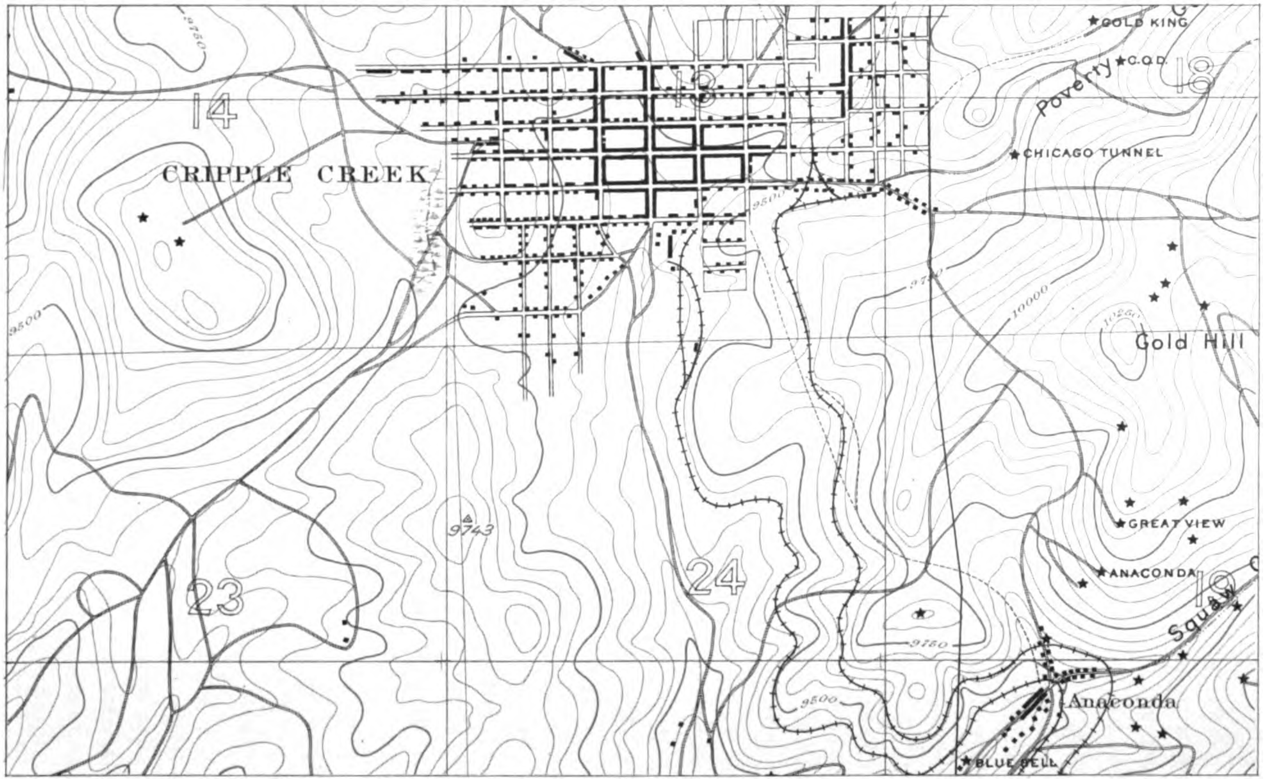


FIG. 22.—PORTION OF U. S. GEOLOGICAL SURVEY MAP ABOUT CRIPPLE CREEK, NEAR PIKE'S PEAK, COLO. 1894.
Scale 1 to 25,000. Contour interval 50 ft.

determine astronomic position by transit, sextant, and similar instruments. In addition, at least one other member of the party should be versed in the sciences of geology and biology in order that he may understand how to collect information of the mineral resources, the flora and fauna of the region traversed. A photographic camera should be carried for record of the aspect of the landscape or of details seen. The results of the work of this member of the party will accompany the map in the form of an illustrated report.

The *instrumental equipment* of such a party should include various forms of surveying instruments, at least two or three methods of measuring directions and distances being duplicated lest any of the instruments be lost or destroyed. There should also be carried aneroids and mercurial barometers for the determination of heights (Arts. 174 and 170). The instruments for determining direction should include, if possible, a light mountain transit (Art. 85) specially provided with prismatic eyepiece and striding-level for the determination of latitude and azimuth; a sextant for astronomic observations and the measurement of horizontal and vertical angles (Art. 336); a light plane-table with sight alidade (Arts. 56 and 62), to supersede the transit in rougher surveys when necessary, and a prismatic compass (Art. 91) to replace either of the above. Distances may be measured by means of the stadia hairs in the transit, a light pole being marked with bands of white cloth or string or other device as a stadia-rod (Arts. 101 and 112). Distances will be obtained in addition by triangulation (Chap. IX) or pacing, or by means of the linen tape, or by time estimate on land or in floating down streams (Arts. 65, 97, and 96).

Two examples of exploratory surveys are illustrated in Figs. 23, 24, and 25. The first is that of a route traverse made in Alaska in 1898 by Mr. W. J. Peters of the United States Geological Survey. This is the first authentic survey of that region and was made with plane-table and ali-

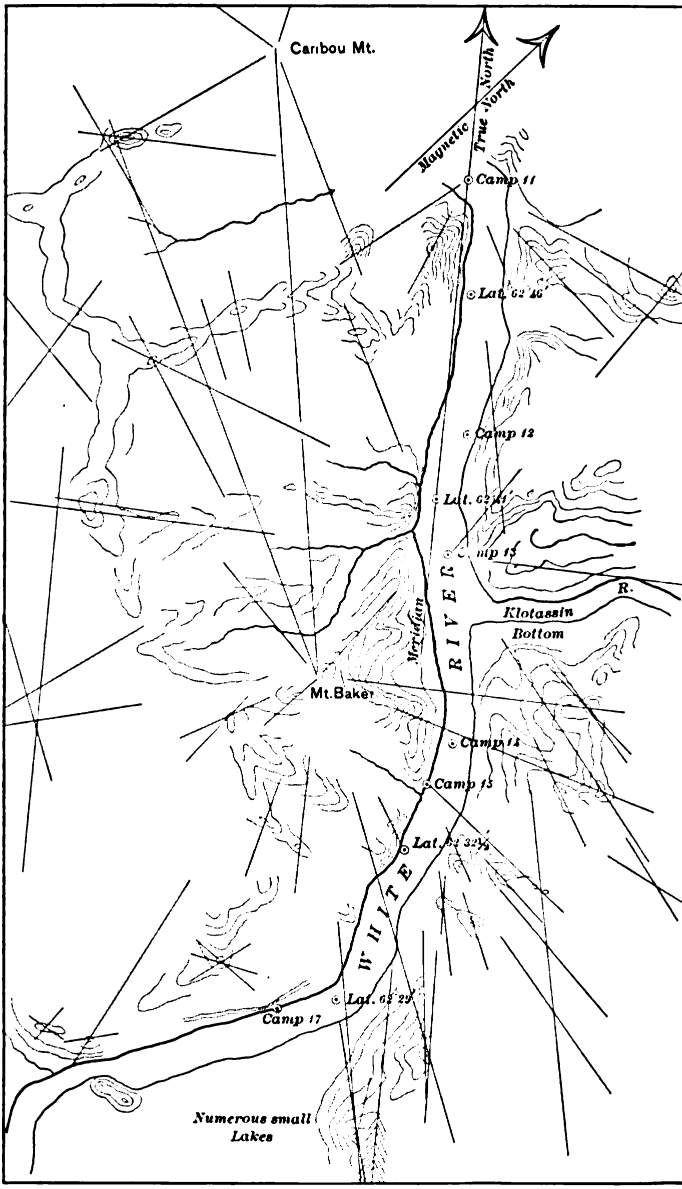


FIG. 23.—FIELD PLANE-TABLE SHEET. EXPLORATORY ROUTE SURVEY. ALASKA.
W. J. Peters, Topographer. Scale 1 to 180,000.

FIG. 24.—EXPLORATORY ROUTE SURVEY, ALASKA. FINAL DRAWING.
Scale 10 miles to 1 inch. Sketched contour interval 100 feet.

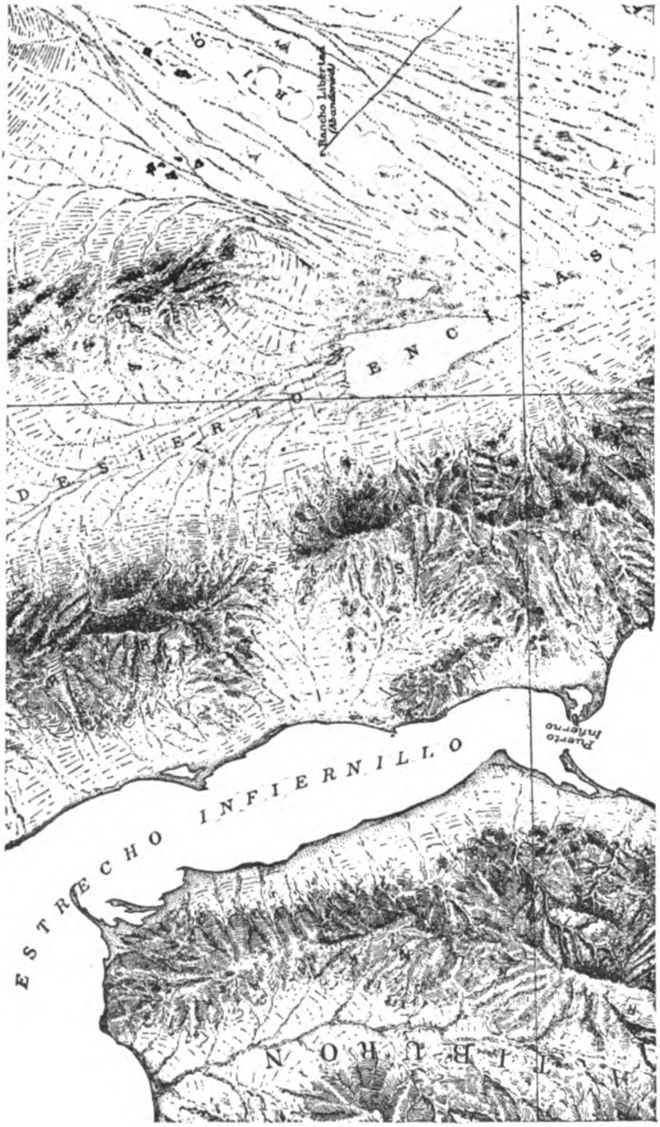


FIG. 25.—SERILAND, SONORA, MEXICO.
Exploratory Survey by W. D. Johnson. Scale 7 miles to 1 inch.

dade for directions, transit for latitude, and distances by stadia and pacing on the portages, and time and eye estimate on the water. It is complete in that it shows by sketch contours the shapes of the surrounding hills and gives from hearsay and other sources some of the detail of the region included within the route of exploration. Fig. 24 is the final office drawing of the same.

Fig. 25 is an exploratory map, not of a route, but of a compact area resulting from surveys by Mr. Willard D. Johnson of the U. S. Geological Survey for the Bureau of American Ethnology in the year 1895. This survey was made with a light traverse plane-table, oriented by compass and alidade. It was executed by plane-table triangulation, started at the international boundary line, and the map is dependent thereon, but is checked by the coast line of the U. S. Hydrographic Survey. The area mapped covers 10,000 square miles. The geographic position of the map is dependent upon the work of the boundary and coast-line surveys. This is a beautiful example of a well-executed sketch map by an expert topographer, although but sparsely controlled by instrumental locations.

CHAPTER V.

MILITARY AND CADASTRAL SURVEYS.

38. Military Surveys.—Ordinary maps are sufficient to enable one to follow the operations of a campaign, but for planning military operations detailed topographic maps are essential, because the merest trail or smallest stream or valley or undulation of the ground may for a time become of the greatest importance either for offensive or defensive purposes.

Topography for such uses, however, calls for the most simple problems of mathematical surveying, rarely has recourse to plane trigonometry, and never employs the principles of spherical trigonometry, because the areas included within any separate map sheet are small. *Military topography* is in fact the art of obtaining a detailed representation of but a moderate extent of the earth's surface. The resulting map should exhibit the important lines and characteristic objects on the ground, not only including main streams, railways, and mountains, but smaller watercourses, roads, and foot-paths, houses, enclosures, ditches, excavations, embankments, fences, hedges, walls, and the minor undulations of the surface, especially abrupt ledges.

The *preparation of the military map* consists of two operations:

1. The projection in proper relative position on a plane surface of the main outlines of the country; and

2. Of leveling, by means of which may be represented the slopes, elevations, and depressions of the ground.

In addition there must be prepared a *memoir* including essential information which it is impossible to exhibit in graphic form, such as kind of road, its surface and state of repair, descriptions of bridges, character of their approaches, depth and rapidity of current in watercourses, nature of bottom, statistics of number of inhabitants, supply of provisions and animals, etc.

The *basal outline map* upon which the military topography is to be exhibited may be a good topographic map, which should ordinarily be constructed in a manner similar to that described for the making of small-scale topographic maps (Chap. II). Having such a base, it is then possible to enlarge it to a sufficient scale to permit of representing upon it those details of information which are essential to military maps, and these may ordinarily be obtained by less accurate methods of survey, by the use of the plane-table or cavalry sketch-board (Arts. 57 and 64), supplemented by odometer, stadia, range-finder, or by pacing or by counting the paces of a horse (Arts. 98, 102, 117, and 95). With such instruments it becomes possible to sketch on the base map with some accuracy the positions of the hedges, minor watercourses, etc., and to enter in a note-book the data which go to make up the memoir.

That form of surveying which produces a military map may be classed as *irregular surveying*, and consists ordinarily of rapid, interrupted journeys having for their object the representation of the natural and artificial features of the country with the maximum exactitude consistent with the greatest rapidity of execution, and it is therefore evident that they are based upon the same principle as are more elaborate surveys (Art. 9). The differences between them consist chiefly in the use of more portable and less bulky instruments, in the substi-

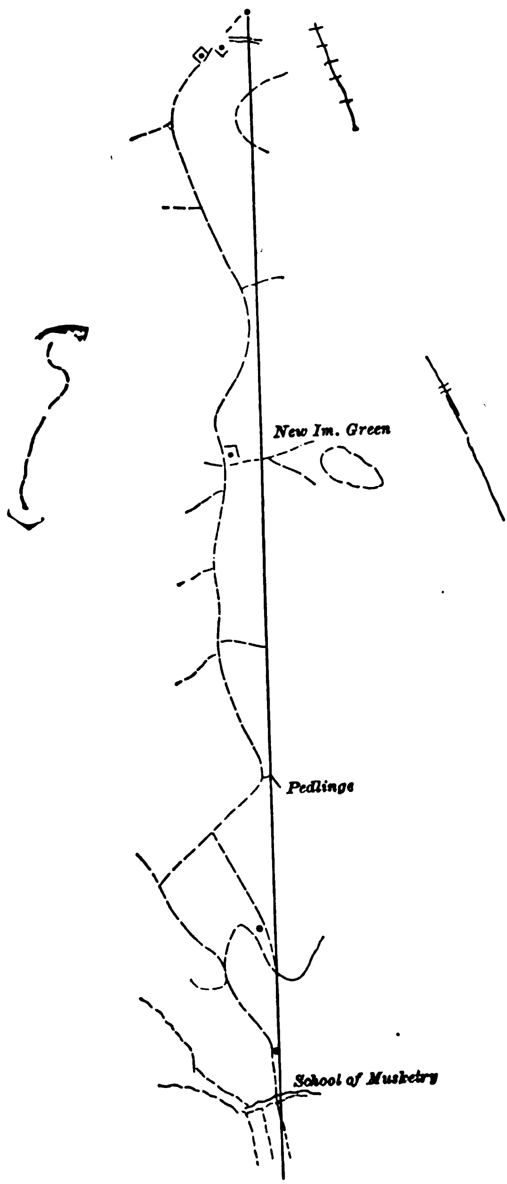


FIG. 26.—SKELETON OF ROUTE FROM BEST AVAILABLE MAP. After Capt. Willoughby Verner. Scale $1\frac{1}{4}$ inches to 1 mile.

tution of pacing or range-finding for the chain, and often in the estimation of distances and details by the eye. Such surveys are commenced by the determination of principal points by triangulation (Chap. IX) if such does not already exist, or by identifying triangulation points already existing. To these triangulation points further details are referred, and further irregular surveys are planned by a general glance at the field from them. Vertical measurements, which are essential in the representation of surface slopes, are but relative, and may be best had by the use of the aneroid.

39. **Military Reconnaissance with Guide Map.**—The following examples from Captain Willoughby Verner show the mode of converting a small-scale geographic map into a detailed military map. In Fig. 26 is shown the outline of the route of the proposed reconnaissance. This is an enlarged copy of road and drainage crossings taken from a one-mile British ordnance map. In the following figure (No. 27) is shown the mode of filling in important military information on such a base, the notes all being made on the map in the course of a quick cavalry ride. In Fig. 28 is shown the final drawing made in camp from the notes taken on the preceding cavalry sketch map. All of this information was obtained without the use of instruments other than a sketch-board carried on the wrist of the topographer.

40. **Military Reconnaissance without Guide Map.**—In Fig. 29, taken from Willoughby Verner, is shown a portion of a river reconnaissance on the Nile executed from a river steamer. The distances were reckoned by time, and the directions by magnetic bearings. The important military information accompanying this is in the form of marginal notes.

An extended reconnaissance sketch was made in the Soudan by Captain Verner, with the range-finder for distances, and light plane-table board for directions. This was more than

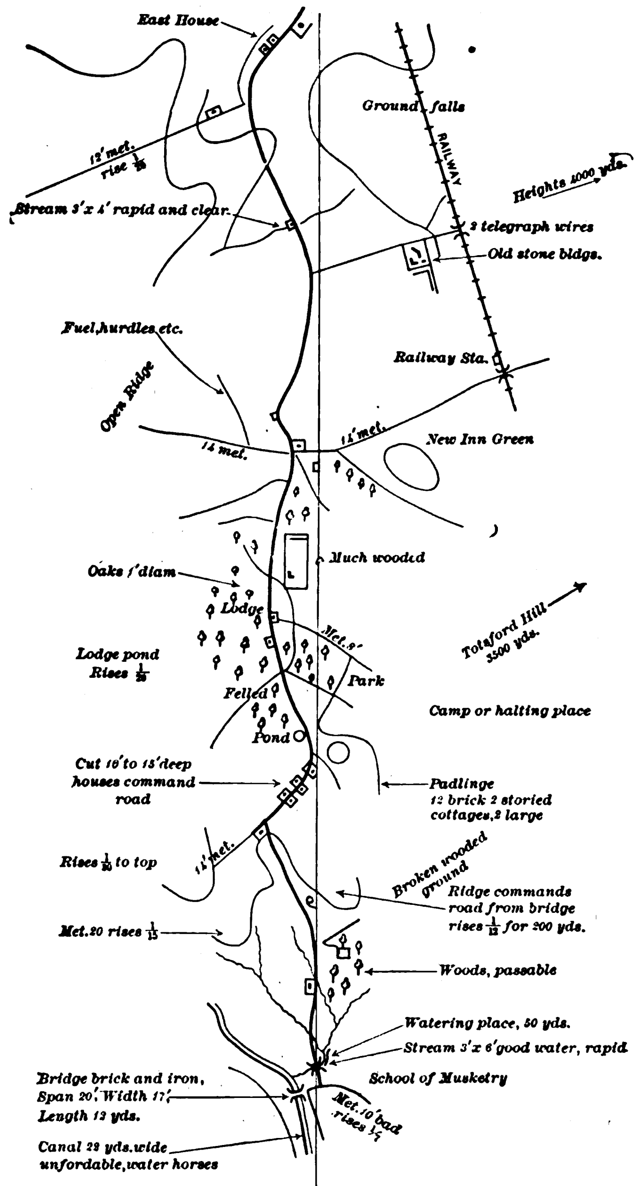


FIG. 27.—SKETCH ROUTE OF FIG. 26 FILLED IN WITH FIELD NOTES. After Capt. Willoughby Verner.

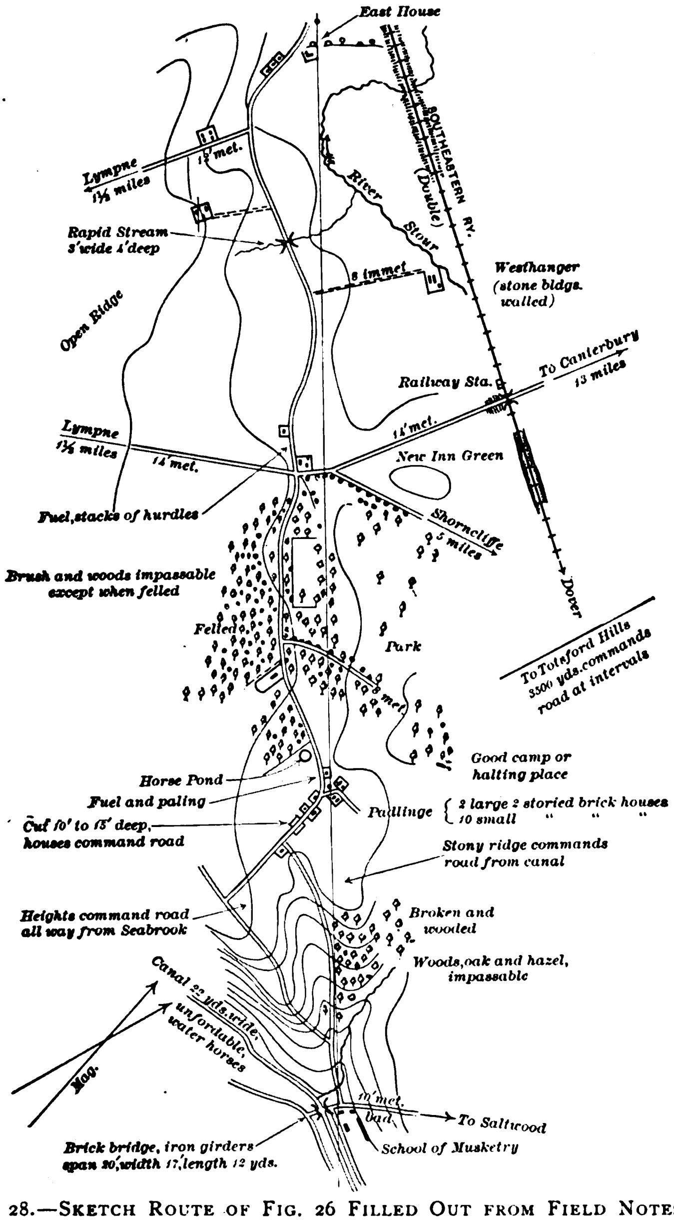


FIG. 28.—SKETCH ROUTE OF FIG. 26 FILLED OUT FROM FIELD NOTES OF FIG. 27.
After Capt. Willoughby Verner.

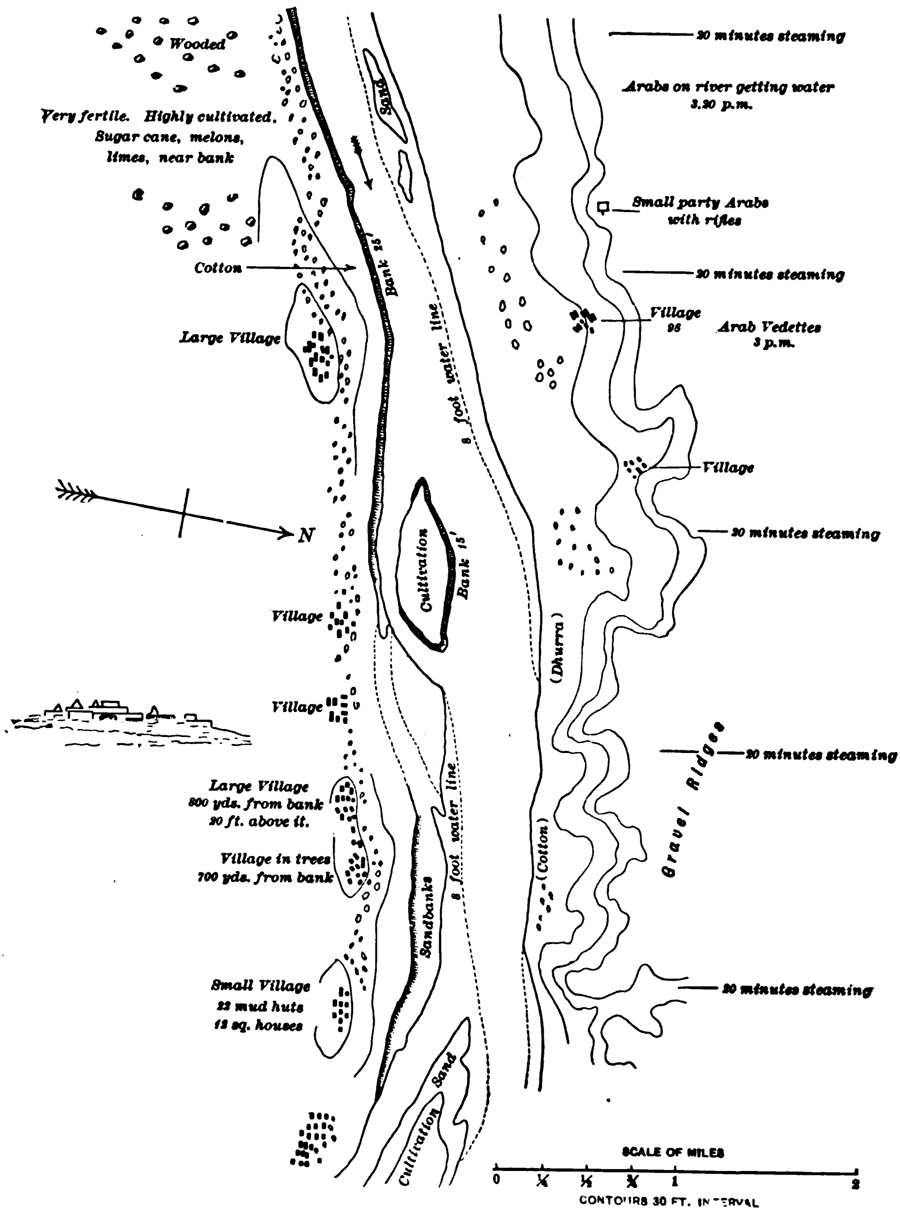


FIG. 29.—RECONNAISSANCE ON NILE RIVER FROM GORDON'S STEAMER. DISTANCE BY TIME.
After Capt. Willoughby Verner.

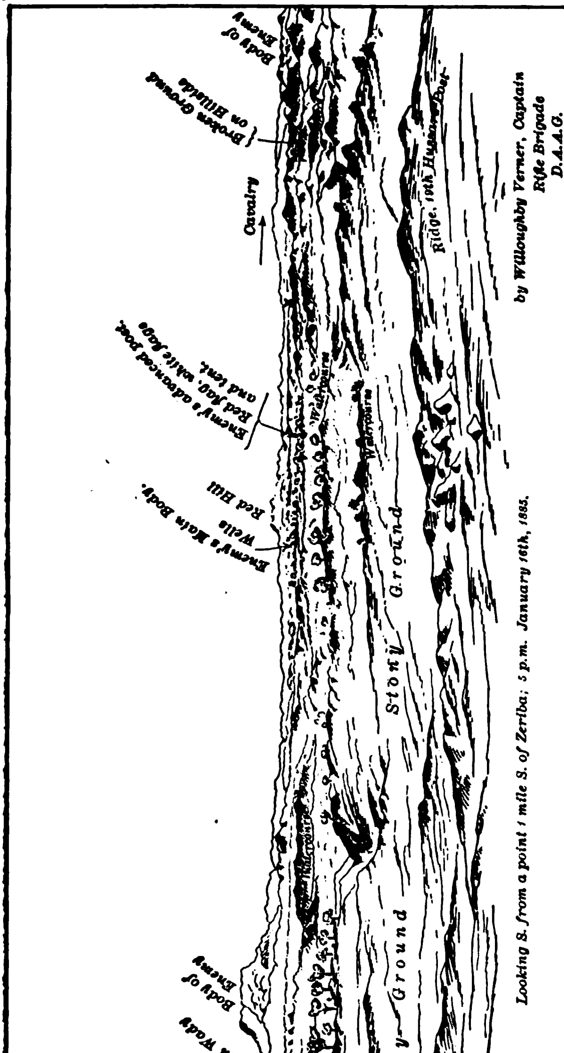


FIG. 30.—RECONNAISSANCE SKETCH OF ARAB POSITION AT ABU KLEA.
After Capt. Willoughby Verner.

a route reconnaissance, the territory covered being developed by plane-table triangulation (Chap. IX) and range-finder (Art. 116), so as to cover a fairly extended area of the country. Such reconnaissances need not be accompanied by memoir, since all necessary notes are placed upon the margin of the map. They should, however, when made to develop the position of an enemy, be accompanied by perspective sketches similar to that illustrated in Fig. 30.

41. Detailed Military Map.—In many of the more important operations of the Civil War, such as the final action about South Mountain, time was afforded for the making of careful topographic surveys for the procurement of military information. Advantage was taken of such opportunity by making detailed surveys, in which roads were traversed with transit and chain or odometer (Arts. 87, 99, and 98), elevations measured with level as well as aneroid (Arts. 129 and 174), and the resulting map gave all the detail of relief and cultivation which could be of use to the commanders. Fig. 31, reproduced from one of these maps, is an excellent example of the character of detail procured in making such surveys.

Where a number of persons are employed in surveying a region not previously mapped, as that described, accurate results can be obtained only by extension of a brief skeleton triangulation, locating a number of conspicuous points, or by running a few very careful transit and chain traverses of important roads. This laid down on a large sheet is the basis of the survey, securing for it accuracy independent of the minor errors which must pervade the more detailed surveys executed with inferior instruments and in haste. Such a survey can only be made when there is ample time and protection and suitable instruments are available. Moreover, such survey would only be necessary in unknown country. Where pressed by lack of time or protection only the crudest sketching implements and methods can be employed, and the survey must be a rapidly executed sketch accompanied by

FIG. 31.—MILITARY MAP OF OPERATIONS ABOUT SOUTH MOUNTAIN.

memoranda of the country immediately adjoining the line of march.

42. Military Siege Maps.—In siege operations sketch maps, photographs, and, when possible, more accurate surveys based on instrumental observations, are made of the position attacked. These, so far as possible, are executed in such detail as to show not only the immediate surroundings with a view to their availability for the purposes of the attack, but especially the details of fortified places in order to develop

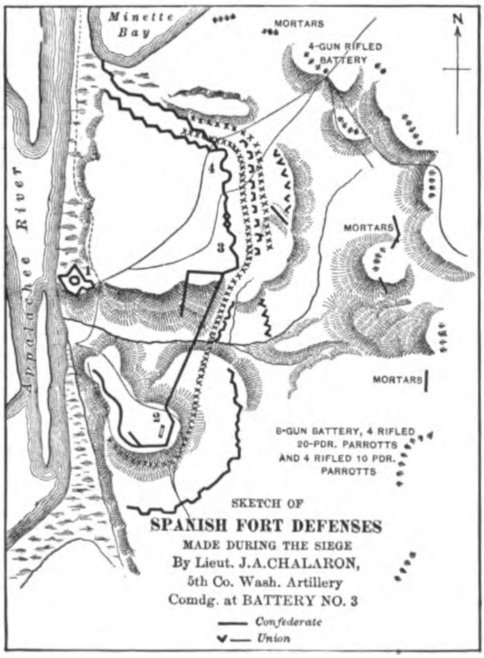


FIG. 32.—MILITARY SIEGE MAP.

their weak points and their strength. Fig. 32, reproduced from maps accompanying the Records of the War of the Rebellion, is a map of this sort executed during the siege of Spanish Fort.

43. **Military Memoirs.**—Information must be procured from the inhabitants, spies, or other sources, and the military map filled out as well as may be from verbal descriptions. Itineraries of routes should be plotted and kept in memoir form for the guidance of bodies or troops in marching, and for

resting and camping places for convoys and supply trains. In the memoir various streams must be noted, their number, position, depth, banks, fords, bridges, etc.; ponds, marshes, canals, and springs must all be described, with a statement as to how they are formed, whether subject to overflow, and if crossed by roads, how and where. Bodies of woodland and forest must be described, as to their shapes, positions, etc. The classes of roads, their condition, facilities for passage of heavy wagons or troops, and for repair must also be noted.

Villages and fortified places must be described, with notes of houses, materials of construction, supply depots, workshops, and fortifications. Statistics must also be gathered of modes of transportation of horses, wagons, cattle, sheep, etc., also of available provisions, including corn and hay for forage. Mountains and hills must be described, with regard to their continuity, direction, nature of slopes, how covered, their area; also whether rocky or smooth, practical for occupation by either arm of the service, and if so, where; also, the passes across them, their relation to the main chain or ridge, etc., etc.

44. Cadastral Surveys.—This class of surveys takes no account of the surface of the earth outside of the limited area covered by the immediate route touched in the actual process of fixing located points upon it. *A cadastral survey is prosecuted* for the sole purpose of determining political or property lines, and merges on the one hand into surveys crudely executed with chain and compass (Arts. 99 and 91), and on the other hand into field methods of the most refined geodetic nature (Art. 201). Cadastral surveys are mentioned here because they are frequently made with such thoroughness as to result in the production of a topographic map of the entire area inclosed within the boundary lines.

The word *cadastral* is one which is not familiarly used in surveying nomenclature and the meaning of which is variously and frequently erroneously interpreted. It is probably of French origin and was apparently first applied with any definiteness at the statistical conference held in Brussels in 1853, as referring to national maps on very large scales, approximating 1:2500. At the same time the term "tableau d'assemblage" was applied to large-scale general maps of, say, about 1:10,000. The word *cadastre* has been accepted in Great Britain as being referred to a map or survey on a large scale, because the scale of the map corresponds with a *cadrer*, being that scale in nature which will permit of representing accurately the width of a road and the dimensions of a building. More recently on the Continent the expression "cadastral survey" is applied to a plan from which the area of land may be computed and from which its revenue may be valued.

As now more generally understood, *a cadastral survey* is one which includes several of the above features. It is not a topographic survey for representation of a terrane on a very large scale, nor is it any form of a topographic survey, as defined by the English interpretation of the meaning of the word cadrer. It is essentially a property survey as expressed in the more recent Continental definition, but it is executed not only that the areas of lands may be computed for the valuation of revenue, but also and primarily for the purpose of fixing public and private property lines by marks and monuments. A secondary result of a cadastral survey of such thoroughness as to closely cover the entire area is the procurement of such notes as will permit of the making of a large-scale topographic map, such as would come under the British or Brussels definition. Examples of topographic maps resulting from or executed in the progress of cadastral surveys are instanced in Article 27, describing the surveys of the cities of Baltimore and St. Louis. In both of these cases the cadastral surveys are based on control of geodetic accuracy.

Near the other extreme are the cadastral surveys executed under the direction of the U. S. Land Office in the subdivision of the public lands of the West. The primary object of these surveys is the division of the public lands into areas called townships and sections by means of comparatively crude transit and chain traverses (Art. 87). The result is depicted in maps or plots showing the property outlines with their dimensions. A secondary result is the furnishing of data for the computation of the areas of the various subdivisions. A tertiary result is the representation of the terrane covered by a crude topographic map of such quality as to be exploratory only in its accuracy. The only information accurately depicted on such maps is that lying immediately under the route covered by the boundary survey, the information contained between such traverse lines being largely interpolated by estimation and guess.

Another step toward the attainment of a higher grade of cadastral survey in the *subdivision of the public lands* is instanced by the mode of subdivision employed in the public-land surveys of the Indian Territory as executed by the U. S. Geological Survey (Fig. 33). In addition to being performed in a manner similar to that of other public-land surveys, these surveys are controlled and checked by means of a geodetic survey executed by trigonometric methods, thus giving it a far more permanent character and fixing with accuracy its position upon the face of the earth. Moreover, the resulting map is a true topographic or geographic map, because the terrane between the property lines was surveyed and its elevations determined as an adjunct to the execution of the cadastral survey.

A still more accurate cadastral survey prosecuted solely for the purpose of marking political boundary lines is that executed by the Massachusetts Topographic Commission. The purpose of this survey is the demarkation with exactitude upon the ground-surface of town and county boundary lines.

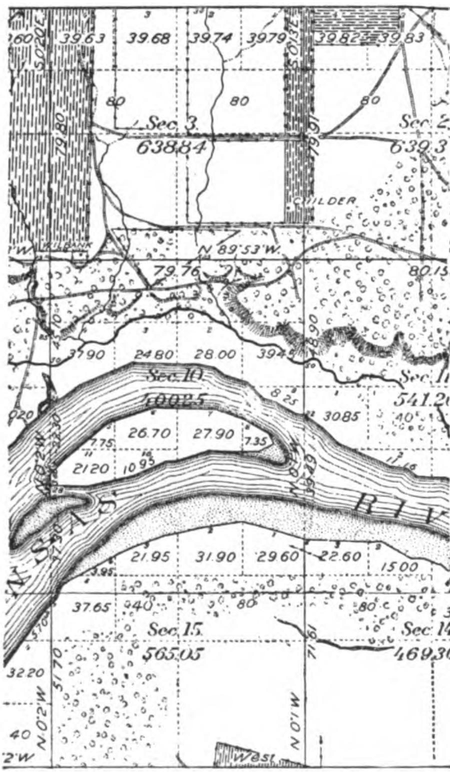


FIG. 33.—CADASTRAL MAP OF U. S. PUBLIC LAND SURVEY. INDIAN TERRITORY.
Original scale 2 inches to 1 mile.

The field-work of this survey is based upon and controlled by a trigonometrical survey of geodetic accuracy, and as a result the positions of the various monuments and their connecting lines are fixed on the surface of the earth in proper astronomic position. The primary result of this survey is the accurate demarkation of the boundaries. A secondary result is the procurement of data by which the exact area of each township may be computed. The amount of data gathered, however, does not permit of the making of a topographic map of the area included within the bounds.

TABLE III.

SCALE AND COST OF CADASTRAL SURVEYS.

Country.	Scale.	Relief.	Cost per sq. mi.
Great Britain; Ordnance Survey......	1:2,500	Hachures and contours	$ 294.00
City of St. Louis, Mo. (Cadastral)...	1:2,400	Contours 3 ft.	739.00
City of Baltimore, Md. (Cadastral)....	1:2,400	" 5 "	4070.00
Public Land Subdivisions, U. S. G. S.	1:31,680	" 50 "	31.00

CHAPTER VI.

TOPOGRAPHIC FORMS.

45. Relations of Geology to Topography.—The claim is not infrequently made by geologists that a knowledge of this science is essential to the topographer in the prosecution of his work. On the other hand, the not infrequent contention of the topographer is that no amount of knowledge of the sciences will add one jot to his ability to accurately represent the topography. Both contentions are correct under various circumstances, dependent upon the purpose of the resulting map and the accuracy desired. A *knowledge of geology* may influence the plans but not the operations of a topographic survey. It may aid in the execution of the topography, but cannot exercise a control over it. It has been stated by a topographer that the sphere of the sciences is to follow after topographic surveying and not to precede it; that they furnish resulting utilities dependent upon the topography, but are not essential factors controlling its method. This latter view is no more correct than the former.

Where topographic surveying is to be executed with only one or two specific objects in view, as the furnishing of a base map for a geologic survey, or of a small-scale topographic map for general uses as a guide or sketch map, a knowledge of the science of geology may be of the greatest utility. Under such circumstances the greatest accuracy is not essential, and such accuracy as is required the topographer will attain. His work may, however, be greatly facilitated, and the expression of the resulting map improved, if he has sufficient knowledge of both geography and of geology to appreciate the origin of the topographic forms which he is sketching, and the way in which the various rocks have been upheaved, eroded, folded, or deposited. By a careful study of such forms as he first encounters the topographer is able to bring out with less effort the same characteristics in such similar formations as he may encounter as his work progresses.

On the other hand, if the survey is to *result* in *an accurate topographic map* useful as a base-map for all scientific and engineering purposes, the topographer must obtain such amount of control, and must see at such close range every feature which he sketches, that any amount of knowledge of geology or geography will add little to the quality of his representation of the terrane or the accuracy with which the result is depicted. If his work is done with that detail which is essential to the making of an accurate map, he will locate contours with such frequency that the resulting map will depict them as they actually exist, regardless of theories as to the origin of the forms sketched. Such a map is a topographic map *per se*. It is the mother map, and from a study of it the geologist or geographer learns to interpret the origin of topographic forms and is enabled to devise a correct scientific hypothesis.

46. Origin and Development of Topographic Forms.— A knowledge of the laws governing the origin and development of topographic forms is desirable in those who would intelligently depict them. The new topographic method demands such a knowledge in order that the surveyor may attain the highest skill, not only in the representation of the relief, but in the speed and methods, and consequently the cost, of such representation. The various rules for the classification of topographic forms hold good only in limited areas, and are subject to so many exceptions that any attempt at their general application utterly fails. Nevertheless a knowledge of these may frequently aid in mapping a region to which they are found to apply.

Prof. John C. Branner has aptly described topography as the "*expression of geologic structure*, much as the outlines of the human body express anatomical structure." As topographic form is the resultant of eroding agencies and the resistance of rocks, their study, he says, belongs fundamentally to the province of geology. It follows that for a thorough understanding of topographic forms the surveyor should have a knowledge of geology. This is true in topographic surveying, because a large part of every map must be sketched in (Arts. 9 and 13), and this sketching cannot be properly done unless the surveyor possesses some knowledge of the formations which he is depicting. Unless he knows what to look for he does not find it all, but only a part of it; consequently it is of importance to the topographer that he should know what kind of topography to expect, and to this end the more he knows of the materials in which the topography is carved, and the agencies which shaped it, the clearer will be his insight and the less the waste of energy and time required for the representation of the relief.

47. Physiographic Processes.—Before an intelligent understanding can be had of the origin of topographic forms we must first look to the processes in nature by which such forms are created. Major J. W. Powell defines *Physiography* as a description of the surface features of the earth, and a study of Physiography as including an explanation of their origin.

The earth has three moving envelopes:

1. The atmosphere which covers it to a great depth;

2. Water, which covers more than three-fourths of its surface; and

3. A garment of rock

There are two general classes of *topographic agencies*, which may be called *constructive* and *destructive*. An example of the former is a volcanic cone built of ejecta from the vent of a volcano which may have burst forth upon a level plain, or of a plain resulting from the flow of fluid lavas, which form a flat surface by filling up existing irregularities. The destructive agencies, chief among which are erosion by running water, wave action, wind, and frost, would be illustrated by a part of the ocean's bottom, which being uncovered and left as dry land would possess certain irregularities, but of a smoothed-out and easily rolling character. Erosion and wave action soon begin to attack such a surface and to cut stream-beds and produce topographic forms altogether different from its original surface.

Among the chief *constructive agencies* are:

1. *Disastrophic processes* by which regions sink and rise;

2. *Vulcanic processes*, due to ejectment from rents in the earth's surface of material brought from the interior.

Among the chief of the *destructive agencies* are:

1. *Aqueous erosion*, due to water flowing over the surface of the earth, as from rain, springs, or streams;

2. *Aerial erosion*, from wind-driven sand;

3. *Corrasion*, due to ripple and wave action and to glaciers; and

4. *Disintegration*, due to changes in temperature and to frost.

Horizontal changes are produced primarily by aqueous agencies, and the action of water is the chief agency in shaping topographic forms. *Aqueous agencies* act by erosion, transportation, and corrasion, and of these erosion has produced nine-tenths of the topographic forms in the United States. To the topographer the forms produced by aqueous erosion are those commonly seen, and have been classed by Mr. Henry Gannett as *regular forms*, while those shaped by other agencies he calls *irregular*. Aqueous erosion, being produced by simple actions of a kind which can be seen and comprehended, produces forms which can be to a certain extent predicted or foreseen. The forms produced by other agencies, being unseen, can rarely be predicted. Such agencies have produced the complicated system of mountain-folds of the Appalachian region. Acting on these complicated forms, aqueous erosion has in the same regions produced a remarkably complex drainage system.

48. Classification of Physiographic Processes.—Physiographic processes, by which is meant the operations of nature by which topographic forms are produced, may be divided into four classes:

1. Diastrophism,
2. Vulcanism,
3. Weathering,
4. Gradation.

These various primary physiographic processes have been well classified by Doctor C. Willard Hayes in the following tabular form:

I. *Diastrophism.*
- 1. Tangential forces, producing deformations of the strata.
- 2. Radial forces, producing vertical oscillations.

II. *Vulcanism.*
- 1. Intrusions.
  - (1) Plutonic plugs.
  - (2) Laccolites.
  - (3) Volcanic necks.
  - (4) Dikes.
- 2. Eruptions.
  - (1) Lava flows.
  - (2) Explosive ejections.

III. *Weathering.*
- 1. Agencies, Chemical.
  - (1) Hydration.
  - (2) Oxydation.
  - (3) Solution.
- Agencies, Mechanical.
  - (1) Heat.
  - (2) Moisture.
  - (3) Vegetation.
- 2. Conditions.
  - (1) Altitude.
  - (2) Temperature.
  - (3) Humidity.
- 3. Effects upon.
  - (1) Igneous and crystalline metamorphic rocks.
  - (2) Calcareous rocks.
  - (3) Argillaceous rocks.
  - (4) Siliceous rocks.

IV. *Gradation.*

1. Agencies.
   - (1) Running water.
     - *a.* Erosion.
     - *b.* Corrasion.
     - *c.* Planation.
   - (2) Glaciers.
   - (3) Winds.
2. Processes.
   - (1) Disintegration.
     - *a.* Chemical.
     - *b.* Mechanical.
   - (2) Transportation.
     - *a.* By solution.
     - *b.* By suspension.
     - *c.* By rolling.
   - (3) Deposition.
     - *a.* Alluviation.
     - *b.* Sedimentation.
     - *c.* Chemical precipitation.
3. Conditions modifying Gradation.
   - (1) Rainfall.
     - *a.* Amount.
     - *b.* Distribution.
   - (2) Declivity—effect on.
     - *a.* Corrasion.
     - *b.* Transportation.
   - (3) Vegetation—effect on.
     - *a.* Erosion.
     - *b.* Rainfall.
4. Drainage development.
   - (1) The normal cycle—characteristics of.
     - *a.* Youth. *b.* Maturity. *c.* Old age.
   - (2) Consequent streams—courses determined;
     - *a.* By accidental irregularities.
     - *b.* By deformation before emergence.
     - *c.* By deformation after emergence.
   - (3) Antecedent streams.
   - (4) Superimposed streams;
     - *a.* From unconformable horizontal strata.
     - *b.* From planation and alluvial deposits.
   - (5) Subsequent streams.
     - A. Conditions favoring stream adjustments.
       - *a.* Successive periods of base leveling.
       - *b.* Strata of diverse resistance.
       - *c.* Folded structure.
       - *d.* Local deformations of base level.
     - B. Process of stream diversion.

49. **Erosion, Transportation, and Corrasion.**—The *erosive action* of water on rocks has been enormous in amount and has continued through such extended periods of time a

to carve the giant ranges of Colorado from enormous plateaus. From these plateaus the drainage systems of the Colorado and Arkansas rivers have been worn away to such depths as to produce canyons and cliffs of thousands of feet in depth (Fig. 34). The action of water through weathering is illustrated in the disintegration of rock and its conversion into soil. *Transportation* has carried the material thus loosened. In the movement of this transported material by streams it has corraded other materials from their channels. It is thus seen that *corrasion* is effected by the detritus which running water holds in suspension. The rate of corrasion is increased in proportion to the volume of the stream, its velocity, and the amount of detritus borne, as well as by the coarseness of that detritus.

If a stream have its source at a high altitude and be assumed to have a uniform slope thence to its mouth, its volume, velocity, and amount of detritus borne will be greatest near its mouth, and there corrasion will be most rapid. As a result the slope of the stream will be reduced rapidly near its mouth, producing as a normal profile of its bed a *curve concave upward.* While the slope of the bed remains great and the velocity consequently great, the stream has a comparatively straight channel. As the slope is reduced the course of the stream becomes crooked and winding and its corrasive agencies are diverted from its bottom to its sides. Therefore, *swift streams flow in straight channels, sluggish streams in crooked channels.* This operation is being performed not only in the main stream, but in its numerous affluents to the minutest rill, but with different intensity. Accordingly, the higher branches have less power to cut and level the surface, and there the *curves* of their bed are *convex upward.* Such a curve is the curve of the terrane, while the concave curve is that of the watercourse. The former is the curve of the upper relief of high slopes, and the latter of the valleys.

In an *arid region* rainfall is scanty and spasmodic, streambeds are few in number, and the drainage system is consequently imperfectly developed. There the erosion of the

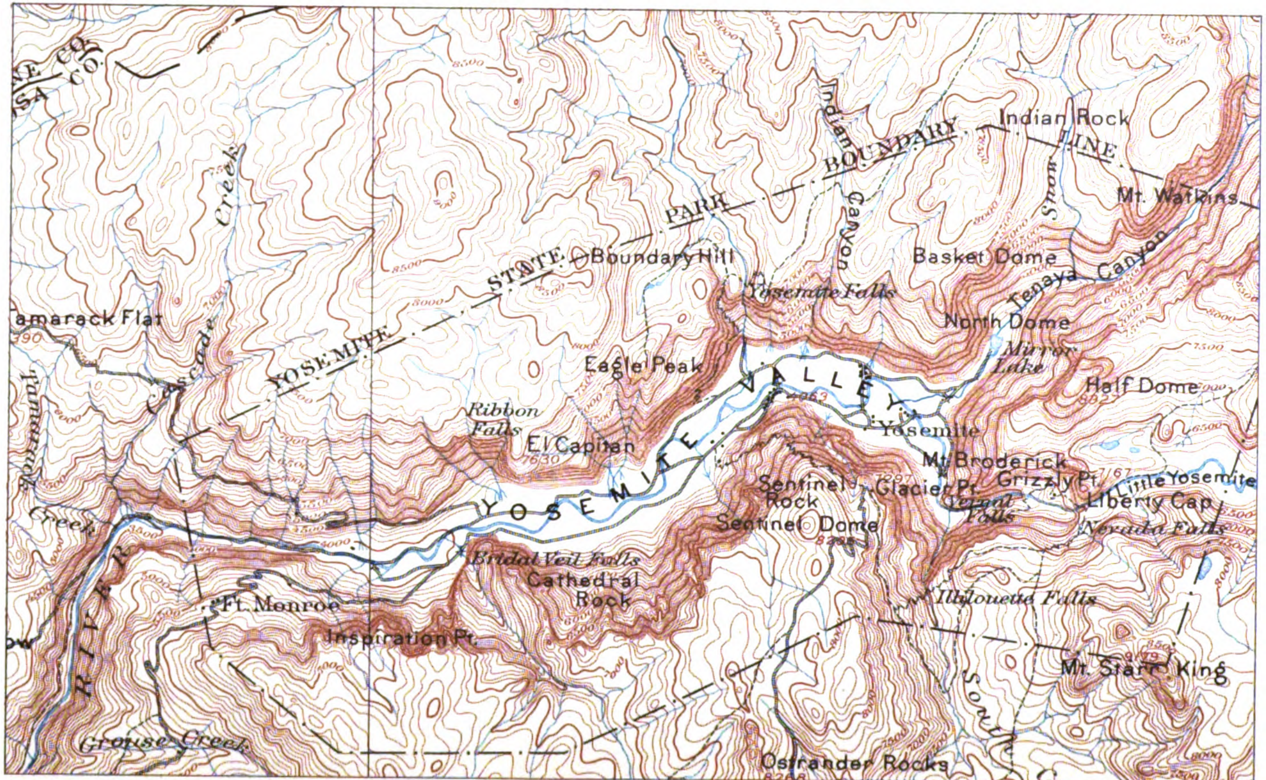


FIG. 34.—CANYON IN HOMOGENEOUS ROCK; ALSO VALLEY, CLIFF, DOME, MOUNTAIN, RIVER, CREEK, POND, AND FALL, YOSEMITE PARK, CAL.
Scale 2 miles to 1 inch. Contour interval 100 ft.

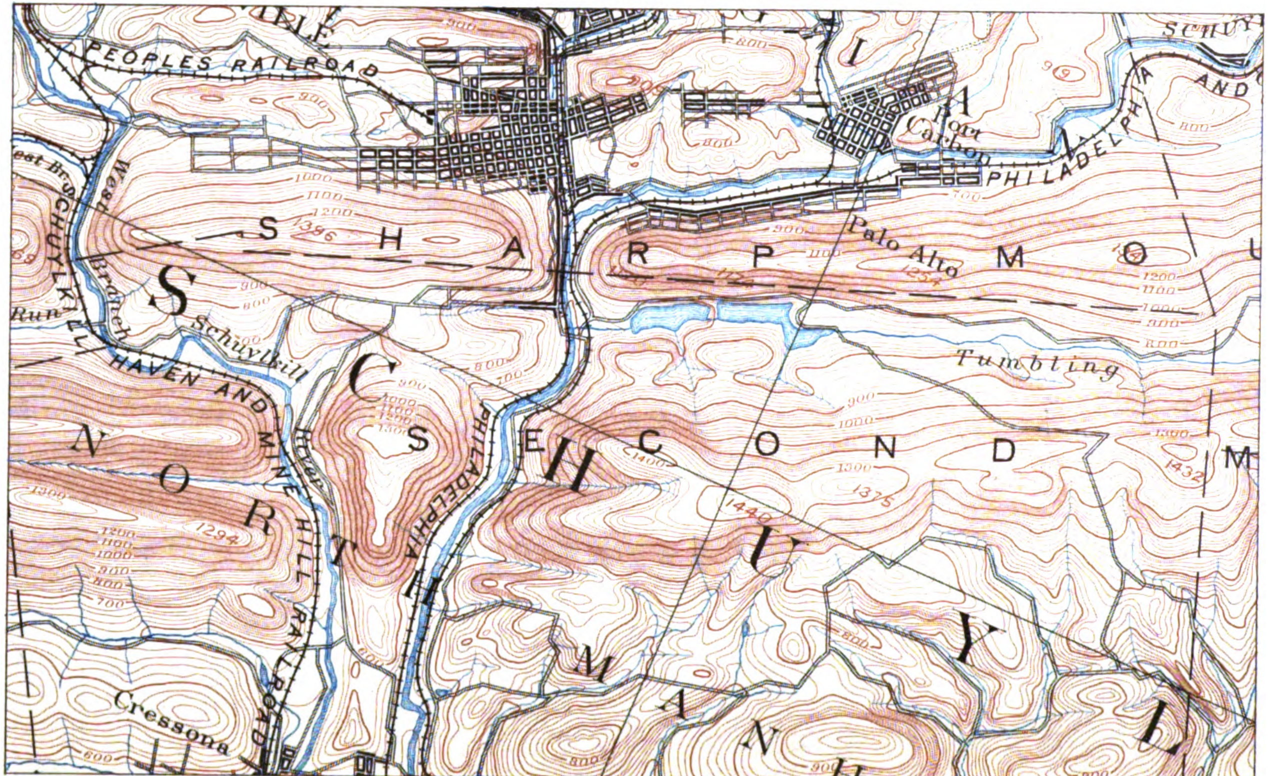


FIG. 35 —WATERGAPS AND PIRATING STREAMS IN MOUNTAIN RIDGE, POTTSVILLE, PA.
Scale 1 mile to 1 inch. Contour interval 20 ft.

terrane is slow, while stream corrasion is proportionately rapid because such rainfall as occurs is in sudden showers of great volume but short duration. It is thus that the canyons of the arid regions have been formed.

There is a tendency in every stream to extend its drainage area by erosion on all sides, the stream having the most rapid fall eroding its margin most quickly. Hence the stream having the most rapid descent draws area from others. This extension of drainage basins is called *piracy* and is in active progress in the Appalachian Mountains (Fig. 35). While under some circumstances the courses of streams are changeable, under others they maintain their courses with great persistency. An example of the latter condition is seen in water-gaps (Fig. 35) and canyons. A *canyon* illustrates the persistency of stream channels (Fig. 34); it is the result of the uplift of a mountain range across the course of an existing stream. The rate of uplift has been such that the stream has been able to maintain its course by corrasion as the mountain rose. *Wind-gaps* illustrate the changeability of stream channels, since they are abandoned water-gaps from which the stream has been drained by a more powerful pirating neighbor. These are to be clearly distinguished from passes in mountain ranges caused by the erosion of divides at stream-heads.

Since disintegration of hard or of insoluble rocks goes on slowly, and of soft or soluble rocks rapidly, elevated areas due to erosive action are as a rule composed of the former, and depressed areas resulting from the same kinds of action are generally composed of the latter class of rocks. Streams usually make their channels along lines of least resistance. The level surface of the plateau is generally the summit of the hard stratum of rock from which, perhaps, softer strata have been eroded. Other things being equal, *the harder the rock the steeper the slope, the softer the rock the more gentle the slope.* Applying this principle to the cross-section of two stream-beds, one in soft rock, the other in hard rock, they will be carved into forms shown in Figs.

36 and 37, in which the lines indicate progressive stages. Where the rock strata are laid in horizontal beds, alternately soft and hard, the forms resulting from stream corrasion will be similar to those represented in Figs. 38 and 39. Where one or more horizontal beds are of nearly equal hard-

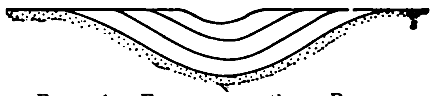


FIG. 36.—EROSION IN SOFT ROCK.

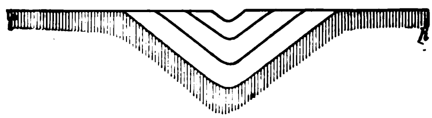


FIG. 37.—EROSION IN HARD ROCK.

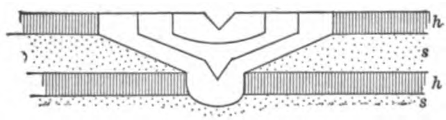


FIG. 38.—EROSION IN HORIZONTAL BEDS OF HARD AND SOFT ROCK.

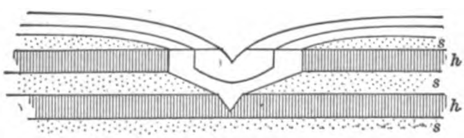


FIG. 39.—EROSION IN ALTERNATE BEDS OF SOFT AND HARD ROCK.

FIG. 40.—EROSION IN SOFT ROCK UNDERLAIN BY HARD.

ness, as sandstone underlain by granite, the canyon resulting from erosion will be similar in form to that shown in Fig. 40. If the stream flow parallel to the strike of the inclined bed, similar forms will be produced, but on sloping surfaces.

50. Topographic Forms.—As there are two general classes of physiographic processes, the constructive and the destructive, so are there two classes of topographic forms resulting from these kinds of agencies.

Among the important *constructive features* are:

1. *Subaqueous forms*, resulting from depositions of whatever kind in lakes and oceans, as spits and bars erected by wave action;

2. *Emergent forms*, or those partly built up beneath the water but gradually rising above it, as deltas and storm-spits;

3. *Subaërial forms*, or those produced by volcanic ejectment of dust and ashes which may be blown about, and the outpouring of geysers and deposits from certain saline springs; and

4. *Subsurface forms*, as faults and folds, produced by slipping and distortion and crumbling of the earth's crust.

Among the chief *destructive features* are:

1. *Vertical diastrophic forms*, resulting from upheaval and subsidence, as massive mountain-ranges, plateaus, and emerged plains.

2. *Horizontal diastrophic forms*, resulting from deformations of the strata due to tangential forces, as faults, flexures, throws, and folds.

3. *Vulcanic intrusions*, resulting from materials brought to the surface from the interior, as dikes, laccolites, and volcanic necks.

4. *Vulcanic ejecta*, due to materials of the interior violently erupted by volcanoes or geysers, as lava-plains, cinder-cones, deposits from spring-waters.

Topographic forms are modeled—

1. By the character and hardness of the rocks;
2. By the geologic structure or position of bed-rock;
3. By the slope of the surface;
4. By climatic conditions;
5. By accidents during development;
6. By length of time during which eroding agents have acted; and
7. By the nature and working methods of these agencies.

Topographic forms vary according to the physiographic processes which have produced them. According as these act they may be divided into two great classes:

1. By vertical change;
2. By horizontal change.

As a result of the former the surface of the earth moves up and down, producing the general forms which are said to result from *uplift* and *downthrow*. As a result of horizontal change *land is transported* from one locality to another. This form of change acts generally through the agency of water and, to a minor extent, of the wind. It produces the general forms resulting from aggradation and degradation, and through the action of erosion, corrasion, and transportation.

51. **Classification of Topographic Forms.**—Doctor Hayes has classified topographic forms in the following simple tabular manner:

I. *Constructional forms* due to
- 1. Diastrophism. (Fig. 21.)
  - (1) Emerged plains.
  - (2) Plateaus.
  - (3) Block ranges.
  - (4) Lake basins.
- 2. Vulcanism. (Fig. 41.)
  - (1) Lava plains and coulees.
  - (2) Volcanic cones.

II. *Gradational forms* due to
- 1. Aggradation by
  - (1) Water. (Fig. 42.)
    - *a.* Alluvial cones.
    - *b.* Alluvial plains.
    - *c.* Deltas.
  - (2) Ice. (Fig. 41.)
    - *a.* Moraines—(*a*) terminal, (*b*) drift-sheets.
    - *b.* Eskers.
    - *c.* Sand plains.
    - *d.* Drumlins.
  - (3) Wind. (Fig. 43.)
    - *a.* Dunes.
    - *b.* Loess plains.
- 2. Degradation.
  - (1) Sculptured forms. (Figs. 4, 34 and 44.)
    - *a.* Canyons and gorges.
    - *b.* Valleys.
    - *c.* Plains and peneplains.
    - *d.* Lake basins.
  - (2) Residual forms. (Figs. 6, 44 and 45.)
    - *a.* Plateaus.
    - *b.* Mesas.
    - *c.* Mountains.
    - *d.* Ridges, hogbacks.
    - *e.* Hills.

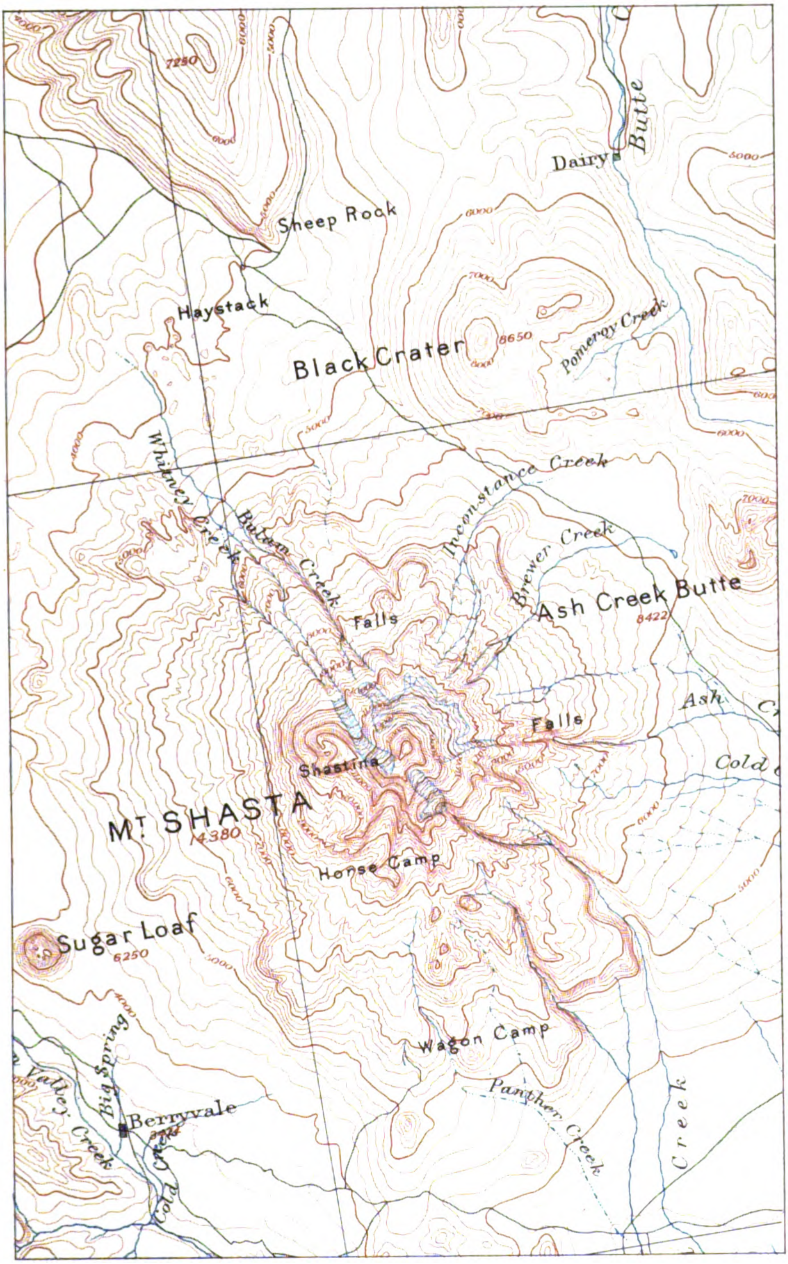


FIG. 41. — VOLCANIC MOUNTAIN, CRATER, CINDER-CONE, GLACIER, AND MORAINES, MT. SHASTA, CAL.
Scale 4 miles to 1 inch. Contour interval 200 ft.

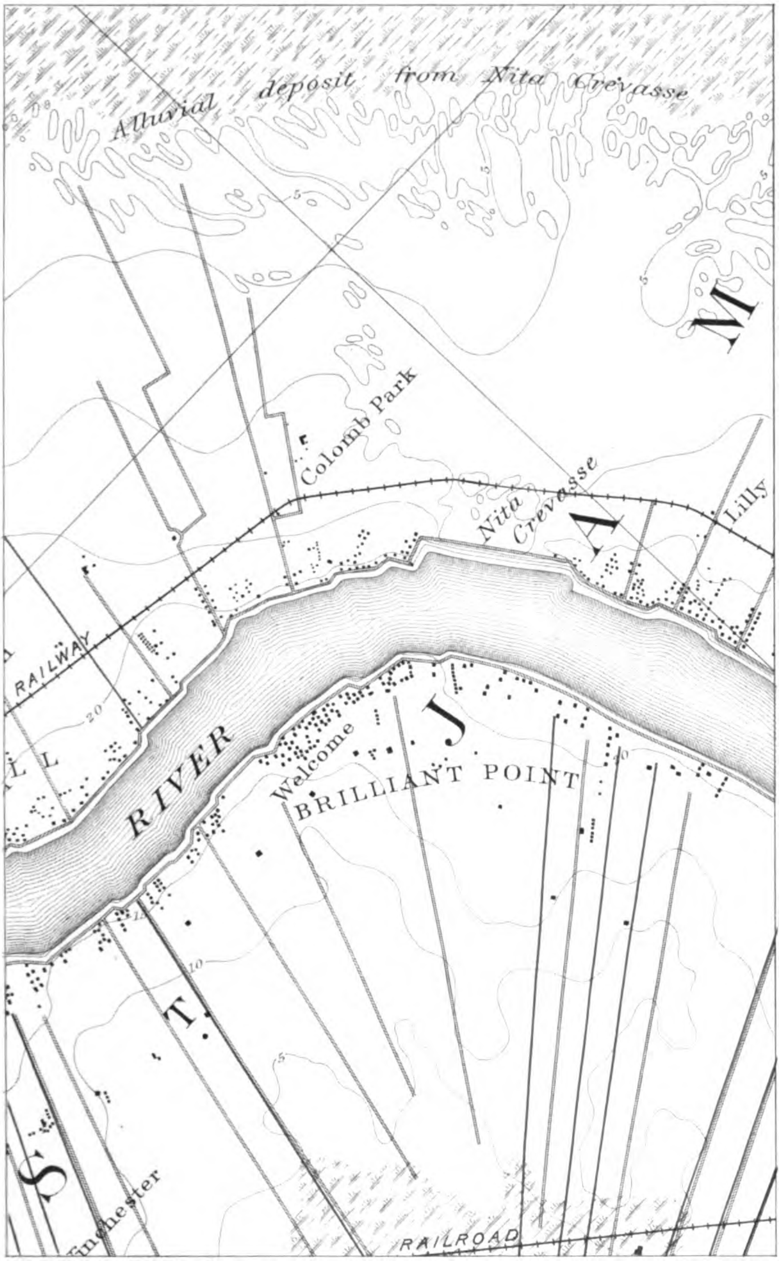


FIG. 42.—ALLUVIAL RIDGE, CREVASSE, SWAMP, RIVER, AND FLOOD PLAIN LOWER MISSISSIPPI RIVER.
Scale 1 mile to 1 inch. Contour interval 5 ft.

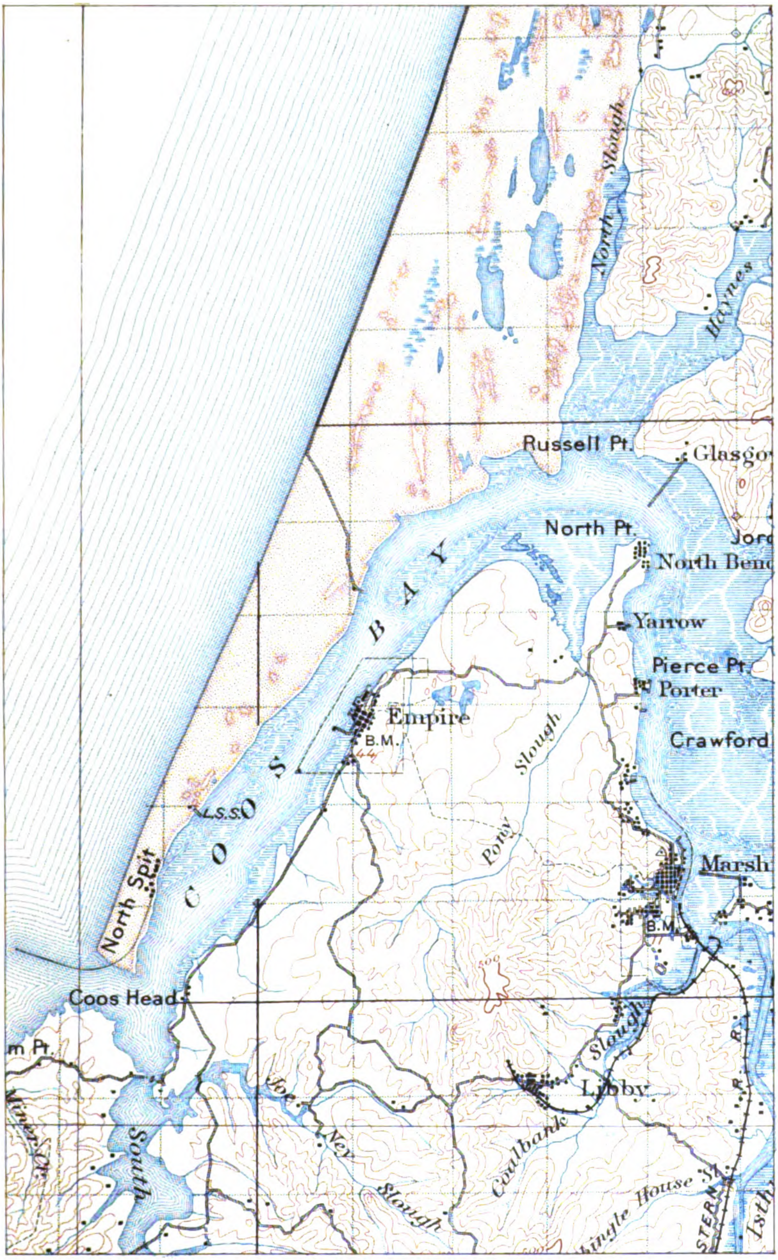


FIG. 43.—SAND-DUNE, SPIT, OCEAN, BAY, LAGOON, SLOUGH, TIDAL FLAT, SWAMP, AND MARSH COOS BAY, ORE.
Scale 2 miles to 1 inch. Contour interval 100 ft.

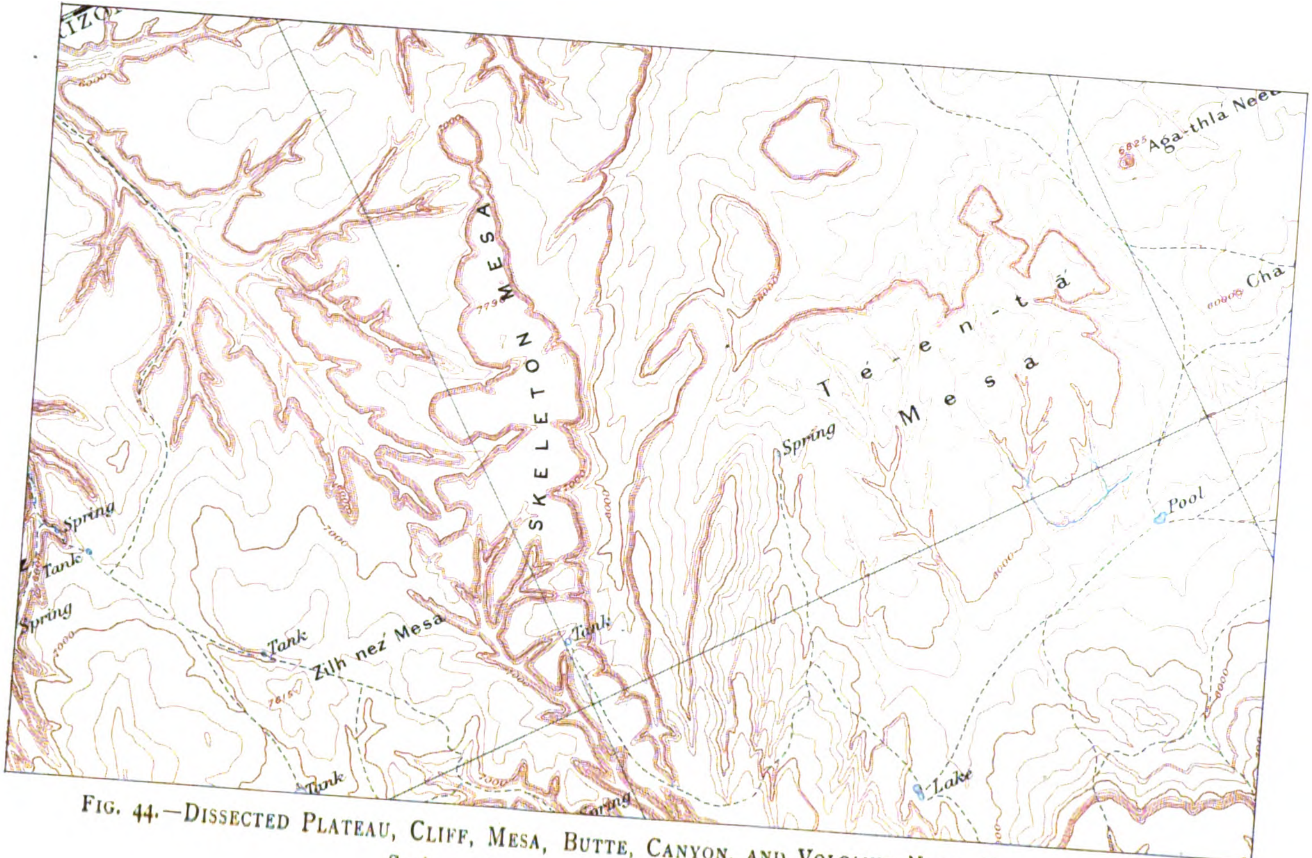


Fig. 44.—Dissected Plateau, Cliff, Mesa, Butte, Canyon, and Volcanic Neck, Northern Arizona.
Scale 4 miles to 1 inch. Contour interval 200 ft.

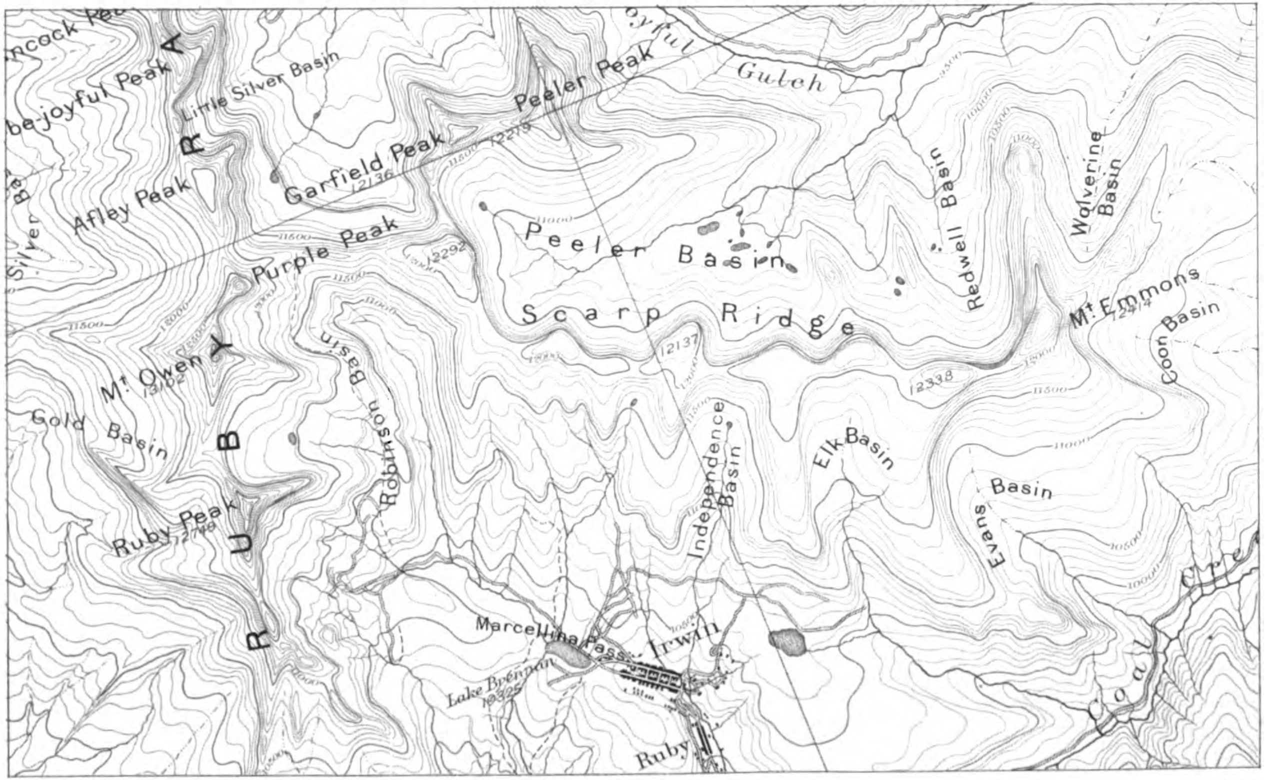


FIG. 45.—MOUNTAIN RANGE, AMPHITHEATRE, SCARP, BASIN, PEAK, AND CREEK, IRWIN, COLO. Scale 1 mile to 1 inch. Contour interval 100 ft.

GLOSSARY OF TOPOGRAPHIC FORMS.

The following list of definitions is intended to include all those terms employed popularly or technically in the United States to designate the component parts of the surface of the earth. None of the words similarly applied in other portions of the world are given. So far as practicable, the endeavor has been to refrain from defining such words or using such definitions as refer merely to the origin of the various topographic forms. At the same time it has been found necessary in a few instances to define forms according to their variety or origin, as those resulting from volcanic or glacial action. In the case of names which are locally peculiar to a limited portion of the country, the effort has been to indicate the regions in which they were employed. The language whence derived is denoted by Sp. for Spanish, Fr. for French, etc.; the word "origin" following indicates that it has been generally adopted in American nomenclature.

Acclivity: An ascending slope as opposed to declivity.

Alkali Flat: A playa; the bed of a dried up saline lake, the soil of which is heavily impregnated with alkaline salts.

Alpine: Pertaining to mountains of great height and ruggedness of outline and surface, and containing glaciers. Resembling a great mountain range of southern Europe called the Alps.

Amphitheater: A cove or angle of glacial origin near the summit of a high mountain and nearly surrounded by the highest summits. A small flat valley or gulch-like depression at the head of an alpine mountain drainage. Local in far West.

Arete: A sharp, rocky crest; a comb-like secondary crest of rock which projects at a sharp angle from the side of a mountain. (Fr. origin.)

Arroyo: The channel of an intermittent stream steep cut in loose earth; a coulee. Local in southwest. (Sp.)

Artesian Well: A well which has been excavated or drilled through impervious strata to a subterranean water supply which has its source at a higher level. The resulting hydrostatic pressure causes the water to rise in the bore to a sufficient height to overflow at the mouth of the well.

Atoll: A ring-shaped coral island nearly or quite encircling a lagoon.

Badlands: Waste or desert land deeply eroded into fantastic forms. Local in arid northwest.

Bald: A high rounded knob or mountain top, bare of forest. Local in Southern States.

Bank: A low bluff margin of a small body of water.

A mound-like mass of earth.

Bar: An elevated mass of sand, gravel or alluvium deposited on the bed of a stream, sea or lake, or at the mouth of a stream.

Barranca: A rock-walled and impassable canyon. Local in southwest. (Sp.)

Barrier Beach: A beach separated from the mainland by a lagoon or marsh.

Barrier Island: A detached portion of a barrier beach betweeen two inlets.

Base-level Plain: A flat, comparatively featureless surface or lowland resulting from the nearly completed erosion of any geographic area.

Basin: An amphitheater or cirque. Local in Rocky Mountains.

An extensive, depressed area into which the adjacent land drains, and having no surface outlet. Use confined almost wholly to the arid West.

The drainage or catchment area of a stream or lake.

Bay: An indentation in the coast line of a sea or lake; a gulf.

Baygall: A swamp covered with growth of bushes. Local on south Atlantic coast.

Bayou: A lake or intermittent stream formed in an abandoned channel of a river; one of the half-closed channels of a river delta. Local on Gulf Coast. (Fr. origin.)

Beach: The gently sloping shore of a body of water; a sandy or pebbly margin of water washed by waves or tides.

Bed: The floor or bottom on which any body of water rests.

Bench: A strip of plain along a valley slope.

A small terrace or comparatively level platform on any declivity.

Bight: A small bay.

Bluff: A bold, steep headland or promontory.

A high, steep bank or low cliff.

Boca: A mouth; the point at which a streamway or drainage channel emerges from a barranca, canyon or other gorge, and debouches on a plain. (Sp.)

Bog: A small open marsh.

Bolson: A basin; a depression or valley having no outlet. Local in southwest. (Sp., meaning "purse.")

Bottom: The bed of a body of still or running water.

Bottom Land: The lowest land in a stream bed or lake basin; a flood plain.

Boulder: A rounded rock of considerable size, separated from the mass in which it originally occurred.

Box Canyon: A canyon having practically vertical rock walls.

Branch: A creek or brook, as used locally in Southern States. Also used to designate one of the bifurcations of a stream, as a fork.

Breaks: An area in rolling land eroded by small ravines and gullies. Local in Northwest.

Bridal-veil Fall: A cataract of great height and such small volume that the falling water is dissipated in spray before reaching the lower stream-bed.

Brook: A stream of less length and volume than a creek, as used locally in the Northeast.

Brow: The edge of the top of a hill or mountain; the point at which a gentle slope changes to an abrupt one; the top of a bluff or cliff.

Butte: A lone hill which rises with precipitous cliffs or steep slopes above the surrounding surface; a small isolated mesa. Local throughout far West. (Fr.)

Cajon: A box canyon. Local in Southwest. (Sp., meaning "box.")

Cala: A creek. Local in Southwest. (Sp.)

Camas: A small upland prairie; a glade; a small park; a small, gently sloping prairie, partly wooded and surrounded by high mountain slopes. Local in Pacific northwest. (Sp. meaning "bed.")

Cañada: A very small canyon. Local in Southwest. (Sp.)

Canal: A sluggish coastal stream. Local on Atlantic Coast.

Candelas: A group of candel-like rocky pinnacles. Local in Southwest (Sp.)

Canyon: A gorge or ravine of considerable dimensions; a channel cut by running water in the surface of the earth, the sides of which are composed of cliffs or series of cliffs rising from its bed. Local throughout the far West. (Sp. origin.)

Cape: A point of land extending into a body of water; a salient of a coast.

Cascade: A short, rocky declivity in a stream-bed over which water flows with greater rapidity and higher fall than over a rapid; a shortened rapid, the result of the shortening being to accentuate the amount of fall.

Col: A low divide or pass forming a depression between two mountains and joining two valleys.

Cataract: A waterfall, usually of great volume; a cascade in which the vertical fall has been concentrated in one sheer drop or overflow.

Cave: A hollow space or cavity under the surface of the earth.

A depression in the ground, by abbreviation from a "cave in," as used colloquially.

Cavern: A large, natural, underground cave or series of caves.

Cay: A key; a comparatively small and low coastal island of sand or coral. Local in Gulf of Mexico. (Sp. origin.)

Ceja: The cliff of a mesa edge; an escarpment. Local in Southwest. (Sp.)

Cerro: A single eminence intermediate between hill and mountain. Local in Southwest. (Sp.)

Cerrito, or **Cerrillo**: A small hill. Local in Southwest. (Sp.)

Channel: A large strait, as the British Channel.

The deepest portion of a small stream, bay or strait through which the main volume or current of water flows.

Chasm: A canyon having precipitous rock walls; a box-canyon.

Cienega: An elevated or hillside marsh containing springs. Local in Southwest. (Sp.)

Cone: A low, conical hill, built up from the fragmental material ejected from a volcano.

Cirque: A glacial amphitheater or basin. (Fr. origin.)

Cliff: A high and very steep declivity.

Clove: A gorge or ravine. Local in Middle States. (Dutch origin.)

Coast: The land or the shore next to the sea.

Coastal Marsh: A marsh which borders a seacoast and is usually formed under the protection of a barrier beach.

Coastal Plain: Any plain which has its margin on the shore of a large body of water.

Continental Shelf: A comparatively shallow marginal ocean bed or floor bordering a continent; a submerged terrace bordering a continent.

Cordillera: A group of mountain ranges, including valleys, plains, rivers, lakes, etc.; its component ranges may have various trends but the cordillera will have one general direction. (Sp. origin.)

Coteau: An elevated, pitted plain of rough surface. Local in Missouri and neighboring States. (Fr. origin.)

Coulee: A cooled and hardened stream of lava. Coulees occur as ridges of greater or less length and dimensions, but rarely of great height. Local in Northwest.

A wash or arroyo through which water flows intermittently. Local in Northwest. (Fr. origin.)

Cove: A small bay.

An amphitheater or indentation in a cliff. It may be the abrupt heading of a valley in a mountain.

PLATE III.—SAND HILLS, BENCH, TERRACE, CREEK, AND RIVER, ABOVE ALBANY, N. Y.
Scale 1 mile to 1 inch. Contour interval 20 ft.

Crag: A rough, steep or broken rock standing out or rising into prominence from the surface of an eminence; a rocky projection on a cliff or ledge.

Crater: The cup-shaped depression marking the position of a volcanic vent; its margin is usually the summit of the volcano.

Creek: A stream of less volume than a river.

A small tidal channel through a coastal marsh.

Crest: The summit land of any eminence; the highest natural projection which crowns a hill or mountain.

Crevasse: A fissure in a glacier.

A break in a levee or other stream embankment. (Fr. origin.)

Cuesta: An ascending slope; a tilted plain or mesa top. Local in Southwest. (Sp)

Current: A continuous movement or flow, in one direction, of a body of water; a stream in or portion of an ocean which has continuous motion or flow in one direction.

Dalle: A rapid. Local in Northwest. (Fr. origin.)

Declivity: A descending slope, as opposed to acclivity.

Deep: A profound or abysmal depression in the ocean bottom.

Defile: A deep and narrow mountain pass.

Delta: The mouth of a river which is divided down stream into several distributaries.

Depression: A low place of any dimensions on a plain surface; the negative or correlative of elevation or relief.

Desert: An arid region of any dimensions, barren of water other than in occasional flood streams or springs, frequently covered with considerable growths of cacti, coarse bunch grass, mesquite and other shrubs. A desert is not necessarily a plain surface, as most deserts are broken by the sharp escarpments and buttes which are common to the arid regions, by sand dunes or volcanic ejecta. A desert may include canyons and mountains of considerable differences of elevation.

Dike: A ridge having for its core a vertical wall of igneous rock.

Divide: The line of separation between drainage systems; the summit of an interfluve.

The highest summit of a pass or gap.

Dome: A smoothly rounded rock-capped mountain summit. Local in far West.

Draft: A draw.

Draw: A very shallow and small gorge, gulch or ravine; the eroded channel through which a small stream flows.

Drift: A slow, great ocean current.

Drumlin: A smooth, oval or elongated hill or ridge composed chiefly of glacial detritus.

Dry Wash: A wash, arroyo or coulee in the bed of which is no water.

Dune: A hill or ridge of sand formed by the winds near a sea or lake shore, along a river-bed or on a sandy plain.

Eminence: A mass of high land.

Escarpment: An extended line of cliffs or bluffs.

Esker: A long, winding ridge of sand or gravel, the deposit from a stream flowing beneath a glacier.

Estuary: A river-like inlet or arm of the sea.

Everglade: A tract of swampy land covered mostly with tall grass. Local in South.

Fall: A waterfall or cataract.

The flow or descent of one body of water into another.

Fan: A mountain delta; a conical talus of detrital material.

Fiord: A narrow inlet with high, rocky walls; a glacial gorge filled by an arm of the sea.

Flat: A small plain usually situated in the bottom of a stream gorge; often applied to a small area of tillable land in the bend of a bluff-walled stream.

Floodplain: Any plain which borders a stream and is covered by its waters in time of flood.

Floor: The bed or bottom of the ocean.

A comparatively level valley bottom.

Fly: Corrupted from Vly.

Foot: The bottom of a slope, grade or declivity.

Foothill: One of the lower subsidiary hills at the base of a mountain.

Fork: One of the major bifurcations of a stream; a branch.

Fountain: A flow of water rising in a jet above the surrounding surface. Artesian wells, geysers and springs may be fountains.

Fumarole: A spring or geyser which emits steam or gaseous vapor; found only in volcanic areas.

Gap: Any deep notch, ravine or opening between hills, or in a ridge or mountain chain.

Geyser: A hot spring, the water of which is expelled with steam in an accumulated volume in paroxysmal bursts.

Glacier: A permanent body or stream of ice having motion.

Glacial Gorge: A deeply cut valley of U-shaped cross-section, the result of glacial erosion.

Glacial Lake: A lake, the basin of which has been carved by glacial action, or is dammed on one side by glacial detritus.

Glade: A grassy opening or natural meadow in the woods; a small park. Applied in western Maryland to a brushy, grassy, or swampy opening in the woods.

Gorge: A canyon; a rugged and deep ravine or gulch.

Grade: A slope of uniform inclination.

Grotto: A small, picturesque cave.

Gulch: A small ravine; a small, shallow canyon with smoothly inclined slopes. Local in far West.

Gulf: A gorge or deep ravine; a short canyon. Local in Southern States and New York.

A bay, usually of great dimensions.

Gully: A channel cut by running water; less than a gulch or ravine.

Gut: A narrow passage or contracted strait connecting two bodies of water.

Hanging Valley: A high glacial valley, tributary to a more deeply eroded glacial gorge or fiord.

Headland: A promontory.

Height of Land: The highest part of a plain or plateau; or, on a highway, a pass or divide. Local in Northeast and British America.

Highland: A relative term denoting the higher land of a region; it may include mountains, valleys, and plains.

Hill: An eminence less than a mountain rising above the surrounding land.

Hogback: A steep-sided ridge or long hill; used to describe a group of sharply eroded low hills.

A steep foothill having parallel trend to the associated mountain range. Local in the far West.

Holl: A small bay, as Wood's Holl, Mass. Local in New England.

Hollow: A small ravine; a low tract of land encompassed by hills or mountains.

Hook: A low, sandy peninsula terminating in curved or hook-shaped end forming a bay.

Hot Spring: A spring, the water of which has a temperature considerably above that of its surroundings.

Huerfano: A solitary hill or cerro. Local in Southwest. (Sp., meaning "orphan.")

Inlet: A small narrow bay or creek; a small body of water leading into a larger.

Interfluve: The upland separating two streams having an approximately parallel course.

Island: An area of land entirely surrounded by water. In dimensions islands range from a point of rock rising above the surface of the water to an area of land of continental dimensions, as Australia.

Isthmus: A narrow strip of land connecting two considerable bodies of land.

Kame: A small hill of gravel and sand made by a glacier.

Kettle Hole: A steep-sided hole or depression in sand or gravel; a hole in the bottom of a stream or pond.

Key: A cay, as the Florida Keys.

Kill: A creek. Locally in Middle States. (Dutch origin.)

Knob: A prominent peak with rounded summit. Local in Southern States.

Knoll: A low hill.

Lagoon: A shallow bay cut off from a sea or lake by a barrier; often stagnant with ooze bottom and rank vegetation. It may be of salt or fresh water. Locally in South and Southwest. (Sp. origin.)

Lake: Any considerable body of inland water.

Landslide: Earth and rock which has been loosened from a hillside by moisture or snow, and has slid or fallen down the slope.

Landslip: A landslide of small dimensions.

Lateral Moraine: A moraine formed at the side of a glacier; usually ridgelike in shape.

Ledge: A shelf-like projection from a steep declivity; a rocky outcrop or reef.

Lenticular Hill: A short drumline.

Levee: An artificial bank confining a stream channel. (Fr. origin.)

Littoral: That portion of a shore washed by, or between high and low water.

Malpais: A badland. Local in Northwest. (Fr.)

Marsh: A tract of low, wet ground, usually miry and covered with rank vegetation. It may at times be sufficiently dry to permit of tillage or of having hay cut from it. It may be very small and situated high on a mountain, or of great extent and adjacent to the sea.

Meadow: A bit of natural grassland in wooded mountains; a glade or small park. Local in far West.

Mesa: A tableland; a flat-topped mountain bounded on at least one side by a steep cliff; a plateau terminating on one or more sides in a steep cliff. Local in Southwest. (Sp. origin.)

Mesita: A small mesa. Local in Southwest. (Sp.)

Mire: A small, muddy marsh or bog.

Monadnock: An isolated hill or mountain rising above a peneplain, after the removal by erosion of its surrounding features.

Monument: A column or pillar of rock. Locally in Rocky Mountain region.

Moraine: Any accumulation of loose material deposited by a glacier.

Morass: A swamp, marsh or bog having rank vegetation and muddy or offensive appearance.

Mound: A low hill of earth.

Mountain Chain: A series or group of connected mountains having a well defined trend or direction.

Mountain: An elevation of the surface of the earth greater than a hill and rising high above the surrounding country.

Mountain Range: A short mountain chain; a mountain much longer than broad.

Mountain System: A cordillera.

Mouth: The exit or point of discharge of a stream into another stream or a lake or sea.

Muskeg: A bog or marsh. Local in Northwest and British America.

Neck: The narrow strip of land which connects a peninsula with the mainland.

Nevé: The consolidated granular snow on a mountain summit in which glaciers have their source.

Notch: A short defile through a hill, ridge or mountain.

Nunatak: A rock island in a glacier.

Ocean: The great body of water which occupies two-thirds of the surface of the earth. The sea as opposed to the land.

Oceanic Plateau: An irregularly elevated portion of the ocean bed, of considerable extent and perhaps rising in places above the water surface.

Outlet: The lower end of a lake or pond; the point in which a lake or pond discharges into the stream which drains it.

Paha: A long ridge of fine, loamy material deposited from a stream which has cut a channel in a melting glacier. Local in Iowa and vicinity. (Indian.)

Palisade: A picturesque, extended rock cliff rising precipitately from the margin of a stream or lake, and of columnar structure.

Park: A grassy, wide, and comparatively level open valley in wooded mountains. Local in Rocky Mountains.

Pass: A gap or other depression in a mountain range through which a road or trail may pass; an opening in a ridge forming a passageway.

A narrow, connecting channel between two bodies of water.

Peak: A pointed mountain summit; a compact mountain mass with single conspicuous summit.

Peneplain: A land surface which has been reduced to a condition of low relief by the erosive action of running water.

Peninsula: A body of land nearly surrounded by water.

Picacho: A peaked butte. Local in Southwest. (Sp.)

Pinnacle: Any high tower or spire-shaped pillar of rock, alone or cresting a summit.

Pitted Plain: A plain of gravel or sand with kettle holes.

Plain: A region of general uniform slope, comparatively level, of considerable extent and not broken by marked elevations and depressions; it may be an extensive valley floor or a plateau summit.

Plateau: An elevated plain. Its surface is often deeply cut by stream channels, but the summits remain at a general level. The same topographic form may be called a plain and a plateau, and be both. An elevated tract of considerable size and diversified surface. (Fr.)

Playa: An alkali flat; the dried bottom of a temporary lake, without outlet. Local in Southwest. (Sp. origin.)

A small area of land at the mouth of a stream and on the shore of a bay; an alluvial flat coast land as distinguished from a beach. Local in Southwest. (Sp.)

Playa Lake: A shallow, storm-water lake. When dried it forms a playa. Local in Southwest. (Sp. origin.)

Plaza: An open valley floor; the flat bottom of a shallow canyon. (Sp.)

Pocason: A dismal swamp. Local on South Atlantic coast. (Indian.)

Point: A small cape; a sharp projection from the shore of a lake, river, or sea.

Pond: A small, fresh-water lake.

Pool: A water-hole or small pond.

Pothole: A basin-shaped or cylindrical cavity in rock formed by a stone or gravel gyrated by eddies in a stream.

Prairie: A treeless and grassy plain.

Precipice: The brink or edge of a high and very steep cliff.

Promontory: A high cape with bold termination; a headland.

Puerto: A pass or defile through an escarpment or sierra. Local in Southwest. (Sp., meaning "gate.")

Quagmire: Any mire or bog.

Quebrado: A canyon of rugged aspect; a fissure-like ravine or canyon. Local in Southwest. (Sp.)

Rapid: Any short reach of steep slope between two relatively quiet reaches in a stream-bed. The water flows over a rapid with greater velocity than in adjacent portions of a stream.

Ravine: A gulch; a small gorge or canyon, the sides of which have comparatively uniform slopes.

Reef: A ridge of slightly submerged rocks.
A ledge of rock on a mountain.

Relief: Elevation as opposed to depression; the elevated portions of the land surface; the irregularities of the earth's surface.

Ridge: The narrow, elongated crest of a hill or mountain; an elongated hill.

Riffle: The shallow water at the head of a rapid; a rapid of comparatively little fall.

Rift: A narrow cleft or fissure in rock.

Rill: A very small trickling stream of water, less than a brook.

Rincon: Corner or cove; the angular indentation in a mesa edge or escarpment in which a canyon heads. Local in Southwest. (Sp. origin.)

Rio: A river. Local in Southwest. (Sp. origin.)

River: A large stream of running water. A stream of such size as to be called a river in one locality may be called a creek or brook in another.

Rivulet: A small river.

Rock Cave: A shelter cave.

Rolling Land: Any undulating land surface; a succession of low hills giving a wave effect to the surface.

Run: A brook or small creek. Local in South.

Salient: An angle or spur projecting from the side of the main body of any land feature.

Sand Dune: Any dune.

Sandia: An oblong, rounded mountain mass. Local in Southwest. (Sp., meaning "watermelon.")

Scarp: An escarpment.

Sea: A large body of salt water.

Seep: A small, trickling stream. Local in Southwest.

Serrate: The rocky summit of a mountain having a sawtooth profile; a small sierra-shaped ridge. Local in Southwest. (Sp.)

Shelter Cave: A cave only partially underground, which is formed by a protecting roof of overlying rock; generally open on one or more sides.

Shoal: A shallow place in a stream or lake; an elevated portion of the bed of a stream, lake, or sea, which rises nearly to the water surface; a bar.

Shore: The land adjacent to any body of water.

Sierra: A rugged mountain range with serrate outline. Local in Southwest and Pacific States. (Sp. origin.)

Sink: The bottom of an undrained basin.

Slide: The exposed surface left in the trail of a landslide; the place whence a landslide has departed. Local in Northeast.

Slope: The inclined surface of a hill, mountain, plateau, or plain or any part of the surface of the earth; the angle which such surfaces make with the level.

Slough: A freshet-filled channel or bayou; a depression in an intermittent stream channel filled with stagnant water or mire.

Sound: A relatively shallow body of water separated from the open sea by an island and connected with it at either end so that through it there is clear tidal flow.

Spit: A low, sandy point or cape projecting into the water; a barrier beach.

Spring: A stream of water issuing from the earth.

Spur: A sharp projection from the side of a hill or mountain; a radial ridge of subordinate dimensions.

Stillwater: Any reach in a stream of such level inclination as to have scarcely any preceptible velocity of flow; a sluggish stream, the water of which appears to be quiet or still. Local in Northeast.

Strand: The shore or beach of the ocean or a large lake.

Strait: A relatively narrow body of water connecting two larger bodies.

Stream Channel: The trench or depression washed in the surface of the earth by running water; a wash, arroyo, or coulee.

Stream: Any body of flowing water. It may be of small volume, as a rill, great as the Mississippi or mighty as the Gulf Stream in the Atlantic Ocean.

Sugarloaf: A conical hill comparatively bare of timber. Local in far West.

Summit: The highest point of any undulating land, as of a rolling plain, a mountain or a gap or pass in a mountain.

Swale: A slight, marshy depression in generally level land.

Swamp: A tract of stillwater abounding in certain species of trees and coarse grass or boggy protuberances.

Table: An elevated, comparatively level bit of land between two streams. Local in Northwest.

Table Land: A mesa.

Table Mountain: A mountain having comparatively flat summit and one or more precipitous sides. A mesa.

Talus: A collection of fallen disintegrated material which has formed a slope at the foot of a steeper declivity.

Tank: A pool or water-hole in a wash. Local in arid West.

Terminal Moraine: A moraine formed across the course of a glacier, irregularly ridge-like in shape.

Terrace: A relatively narrow level plain or bench on the side of a slope and terminating in a short declivity

Terrain: See Terrane.

Terrane: An extent of ground or territory; a portion of the surface of the earth; the land. (Fr.)

Terrene: Pertaining to the earth. (Fr.)

Teton: A rocky mountain-crest of rugged aspect. Local in Northwest.

Thalweg: A watercourse; a valley bottom; the deepest line or part of a valley sloping in one direction. (Ger.)

Tidal Marsh or **Flat**: Any marsh or flatland which is wetted by a tidal stream or sea.

Tongue: A narrow cape.

Tower: A peak rising with precipitous slopes from an elevated table land. Local in Northwest.

Tundra: An upland or alpine marsh, the ground beneath which is frozen. There are great areas of tundra in the Arctic. (Rus.)

Upland: A highland.

Valley: A depression in the land surface generally elongated and usually containing a stream.

Vlei: See Vly.

Vly: A small swamp, usually open and containing a pond. Local in middle Atlantic States. (Dutch origin.)

Volcano: A mountain which has been built up by the materials forced from the interior of the earth, piling about the hole from which they were ejected. These may be lava, cinders or dust.

Volcanic Neck: The solid material which has filled the throat or vent of a volcano, and has resisted degradation better than the mass of the mountain. It thus finally stands alone as a column or crag of igneous rock.

Wash: The broad, dry bed of a stream; a dry stream channel. Local in arid West.

Waterfall: Any single cataract. Both the terms waterfall and cataract may be applied to falls of like magnitude.

Water Gap: A gap through a mountain occupied by an existing stream.

Watershed: The ridge of high land or summit separating two drainage basins; the summit of land from which water divides or flows in two or more directions.

The area drained by a stream.

Well: Any excavation in soil or rock which taps underground water.

Wind Gap: An elevated gap not occupied by a watercourse.

PART II.

PLANE AND TACHYMETRIC SURVEYING.

CHAPTER VII.

PLANE-TABLES AND ALIDADES.

52. Plane and Topographic Surveying.—*Plane surveying* consists of the representation of any portion of the surface of the earth in horizontal plan as it would appear viewed from vertical positions over every point on the surface. The resulting map may be considered as consisting of an infinite number of points, the positions and relations of only so many of which are established as may be necessary to define the features which it is desired to represent, and this constitutes the instrumental work of the survey. The prime element of position in the construction of such a map is a point in space. Such a position is indefinite, however, and to introduce the definite elements of direction and distance it is necessary to add at least one other point. The addition of a third point introduces trigonometric functions by which any three elements of a triangle, except the three angles alone, serve to determine all others. These trigonometric functions may be solved or determined mathematically in figures from linear measures or angles, and graphically by means of the plane-table.

The element of direction or *azimuth* is the deflection from a true north and south line, or it may be a compass bearing, which is a deflection from the magnetic north and south line, or it may be the amount of deflection from any assumed line. The amount of such deflection of one line from another is measured by the angle formed at their intersection. The element of *distance* is measured by any unit conventionally established, the most definite of which in present use are the yard and meter (Art. 293).

The representation of any portion of the earth's surface by a *topographic map* requires that, in addition to the projection upon a horizontal plane of a sufficient number of points to reproduce in plan the surface of the land, there shall also be indicated in some way the relief of the surface or its changes of height above, within, or below a fixed level surface. Such representation is made by determining instrumentally the elevation of such a number of points that the survey may be completed by drawing in the details between them. The new method of topographic surveying is by the determination of the least number of such points, and therefore calls for the greatest display of artistic and topographic skill, perception, and judgment on the part of the topographer.

53. Plane-table Surveying.—The plane-table (Art. 56) is peculiarly well adapted to the mapping of topography, not only because it permits of quickly and graphically obtaining all the instrumental data which is requisite, but also because it has the added advantages,

First, of having the map made in the field while the terrane is in view of the topographer;

Second, the topographer can see at all times whether he has obtained all data necessary for the representation of the country; and

Third, any insufficiency of instrumental or interpolated data can at once be supplied before leaving the field.

In *surveying with the plane-table* the errors in measurement of horizontal angles can be so far eliminated in practice that they may be neglected. In practice the horizontal projections of existing angles are recorded graphically and are therefore free from errors of record, adjustment, or platting. In using the plane-table a number of points may have had their positions previously determined and platted on the map sheet. After the plane-table has been oriented and clamped each of these should be sighted from the first position occupied, and all other points in view should appear in vertical planes passing through the station and corresponding points on the sheet. Such points as do not meet this geometric test should be rejected until corrected or relocated graphically.

In locating from a given station positions which are to be used in controlling the details of the sketching, a series of *radial lines* (Fig. 46) should be drawn from the station in all

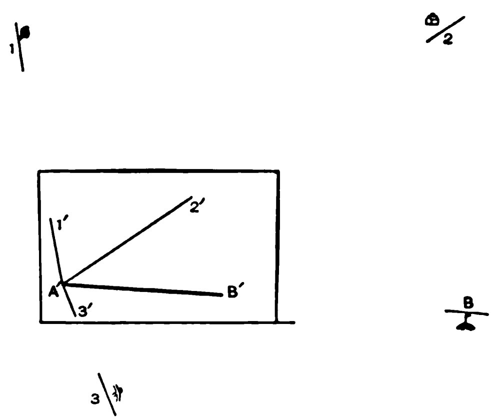


FIG. 46.—DRAWING RADIAL SIGHT LINES.

directions to salient points. This operation should be repeated at other stations, and the *intersection* (Fig. 47) of any two on the same object gives its elementary location, a third line through the same point placing beyond doubt the accuracy of its location. Accordingly, in this mode of surveying, constant opportunities occur for checking locations without calculation. The causes of failure to check may be immediately tested, and the scale being ever present, it controls the amount of detail which it is necessary to gather.

Another advantage of the plane-table as a surveying instrument lies in the fact that, any two points being taken on the sheet as a base, a map may be constructed therefrom independent of scale, yet perfect in its proportion, by the method of intersection alone. If the base chosen be a very long one, as the two points of a trigonometric survey, and

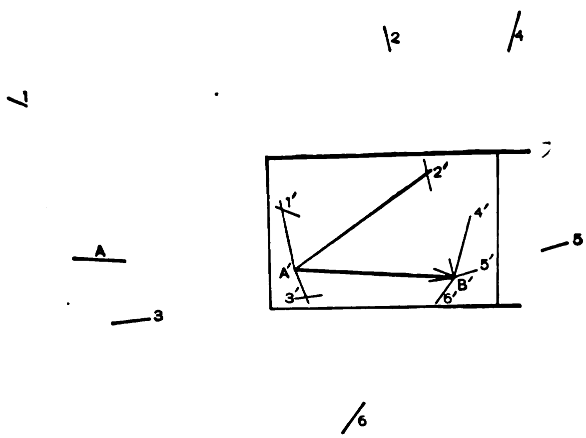


FIG. 47.—INTERSECTING ON RADIAL LINES.

the details of the plane-table survey be included within the limits of the base, then the survey is a contracting one with a diminishing chance of error, and each pair of intersections which has been tested geometrically becomes in turn a base for further triangulation. Ultimately the length and azimuth of some line in this survey may be determined and the whole plane-table survey thus be reduced after the completion of the field-work to any desired map scale.

54. Reconnaissance and Execution of Plane-table Triangulation.—Having located on the plane-table sheet (Art. 188) two intervisible and well-defined points, the topographer should visit one of these and erect a signal of sufficient size to be visible from the most distant part of the territory corresponding to the plane-table sheet. Then selecting a number of points visible from his position which may furnish satisfactory stations, a hasty reconnaissance trip is made over the territory, covering it all, if convenient, in the first reconnaissance, or perhaps only a portion, and returning to the reconnaissance after the plane-table work has caught up with it.

This *reconnaissance* should consist in the selection of a few commanding and well-distributed stations, preferably on the highest eminences in the region under survey, and on each of these a signal must be erected on the point from which the greatest command of the surrounding country may be had. The distance apart of such secondary stations is chiefly dependent on the character of the topography and upon the scale of the map, and may be such as to correspond for the scale of the map chosen to a distance on the paper of five or six inches. In the course of this reconnaissance, a number of other stations may be selected by merely noting their positions and appearance, these being on prominent cleared points, such as bare rocks on mountain summits or slopes, a high lone tree, a building in a field, etc., or a few signal-flags may be placed, providing a sufficient number of such objects are not discovered.

The topographer then begins his *plane-table triangulation* by occupying one of the two primary points and orienting on the other, lines being drawn to the secondary stations and to such other possible tertiary points as may be easily recognized (Fig. 1). He then occupies the second of the primary stations and orients from the first, intersecting on such of the flags established as are visible from his position. Or, he may make his second station on one of the points sighted from the first, determining his position by resecting (Art. 74) from the second primary station. He continues thus until he has carried a secondary or skeleton triangulation over the entire area under survey, taking care not to spend too much time in sighting and attempting to locate minor and unimportant objects, but devoting his attention primarily to the location of his secondary stations and such other prominent objects as are readily recognizable and as may be of service in the further conduct of the plane-tabling.

To this *skeleton scheme of triangulation* the topographer adjusts the traverse lines and the level elevations (Arts. 80 and 129) and then proceeds to fill in the details of his map by further triangulation and the sketching of topographic forms, which should progress together hand in hand (Art. 13).

55. Tertiary Triangulation from Topographic Sketch Points.—While the topographer is executing this outline triangulation, one or more assistants may be engaged in traversing roads and running lines of careful levels (Arts. 80 and 129) over the area under survey, so planning the latter as to well distribute the primary elevations to which levels of less accuracy may be tied and adjusted. On the completion of the secondary or skeleton outline of triangulation, the topographer will then have at his command a network of instrumental control to which to tie future instrumental and topographic details. He should adjust his traverses to his located points, add the elevations obtained by leveling or by vertical triangulation, and this network of control he proceeds to fill in with the topographic details (Arts. 13 and 15).

As this filling in of details progresses he occasionally occupies tertiary plane-table stations, which should now be selected preferably at elevations midway of the slopes of the country, or, in other words, on lower elevations than the main plane-table stations, as in a vacant field on a hillside, a point on a road, or on a low bare summit. From such points he should not only draw lines and obtain intersections for the location of additional objects, as road intersections, buildings, hill-tops, etc., to which vertical angles must also be measured, but he should also sketch in as much of the detail of the topography as is clearly visible from his position and as is satisfactorily controlled by locations already obtained.

The above method of conducting a plane-table survey is not always practicable of execution. Frequently the country is unfavorable for the conducting of plane-table triangulation, owing to its being too wooded for the occupation of many stations, or because it is of such a generally uniform level as to offer few salient objects which may be located by triangulation. In such cases other methods must be employed to fill in the topographic details, chiefly traversing of various kinds (Art. 80), but in any event it is desirable when practicable to control and tie these in by a skeleton plane-table triangulation executed between a few scattered points, which may be prepared for such triangulation by clearing, or by the erection of signals even at considerable expense of time and money (Art. 243).

56. Varieties of Plane-tables.—The plane-table may be divided into four parts :

1. The tripod;
2. The movements;
3. The plane-table board;
4. The alidade.

There are in use in this country three general types of plane-tables, which, in the order of their rigidity and delicacy of mechanism, may be classified as follows :

1. The Coast Survey plane-table ;
2. The Johnson plane-table ;
3. The Gannett plane-table.

The first two differ little in the form of tripod and tripod legs, plane-table board and alidade, and greatly in the movement. The latter differs in all respects from the other two and is adapted only to crude or rough reconnaissance work. To these may be added a fourth type, little used but of some value in exploratory or geographic surveys extending over a large scope of rough country. This is the folding plane-table, which has been chiefly used in the rough map-work of the Powell Survey and the early Geological Surveys. Its chief advantages are its extreme portability, the tripod and board folding so as to occupy the least space. To the above instruments may still be added a fifth, also of the rougher reconnaissance kind; namely, the English Cavalry sketching board, which is rarely used with a tripod, being attached to the wrist, but oriented and used as a plane-table; this instrument is sometimes erected on a Jacob's- staff.

57. **Plane-table Tripods and Boards.**—The tripod legs of the Coast Survey and Johnson plane-tables (Figs. 48 and 49), which are the only two forms adapted to accurate work, are as lightly made of wood as is consistent with requisite strength, shod with brass, and at the tripod head are of sufficient width to reduce lateral motion to a minimum.

The plane-table board is made of well-seasoned pine, paneled with the grain at right angles, or more usually with a binding strip of wood dovetailed on its two ends at right angles to the grain, so as to counteract as much as possible the tendency to warp. The upper surface should be finished as nearly as possible in a plane, and when attached to the movement this surface should always be as nearly parallel as possible to the plane of revolution of the movement, so that the two planes shall remain parallel in all positions of the board.

58. **Plane-table Movements.**—The movements of both forms of plane-table are of different construction, but both are of brass. Their design involves the same essentials, namely, sufficient strength for solidity; horizontal revolving faces of large enough diameter and sufficiently accurate fit to prevent vertical motion when clamped together; and a means of clamping the axis of revolution to the tripod head when the revolving faces have been made horizontal by the leveling apparatus.

The details of the Coast Survey plane-table are well illustrated in Fig. 48, from which it will be observed that the tripod legs are split, widely separated, and attached to the tripod

FIG. 48.—COAST SURVEY PLANE-TABLE.

head by binding-screws and clamps; that the movement is attached to the tripod head by means of a center clamping-screw as in ordinary surveying-instruments; also that the leveling is effected by the usual form of three leveling-screws; and that the horizontal motion is obtained by two heavy

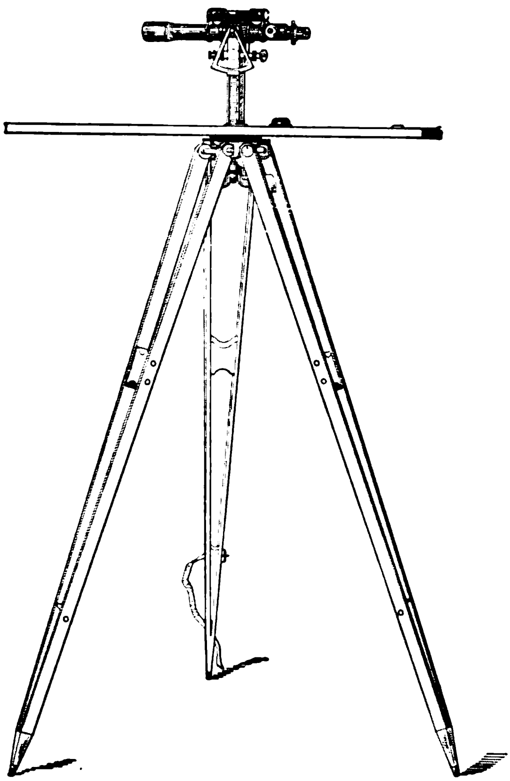
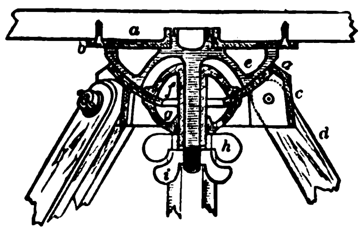


FIG. 49.—TELESCOPIC ALIDADE AND JOHNSON PLANE-TABLE.

circular plates sliding one upon the other, the lower attached to the tripod head by the center clamping-screw, and the upper to the plane-table board by clamping-screws, and both fastened together by a center axis of revolution. There is in addition a clamping-screw for fixing the instrument in orientation, and a tangent screw for slow motion. It will be observed that this instrument is very heavy, is rather difficult of manipulation because of inaccessibility of leveling and clamping-screws, and is in fact too cumbersome for convenient use, excepting where travel is easy.

The Johnson plane-table (Fig. 49), so named after its inventor, Mr. Willard D. Johnson, is used by the United States Geological Survey, and though not quite as rigid as the Coast Survey type, is sufficiently so for all practical purposes and is much lighter, more portable, and more easily manipulated. The movement is also more compact and less liable to derangement or injury. It consists of a split tripod securely attached to the head as in the case of the Coast Survey tripod, but the leveling and horizontal movements are entirely unique in surveying-instruments, being essentially an adaptation of the ball-and-socket principle, so made as to furnish the largest practicable amount of bearing surface.

They consist of two cups, one inside the other, the inner surface of the one and the outer surface of the other being



a. Plane-table Board
b. Bearing Plate
c. Tripod Head
d. " Legs
e. Azimuth Cup
f. Upper Level Cup
g. Lower " "
h. Level Clamp
i. Azimuth Clamp

FIG. 50.—JOHNSON PLANE-TABLE MOVEMENT.

ground to fit as accurately as possible. The interior cup consists of two parts or rings, one outside the other, one controlling the movement in level, and the other that in azimuth (Fig. 50). From each of these there projects beneath the movement a screw, and each screw is clamped by a wing nut. These cups and rings are bound together and to the tripod head by the two nuts, and are attached to the plane-table board by screwing it over a center axis or pin projecting from the upper surface of the upper cup. The instrument is first leveled, not by leveling-screws, but by the ball-and-socket motion given by the pair of cups which are clamped by the upper screw when the board is level, the latter being still left free to revolve horizontally for orientation and being clamped by the lower screw. There is no tangent screw for slow motion in azimuth, it being possible owing to the long lever-arm furnished by the outer edges of the board to move it with sufficient slowness for all practicable purposes.

59. **Telescopic Alidades.**—Alidades used with the more rigid plane-tables differ in form according to the character of the work to be executed. Where the instrument is used chiefly in triangulation, the alidade should be of the most approved type and the rule should be of sufficient length to permit its being used as a straight-edge in drawing lines from one extreme of the board to the other. In practice this rarely exceeds 25 inches in length. In using the plane-table in sketching or traversing a smaller alidade and one having a straight-edge not exceeding eighteen inches in length will be found more portable and better adapted to the work required.

The alidade generally employed by the U. S. Coast and Geodetic Survey consists of a brass or steel straight-edge 2½ by 3 inches in width and 12 to 15 inches long, from and perpendicular to which rises a brass column 3 inches in height, surmounted by Y's in which rest the transverse axes of the telescope. To one end of the axis is firmly attached an arm of thirty degrees, graduated to minutes on either side of a central zero, the accompanying vernier being attached to the Y support. On the telescope-tube are turned two shoulders on which rests a striding-level. There is a clamp and tangent screw for slow motion for moving the telescope in vertical arc, and on the straight-edge are two small spirit-levels at right angles to each other. A déclinatoire accompanies the alidade and is carried in a separate box or is sometimes attached as a part of the striding-level. The déclinatoire box is oblong, with the sides parallel to the north and south lines and graduated to about 5 minutes on either side of the zero.

The chief difference between this alidade and the one used by the United States Geological Survey (Fig. 49) is that the straight-edge of the latter is 18 or 24 inches in length, with one edge beveled and graduated to the scale of map-work. The telescope is on a standard 4 inches high, has a focal distance of 15 inches and a power of 20 diameters, with an objective of 1⅜ inches diameter. The telescope revolves horizontally in a sleeve, with a stop for adjustment of vertical collimation. Instead of two small levels attached to the straight-edge, a single detached circular level is carried by the topographer.

The smaller telescopic alidade used by the United States Geological Survey (Fig. 51) on traverse and stadia work is

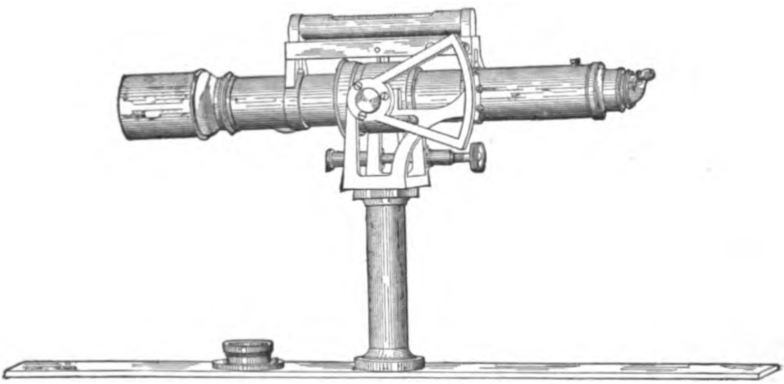


FIG. 51.—TELESCOPIC ALIDADE.

more like the Coast Survey alidade in having a shorter telescope and focal distance and a shorter straight-edge. The vertical arc, instead of being graduated on the side and reading against a vernier as is customary with other surveying instruments, is a sector, which, instead of pointing downwards, points towards the rear or eyepiece of the telescope and is graduated on its outer surface. This is read against a vernier fixed in such position that the reading may be made from one position of the observer at the eyepiece without his moving to the side of the plane-table as in ordinary instruments.

60. Adjustments of Telescopic Alidade.—There are practically no adjustments to the plane-table and alidade excepting the adjustments of the latter for striding-level and collimation. Adjustments or tests may be made of the straightness of the *fiducial edge of the rule* by drawing a line along it, and reversing it, placing the opposite ends upon the marked points and again drawing a line; if the two lines do not coincide, the edge is not true. It is not necessary that the two edges of the straight-edge be exactly parallel, if care is always taken in using the instrument to draw along but one edge.

Attached levels when used may be *adjusted* by placing the alidade in the middle of the table, marking its edges on the paper, and bringing the bubble to center by means of the leveling apparatus; then it is reversed 180 degrees, and if the bubble be not in the center it is corrected one-half by leveling the table, and the other half by adjusting the screws of the attached levels.

The *striding-level* is *adjusted* by placing the alidade in the center of the table, leveling the telescope by the vertical tangent screw, then reversing the level upon the telescope. If the bubble come not to the exact center of the tube, half of the error is to be adjusted by the screws in the level, and the other half by releveling the telescope with the tangent screw.

In addition to the above there are a few *other adjustments*, as that of making the line of collimation perpendicular to the axis of revolution of the telescope, and of making the latter parallel to the plane of the rule, and for parallax, and to correct the zero of vertical arc, etc. None of these need, however, be described, as in the alidade as now made the bearings of the axes are unchangeable and there is either a means of setting the vernier at zero or an index error is to be read.

61. Gannett Plane-table.—Where rough traverses are run in connection with the making of small-scale maps, and a firm board is unnecessary since the telescopic alidade is not used, an exceedingly convenient and portable plane-table is that employed in the U. S. Geological Survey and known as the Gannett plane-table (Fig. 52), after its originator, Mr. Henry

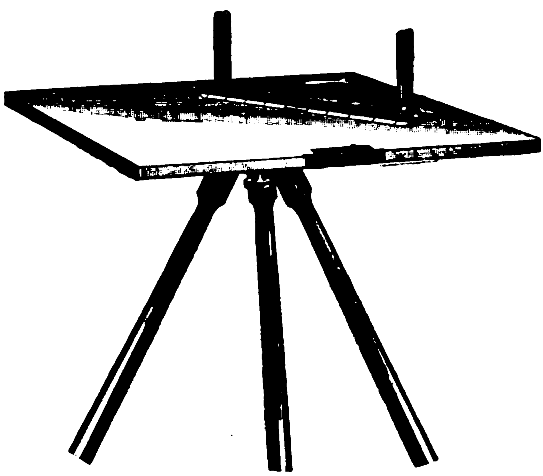


FIG. 52.—GANNETT TRAVERSE PLANE-TABLE AND SIGHT ALIDADE.

Gannett. The tripod of this instrument is very light, consisting of three straight legs made of single pieces of wood. These are shod with metal tips and attached by bolts and nuts to the head, which is a simple plate 3½ inches in diameter. The board, which is 15 inches square by ⅝ of an inch in thickness, is a well-seasoned piece of pine and is attached to the tripod by a single center screw. There is no leveling apparatus, the instrument being leveled by means of the tripod legs, and there is no means of clamping the instrument in azimuth, the movement in azimuth being controlled by friction, and the board being held in place by friction due to the tightness with which it is bound to the tripod head by the center screw. In running traverses the table is not oriented by backsights and foresights, but is adjusted in azimuth by means of a compass needle or déclinatoire having a range of 5° to 8° and placed in a small oblong box 3 to 5 inches in length set into the side of the board.

62. Sight-alidades.—The alidade used with the Gannett plane-table (Figs. 52 and 53) is 6 inches in length, 0.1 inch in thickness, and 0.6 of an inch in width, of brass, with a folding front sight with vertical hair $3\frac{1}{2}$ inches in length, with V sight-notch in the top and a short peep-sight in the rear. The fiducial edge is beveled and graduated to the scale of the map.

To determine elevations of near objects in traversing with light traverse outfit, a small sight-alidade was devised by the author both for sighting directions and for determining elevations by vertical angulation. (Fig. 53.) This consists of a

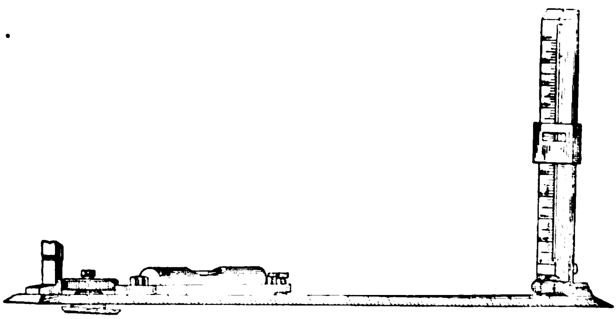


FIG. 53.—VERTICAL ANGLE SIGHT ALIDADE.

ruler nearly 7 inches in length, $\frac{3}{4}$ of an inch wide, and $\frac{1}{10}$ of an inch thick, made of brass with a beveled fiducial edge divided to hundredths of a mile on the scale of the field-work. At the rear end is a fixed sight one-half inch high with a notched gun-sight, and beneath this is a fine peep-sight. At the far end is a hinged sight nearly 3 inches in height, a little over one-half inch in width, and with a slot $\frac{1}{10}$ of an inch in width extending nearly its entire length.

So far it is quite similar to the ordinary sight-alidade used in traversing only. It differs from this in having in addition a small level-bubble attached to it near the rear end, and close to this is a leveling-screw with milled head. The forward sight has ruled on it a tangential scale on which the smallest division is equivalent to 20 feet vertical elevation at the unit distance of 1 mile. Running on this is a slide with horizontal cross-hair,. and the traverseman in sighting any object applies his eye to the peep-sight if the object is above, or to the sliding scale on the hinged sight if the object is below him, and moves the slide up and down until the horizontal cross-hair is in contact with the top of the object sighted. He then notes the reading on the tangent scale, and measuring on his traverse-board in hundredths of miles the distance from his occupied point to the point sighted, he multiplies the reading of the scale by this distance, and the product is the difference in height in feet. The alidade must necessarily be leveled by the small milled-head screw or by the plane-table movement at each sight taken, and the position of the object sighted is determined, in case of traverse, by intersection from various traverse stations.

This instrument is most used in rough traversing and in sketching topography where there are no roads and it is impossssible to carry heavy instruments, because the traverseman moves either on foot or horseback; its use is limited in distance, but for work on a scale of one or two miles to the inch it gives comparatively accurate results for distances not exceeding one or two miles and for elevations less than 8 or 10 degrees. In sketching in details of topography along a road traveled on foot or by conveyance it is also convenient in determining the elevations of unimportant points near by, as it is much more rapidly used than the telescopic alidade.

63. Folding Exploratory Plane-table.—This consists of a folding split-leg *tripod* similar to those made for supporting photographic cameras, but a little more substantial. The three legs are carried in a canvas case 24 inches in length and 3 by 4 inches in cross-section. The tripod head consists of a

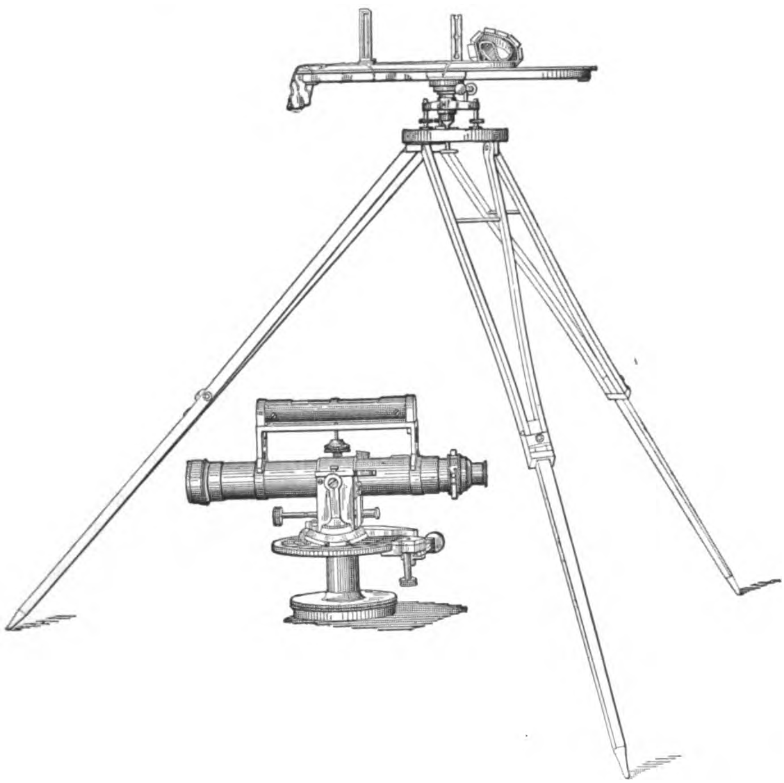


FIG. 54.—FOLDING EXPLORATORY PLANE-TABLE AND SMALL THEODOLITE.

triangular block of wood 7 inches on each side by 1 inch thick, with metal pegs on the under side into which the split legs of the tripod are sprung, and carrying a centre binding-screw for clamping the plane-table movement. This latter consists of three small bronze arms, in general shape like those of a theodolite or transit, supported by three leveling-screws and having a clamp and tangent screw. (Fig. 54.) The top of the movement is a screw 3½ inches in diameter, to which are fastened the cross-braces which support the board. These are two strips of wood, 24 inches in length by 3 inches in width and 1 inch in thickness, and to the four ends of these cross-arms are screwed the outer slats of the folding board.

The *plane-table board* consists of 24 wooden slats, each 24 inches in length, 1 inch in width, and ¼ inch in thickness, and bound together by heavy canvas glued to one surface in such manner that the whole can be rolled into a compact, cylindrical form and carried in a case 24 by 6 inches in diameter or be kept unrolled and clamped to the binding-strips. The surface of this plane-table board is so uneven that good work cannot be carried over any considerable area without appreciable error. Accordingly, there is used in conjunction with this instrument a *small theodolite* with 5-inch circle, which is screwed to the plane-table movement in place of the board. (Fig. 54.) The alidade used with this instrument consists of a simple straight-edge of brass, 18 inches in length, with folding sights, the foresight being a slot with two or three peep-holes. With this apparatus the writer has carried a system of plane-table triangulation, accompanied by vertical and horizontal angles with the gradienter, and has made a complete geographic map on a scale of 4 miles to 1 inch and with sketched contours of 200 feet interval, in a season of seven months, over an area of 11,000 square miles. The extreme error of location on the plane-table, as afterwards corrected by the gradienter angles platted to a primary theodolite triangulation, was a little in excess of ¼ inch in a linear distance of 140 miles.

64. Cavalry Sketch-board.—This is a modified plane-table devised by Captain Willoughby Verner of the British Army. It has an extreme length of 9 to 12 inches and an extreme width of 7 to 9 inches. (Fig. 55.) On either side

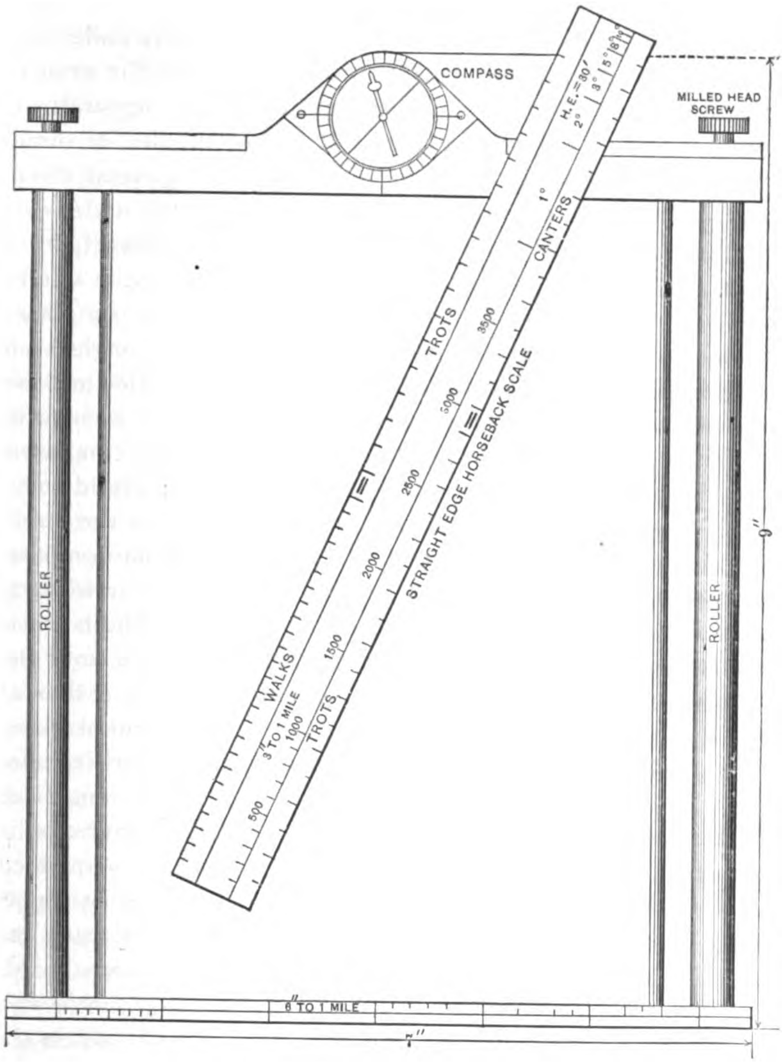


Fig. 55.—Cavalry Sketchboard and Straight-edge.

are two rollers held by friction thumb-screws over which a continuous roll of paper is passed. At one end of the board is a déclinatoire or small box compass, while on its under side is a pivoted strap by which it can be fastened to the wrist of the surveyor and revolved for orientation. This apparatus is used chiefly as a traverse plane-table board, the line of direction through the compass being parallel to the general direction of the route traversed. An attachment to the under side of the board permits of its being fastened to a light tripod or Jacob's-staff, when desired. An adjunct to its use is a light alidade with scale, and it is employed much as is a plane-table excepting that its range is limited by the angle seen ahead when attached to the wrist. Instead of a déclinatoire a small magnetic compass may be counter-sunk in a collar in which it can be revolved, and on the glass of the compass a fine line is engraved which is termed the working meridian.

To use the board the working meridian is set in the direction in which the traverse is being run by turning the compass-box around in its socket, the relative positions of the working meridian and the board being thus determined. The latter is set for sketching by revolving it on the strap pivot until the working meridian coincides with the magnetic needle when at rest. In order to prevent the paper rollers from working loose, a thumb-screw is provided on the end of either roller by which it can be clamped so as to regulate the degree of friction with which it moves. If a half-circle protractor is attached to the side of the board with a plumb-line suspended therefrom, the angle of a slope may be determined by sighting along the edge of the board held on edge and reading the position of the plumb-line on the protractor as a slope board.

This cavalry sketching-board will prove most serviceable in running rough meanders or traverses, and when held in the hand may be used with as much accuracy as the prismatic compass, while there is added to the process of eye-sketching all the data which can be incorporated on a plane-table sketch.

CHAPTER VIII.

SCALES, PLANE-TABLE PAPER, AND PENCILS.

65. Special Scales.—In the execution of any topographic survey some special scale is selected for the field platting. This may be 100 feet or 1000 feet to the inch, or 1 mile to the inch, etc.; but be the scale whatever it may, the work of platting distance will be greatly facilitated by the construction of special scales which will reduce the field measurement directly to relative distances on the map. As in Article 95, in which a special scale for reducing paces of men or animals or time of travel to map scale is shown, scales or tables should be constructed in odometer work, in which a certain number of revolutions of the wheel shall correspond directly with so many divisions of the platting scale (Art. 98). Such scales can be easily prepared by the topographer. They may be so divided that given distances on the scale represent so many revolutions; or a mile or inch scale may be used and a table constructed in which a given number of revolutions for a given sized wheel will correspond to a fixed proportion of the mile or inch.

In such topographic mapping as is executed by large organizations, as the U. S. Geological Survey, standard scales are adopted for field-work, as 1 : 45,000 for the larger-scale topographic maps and 1 : 90,000 for the smaller-scale maps, and boxwood or steel rules are obtained from the various makers on which a distance corresponding to a mile on a scale of 1 : 45,000 is divided into 100 parts. Then if the topographer measures a given fraction of a mile with the odometer, chain, or stadia, he plats the same on the map, not by reducing it to inches (Art. 189), but by his scale of miles. Likewise for computing vertical angles he has but to measure the distances between two points when the result is given him, not in inches, but in tenths and hundredths of a mile, and that quantity can be quickly computed by slide-rule (Art. 66) or table, since these are generally prepared for mile or foot measurements and not for inch measurements.

Similar scales or diagrams greatly facilitate the work of platting triangulation points and projecting maps. A scale has been devised by Mr. A. H. Bumstead of the U. S. Geological Survey for platting projections and triangulation points on a scale of 1 : 45,000 or multiples thereof. This saves all the work of reducing the odd minutes and seconds between platted projection lines to distances on the map scale (Art. 188), as the scale is divided into minutes and their fractions of latitude and longitude on the fixed map scale. Similar scales may be graduated for other map units.

66. Slide-rule.—The slide-rule consists of a number of scales which slide one on the other and are logarithms of numbers platted to scale. These scales are so arranged that the corresponding logarithms may be brought opposite each other so as to mechanically add or subtract. By its use nearly all forms of multiplication and division, involution and evolution, including trigonometric operations and computations, may be performed. Slide-rules are made not only for the ordinary operations of multiplying and dividing, but also for special use in computing stadia measures and for computing engineering quantities of various kinds.

A topographer who has much *stadia work* to compute and who has no specially prepared tables or diagrams for his scale of work should use a slide-rule. Likewise in computation of *vertical angulation* a slide-rule should be used where tables to scale are not at hand. The instrument performs accurately and without mental effort a mass of tiresome calculations, multiplications, and divisions, which could not possibly be worked out by ordinary methods in nearly so short a time as by its use. Where accuracy is desired slide-rules may now be procured made with the graduations on celluloid facings. As a result very fine readings can be made, especially as the brass runner of the older forms is superseded by a glass plate on which fine lines are ruled.

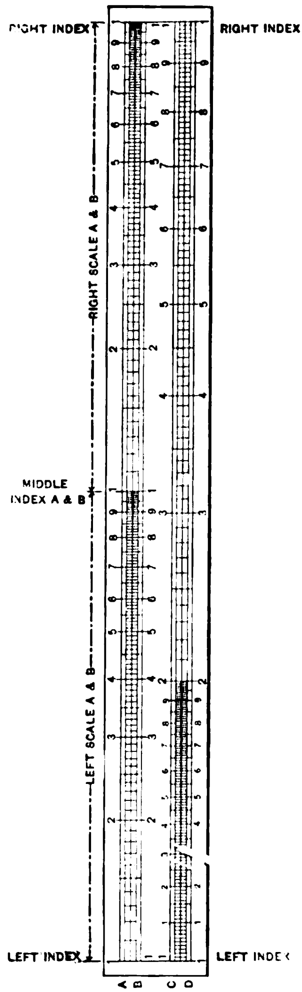


FIG. 56.—SCALES OF THE SLIDE RULE.

67. Using the Slide-rule.—The following simple explanations of the use of the slide-rule in such operations as the topographer has to perform are extracted from an article by Mr. G. B. Snyder published in the Engineering News. For better understanding of this explanation the four scales on the slide-rule shown in Fig. 56 are marked *A*, *B*, *C*, and *D*.

Multiplication.—To multiply, set the index of the slide opposite the multiplicand on the rule; the result will then be found on the rule under the multiplier on the slide.

Example: Multiply 2 by 3.

Using scales *A* and *B*, set the left index of *B* under 2 on *A*; then over 3 on *B* will be found 6 on *A*, and all the other numbers on *B* will be found to be in the same proportion with those on *A*. Thus, 4 will be found under 8, 6 under 12, 7 under 14, etc., or, considered as a proportion, 1 : 2 : : 3 : 6 : : 4 : 8 : : 7 : 12, etc., or $\frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{6}{12}$, etc. These results will be proportionally the same whatever value we assign to the numbers, which can be considered as 200, 600, 800, etc., as 20, 60, 80, etc., or as .2, .6, .8, etc.

Rules for the position of the decimal-point are given in the pamphlet that accompanies the rule, but usually its position can be obtained by inspection.

The above multiplication can be performed on the lower scales, when finer readings can be obtained. If the left-hand index of *C* is set over 2 on *D*, it will be found that numbers above 5 on the slide protrude beyond the rule. To obtain these results the right-hand index of *C* must be set over 2 on *D*, when 12 will be found under 6, 14 under 7, 16 under 8, etc.

Division.—The process of division is merely the reverse of multiplication. The divisor is set opposite the dividend, and opposite the index is found the quotient.

Example: Divide 20 by 8.

Using the lower scales, set 8 over 20; under the index will be found 2.5.

Squares and Square Roots.—On scales *A* and *B* there are in the length of the rule two complete sets of numbers, while there is only one set of numbers on scales *C* and *D*, the numbers on the lower scales taking up twice the distance they do on the upper. To square a number its logarithm must be multiplied by 2, and to obtain its square root its logarithm must be divided by 2, and as the distances on the rule represent logarithms of the numbers affixed to it, the numbers on the upper scales are the squares of those on the lower.

To square a number, set the runner to the number on the lower scale, and the coinciding number on the upper scales will be its square. Thus, over 2 will be found 4; over 5 will be found 25; over 15 will be found 225, etc.

To obtain the square root the above operation is reversed by setting the runner to the number on the upper scale; the coinciding number on the lower scales will be the square root. Thus, under 9 will be found 3, under 16 will be 4, under 625 will be 25, etc. As there are two sets of figures on the upper scale, care must be taken that the proper one is used; thus, in obtaining the square root of 9, the 9 on the left scale must be used, for if the runner is set to the 9 on the right-hand scale, its coincident number will be found to be 9.48 +, which is the square root of 90.

If the number whose square root is to be taken has an odd number of figures in it, counting the figures in front of the decimal-point, use the left-hand scale; if an even number, use the right-hand scale. Thus, with 625 use the left-hand scale; with 62.5 use the right-hand. If the number is all decimal, use the right-hand scale.

The Solution of Plane Triangles.—The under side of the slide is graduated to a scale of sines and a scale of tangents, so that trigonometric calculations can be made on the rule. When the under side of the slide is uppermost, the scale of sines will be along scale *A* and the scale of tangents along scale *D*. By referring to a table of natural sines, it will be found that 1.000 is the sine of 90°, that .100 is the sine of 5° 44′, and that .010 is the sine of about 0° 34½′, so the right index of the scale is 90°, the middle index is 5° 44′, and the left index is 0° 34½′. Sines of less than 0° 34½′ can be found by setting 34½ on *B* under the right index of *A*; then over any number of minutes on *B* will be found the corresponding natural sine on *A*. Note the graduations on the scale of sines; as rules are usually graduated, every degree is marked between 40° and 70°. Above 70° the shorter marks are every 2° until the first long mark is reached, which is 80°. There is only one mark (85°) between 80° and the index.

By referring to a table of natural tangents, 1.00 will be found to be the tangent of 45°, and .100 to be the tangent of 5° 43′, so the right index of the scale of tangents is 45°, and the left index is 5° 43′. To obtain tangents less than 5° 43′, set 5° 43′ = 5.72 on *C*, over the right index of *D*; then under the angles expressed in degrees and decimals on *C* will be found their corresponding natural tangents on *D*. With the slide set as above, the tangent of 1° will be found to be .01745. To find the tangent of angles less than 1°, set 60 opposite 1745, and minutes on the slide will be opposite their corresponding tangents on the rule. Tangents of angles greater than 45° can be obtained by dividing 1 by the tangent of the complement of the angle.

Triangles can be solved very readily on the slide-rule and with considerable accuracy if not more than three or four figures are necessary in the results.

Right-angled Triangles.—Example: What is the altitude of a right-angled triangle, with an angle at the base of 0° 25′ and a hypothenuse of 1240? Here the angle is smaller than can be read on the scale of sines. On the scale of sines will be found a mark for single minutes near the 2° mark. Set this mark to the under index of the rule, then minutes can be read along *B* and their corresponding sines will be found on *A*.

As noted before, 34½′ is about as low as can be read on the scale of sines. With the slide set as above, 34½′ will nearly coincide with the index, and 1′ will be found under .000291, which is its sine. Set the runner to 25′ on *B* and move index to runner; over 1240 will be found 9.0. The position of the decimal-point can be found by a mental calculation, thus: As noted before, the indexes of scale *A* correspond with 0° 34½′, 5° 44′, and 90°, respectively; if the angle had been 90°, the altitude would have been 1240; if it had been 5° 44′, the altitude would have been 124.0; if it had been 0° 34½′, the altitude would have been 12.40; the angle is 0° 25′, therefore the result must be less than 12.40.

Example: Given a right-angled triangle with a base 64 ft. and an angle at the base of 42° 31′; what is the altitude? Set index over 64. Under 42° 30′ on the scale of tangents will be found 58.6 ft.

Example: What is the altitude of a triangle with a base of 24.5 ft. and an angle at the base of 72° 15′?

Here the angle is greater than 45°, and cannot be read on the scale of tangents, so the complement of the angle is used and divided into the base instead of multiplying. 90° − 72° 15′ = 17° 45′. Set 17° 45′ over 24.5; under index will be found 76.5 ft., the altitude required.

Owing to the ease with which numbers can be squared on the slide-rule, work can readily be checked by seeing if the square root of the sum of the squares of the two legs is equal to the hypothenuse. One of the simplest ways of avoiding mistakes is to bear in mind that sines and cosines are merely percentages of the hypothenuse, and that tangents and co-tangents are percentages of the base or altitude.

Plane Triangles.—The preceding examples have been applied to right-angled triangles only. The following are applied to plane triangles in general:

Example: Given one side and the angles of a triangle to obtain the remaining sides.

Here we use the proposition: Sine of the angle opposite the given side : sine of the angle opposite the required side : : the given side : the required side.

To solve the above problem, set 64°, the given angle, on scale of sines, under 117, the given side on *A*; then over 76° will be found 126.3, the length of its opposite side, and over 40° will be found 83.7, the length of its opposite side.

With the three sides given and one of the angles the remaining angles can be found in the same way, and with two sides given and the angle opposite to one of them, the solution is equally simple.

Example: Given a triangle with a side of 81 ft. and a side of 60 ft., with an opposite angle of 40°. Required the remaining side and the remaining angles.

Set 40° on scale of sines under 60 on scale *A*; then under 81 will be found 60°, being the opposite angle, and the remaining angle will be $180° - (60° + 40°) = 80°$; over 80° will be found 92, the remaining side.

68. Plane-table Paper.—In conducting an accurate plane-table survey the paper employed is as important an instrument and should be selected and handled with as great care as other portions of the outfit. An accurate scheme of plane-table triangulation cannot be developed and delicate intersection obtained from lines drawn on inferior paper or on paper that presents an uneven surface. The practice of using large sheets of paper only a portion of which is attached to the board at one time, the remainder being rolled up and retained in position by clamps, is to be discouraged. The *rolling of the paper* produces cracks and causes it to buckle in such manner as to render it impossible to obtain the most satisfactory surface on which to rest the alidade. Moreover, the cumbersome roll at one or both ends of the board presents a large surface to the winds and renders it difficult to keep the table steady from vibration even in winds of moderate velocity. Finally, paper is very sensitive to atmospheric changes; especially is it affected by the moisture in or dryness of the atmosphere, and points plotted twenty to thirty inches apart will frequently be found in error after a lapse of but a few days, and by a very appreciable amount if any but the best paper is used.

The best *plane-table paper* is *double-mounted*, and is prepared in the following manner: A rectangular wooden frame a little larger than the size of the sheet required is made, and over it is tightly stretched, by means of tacks, a piece of the ordinary muslin or cotton cloth used in map-mounting. To each side of this is pasted, with the right surface out, a sheet of the best drawing-paper, so oriented that the grain of the two sheets will be crossed at right angles. The result is a sheet of "double-mounted" drawing-paper; one which is least affected by atmospheric changes, and it has been found by experiment that such changes affect it almost uniformly in all directions. Therefore, if variations take place in its dimensions, they are of such kind as may be largely eliminated by a uniform reduction or enlargement of scale. Such double-mounted paper can now be purchased of most of the larger dealers in drawing and surveying instruments, and the best paper for this purpose has been found to be paragon grade of heavy eggshell or double elephant paper. Such plane-table sheets cannot be rolled, and must be transported in flat wooden boxes, or else be laid against the surface of the plane-table board and carried in a suitable canvas or leather case.

For less important plane-table work, especially where plane-table triangulation is to be frequently checked by instrumental triangulation, or for plane-table traverse, ordinary *single-mounted* drawing-paper of good quality may be used, and this may be rolled, though even in such classes of work it is preferable to use single sheets and transfer from one to another by long orienting marks on the board and on the paper. For plane-table work in a region where the sun is so very bright that the glare affects the eyes, as in the arid regions of the Southwest, it has been found desirable to use *tinted* drawing-paper in preference to plain white, and the most satisfactory tints and those which appear to affect the texture of the paper itself least are the neutral tints between Paine's gray and slate-blue. *Celluloid* sheets are very useful in regions like the Adirondacks or the Northwest, where there is much rain and dew. With this, work can often be done on fair mornings, when the wet from brush and leaves of trees would soon soak common paper.

69. Preparation of Field Sheets.—In planning a plane-table survey of a given region a *number of plane-table sheets* should be prepared of such a size as will fit the board. On these the work should be so planned as to leave ample margin on each edge to permit of transferring from and connecting between the various sheets. On each sheet should then be platted at least two points (Art. 188), the relative positions and distances between and azimuths of which have been previously determined by instrumental triangulation of primary or secondary order. In case no such prior triangulation exists, a suitable location should be chosen and a temporary base line carefully measured with long steel tapes or wire (Art. 204), and plotted on the sheet as nearly as possible in correct azimuth and in the relative position which its location on the ground bears to the area under survey. From the ends a plane-table triangulation may be expanded as from located trigonometric positions. In extreme cases, where absolute distances are not essential or where work is to be checked by an after-primary triangulation, two points may be selected as initial stations and their relative positions be fixed on the plane-table sheet, the distance being estimated and the azimuth marked by means of a magnetic needle. If later a geodetic triangulation locates two connected points and an azimuth within the surveyed area, the map may be adjusted.

Where careful plane-table triangulation is being conducted, *points should not be transferred* from one plane-table sheet to another. Each sheet should have located upon it at least two points, the positions of which have been determined and computed by geodetic methods. If for any reason this is impossible, it should be assumed that the act of transferring from one sheet to another has distorted or affected unfavorably the plane-table triangulation, and in order that this shall be in one direction only, and therefore susceptible of after-correction, only two points should be so transferred, with the intention that ultimately a scheme of instrumental triangulation may be extended over the area under survey and the plane-table work be adjusted thereto.

The first desideratum in *fastening plane-table paper* to the board is that it shall be held firmly and equally, and so as not to be disturbed in its position by the friction of the alidade or by ordinary winds. One means of effecting this is by brass spring-clamps; a second is by ordinary thumb-tacks; and a third by screw-tacks. The latter are decidedly the better. Clamps, being large, are liable to accidental disturbance, they do not hold the paper firmly, and are at all times therefore liable to permit a movement of the paper. The ordinary thumb-tacks hold the paper firmly when in place, but are easily loosened and lost, while in high winds the whole paper may be suddenly ripped from the board.

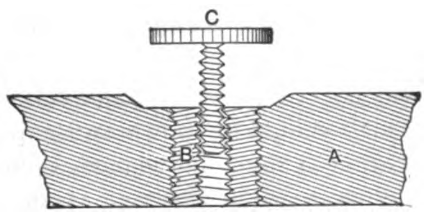


FIG. 57.—DOUBLE SCREW TO HOLD PLANE-TABLE PAPER.
A, plane-table board. *B*, hollow brass wood screw.
C, milled head brass clamping screw.

The paper should bear at all times the same relation to the board and should be so immovable as to form practically a part of it. A thumb-tack which fairly fills the requirements has a screw-thread cut on the spike, and the head has holes sunk into it so that these may be clamped by a spanner and the tacks screwed into the wood. These, however, project so as to interfere with the free movement of the alidade. The plane-table boards of the Geological Survey have a special attachment set into each of the corners and sides, which consists of a brass cylinder having a screw-thread on the outside by which it is sunk into and flush with the surface of the board, and the inner surface has a female screw, into which a milled-head clamping-screw is fastened through the paper. (Fig. 57.)

70. Manipulation of Pencil and Straight-edge.—All lines drawn on the plane-table board should be made with the hardest of pencils sharpened to a very fine point, and the lines should be drawn lightly and carefully and close to the edge of the rule. Great care should always be taken to hold the pencil in the same position, either very close under the edge of the alidade or vertically, so that its point shall be invariably at the same distance from the edge, and the same side of the straight-edge should always be employed, lest the two sides be not truly parallel or bear a wrong relation to the axis of the telescope. If any part of the straight-edge is raised from the paper, especial care must be observed that the pencil does not run under its edge and thus deviate from the straight line.

It is desirable not to draw lines the full length of the sight, but short lines should be drawn on the paper approximately at the location of the point which is sighted, and other short lines should be drawn at each end of the straight-edge, so that the latter may at any time be laid correctly on the line sighted (Fig. 47). The alidade should never be moved by sliding it over the surface of the table, but in changing its position it should be lifted up and carefully set down again on the table, so as not to rub the lines or soil the paper. When an intersection of two or more lines is obtained, the point located should not be pricked with a pin or pencil point, but the location should be pricked lightly and finely with a delicately pointed needle. A needle-point should never be inserted in the paper at the point located so as to be used as the fulcrum about which to rotate the alidade, but the latter should always be lifted up and laid down with its edge against the located point and in the same relation thereto as were the lines drawn to the point with the pencil; that is to say, the under edge of the rule must bisect or be tangent to the point according as was the pencil-point in drawing the line which produced the location.

71. **Needle-points, Pencil Holders and Sharpeners.**—In running a traverse, and in the execution of plane-table triangulation, the little devices and tools with which the topographer is provided aid greatly in facilitating his work. A fine needle-hole may be made to mark the location of a triangulation station. In traversing, however, the work is greatly expedited by sticking a very fine needle into the board around which to revolve the light sight-alidade. In this manner the topographer has not to watch the point on the paper to see that his alidade is tangent to it, but has simply to press the alidade edge against the needle-point. Such *needle-points* are made by taking a No. 10 needle, breaking it in half, and melting a sealing-wax head upon it. In this manner the short stem renders it less liable to be broken, and the head gives something large enough for the topographer to handle readily and press with force into the paper.

It is slow work attempting to get a sufficiently sharp and satisfactory edge on a pencil with a penknife, and as the pencil must be sharpened frequently in order to keep it in condition for fine work, *sand-paper sharpeners*, preferably in the form of pads, as furnished by dealers, should be provided, and these should be carried attached to the board by a string, so as to be always at hand for rapid renewal of the pencil-point.

In order that the rubber eraser and the pencil shall be always in the most accessible places, *leather pencil pockets* or holders should be provided in which pencils can be carried by attaching the holders to the outer garment of the topographer. These holders help protect the pencil-point. The rubber eraser should either be tied by a string to the board, or, better, metal tips provided with rubber should be supplied for all pencils. A sufficient number of these should be carried for renewals, and thus the rubber is always handy when it is attached to the reverse end of the pencil.

CHAPTER IX.

PLANE-TABLE TRIANGULATION.

72. Setting up the Plane-table.—In sighting signals these should be bisected as near the base as possible, and signal-poles should be straight and perpendicular, and the flags upon them white or black according to the color of the background against which they are to be seen. They should be of such size as to be visible at the greatest distance from which they must be observed. The positions of the stations should be well marked with a small cairn of rocks and by measurement to some near-by witness-mark, so that if the signals are disturbed their positions can be readily found.

The theoretic requirements of setting up a plane-table at a station are:

1. The plane of the board should be horizontal.

2. The projection of the station on the map should be vertically over its position on the ground.

3. The meridian of the point on the plane-table sheet should be in the plane of the meridian of the station.

The first of these requirements is met by a proper construction of the instrument. For small-scale maps, as those of more than 1000 feet to the inch, the second requirement does not necessitate the plumbing of the platted point exactly over the station, since the instrument can generally be set up near enough by eye. On maps of larger scales the location on the plane-table corresponding with the point occupied must be plumbed over the latter; that is to say, the center of the board is not plumbed over the station-mark, but the platted point itself. If the plane-table be set up by eye, it can easily be fixed within six inches of its true position. At a range of half a mile such an error would subtend an angle of less than a minute, and angular errors of such small amount may easily be neglected.

The third of the above requirements is met by *orientation* of the plane-table board. This is its adjustment in azimuth, by which all lines joining points on the sheet are made parallel to corresponding lines in nature.

The *inclination of the board* from the true horizontal plane or the amount which it is out of level affects the location in azimuth far less than would be at first estimated. This is well illustrated in the following table, prepared by Mr. Josiah Pierce, Jr.

TABLE IV.

ERROR IN HORIZONTAL ANGLE DUE TO INCLINATION OF PLANE-TABLE BOARD.

Inclination of Board. θ	Angle when Level. α			Angle when Inclined. β			Maximum Errors.	
°	°	′	″	°	′	″	′	″
1	45	00	08	44	59	52	0	16
2	45	00	36	44	59	53	1	03
3	45	01	10	44	58	49	2	21
4	45	02	06	44	57	54	4	12
5	45	03	16	44	56	43	6	33
6	45	04	43	44	55	16	9	27
7	45	06	26	44	53	34	12	52
8	45	08	24	44	51	33	16	51
9	45	10	38	44	49	20	21	18
10	45	13	09	44	46	50	26	19
11	45	15	57	44	44	03	31	54
12	45	18	59	44	41	01	37	58
13	45	22	12	44	37	42	44	36
14	45	25	55	44	34	05	51	50
15	45	29	47	44	30	14	59	33

From the above it appears that a plane-table or theodolite may be 15° out of level before the maximum error in the measurement of a horizontal angle will approach 1°.

Also, the error in azimuth is a maximum when $\alpha + \beta = 90°$.

The above results may be obtained by the following simple formula:

$$I = \frac{\theta^2}{230} \text{ approximately}, \quad . \quad . \quad . \quad . \quad (1)$$

in which θ is the inclination of the board or angle which it makes with the horizontal.

Thus, if the board were out of level 1°, the maximum error in azimuth which would be produced would be about 16″, an amount scarcely appreciable at 12 feet. An error in level of 3° would only produce an appreciable maximum error of 2′ 21″.

73. Location by Intersection.—There being platted upon the plane-table paper (Arts. 69 and 188) the known positions of at least two points which are in view from the station over which the plane-table is set up, the succeeding plane-table triangulation consists in the determination of the relative positions on the paper of additional points in nature. This should, so far as practicable, be accomplished by the method of *intersections*. This is accomplished by previously occupying known positions and by constructing a graphic triangulation on the plane-table board from these, including unknown positions which are platted in the course of the work. Where this is not practicable, as is occasionally the case, because of the impossibility of occupying some of the known positions, the work must be performed by the method of *resections* (Art. 74), by which unknown points are occupied and positions determined and platted on the paper by sighting to known points.

The controlling condition in the conduct of plane-table triangulation is that the board shall be in *orientation* (Art. 72). Let the station P be occupied, and p be its platted position on the plane-table board (Fig. 58, A). Let a, b, c be the platted positions on the board of the signal A, the church-spire B, and the flag C. The plane-table board being leveled

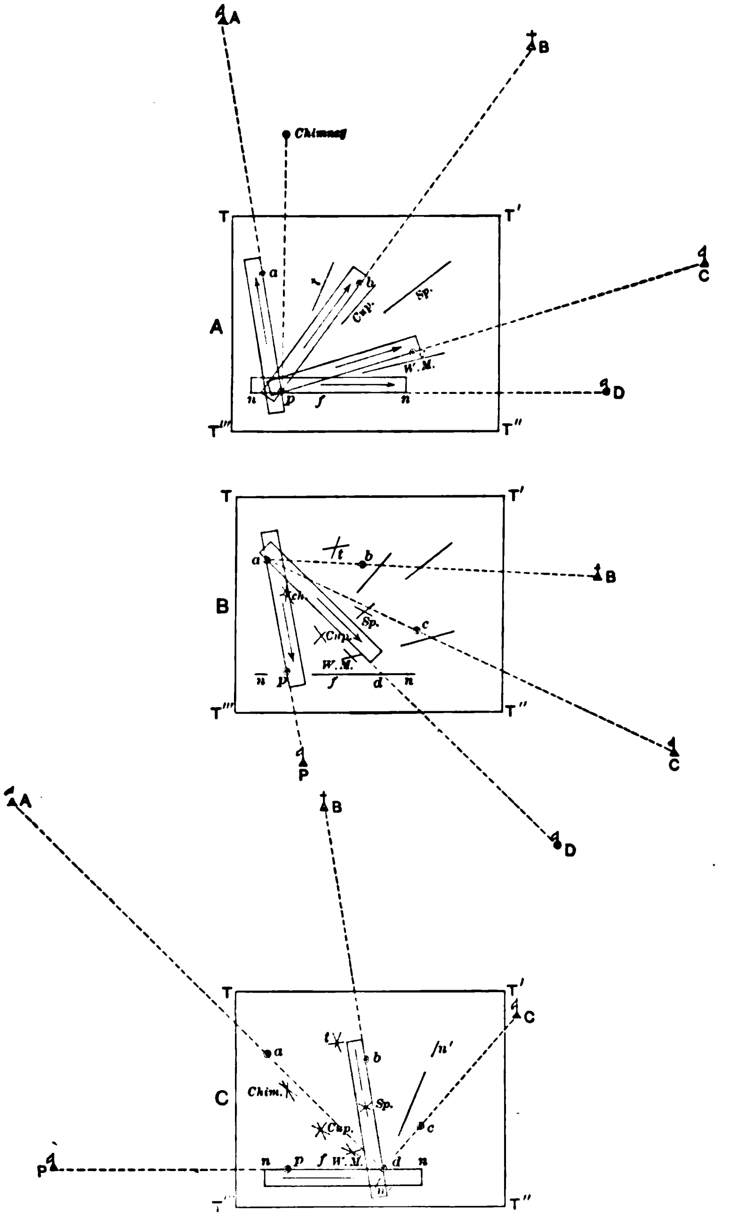


FIG. 58.—INTERSECTION WITH PLANE-TABLE.

and oriented approximately by eye by ranging the lines from *p* towards *a*, *b*, *c* in the directions of the corresponding signals, *A*, *B*, *C*, the edge of the alidade is placed on the line *pa*, and, the horizontal motion being unclamped, the board is swung in azimuth until the cross-hairs bisect the signal *A*, when the horizontal motion is clamped. The alidade is now placed successively on the lines *pb* and *pc*. If in sighting the signals *B* and *C* the cross-hairs bisect these, the instrument is oriented. If it does not exactly bisect them, there is something wrong with the platting on the known points or with the observation of one or more of the signals. If the positions of the points have been accurately determined and platted, the cross-hairs must bisect all of the signals on known stations when observed from any known position.

The orientation of the board being now verified, *plane-table triangulation is extended* by placing the edge of the alidade on the point *p* and swinging it until the cross-hairs bisect a new signal, *D*, towards which a line is drawn. Lines are also drawn along the edge of the ruler when it is pointed to a chimney, a cupola, or to other visible and easily distinguishable objects, the near end of the alidade being of course on the point *p*. Everything which is observable from this station and which may possibly be recognized from succeeding stations being now indicated on the paper by lines drawn from *p*, the alidade may be moved and sighted successively to each of the points observed, and the vertical angle read to them and recorded (Art. 160). The work of this station is then completed, and the topographer moves to the next station, *A*.

Having oriented the board on the second station as before, by placing the edge of the alidade against the known and occupied point *a* and sighting successively to the known points *P*, *B*, *C*, etc., the orientation is verified by observing if the edge of the alidade passes through the located points *p*, *b*, *c*, etc. If so the topographer proceeds to *intersect* some of the lines previously drawn from the first station, *P*. The line drawn towards the new point, D, intersects the line drawn from P in the point d, which is its position (Fig. 58, B). Likewise intersections are made on the cupola, the chimney, etc., by sighting these and drawing lines along the edge of the ruler.

The positions of the points thus determined are not considered sufficiently well established for the propagation of triangulation unless a third intersection is had on them for the purpose of *verification*. Where it is difficult to get a third intersection, locations by two lines will answer sufficiently well for intermediate or tertiary points, but every effort should be made to get a third intersection (Fig. 58, C), providing anything of moment is dependent upon the position. The third intersection is had as in the case of the previous ones by occupation of one of the remaining points, B or C, or perhaps by occupation of the new point, D. In the latter event, only two lines having been previously drawn through d, its position is more accurately verified after orientation on the previously occupied stations P and A by resection from the occupied stations B and C. In this event it may be necessary to unclamp the board and swing it a trifle in azimuth as described in the three-point problem (Art. 75), in order to get a more exact location than is given by two intersections.

74. Location by Resection.—The *three-point problem* calls for the finding of distances from an unknown and occupied point to three others whose relative positions and distances are known. Only the constructive or graphic solutions of the problem are here given, and not the theoretic or trigonometric, since the operation of locating a point on the plane-table is graphic and not trigonometric.

The determination of an unknown point graphically on the plane-table is performed by the method of *resection*, which consists in the occupation of the unknown point with the plane-table and the sighting from it to the three known points, on which well-defined signals must be erected and the positions of which are previously plotted upon the plane-table sheet. The determination of the unknown position may be accomplished by several methods, the earlier of which is known as Bessel's, from its inventor, though the most satisfactory method, and that now almost universally employed, is known as the Hergesheimer or Coast Survey method. In addition there is an approximate but rapid and practical method by means of tracing-paper, generally known as the graphic method, and there are also the less well-known and rarely employed Lehmann's and Netto's methods.

75. Three-point Problem Graphically Solved.—Three simple, practical rules for determining the location of an un-

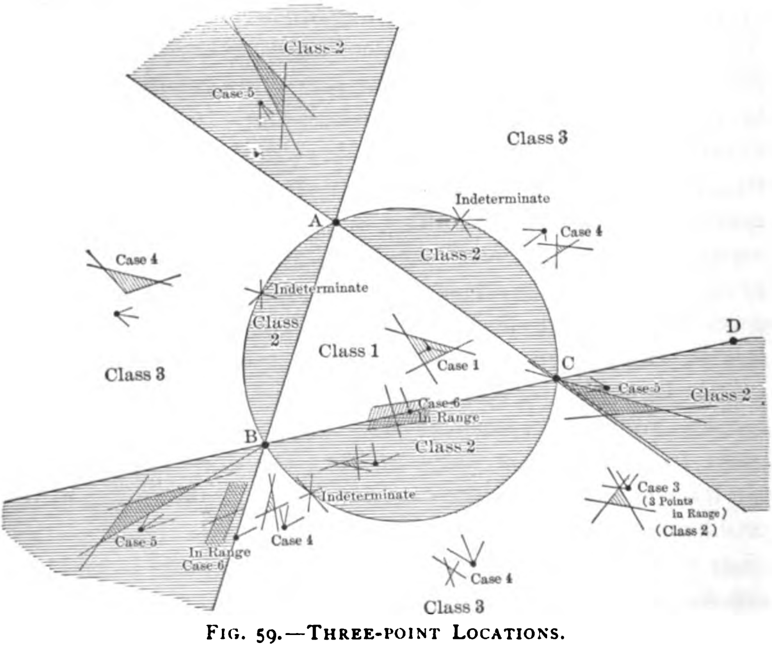


FIG. 59.—THREE-POINT LOCATIONS.

known point on the plane-table by means of the three-point problem are the following (Fig. 59):

1. When the new point is on or near the circle passing through the other points, the location is uncertain.

2. When the new point is within the triangle formed by the three points, the point sought is within the triangle of error.

3. When the new point is without the triangle, orient on the most distant point, then the point sought is always on the same side of the line from the most distant point as the point of intersection of the other two lines.

The last rule is that most usually called into requisition, and is perhaps the most important in aiding in the quick determination of the unknown position.

76. Tracing-paper Solution of the Three-point Problem.—By the use of tracing-paper the three-point problem is solved approximately with great rapidity. Setting-up the table on the unknown point *P* (Fig. 58), fasten on it a piece of tracing-paper of sufficient size to include the positions of all four points. A fine point is marked upon it to represent the position of *p* and as near the actual location of that point on the paper can be estimated by eye. The alidade is then centered about the point *p* and pointed successively at the three known points *A*, *B*, and *C*, and the lines *pa*, *pb*, and *pc* are drawn on the tracing-paper. The alidade being then removed and the tracing-paper released, this is so shifted over the plane-table sheet that the line *pa* shall always pass through the located point *a*, the line *pb* through the located point *b*, and the line *pc* through the located point *c*. Then, with all three lines passing through the known points, the point *p* is exactly over its correct position on the plane-table paper, and may be pricked through to the latter.

As this method is approximate only because of the little inaccuracies introduced in stretching the tracing-paper; or because of its wrinkling and the difficulty of drawing very fine lines on the tracing-paper and properly superimposing this, it is well, where an exact location of *d* is desired, to then test the position of the latter by resection from the known points, when a small triangle of error may be found. This will be so small, however, that a trifling movement of the board will bring the table into exact orientation, and frequently with much greater accuracy and ease than by using the graphic three-point method only.

77. Bessel's Solution of the Three-point Problem.—Bessel had two methods of solving this problem, only the first of which will be described, as the other is less practical. The plane-table is put in position at the unknown station from which the three known points must be visible, and the position of the unknown point can then be found as follows, providing it be not in the circumference of a circle passing through the three fixed points:

A quadrilateral is constructed with all the angles within the circumference of a circle, one diagonal of which passes through the middle one of the three fixed points and the point sought. On this line the alidade is set, the telescope directed to the middle point, and the plane-table oriented. Resections upon the extreme points intersect on this line and determine the position of the point sought. In Fig. 60, let *a*, *b*, *c* be the platted position of the known points; the plane-table being set up on the unknown station *D* and leveled, the alidade is set on the line *ca*, and the end at *a* is directed, by revolving the table, to its corresponding signal *A*, and the table clamped; then, with the alidade centered on *c*, the middle point *B* is sighted with the alidade and the line *ce* drawn along the edge of the rule; the alidade is then set upon the line *ac*, and the telescope directed to the signal *C* by revolving the table, and the table clamped. Then, with the alidade centering on *a*, the telescope is directed to the middle signal *B*, and the line *ae* is drawn along the edge of the rule. The point *e* (the intersection of these two lines) will be in the line passing through the middle point and the point sought. Set the alidade upon the line *be*, direct *b* to the signal *B* by revolving the table, and the table will be in position. Clamp the table, center the alidade upon *a*, direct the telescope to the signal *A*, and draw along the rule the line *ad*. This will intersect the line *be* at the point sought. Resection upon *C*, by centering the alidade on *c* in the same manner as upon *A*, will verify its position.

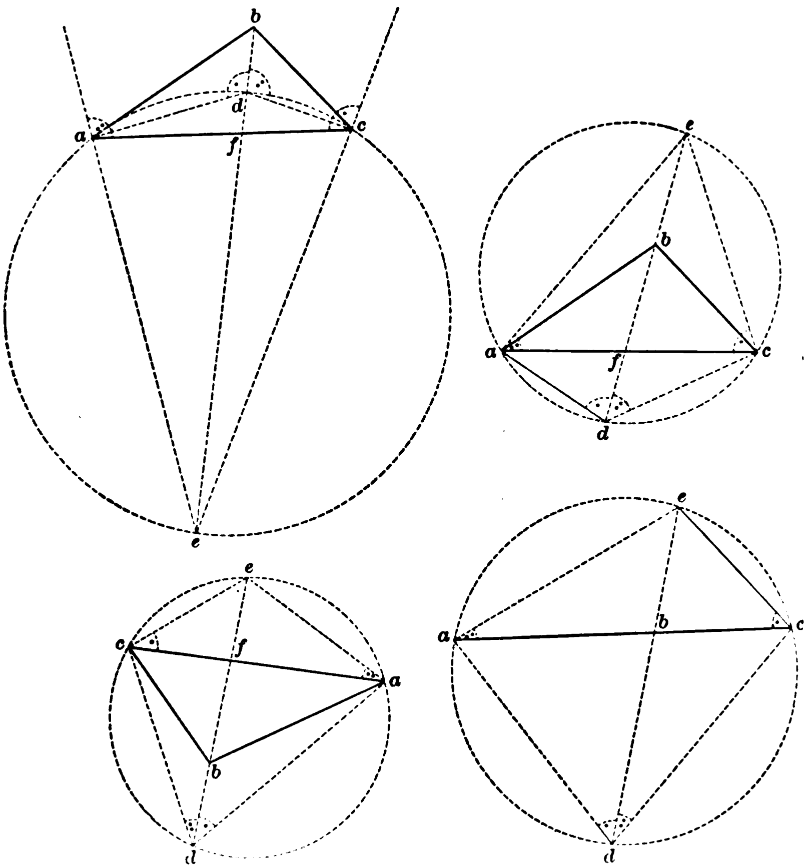


FIG. 60.—BESSEL'S GRAPHIC SOLUTION OF THE THREE-POINT PROBLEM.

In the use of the Bessel methods for the determination of position, the triangle formed by the three fixed points can be contracted or extended as may be desirable, by drawing a line parallel to the one joining the two extreme points, terminated by those joining the extremes with the middle point. The lines laid off at these representative extreme points, in the manner described for the extremes, will intersect in the line passing through the middle point and the point sought.

This affords the means of using a point in view which would not be within the size of the table when the other two points are shown, by contracting the triangle formed by the three points until both extremes are brought within the table size and within reach of the alidade. A resecting line for the point off the table can be drawn from its signal near the estimated position of the point sought, and a line drawn through the corresponding point off the table, parallel to this, will determine the precise position of the point sought, to be verified by resection on the other extreme point.

78. Coast Survey Solution of the Three-point Problem.—This method depends upon the fact that when the plane-table is set up and is not in orientation, resection from any three known points, except from a point on the circumference of a circle passing through these points, will form a triangle called the *triangle of error*, or two of these lines will be parallel and intersected by the third. The position of the true point can then be determined graphically from these several intersections, and is always at the point of intersection of the arcs of the circles drawn through each two points and the point of intersection of the lines drawn from them. There are numerous practicable modes of locating the point sought, and these have been divided into several classes, and these again into several cases or subdivisions for convenience of description (Fig. 59). This classification is based upon the location of the true point in relation to the triangle of error, the triangle formed by the three fixed points being called the great triangle, and the circle passing through these points the great circle. The topographer is supposed to face the signals, and directions right and left are given accordingly.

Class 1. When the point sought falls within the great triangle, the true point is within the triangle of error. If the line from any of the view-points falls to the right of the intersection of the other two points, turn the table to the left; and if to the left, turn it to the right.

When the point sought is without the great triangle, the true point is also without the triangle of error and is situated to the right or left of it, according as the table is out of position to the right or left.

Class 2. When the point sought falls within either of three segments formed between the great circle and the sides of the great triangle, the true point is on the side of the line from the middle point opposite to the intersection of the lines from other points. Also, where the three fixed points are in a straight line, in which case the three points are considered as being on the circumference of a circle of infinite diameter, the true point always lies within one of the segments of the great circle.

If the line from the middle point is to the right of the intersection of the other two, turn the table to the right, and if to the left, turn it to the left.

Class 3. When the point sought falls without the great circle and within the sector of either angle of the great triangle, the true point is on the same side of the line from the middle point as the intersection of the lines from the other two points.

If the line from the middle point is to the right of the intersection of the other two, turn the table to the left, and if to the left, turn it to the right.

Class 4. When the point sought is without the great circle and the middle point is on the near side of the line joining the other two points, the true point is without the triangle of error, and the line drawn from the middle point lies between the true point and the intersection of the other two lines. Also, when the point sought is on the range of either of the two points, and the table deflected from the true position, the lines drawn from these points will not intersect, but will be parallel to and intersected by the line drawn from the third. The true point is then between the two parallel lines.

When the line from the right-hand station is uppermost, turn the table to the right, and when that from the left is uppermost, turn the table to the left.

79. Ranging-in, Lining-in, and Two-point Problem.—It is sometimes desirable to place the plane-table in position at an unknown point from which only two known points are visible. This may be easily done in the following two cases by methods known as "ranging-in" and "lining-in."

Ranging-in consists in determining the position of a point on a line already drawn on the plane-table, but elsewhere on that line than at the position of the point sighted. In Fig. 61 let *A* and *B* be the positions of the two known points, and let *AC* be a line drawn from *A* towards the point *C*. When

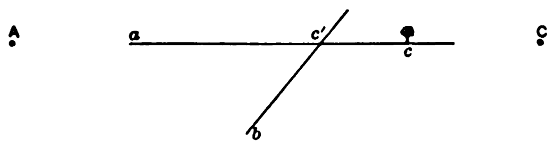


FIG. 61.—RANGING-IN.

the topographer reaches *C* let him find it, for some reason, inaccessible; it may be a tree, a building, or some other object near which, but not over which, he may set up. Aligning himself, therefore, by eye in the direction *AC* by means of range-poles or by sighting over the top of *C* at *A*, he sets up the plane-table on the line thus sighted by placing the alidade on the line *ac* and resecting on *A*, and clamping the table, when it will be in orientation. Placing the end of the alidade now on the point *b* and resecting on *B*, the line drawn along the edge of the rule will intersect the line *ac* at the point *c'*, the position sought.

In *lining-in*, the positions of the points *A* and *B* (Fig. 62) are known and located on the plane-table sheet at *a* and *b*, and a line having been drawn from one of the stations A towards an undetermined point, C, it is desired to locate another undeter-

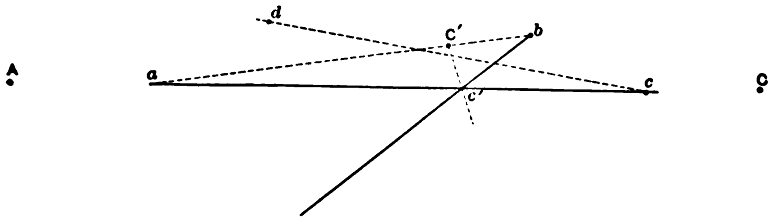


FIG. 62.—LINING-IN.

mined point, C', on the line ab, but at a considerable distance from either point. The topographer, finding a suitable position at C', proceeds with the aid of an assistant, d, to place himself in a line between A and B. Standing some distance apart, they line one another in, the topographer, c', by sighting over his assistant, d, at A; the assistant, d, sighting over the topographer, C', at C; then they motion each other backwards and forwards at right angles to the line ac until each finds the other exactly in line with his range-point. The topographer is then on the line sighted from A to C, and may set up his plane-table and, placing the alidade on ac, resect on A, when the board will be in orientation. Now, setting the alidade on point b and resecting on B, a line drawn along the edge of the rule will intersect the line ac in the undetermined point c'.

A more difficult case of making a location by the *two-point problem* is the following: Two points A and B (Fig. 63), not conveniently accessible, being located on the paper at a and b, it is desired to put the plane-table in position at a third point, C. A fourth point, D, is selected, such that the intersection from C and D upon A and B make sufficiently large angles for good determinations. Put the table approximately in position at D, by estimation or compass, and draw the lines Aa, Bb, intersecting in d; through d draw a line directed to C. Then move to and set up at C, and assuming the point c on the line dC, at an estimated distance from d, and putting the table in a position parallel to that which is occupied at D, by means of the line cd draw the lines from c to A, and from c to B. These will intersect the lines dA, dB at points a' and b',

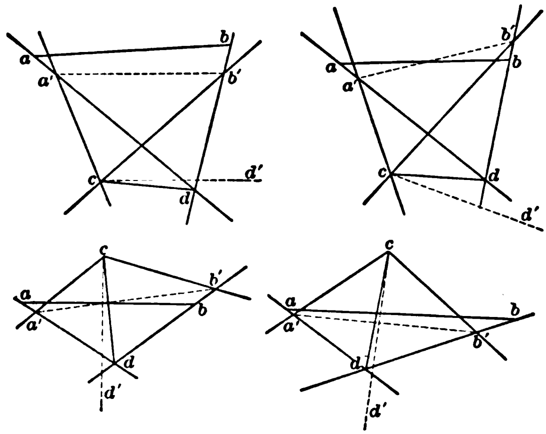


FIG. 63.—TWO-POINT PROBLEM.

which form with c and d a quadrilateral similar to the true one, but erroneous in size and position.

The angles which the lines ab and $a'b'$ make with each other is the error in position. By constructing now through c a line cd' making the same angle with cd as that which ab makes with $a'b'$, and directing this line cd' to D, the table will be brought into position, and the true point, c, can be found by the intersections of aA and bB. Instead of transferring the angle of error by construction, it may be convenient to proceed as follows, observing that the angle which the line $a'b'$ makes with ab is the error in the position of the table. As the table now stands, $a'b'$ is parallel with AB, but it is desired to turn it so that ab shall be parallel to the same. If, therefore, the alidade be placed on $a'b'$ and a mark set up in that direction, then placing the alidade on ab and turning the table until it again points to the mark, ab will be parallel to AB and the table be in position.

CHAPTER X.

TRAVERSE INSTRUMENTS AND METHODS.

80. Traverse Surveys.—In making topographic surveys—

1. The area mapped may of necessity be surveyed by running meander or traverse lines where it is impossible or impracticable to conduct the work by triangulation; or

2. Traverse lines may be run in conjunction with a trigonometric survey to fill in the details which cannot be economically reached by such methods.

Rarely can a topographic survey be made in the most satisfactory manner by trigonometric methods alone and without the aid of traverse work. Such conditions may be met in country of bold features, quite open, where numerous natural objects may be at all times visible for triangulation intersection or where stadia-rods or flags may be readily seen from every station occupied. Ordinarily, in any country the lower lines of the terrane are not visible from the triangulation stations, and therefore their topography is most easily obtained by means of traversing.

In running traverse surveys the errors naturally due to the measurement of distances and azimuths are of such amount as to be perceptible in maps of almost any scale, and they must therefore be adjusted or eliminated by tying either to traverses of greater refinement (Arts. 82 and 226) or to positions located by the trigonometric survey (Art. 73). Traverses made in connection with topographic mapping are of several degrees of accuracy, according to the amount of trigonometric or other control available for their adjustment. Where the summits are of comparatively uniform elevation and are timbered, and it is therefore difficult to conduct triangulation, it may be more economical to control the surveys by traverses. In making surveys in this manner, covering large areas on small scales, as 1 or 2 miles to the inch, primary traverses (Art. 226) are run about the area to be surveyed, and these are executed with the greatest care, almost as in the measurement of base lines (Art. 202), and they are adjusted to one or more astronomic positions (Part VI). Between such primary traverses the topographer will run secondary traverses with transit (Art. 85) or plane-table (Art. 81), preferably the latter; distances being measured by stadia, chain, or odometer, according to circumstances. Where the roads are level, have few short bends, are mostly in long tangents and are open, measurements may be made with nearly as great accuracy by means of the odometer (Art. 98) as by stadia or chain. Where the roads are crooked or it is necessary to run traverses off them and across country, stadia measurement (Art. 102) should have the preference, providing the timber is not so dense as to preclude its use. In densely wooded country the chain or tape (Arts. 97 and 99) must be employed to measure distances. When it becomes necessary to procure additional elevations in conjunction with the traverse, the stadia is most economical, since vertical angulation may be carried on at the same time.

Where traverses are run in connection with small-scale geographic mapping (Art. 29), merely to get the directions and bends in roads and trails, the crudest methods are permissible, because of the numerous points on these which will be sketched-in from the plane-table stations. Under such circumstances the prismatic compass (Art. 91) and measurement of distance by odometer, by pacing, or by counting the paces of animals (Art. 95) with notes, kept in a book, will furnish sufficiently ample results. Where the command of the terrane from the plane-table stations is incomplete, and traverses must be run either to obtain the positions and directions of roads or to map adjacent topographic data, traverses should be run with light plane-table and sight-alidade (Arts. 61 and 62), accompanied by distances measured with odometer, stadia, chain, or pacing. Where traversing is done not only to get roads and topographic detail, but also to furnish secondary and tertiary control, plane-tables of the Johnson pattern (Arts. 58 and 59) should be employed, with telescopic alidade for vertical angulation, to surrounding hills. Along the line of the traverse the sight-alidade (Art. 62) should be used on most of the intermediate locations, and distances should be measured as in the previous case.

81. Traversing by Plane-table and Magnetic Needle.—In all traverses for small-scale maps, the plane-table can be most satisfactorily oriented by means of a compass-needle. In work of this character a substantial plane-table is not necessary, a light portable one being most satisfactory. This may be either of the Johnson form (Art. 58), where a telescopic alidade is to be used in order that vertical angles and stadia measurements may be taken (Arts. 160 and 102); or if a sight-alidade will suffice for the work to be performed, the traverse-table should be of the simplest form possible (Art. 61).

Traverses run with this apparatus in conjunction with odometer or stadia measurements (Arts. 98 and 102) will usually close in short circuits of ten to thirty miles perimeter, with errors so small as to be readily adjusted by connection to better traverse or triangulation locations. In conducting traverses by this method back-flags are unnecessary, and fore-flags are only necessary in large-scale work (Chap. III). Work on scales smaller than one mile to the inch and where the traversing is on roads requires no fore-flags, as the direction of the road itself affords sufficient guide to the direction of the sight taken. Where traversing is across country and without guide of road or stream line, some signal, as a rod or man, is necessary to serve as a foresight and mark the fore-station. In traversing in this way it is unnecessary to set up the instrument at every station, for, as the orienting is done by the needle, it can be done with greater satisfaction by the occupation of every alternate station only, whereas the speed of thus setting up only at alternate stations is greatly increased.

Set up the plane-table at the first station, *A*, and *orient* (Art. 72) by swinging the board into such position that the needle will point to the north and south marks. The

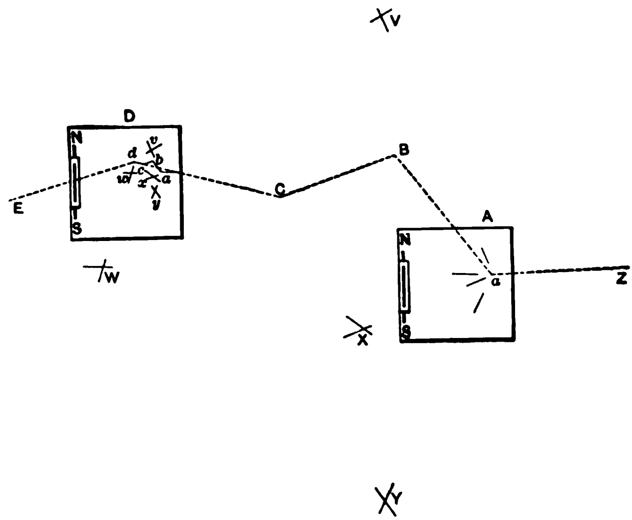


FIG. 64.—TRAVERSING WITH PLANE-TABLE.

foresight is taken by placing the near end of the alidade against the known point, *a*, and sighting in the direction of the road or to a fore-flag, *B* (Fig. 64), a short pencil-line being drawn along the edge of the ruler. Moving forward now to the foresight point *B*, the traverseman notes the distance and continues on to the next bend in the road or traverse line, where he sets up his instrument at *C*, again orients by the compass-needle, and at once plats on the first foresight the distance to the first fore-station *b*. Now placing the far end of the alidade against the second location *b*, and revolving the alidade about it, he backsights on the station B, and draws a line towards his present station, from which he measures off the distance from the second to the third or present station, the position of which is then determined and platted at *c*. The result is to give him on his board two lines and three points in the traverse. He then proceeds as before, by observing a foresight on the next forward station, *D*, and moving on beyond it to the next station, *E*.

82. Control by Large-scale Magnetic Traverse with Plane-table.—When it is necessary to secure secondary control quickly and for but limited areas, this may be graphically done on the plane-table more conveniently than by using a transit and computing latitudes and departures (Art. 90). The process is by traversing with compass-needle for scales less than 1 : 10,000, as above described, but the *scale* of platting must be *increased* to two or three times that chosen for the field map. This is in order to eliminate the errors in measurement of distances, and also those due to the graphic platting of the azimuths. Such errors as occur in running the traverse will be largely eliminated by its reduction to the smaller scale on which the remaining field-work is to be done. As described in Art. 69, the location of the initial point may be platted on the plane-table sheet, or, if not known, may be assumed, in which case the work will be started in such position on the board as will permit of the greater extent of the traverse coming within its area.

In this way a number of traverse lines on the larger scale are run back and forth across the board from one known point to a terminus at another known point, perhaps thirty or fifty miles distant. The work must be performed on a large plane-table board with a telescopic alidade (Arts. 57 and 59). Distances must be measured with a stadia or chain (Arts. 102 and 99), and the former or a flag must be sighted to give directions more accurately. A long azimuth line parallel to the compass-needle is drawn the full length of the sheet. The compass should be a déclinatoire of about 5 degrees range, the needle being not less than 6 inches in length, and this should be set in a brass box let into the board and parallel to one of its sides (Art. 61).

On completion of the traverse, a projection is made (Art. 184) on the same scale, and on it are platted the initial and closing known points (Art. 188). The *traverse* is then *transferred* to this projection by means of the long orienting lines, and if run with care will close between the two known points within a reasonably small limit of error, perhaps a tenth of an inch. Controlling points on this traverse, as road-crossings, buildings, etc., are then transferred by proportional dividers or by measurement of their positions with relation to projection lines to another projection which is platted on the scale of the topographic field-work, probably two or three times smaller than that of the traverse. This reduction will diminish the closure error to such an extent that on the topographic field scale it may be a twentieth of an inch or less. This will probably be sufficiently close to serve all the purposes of secondary control on which to tie additional traverses. These may now be run with less accuracy (Art. 81), as they are only to obtain details of topography (Arts. 12 and 16).

83. Traversing by Plane-table with Deflection Angles.—Where plane-table work is being executed on a scale larger than 1000 feet to 1 inch, directions should be by deflections from back-flags. Where, however, traversing is platted to smaller scales than, say, 1 : 10,000, they can be executed with greater precision by means of a plane-table oriented by magnetic needle.

In traversing with plane-table and deflection angles on a large scale, the plane-tabler will set up at the first traverse station. If this is located on his map by intersection from triangulation, or is a point on a line the azimuth of which is known, he is at once prepared to proceed with his traverse. The position of his station may not be known on paper, in which case it may be obtained by resection from three visible platted points (Art. 74), or he may have no way of fixing the position on the plane-table. Making a fine mark on the paper by means of a sharp-pointed needle, and accepting this as the position of his station, he proceeds with the traverse in the anticipation that the latter may ultimately connect with some known point, thus furnishing data from which to make adjustment.

Setting up the plane-table at the first station, *A* (Fig. 64), and accepting its known or assumed location, *a*, on the board, the traverseman proceeds by orienting (Art. 72) on some known point or azimuth line, *Z*, if he has such, or by placing his plane-table as nearly as possible in magnetic meridian by needle or by eye estimate. He then rotates the near end of his alidade about the occupied point *a*, and sights over its far end at a stadia-rod or other flag, *B*, for the first foresight. Moving ahead now to the new position, he leaves either a rodman or a stake or sapling with a piece of cloth or paper on it as a back-flag at *A*. Setting up now at this first foresight station, *B*, and carefully plumbing over it, he orients by placing the alidade on the line just drawn and sights back with the alidade to the rear flag *A* by revolving the table, the undetermined end of the line on the plane-table sheet being towards him. Knowing the distance from the first station to the point *B* now occupied, either by stadia chain (Arts. 102 and 99), or other measure, he plots this to scale on the line first sighted, and the resulting point is the new position, *b*, on the plane-table. The traverseman, now having his present station located and the table oriented in relation to the first foresight, revolves the alidade about the present point, *b*, sights the next fore-flag, *C*, and draws a line along the edge of the ruler. He now moves to the next station, *C*, and proceeds as before.

84. Intersection from Traverse.—In running traverses to obtain minor control and to furnish details of topography, it is necessary that the traverseman locate by intersection as many of the near-by features as practicable, that these may act as guides for the control of the sketching and aid in the determination of additional elevations. These intersections are also essential as aids in the adjustment of traverses (Art. 12), for some of the neighboring summits and prominent objects located from the traverse will also be located by the plane-table triangulation (Art. 73), and they thus furnish a means of adjusting the traverse to the triangulation.

The mode of obtaining these intersections is as follows: The traverseman having set up and oriented his plane-table (Art. 81), either by backsight or by compass-needle, according to the mode of traversing, and having completed the observing and platting of the necessary fore and back sights for the continuation of his traverse, he places the needle at the occupied station, *a* (Fig. 64), and swinging the alidade about this, sights consecutively to such prominent objects, *v*, *w*, *x*, and *y*, as may be in view and may possibly be seen from some of the succeeding traverse stations. To each of these he rules a short, light line. Moving on now to the succeeding stations, *B*, *C*, and *D*, as any of the points previously sighted or additional useful points come into view, as at *D*, radial lines are drawn to them from *d*, and the intersections of these with the lines from *a* gives the positions of the points *v*, *w*, etc. The location of some of these points having been fixed by one or more intersections from the traverse, their elevations may be determined by the vertical angle read to them with the telescopic alidade or the vertical-angle sight-alidade (Arts. 59 and 62); the angle read with the distance which can be measured from the plane-table furnishing data from which to compute, or obtain from tables, differences of elevation (Art. 163).

85. Engineers' Transit.—This is the instrument commonly employed by surveyors for the angular measurement of directions. It consists of a telescope supported in axes, called wyes, in which it can revolve in a vertical plane while the whole revolves in a horizontal plane, the amount of both movements being measured on graduated circles read with verniers. There are suitable attachments for clamping the telescope and the horizontal circle, and for moving them slowly by means of an apparatus called a tangent screw. Finally the whole may be revolved about a second horizontal axis (Fig. 65). The transit is an instrument but little used

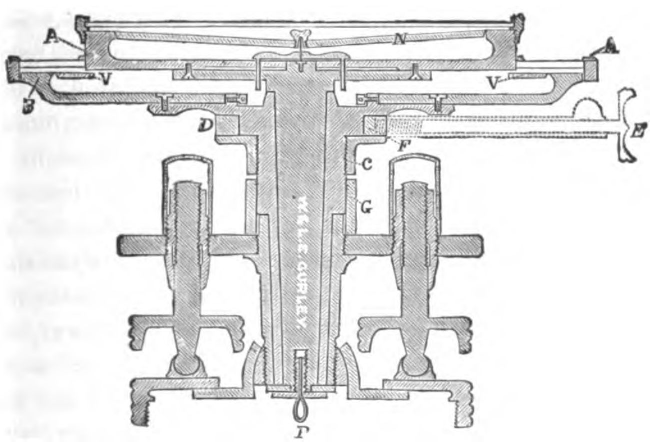


FIG. 65.—SECTION OF ENGINEERS' TRANSIT.

by topographic surveyors, and is so commonly employed in ordinary surveying and described in text-books and catalogues that its description will not be elaborated here. There are various forms, sizes, and patterns of transits, differing with the ideas of the makers and the work for which they are intended, and the catalogues furnished by the makers thoroughly describe the modes of adjusting and using these instruments.

The chief points to be remembered in *selecting a transit* are the work for which it is to be used. If the best work is not to be executed, and portability is an object, a light mountain transit with circles reading to but one minute will be sufficiently accurate. If the highest grade of work is to be performed, large, heavy instruments having circles reading to twenty or thirty seconds, shifting centers and large bubbles, should be employed. If the instrument is to be used for trigonometric work, the most important points, aside from the graduation of the horizontal limb or circle, are the size of the objective of the telescope and its magnifying power. That sights may be observed in hazy weather, the objective should admit the greatest possible amount of light, and it should therefore have but two glasses and be inverting.

86. Adjustments of the Transit.—The transit is employed primarily for measuring horizontal angles between two objects which may not be at the same elevation. Therefore, after pointing at one of these, the telescope has to be moved through a vertical arc to bring the line of sight from the first point to the second. Any error in the instrument which throws the line of sight or line of collimation of the telescope out of plumb in performing this operation will affect the horizontal angle read. It is therefore evident that this adjustment, known as the collimation adjustment, which makes the telescope revolve in a true vertical plane, is one of the most important. Likewise the vertical axis of the transit must be truly vertical in order that when the instrument is turned in azimuth the line of sight projected into the horizontal plane may move horizontally.

The various adjustments of the transit consist each of two operations: (1) the test to determine the error, and (2) the correction of the error found. If the transit were in perfect adjustment—

1. The object-glass and eyeglasses would be perpendicular to the optical axis of the telescope at all distances;

2. The line of collimation would coincide with the optical axis, and

3. It would be parallel with the telescope-level, and

4. It would pass through and be perpendicular to the horizontal axis of revolution.

These salient facts should be ascertained to assure the perfect adjustment of the transit.

The first *adjustment* is that *of the level-bubbles.* After setting up the instrument make the two small levels each parallel to a line joining two opposite leveling-screws; then, by turning the leveling-screws so that both thumbs move inwards or outwards, bring the bubbles to the center of the tubes.

Turn the instrument 180 degrees in azimuth, and if the bubbles still remain centered, the levels are in adjustment. If they do not remain in the centers of their tubes, bring them back half-way by means of the leveling-screws, and the remaining half-way by means of the adjusting-screws at the end of each leveling-tube. Repeat the operation several times, until the bubbles remain in the centers of their tubes when the instrument is revolved.

To make the vertical cross-hair perpendicular to the plane of the horizontal axis, focus the cross-hairs by the apparatus at the eye of the telescope; then adjust the objective upon some well-defined object at a distance of a few hundred feet. Move the horizontal limb so as to bring the vertical wire against the edge of a building or of a plumb-line or other vertical object. Clamp the instrument and note if the vertical wire is everywhere parallel to the vertical line. If not, loosen the cross-wire capstan-screws and, by lightly tapping their heads, move the cross-wire ring around until the error is corrected.

To adjust the line of collimation, which brings the intersection of the wires into the optical axis of the telescope, point the instrument at some well-defined object at a distance of several hundred feet and, having made the previous adjustments, clamp the lower horizontal motion and revolve the telescope completely over, so as to point in the other direction. Place there some well-defined object, as a tack in the end of a stake, and at practically the same distance from the instrument as the first object selected. Unclamp the upper plate and turn the instrument half-way round or through 180 degrees, as indicated by the vernier, and direct the telescope to the first object sighted, *B* (Fig. 66). Again bisect

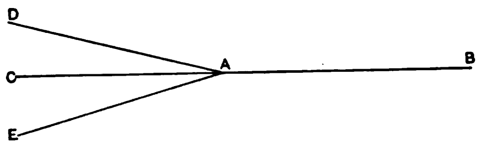


FIG. 66.—COLLIMATION ADJUSTMENT.

this with the wires, clamp the instrument, and revolve the telescope over and observe if the vertical wire bisects the second object, *C*, when the telescope is now pointed at it from the reverse position. If it does, the line of collimation is in adjustment. If not, the second point observed, as *E*, will be double the deviation of that point from the true straight line, as the error is the result of two observations made when the wires were not in the optical axis of the telescope. In the last pointing of the instrument, after the telescope was directed the second time to *B*, the point bisected at *E* was situated as far to one side of the true straight line, *BAC*, as was the point first sighted, *D*, on the other side. To correct the error, use the capstan-head screws on the side of the telescope and move the vertical wire to one side or the other by one-fourth the distance, keeping in mind the fact that the eyepiece inverts the position of the wires, and that in moving these screws the observer must operate them as if to increase the error noted. Unclamping the instrument and swinging it around so as once more to bisect *B*, again revolve the telescope, and if the adjustment has been correctly made the wires will now bisect the central point, *C*. Test the adjustment by revolving the instrument half-way round again, fixing the telescope on *B*, clamping the spindle, and once more revolving the telescope on *C*, and repeat the observations and adjustment of the wires until the correction of the collimation is completed.

The *adjustment of the standards* is the next and last important adjustment of the transit, and this is made in order that the point of intersection of the wires shall trace a vertical line as the telescope is moved up and down. This result is only obtained when the two standards which support the axis of the telescope are at the same height. Point the telescope to some object which will give a long vertical range, as at a star and its reflection in a bath of mercury, or the top of a tall church spire and the center of its base, or a long plumb-line. Fix the wires on the top of the object and clamp the spindle, then bring the telescope down until the wires bisect some good, well-defined point at the base. Turn the instrument half-way round or through 180 degrees, revolve the telescope, and focus the wires in the lower point. Clamp the spindle and raise the telescope again to the highest point. If the cross-hairs again bisect it, the adjustment is perfect; if they pass to one side, the standard opposite to that side is highest, the apparent error being double. This is corrected by turning a screw underneath one of the axes which is made movable, the correction being made for half of the amount of the apparent error.

87. Traversing with Transit.—A traverse line executed with the transit differs from one executed with the plane-table or the theodolite because of the ability to *transit the telescope* or revolve it through 180 degrees vertically. As a result of this construction of the instrument the angle between backsight and foresight which is read and recorded is not the full horizontal angle observed by swinging the instrument in azimuth, but it is the deflection of the new direction, or of the foresight, to the right or left of the backsight prolonged.

Having set up the instrument at *A* (Fig. 67), direct the telescope at the first point in the traverse *B*, with the graduated circle set at zero and by using the lower motion.

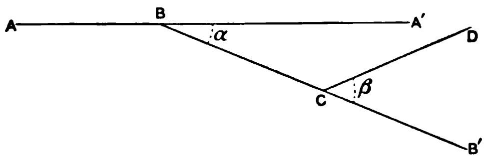


FIG. 67.—TRAVERSING WITH TRANSIT.

Record the angle zero and the distance *AB* measured by chain, stadia, or other method. Move to *B*, and setting up and plumbing the instrument at that point, backsight on the point *A*, using the lower motion and with the circle still at zero. Clamp the lower motion and transit the telescope. The instrument will now point in the direction of *A′*, which is the prolongation of *AB* if the collimation be in perfect adjustment. Loosen the upper clamp and point at the new foresight *C*, and then reclamp the vernier. The angle α is the deflection from the straight line *AA′* to the right towards *C*. In like manner the instrument is moved to *C*, and the line *BC* prolonged to *B′* by transiting the telescope, and the angle β, from *B′* to *D*, is recorded as a left deflection.

EXAMPLE OF TRANSIT NOTES.

Sta.	Distance.	Deflections.	True Bearing. (Azimuth)	Mag. Bearing.	Remarks.
	Feet.				
78 + 80	765	− 18° 42′	187° 09′	N. 4° 30′ E.	Road crossing. House.
71 + 15	1180	+ 27° 06′	205° 51′	N. 22° 30′ E.	
59 + 35	435	+ 6° 27′	178° 45′	N. 5° 00′ W.	Stream to right.
55			172° 18′	N. 10° 30′ W.	House.

The *notes* of such a *transit traverse* are kept in the following manner: In the first or "Station" column is recorded the total distance in hundredths plus single feet from the initial point. In the second or "Distance" column is recorded the distance between two stations *A* and *B*. In the third column is recorded the deflection angle with plus or minus signs, according as the deflection is to right or left. In running a simple traverse nothing further is requisite than the above. If, however, as is likely to be the case in topographic surveying, it is desired to know also the bearing of the line, the true azimuth of some sight, preferably the first line, should be determined by observation on Polaris (Art. 312), and the magnetic declination (Art. 92) should be noted, as well as the true or transit declination, by reading the angle between the azimuth line and the first line of the traverse. This angle should be recorded in the fourth column, "True Bearing." Then, as the traverse is run, the deflection right or left should be added to or subtracted from the last true bearing and thus give the new true bearing. For a check the compass-needle should also be read and recorded in the column "Magnetic Bearing," and the true bearing should agree with this approximately by the difference of the declination observed. There should be a last column in which to record remarks of streams passed, road junctions, etc.

On the opposite page of the note-book, facing the notes, there should be ruled a vertical line through the center of the page, and the customary process of recording the objects encountered on the traverse line is to use this vertical line as the line of the traverse. Beginning at the bottom of the page, plat the first station, *A*; then, at the proper distance above *A*, plat, still on the center of the line and disregarding the deflections, the second station, *B*. Crossing this line of the traverse, note the topographic features, as streams, roads, houses, etc. Where topographic notes are taken in detail it is practically impossible to keep a proper record of the traverse by considering it as a straight line; in which case, instead of using a central line as the traverse line, an irregular line should be drawn up the page, each tangent or deflection line of the traverse making an angle with the last, which shall agree as nearly as possible, by eye estimation or by platting with horn protractor, with the angle made on the ground. By this means the topography of the country will not be distorted in recording it on the sketch page (Fig. 68).

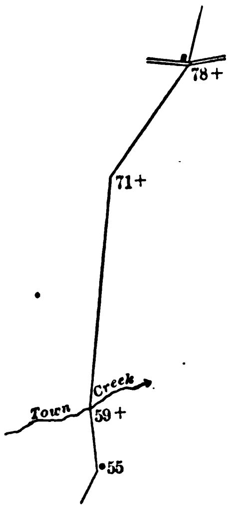


FIG. 68. — PLAT OF TRANSIT ROAD TRAVERSE.

88. Platting Transit Notes with Protractor and Scale.—Transit notes may be platted in two ways:

1. By means of a protractor and scale, and
2. By latitudes and departures (Art. 90).

In platting with a protractor (Art. 89) and scale, set the center of the protractor over the occupied station as platted on the map, set the zero on the prolongation of the last sight, and plat off the deflection to right or left by the proper number of degrees. Then, removing the protractor, plat on this new deflection line the proper distance to scale. This deflection line should be drawn sufficiently long, so that when the protractor is centered over the second station this old deflection line will appear on the map as the zero-point on which to set the protractor for the next following deflection.

89. Protractors.—In the platting of traverses run with a prismatic compass, the simplest form of a semicircular horn protractor will fill the requirements; also in platting reconnaissance triangulation in order to determine the relative positions of stations. Where any attempt is made at accurate platting, as of traverse run with transit, a full-circle vernier arm protractor should be used (Fig. 69). Where angle-reading instruments are used in topographic surveying, it is expected that the work done will be of such high quality

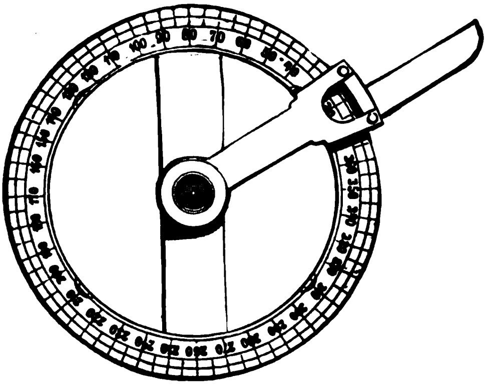


FIG. 69.—FULL-CIRCLE VERNIER PROTRACTOR.

as to call for computation either of latitudes and departures, in the case of traverse, or of geodetic coordinates, in primary triangulation (Chaps. XXIV and XXIX).

Occasionally the topographer, especially if engaged in hydrographic surveying, will need to locate his position by the three-point problem (Art. 74), that is, by angles read from an unknown to three known positions. The location of

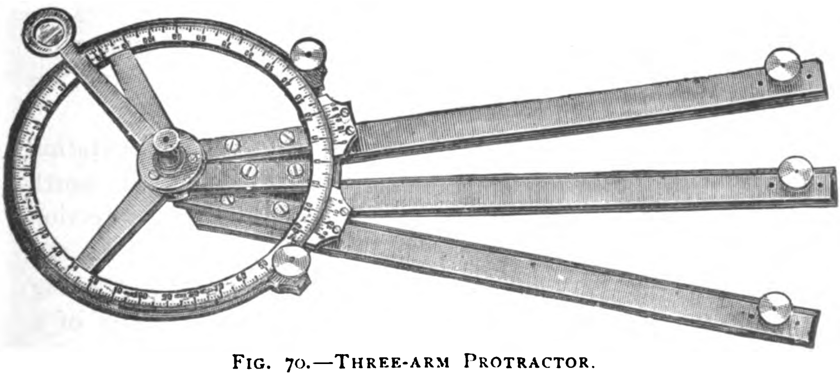


FIG. 70.—THREE-ARM PROTRACTOR.

his unknown and occupied point may be computed (Art. 263), or it may be platted graphically by means of an instrument known as the three-armed protractor, which is very useful and does excellent work of this kind. (Fig. 70.)

90. Platting Transit Notes by Latitudes and Departures.—This is the most accurate method of platting transit notes, and is identical with that employed in platting traverse run with theodolite for primary control (Chap. XXIV). The more common expression "departures" refers to easting and westing, known in astronomical phraseology as "longitude." The computing and platting are not done with the same care and accuracy as for primary traverse. The convergence of the meridians is rarely recorded, nor are the errors of measurement of deflection angles corrected by astronomical azimuths or by checks on known geodetic positions.

The process consists in *platting by rectangular coordinates* to reference lines which are drawn at right angles and correspond approximately to latitude and longitude lines. The horizontal line is assumed as the initial latitude, and is the zero from which differences of latitude are measured up and down. In other words, it is the line of abscissæ, and along it are measured off the differences of longitude or departure from the vertical line, which is zero of longitudes. All northings on the traverse line are measured upwards and all southings downwards, and they are denoted by the signs + and −. Eastings and westings, respectively, are measured to the right or left of the vertical line or initial longitude line, and are denoted also by signs + and −.

The *zero of azimuth* is, as in geodetic computation (Art. 285), supposed to be at the south, while the north is 180°. The azimuth is measured in the same direction as the motion of the hands of a watch, 90° being to the west and 270° to the east. A simple manner of keeping signs in mind during computation is from inspection of a diagram similar to Fig. 71.

The total latitudes and departures are computed only for important points, as crossings of roads, streams, etc. The intermediate bends in the road traverse may be platted by protractor. The total latitudes and departures are determined for each governing point by summation of the partial

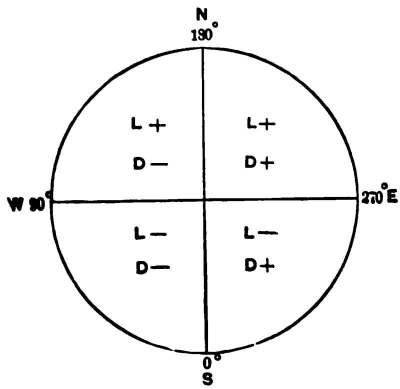


FIG. 71.—SIGNS OF LATITUDES AND DEPARTURES.

latitudes and departures to that point. They are derived by two methods. (1) By adding to the logarithms of the distances (Table V) the logarithms of the sines of the azimuths (Table VI). The total departures are obtained by adding to the logarithms of the same distances the logarithms of the cosines of the corresponding azimuths. The second method of computing latitudes and departures is by means of a table of natural functions (Tables XI and XII).

COMPUTATION OF LATITUDES AND DEPARTURES.

	To Station 59 +.	To Station 71 +.	To Station 78 +.
Log. sin. (Dep.)..	9.5590	9.6394	9.0950
' dist.........	2.6385	3.0719	2.8837
" dep.........	2.1975	2.7113	1.9787
Departure (feet)..	157.5	514.3	9 52
Log. cosin (Lat.).	9.9694	9.9543	9.9966
" dist.........	2.6385	2.0719	2.8837
" dep.........	2.6079	2.0262	2.8803
Departure (feet)..	405.4	106.3	759.2

The details of the computation are given *in extenso* in Chapter XXIV, for primary traverse. The foregoing example is given here, however, as an illustration of the simpler mode of computing latitudes and departures from transit notes, and is taken from the example of such notes given in Article 87.

The table of four-place logarithms of numbers on pages 215 and 216 is derived from Prof. J. B. Johnson's "Theory and Practice of Surveying"; that of similar trigonometric functions on pages 217 to 221 is from Gauss' well-known tables. By their use a traverse run with engineer's transit can be computed by latitudes and departures with sufficient accuracy.

91. Prismatic Compass.—This is a useful instrument for determining directions on reconnaissance traverses of roads, streams, etc. It is unnecessary to mount it on a Jacob's-staff or tripod, as it is easily read while held in the hand. It has a full circle of 360 degrees (Fig. 72) and folded sights.

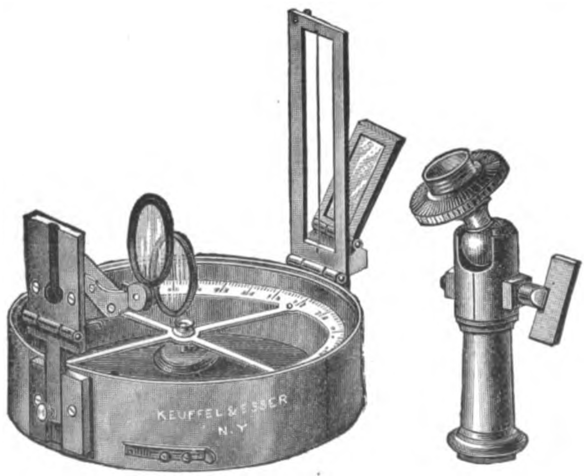


FIG. 72.—PRISMATIC COMPASS.

The foresight has a cross-hair, and the rear- or eye-sight is so provided with a prism that while the instrument is pointed

Table V.

LOGARITHMS OF NUMBERS.

Numbers.	0	1	2	3	4	5	6	7	8	9	Proportional Parts. 1	2	3	4	5	6	7	8	9
10	.0000	.0043	.0086	.0128	.0170	.0212	.0253	.0294	.0334	.0374	4	8	12	17	21	25	29	33	37
11	.0414	.0453	.0492	.0531	.0569	.0607	.0645	.0682	.0719	.0755	4	8	11	15	19	23	26	30	34
12	.0792	.0828	.0864	.0899	.0934	.0969	.1004	.1038	.1072	.1100	3	7	10	14	17	21	24	28	31
13	.1139	.1173	.1206	.1239	.1271	.1303	.1335	.1367	.1399	.1430	3	6	10	13	16	19	23	26	29
14	.1461	.1492	.1523	.1553	.1584	.1614	.1644	.1673	.1703	.1732	3	6	9	12	15	18	21	24	27
15	.1761	.1790	.1818	.1847	.1875	.1903	.1931	.1959	.1987	.2014	3	6	8	11	14	17	20	22	25
16	.2041	.2068	.2095	.2122	.2148	.2175	.2201	.2227	.2253	.2279	3	5	8	11	13	16	18	21	24
17	.2304	.2330	.2355	.2380	.2405	.2430	.2455	.2480	.2504	.2529	2	5	7	10	12	15	17	20	22
18	.2553	.2577	.2601	.2625	.2648	.2672	.2695	.2718	.2742	.2765	2	5	7	9	12	14	16	19	21
19	.2788	.2810	.2833	.2856	.2878	.2900	.2923	.2945	.2967	.2989	2	4	7	9	11	13	16	18	20
20	.3010	.3032	.3054	.3075	.3096	.3118	.3139	.3160	.3181	.3201	2	4	6	8	11	13	15	17	19
21	.3222	.3243	.3263	.3284	.3304	.3324	.3345	.3365	.3385	.3404	2	4	6	8	10	12	14	16	18
22	.3424	.3444	.3464	.3483	.3502	.3522	.3541	.3560	.3579	.3598	2	4	6	8	10	12	14	15	17
23	.3617	.3636	.3655	.3674	.3692	.3711	.3729	.3747	.3766	.3784	2	4	6	7	9	11	13	15	17
24	.3802	.3820	.3838	.3856	.3874	.3892	.3909	.3927	.3945	.3962	2	4	5	7	9	11	12	14	16
25	.3979	.3997	.4014	.4031	.4048	.4065	.4082	.4099	.4116	.4133	2	3	5	7	9	10	12	14	15
26	.4150	.4166	.4183	.4200	.4216	.4232	.4249	.4265	.4281	.4298	2	3	5	7	8	10	11	13	15
27	.4314	.4330	.4346	.4362	.4378	.4393	.4409	.4425	.4440	.4456	2	3	5	6	8	9	11	13	14
28	.4472	.4487	.4502	.4518	.4533	.4548	.4564	.4579	.4594	.4609	2	3	5	6	8	9	11	12	14
29	.4624	.4639	.4654	.4669	.4683	.4698	.4713	.4728	.4742	.4757	1	3	4	6	7	9	10	12	13
30	.4771	.4786	.4800	.4814	.4829	.4843	.4857	.4871	.4886	.4900	1	3	4	6	7	9	10	11	13
31	.4914	.4928	.4942	.4955	.4969	.4983	.4997	.5011	.5024	.5038	1	3	4	6	7	8	10	11	12
32	.5051	.5065	.5079	.5092	.5105	.5119	.5132	.5145	.5159	.5172	1	3	4	5	7	8	9	11	12
33	.5185	.5198	.5211	.5224	.5237	.5250	.5263	.5276	.5289	.5302	1	3	4	5	6	8	9	10	12
34	.5315	.5328	.5340	.5353	.5366	.5378	.5391	.5403	.5416	.5428	1	3	4	5	6	8	9	10	11
35	.5441	.5453	.5465	.5478	.5490	.5502	.5514	.5527	.5539	.5551	1	2	4	5	6	7	9	10	11
36	.5563	.5575	.5587	.5599	.5611	.5623	.5635	.5647	.5658	.5670	1	2	4	5	6	7	8	10	11
37	.5682	.5694	.5705	.5717	.5729	.5740	.5752	.5763	.5775	.5786	1	2	3	5	6	7	8	9	10
38	.5798	.5809	.5821	.5832	.5843	.5855	.5866	.5877	.5888	.5899	1	2	3	5	6	7	8	9	10
39	.5911	.5922	.5933	.5944	.5955	.5966	.5977	.5988	.5999	.6010	1	2	3	4	5	7	8	9	10
40	.6021	.6031	.6042	.6053	.6064	.6075	.6085	.6096	.6107	.6117	1	2	3	4	5	6	8	9	10
41	.6128	.6138	.6149	.6160	.6170	.6180	.6191	.6201	.6212	.6222	1	2	3	4	5	6	7	8	9
42	.6232	.6243	.6253	.6263	.6274	.6284	.6294	.6304	.6314	.6325	1	2	3	4	5	6	7	8	9
43	.6335	.6345	.6355	.6365	.6375	.6385	.6395	.6405	.6415	.6425	1	2	3	4	5	6	7	8	9
44	.6435	.6444	.6454	.6464	.6474	.6484	.6493	.6503	.6513	.6522	1	2	3	4	5	6	7	8	9
45	.6532	.6542	.6551	.6561	.6571	.6580	.6590	.6599	.6609	.6618	1	2	3	4	5	6	7	8	9
46	.6628	.6637	.6646	.6656	.6665	.6675	.6684	.6693	.6702	.6712	1	2	3	4	5	6	7	7	8
47	.6721	.6730	.6739	.6749	.6758	.6767	.6776	.6785	.6794	.6803	1	2	3	4	5	5	6	7	8
48	.6812	.6821	.6830	.6839	.6848	.6857	.6866	.6875	.6884	.6893	1	2	3	4	4	5	6	7	8
49	.6902	.6911	.6920	.6928	.6937	.6946	.6955	.6964	.6972	.6981	1	2	3	4	4	5	6	7	8
50	.6990	.6998	.7007	.7016	.7024	.7033	.7042	.7050	.7059	.7067	1	2	3	3	4	5	6	7	8
51	.7076	.7084	.7093	.7101	.7110	.7118	.7126	.7135	.7143	.7152	1	2	3	3	4	5	6	7	8
52	.7160	.7168	.7177	.7185	.7193	.7202	.7210	.7218	.7226	.7235	1	2	2	3	4	5	6	7	7
53	.7243	.7251	.7259	.7267	.7275	.7284	.7292	.7300	.7308	.7316	1	2	2	3	4	5	6	6	7
54	.7324	.7332	.7340	.7348	.7356	.7364	.7372	.7380	.7388	.7396	1	2	2	3	4	5	6	6	7

TABLE V.

LOGARITHMS OF NUMBERS.

Numbers.	0	1	2	3	4	5	6	7	8	9	Proportional Parts. 1	2	3	4	5	6	7	8	9
55	.7404	.7412	.7419	.7427	.7435	.7443	.7451	.7459	.7466	.7474	1	2	2	3	4	5	5	6	7
56	.7482	.7490	.7497	.7505	.7513	.7520	.7528	.7536	.7543	.7551	1	2	2	3	4	5	5	6	7
57	.7559	.7566	.7574	.7582	.7589	.7597	.7604	.7612	.7619	.7627	1	2	2	3	4	5	5	6	7
58	.7634	.7642	.7649	.7657	.7664	.7672	.7679	.7686	.7694	.7701	1	1	2	3	4	4	5	6	7
59	.7709	.7716	.7723	.7731	.7738	.7745	.7752	.7760	.7767	.7774	1	1	2	3	4	4	5	6	7
60	.7782	.7789	.7796	.7803	.7810	.7818	.7825	.7832	.7839	.7846	1	1	2	3	4	4	5	6	6
61	.7853	.7860	.7868	.7875	.7882	.7889	.7896	.7903	.7910	.7917	1	1	2	3	4	4	5	6	6
62	.7924	.7931	.7938	.7945	.7952	.7959	.7966	.7973	.7980	.7987	1	1	2	3	3	4	5	6	6
63	.7993	.8000	.8007	.8014	.8021	.8028	.8035	.8041	.8048	.8055	1	1	2	3	3	4	5	5	6
64	.8062	.8069	.8075	.8082	.8089	.8096	.8102	.8109	.8116	.8122	1	1	2	3	3	4	5	5	6
65	.8129	.8136	.8142	.8149	.8156	.8162	.8169	.8176	.8182	.8189	1	1	2	3	3	4	5	5	6
66	.8195	.8202	.8209	.8215	.8222	.8228	.8235	.8241	.8248	.8254	1	1	2	3	3	4	5	5	6
67	.8261	.8267	.8274	.8280	.8287	.8293	.8299	.8306	.8312	.8319	1	1	2	3	3	4	5	5	6
68	.8325	.8331	.8338	.8344	.8351	.8357	.8363	.8370	.8376	.8382	1	1	2	3	3	4	4	5	6
69	.8388	.8395	.8401	.8407	.8414	.8420	.8426	.8432	.8439	.8445	1	1	2	2	3	4	4	5	6
70	.8451	.8457	.8463	.8470	.8476	.8482	.8488	.8494	.8500	.8506	1	1	2	2	3	4	4	5	6
71	.8513	.8519	.8525	.8531	.8537	.8543	.8549	.8555	.8561	.8567	1	1	2	2	3	4	4	5	5
72	.8573	.8579	.8585	.8591	.8597	.8603	.8609	.8615	.8621	.8627	1	1	2	2	3	4	4	5	5
73	.8633	.8639	.8645	.8651	.8657	.8663	.8669	.8675	.8681	.8686	1	1	2	2	3	4	4	5	5
74	.8692	.8698	.8704	.8710	.8716	.8722	.8727	.8733	.8739	.8745	1	1	2	2	3	4	4	5	5
75	.8751	.8756	.8762	.8768	.8774	.8779	.8785	.8791	.8797	.8802	1	1	2	2	3	3	4	5	5
76	.8808	.8814	.8820	.8825	.8831	.8837	.8842	.8848	.8854	.8859	1	1	2	2	3	3	4	5	5
77	.8865	.8871	.8876	.8882	.8887	.8893	.8899	.8904	.8910	.8915	1	1	2	2	3	3	4	4	5
78	.8921	.8927	.8932	.8938	.8943	.8949	.8954	.8960	.8965	.8971	1	1	2	2	3	3	4	4	5
79	.8976	.8982	.8987	.8993	.8998	.9004	.9009	.9015	.9020	.9025	1	1	2	2	3	3	4	4	5
80	.9031	.9036	.9042	.9047	.9053	.9058	.9063	.9069	.9074	.9079	1	1	2	2	3	3	4	4	5
81	.9085	.9090	.9096	.9101	.9106	.9112	.9117	.9122	.9128	.9133	1	1	2	2	3	3	4	4	5
82	.9138	.9143	.9149	.9154	.9159	.9165	.9170	.9175	.9180	.9186	1	1	2	2	3	3	4	4	5
83	.9191	.9196	.9201	.9206	.9212	.9217	.9222	.9227	.9232	.9238	1	1	2	2	3	3	4	4	5
84	.9243	.9248	.9253	.9258	.9263	.9269	.9274	.9279	.9284	.9289	1	1	2	2	3	3	4	4	5
85	.9294	.9299	.9304	.9309	.9315	.9320	.9325	.9330	.9335	.9340	1	1	2	2	3	3	4	4	5
86	.9345	.9350	.9355	.9360	.9365	.9370	.9375	.9380	.9385	.9390	1	1	2	2	3	3	4	4	5
87	.9395	.9400	.9405	.9410	.9415	.9420	.9425	.9430	.9435	.9440	0	1	1	2	2	3	3	4	4
88	.9445	.9450	.9455	.9460	.9465	.9469	.9474	.9479	.9484	.9489	0	1	1	2	2	3	3	4	4
89	.9494	.9499	.9504	.9509	.9513	.9518	.9523	.9528	.9533	.9538	0	1	1	2	2	3	3	4	4
90	.9542	.9547	.9552	.9557	.9562	.9566	.9571	.9576	.9581	.9586	0	1	1	2	2	3	3	4	4
91	.9590	.9595	.9600	.9605	.9609	.9614	.9619	.9624	.9628	.9633	0	1	1	2	2	3	3	4	4
92	.9638	.9643	.9647	.9652	.9657	.9661	.9666	.9671	.9675	.9680	0	1	1	2	2	3	3	4	4
93	.9685	.9689	.9694	.9699	.9703	.9708	.9713	.9717	.9722	.9727	0	1	1	2	2	3	3	4	4
94	.9731	.9736	.9741	.9745	.9750	.9754	.9759	.9763	.9768	.9773	0	1	1	2	2	3	3	4	4
95	.9777	.9782	.9786	.9791	.9795	.9800	.9805	.9809	.9814	.9818	0	1	1	2	2	3	3	4	4
96	.9823	.9827	.9832	.9836	.9841	.9845	.9850	.9854	.9859	.9863	0	1	1	2	2	3	3	4	4
97	.9868	.9872	.9877	.9881	.9886	.9890	.9894	.9899	.9903	.9908	0	1	1	2	2	3	3	4	4
98	.9912	.9917	.9921	.9926	.9930	.9934	.9939	.9943	.9948	.9952	0	1	1	2	2	3	3	4	4
99	.9956	.9961	.9965	.9969	.9974	.9978	.9983	.9987	.9991	.9996	0	1	1	2	2	3	3	3	4

TABLE VI.

LOGARITHMS OF TRIGONOMETRIC FUNCTIONS.

° ′	L. Sin.	d.	L. Tang	d. c.	L. Cotg.	L. Cos.	
0 0	—		—		—	0.0000	0 **90**
10	7.4637		7.4637		2.5363	0.0000	50
20	7.7648	3011	7.7648	3011	2.2352	0.0000	40
30	7.9408	1760	7.9409	1761	2.0591	0.0000	30
40	8.0658	1250	8.0658	1249	1.9342	0.0000	20
50	8.1627	969	8.1627	969	1.8373	0.0000	10
1 0	8.2419	792	8.2419	792	1.7581	9.9999	0 **89**
10	8.3088	669	8.3089	670	1.6911	9.9999	50
20	8.3668	580	8.3669	580	1.6331	9.9999	40
30	8.4179	511	8.4181	512	1.5819	9.9999	30
40	8.4637	458	8.4638	457	1.5362	9.9998	20
50	8.5050	413	8.5053	415	1.4947	9.9998	10
2 0	8.5428	378	8.5431	378	1.4569	9.9997	0 **88**
10	8.5776	348	8.5779	348	1.4221	9.9997	50
20	8.6097	321	8.6101	322	1.3899	9.9996	40
30	8.6397	300	8.6401	300	1.3599	9.9996	30
40	8.6677	280	8.6682	281	1.3318	9.9995	20
50	8.6940	263	8.6945	263	1.3055	9.9995	10
3 0	8.7188	248	8.7194	249	1.2806	9.9994	0 **87**
10	8.7423	235	8.7429	235	1.2571	9.9993	50
20	8.7645	222	8.7652	223	1.2348	9.9993	40
30	8.7857	212	8.7865	213	1.2135	9.9992	30
40	8.8059	202	8.8067	202	1.1933	9.9991	20
50	8.8251	192	8.8261	194	1.1739	9.9990	10
4 0	8.8436	185	8.8446	185	1.1554	9.9989	0 **86**
10	8.8613	177	8.8624	178	1.1376	9.9989	50
20	8.8783	170	8.8795	171	1.1205	9.9988	40
30	8.8946	163	8.8960	165	1.1040	9.9987	30
40	8.9104	158	8.9118	158	1.0882	9.9986	20
50	8.9256	152	8.9272	154	1.0728	9.9985	10
5 0	8.9403	147	8.9420	148	1.0580	9.9983	0 **85**
10	8.9545	142	8.9563	143	1.0437	9.9982	50
20	8.9682	137	8.9701	138	1.0299	9.9981	40
30	8.9816	134	8.9836	135	1.0164	9.9980	30
40	8.9945	129	8.9966	130	1.0034	9.9979	20
50	9.0070	125	9.0093	127	0.9907	9.9977	10
6 0	9.0192	122	9.0216	123	0.9784	9.9976	0 **84**
10	9.0311	119	9.0336	120	0.9664	9.9975	50
20	9.0426	115	9.0453	117	0.9547	9.9973	40
30	9.0539	113	9.0567	114	0.9433	9.9972	30
40	9.0648	109	9.0678	111	0.9322	9.9971	20
50	9.0755	107	9.0786	108	0.9214	9.9970	10
7 0	9.0859	104	9.0891	105	0.9109	9.9968	0 **83**
10	9.0961	102	9.0995	104	0.9005	9.9966	50
20	9.1060	99	9.1096	101	0.8904	9.9964	40
30	9.1157	97	9.1194	98	0.8806	9.9963	30
40	9.1252	95	9.1291	97	0.8709	9.9961	20
50	9.1345	93	9.1385	94	0.8615	9.9959	10
8 0	9.1436	91	9.1478	93	0.8522	9.9958	0 **82**
10	9.1525	89	9.1569	91	0.8431	9.9956	50
20	9.1612	87	9.1658	89	0.8342	9.9954	40
30	9.1697	85	9.1745	87	0.8255	9.9952	30
40	9.1781	84	9.1831	86	0.8169	9.9950	20
50	9.1863	82	9.1915	84	0.8085	9.9948	10
9 0	9.1943	80	9.1997	82	0.8003	9.9946	0 **81**
10	9.2022	79	9.2078	81	0.7922	9.9944	50
20	9.2100	78	9.2158	80	0.7842	9.9942	40
30	9.2176	76	9.2236	78	0.7764	9.9940	30
40	9.2251	75	9.2313	77	0.7687	9.9938	20
50	9.2324	73	9.2389	79	0.7611	9.9936	10
10 0	9.2397	73	9.2463	76	0.7537	9.9934	0 **80**
	L. Cos.	d.	L Cotg.	d. c.	L. Tang	L. Sin.	′ °

′	S.	T.
0	—	—
10	6.4637	6.4637
20	6.4637	6.4637
30	6.4637	6.4637
40	6.4637	6.4637
50	6.4637	6.4638
60	6.4637	6.4638
70	6.4637	6.4638
80	6.4637	6.4638
90	6.4637	6.4638
100	6.4637	6.4638
110	6.4637	6.4639
120	6.4636	6.4639
130	6.4636	6.4639
140	6.4636	6.4640
150	6.4636	6.4640
160	6.4636	6.4640
170	6.4635	6.4641
180	6.4635	6.4641
190	6.4635	6.4642
200	6.4635	6.4642
210	6.4635	6.4643
220	6.4634	6.4643
230	6.4634	6.4644
240	6.4634	6.4644
250	6.4633	6.4645
260	6.4633	6.4646
270	6.4633	6.4646
280	6.4632	6.4647
290	6.4632	6.4648

0° .. 5°

$\log \sin \alpha = \log \alpha' + S$
$\log \tan \alpha = \log \alpha' + T$
$\log \alpha' = \log \sin \alpha - S$
$= \log \tan \alpha - T$

85° .. 90°

$\log \cos \alpha = \log (90° - \alpha)' + S$
$\log \cot \alpha = \log (90° - \alpha)' + T$
$\log (90° - \alpha)' = \log \cos \alpha - S$
$= \log \cot \alpha - T$

P. P.

	142	**137**	**134**	**129**
1	14.2	13.7	13.4	12.9
2	28.4	27.4	26.8	25.8
3	42.6	41.1	40.2	38.7
4	56.8	54.8	53.6	51.6
5	71.0	68.5	67.0	64.5
6	85.2	82.2	80.4	77.4
7	99.4	95.9	93.8	90.3
8	113.6	109.6	107.2	103.2
9	127.8	123.3	120.6	116.1

	125	**122**	**119**	**115**
1	12.5	12.2	11.9	11.5
2	25.0	24.4	23.8	23.0
3	37.5	36.6	35.7	34.5
4	50.0	48.8	47.6	46.0
5	62.5	61.0	59.5	57.5
6	75.0	73.2	71.4	69.0
7	87.5	85.4	83.3	80.5
8	100.0	97.6	95.2	92.0
9	112.5	109.8	107.1	103.5

	113	**109**	**107**	**104**	**102**
1	11.3	10.9	10.7	10.4	10.2
2	22.6	21.8	21.4	20.8	20.4
3	33.9	32.7	32.1	31.2	30.6
4	45.2	43.6	42.8	41.6	40.8
5	56.5	54.5	53.5	52.0	51.0
6	67.8	65.4	64.2	62.4	61.2
7	79.1	76.3	74.9	72.8	71.4
8	90.4	87.2	85.6	83.2	81.6
9	101.7	98.1	96.3	93.6	91.8

P. P.

TABLE VI.

LOGARITHMS OF TRIGONOMETRIC FUNCTIONS.

° ′	L. Sin.	d.	L.Tang	d. c.	L.Cotg.	L. Cos.	d.	
10 0	9.2397	71	9.2463	73	0.7537	9.9934		0 **80**
10	9.2468	70	9.2536	71	0.7464	9.9931		50
20	9.2538	68	9.2609	70	0.7391	9.9929		40
30	9.2606	68	9.2680	69	0.7320	9.9927		30
40	9.2674	66	9.2750	68	0.7250	9.9924		20
50	9.2740	66	9.2819	66	0.7181	9.9922		10
11 0	9.2806	64	9.2887	67	0.7113	9.9919		0 **79**
10	9.2870	64	9.2953	65	0.7047	9.9917		50
20	9.2934	63	9.3020	64	0.6980	9.9914		40
30	9.2997	61	9.3085	63	0.6915	9.9912		30
40	9.3058	61	9.3149	63	0.6851	9.9909		20
50	9.3119	60	9.3212	61	0.6788	9.9907		10
12 0	9.3179	59	9.3275	61	0.6725	9.9904		0 **78**
10	9.3238	58	9.3336	61	0.6664	9.9901		50
20	9.3296	57	9.3397	59	0.6603	9.9899		40
30	9.3353	57	9.3458	59	0.6542	9.9896		30
40	9.3410	56	9.3517	58	0.6483	9.9893		20
50	9.3466	55	9.3576	57	0.6424	9.9890		10
13 0	9.3521	54	9.3634	57	0.6366	9.9887		0 **77**
10	9.3575	54	9.3691	56	0.6309	9.9884		50
20	9.3629	53	9.3748	55	0.6252	9.9881		40
30	9.3682	52	9.3804	55	0.6196	9.9878		30
40	9.3734	52	9.3859	54	0.6141	9.9875		20
50	9.3786	51	9.3914	53	0.6086	9.9872		10
14 0	9.3837	50	9.3968	53	0.6032	9.9869		0 **76**
10	9.3887	50	9.4021	53	0.5979	9.9866		50
20	9.3937	49	9.4074	51	0.5926	9.9863		40
30	9.3986	49	9.4127	52	0.5873	9.9859		30
40	9.4035	48	9.4178		0.5822	9.9856		20
50	9.4083	47	9.4230	51	0.5770	9.9853		10
15 0	9.4130	47	9.4281	50	0.5719	9.9849	3	0 **75**
10	9.4177	46	9.4331	50	0.5669	9.9846	3	50
20	9.4223	46	9.4381	49	0.5619	9.9843	4	40
30	9.4269	45	9.4430	49	0.5570	9.9839	3	30
40	9.4314	45	9.4479	48	0.5521	9.9836	4	20
50	9.4359	44	9.4527	48	0.5473	9.9832	4	10
16 0	9.4403	44	9.4575	47	0.5425	9.9828	3	0 **74**
10	9.4447	44	9.4622	47	0.5378	9.9825	4	50
20	9.4491	42	9.4669	47	0.5331	9.9821	4	40
30	9.4533	43	9.4716	46	0.5284	9.9817	3	30
40	9.4576	42	9.4762	46	0.5238	9.9814	4	20
50	9.4618	41	9.4808	45	0.5192	9.9810	4	10
17 0	9.4659	41	9.4853	45	0.5147	9.9806	4	0 **73**
10	9.4700	41	9.4898	45	0.5102	9.9802	4	50
20	9.4741	40	9.4943	44	0.5057	9.9798	4	40
30	9.4781	40	9.4987	44	0.5013	9.9794	4	30
40	9.4821	40	9.5031	44	0.4969	9.9790	4	20
50	9.4861	39	9.5075	43	0.4925	9.9786	4	10
18 0	9.4900	39	9.5118	43	0.4882	9.9782	4	0 **72**
10	9.4939	38	9.5161	42	0.4839	9.9778	4	50
20	9.4977	38	9.5203	42	0.4797	9.9774	4	40
30	9.5015	37	9.5245	42	0.4755	9.9770	5	30
40	9.5052	38	9.5287	42	0.4713	9.9765	4	20
50	9.5090	36	9.5329	41	0.4671	9.9761	4	10
19 0	9.5126	37	9.5370	41	0.4630	9.9757	5	0 **71**
10	9.5163	36	9.5411	40	0.4589	9.9752	4	50
20	9.5199	36	9.5451	40	0.4549	9.9748	5	40
30	9.5235	35	9.5491	40	0.4509	9.9743	4	30
40	9.5270	36	9.5531	40	0.4469	9.9739	5	20
50	9.5306	35	9.5571	40	0.4429	9.9734	4	10
20 0	9.5341		9.5611		0.4389	9.9730		0 **70**
	L. Cos.	d.	L.Cotg.	d. c.	L.Tang	L. Sin.		′ °

P. P.

	99	**95**	**91**	**87**	**84**
1	9.9	9.5	9.1	8.7	8.4
2	19.8	19.0	18.2	17.4	16.8
3	29.7	28.5	27.3	26.1	25.2
4	39.6	38.0	36.4	34.8	33.6
5	49.5	47.5	45.5	43.5	42.0
6	59.4	57.0	54.6	52.2	50.4
7	69.3	66.5	63.7	60.9	58.8
8	79.2	76.0	72.8	69.6	67.2
9	89.1	85.5	81.9	78.3	75.6

	80	**78**	**75**	**71**	**68**
1	8.0	7.8	7.5	7.1	6.8
2	16.0	15.6	15.0	14.2	13.6
3	24.0	23.4	22.5	21.3	20.4
4	32.0	31.2	30.0	28.4	27.2
5	40.0	39.0	37.5	35.5	34.0
6	48.0	46.8	45.0	42.6	40.8
7	56.0	54.6	52.5	49.7	47.6
8	64.0	62.4	60.0	56.8	54.4
9	72.0	70.2	67.5	63.9	61.2

	63	**59**	**56**	**53**	**49**
1	6.3	5.9	5.6	5.3	4.9
2	12.6	11.8	11.2	10.6	9.8
3	18.9	17.7	16.8	15.9	14.7
4	25.2	23.6	22.4	21.2	19.6
5	31.5	29.5	28.0	26.5	24.5
6	37.8	35.4	33.6	31.8	29.4
7	44.1	41.3	39.2	37.1	34.3
8	50.4	47.2	44.8	42.4	39.2
9	56.7	53.1	50.4	47.7	44.1

	50	**49**	**48**	**47**
1	5.0	4.9	4.8	4.7
2	10.0	9.8	9.6	9.4
3	15.0	14.7	14.4	14.1
4	20.0	19.6	19.2	18.8
5	25.0	24.5	24.0	23.5
6	30.0	29.4	28.8	28.2
7	35.0	34.3	33.6	32.9
8	40.0	39.2	38.4	37.6
9	45.0	44.1	43.2	42.3

	46	**45**	**44**	**3**
1	4.6	4.5	4.4	0.3
2	9.2	9.0	8.8	0.6
3	13.8	13.5	13.2	0.9
4	18.4	18.0	17.6	1.2
5	23.0	22.5	22.0	1.5
6	27.6	27.0	26.4	1.8
7	32.2	31.5	30.8	2.1
8	36.8	36.0	35.2	2.4
9	41.4	40.5	39.6	2.7

	43	**42**	**41**	**4**
1	4.3	4.2	4.1	0.4
2	8.6	8.4	8.2	0.8
3	12.9	12.6	12.3	1.2
4	17.2	16.8	16.4	1.6
5	21.5	21.0	20.5	2.0
6	25.8	25.2	24.6	2.4
7	30.1	29.4	28.7	2.8
8	34.4	33.6	32.8	3.2
9	38.7	37.8	36.9	3.6

P. P.

TABLE VI.

LOGARITHMS OF TRIGONOMETRIC FUNCTIONS.

° ′	L. Sin.	d.	L.Tang	d. c.	L.Cotg.	L. Cos	d.	
20 0	9.5341		9.5611		0.4389	9.9730		0 **70**
10	9.5375	34	9.5650	39	0.4350	9.9725	5	50
20	9.5409	34	9.5689	39	0.4311	9.9721	4	40
30	9.5443	34	9.5727	38	0.4273	9.9716	5	30
40	9.5477	34	9.5766	39	0.4234	9.9711	5	20
50	9.5510	33	9.5804	38	0.4196	9.9706	5	10
21 0	9.5543	33	9.5842	38	0.4158	9.9702	4	0 **69**
10	9.5576	33	9.5879	37	0.4121	9.9697	5	50
20	9.5609	33	9.5917	38	0.4083	9.9692	5	40
30	9.5641	32	9.5954	37	0.4046	9.9687	5	30
40	9.5673	32	9.5991	37	0.4009	9.9682	5	20
50	9.5704	31	9.6028	37	0.3972	9.9677	5	10
22 0	9.5736	32	9.6064	36	0.3936	9.9672	5	0 **68**
10	9.5767	31	9.6100	36	0.3900	9.9667	5	50
20	9.5798	31	9.6136	36	0.3864	9.9661	6	40
30	9.5828	30	9.6172	36	0.3828	9.9656	5	30
40	9.5859	31	9.6208	36	0.3792	9.9651	5	20
50	9.5889	30	9.6243	35	0.3757	9.9646	5	10
23 0	9.5919	30	9.6279	36	0.3721	9.9640	6	0 **67**
10	9.5948	29	9.6314	35	0.3686	9.9635	5	50
20	9.5978	30	9.6348	34	0.3652	9.9629	6	40
30	9.6007	29	9.6383	35	0.3617	9.9624	5	30
40	9.6036	29	9.6417	34	0.3583	9.9618	6	20
50	9.6065	29	9.6452	35	0.3548	9.9613	5	10
24 0	9.6093	28	9.6486	34	0.3514	9.9607	6	0 **66**
10	9.6121	28	9.6520	34	0.3480	9.9602	5	50
20	9.6149	28	9.6553	33	0.3447	9.9596	6	40
30	9.6177	28	9.6587	34	0.3413	9.9590	6	30
40	9.6205	28	9.6620	33	0.3380	9.9584	6	20
50	9.6232	27	9.6654	34	0.3346	9.9579	5	10
25 0	9.6259	27	9.6687	33	0.3313	9.9573	6	0 **65**
10	9.6286	27	9.6720	33	0.3280	9.9567	6	50
20	9.6313	27	9.6752	32	0.3248	9.9561	6	40
30	9.6340	27	9.6785	33	0.3215	9.9555	6	30
40	9.6366	26	9.6817	32	0.3183	9.9549	6	20
50	9.6392	26	9.6850	33	0.3150	9.9543	6	10
26 0	9.6418	26	9.6882	32	0.3118	9.9537	6	0 **64**
10	9.6444	26	9.6914	32	0.3086	9.9530	7	50
20	9.6470	26	9.6946	32	0.3054	9.9524	6	40
30	9.6495	25	9.6977	31	0.3023	9.9518	6	30
40	9.6521	26	9.7009	32	0.2991	9.9512	6	20
50	9.6546	25	9.7040	31	0.2960	9.9505	7	10
27 0	9.6570	24	9.7072	32	0.2928	9.9499	6	0 **63**
10	9.6595	25	9.7103	31	0.2897	9.9492	7	50
20	9.6620	25	9.7134	31	0.2866	9.9486	6	40
30	9.6644	24	9.7165	31	0.2835	9.9479	7	30
40	9.6668	24	9.7196	31	0.2804	9.9473	6	20
50	9.6692	24	9.7226	30	0.2774	9.9466	7	10
28 0	9.6716	24	9.7257	31	0.2743	9.9459	7	0 **62**
10	9.6740	24	9.7287	30	0.2713	9.9453	6	50
20	9.6763	23	9.7317	30	0.2683	9.9446	7	40
30	9.6787	24	9.7348	31	0.2652	9.9439	7	30
40	9.6810	23	9.7378	30	0.2622	9.9432	7	20
50	9.6833	23	9.7408	30	0.2592	9.9425	7	10
29 0	9.6856	23	9.7438	30	0.2562	9.9418	7	0 **61**
10	9.6878	22	9.7467	29	0.2533	9.9411	7	50
20	9.6901	23	9.7497	30	0.2503	9.9404	7	40
30	9.6923	22	9.7526	29	0.2474	9.9397	7	30
40	9.6946	23	9.7556	30	0.2444	9.9390	7	20
50	0.6968	22	9.7585	29	0.2415	9.9383	7	10
30 0	9.6990	22	9.7614	29	0.2386	9.9375	8	0 **60**
	L. Cos.	d.	L.Cotg.	d. c.	L.Tang	L. Sin	d.	′ °

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	40	**39**	**38**	**5**
1	4.0	3.9	3.8	0.5
2	8.0	7.8	7.6	1.0
3	12.0	11.7	11.4	1.5
4	16.0	15.6	15.2	2.0
5	20.0	19.5	19.0	2.5
6	24.0	23.4	22.8	3.0
7	28.0	27.3	26.6	3.5
8	32.0	31.2	30.4	4.0
9	36.0	35.1	34.2	4.5

	37	**36**	**35**
1	3.7	3.6	3.5
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	34	**33**	**32**	**6**
1	3.4	3.3	3.2	0.6
2	6.8	6.6	6.4	1.2
3	10.2	9.9	9.6	1.8
4	13.6	13.2	12.8	2.4
5	17.0	16.5	16.0	3.0
6	20.4	19.8	19.2	3.6
7	23.8	23.1	22.4	4.2
8	27.2	26.4	25.6	4.8
9	30.6	29.7	28.8	5.4

	31	**30**	**29**	**7**
1	3.1	3.0	2.9	0.7
2	6.2	6.0	5.8	1.4
3	9.3	9.0	8.7	2.1
4	12.4	12.0	11.6	2.8
5	15.5	15.0	14.5	3.5
6	18.6	18.0	17.4	4.2
7	21.7	21.0	20.3	4.9
8	24.8	24.0	23.2	5.6
9	27.9	27.0	26.1	6.3

	28	**27**	**26**	**8**
1	2.8	2.7	2.6	0.8
2	5.6	5.4	5.2	1.6
3	8.4	8.1	7.8	2.4
4	11.2	10.8	10.4	3.2
5	14.0	13.5	13.0	4.0
6	16.8	16.2	15.6	4.8
7	19.6	18.9	18.2	5.6
8	22.4	21.6	20.8	6.4
9	25.2	24.3	23.4	7.2

	25	**24**	**23**	**22**
1	2.5	2.4	2.3	2.2
2	5.0	4.8	4.6	4.4
3	7.5	7.2	6.9	6.6
4	10.0	9.6	9.2	8.8
5	12.5	12.0	11.5	11.0
6	15.0	14.4	13.8	13.2
7	17.5	16.8	16.1	15.4
8	20.0	19.2	18.4	17.6
9	22.5	21.6	20.7	19.8

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TABLE VI.

LOGARITHMS OF TRIGONOMETRIC FUNCTIONS.

°	′	L. Sin.	d.	L. Tang	d. c.	L. Cotg.	L. Cos.	d.	′	°
30	0	9.6990	22	9.7614	30	0.2386	9.9375	7	0	**60**
	10	9.7012	21	9.7644	29	0.2356	9.9368	7	50	
	20	9.7033	22	9.7673	28	0.2327	9.9361	8	40	
	30	9.7055	21	9.7701	29	0.2299	9.9353	7	30	
	40	9.7076	21	9.7730	29	0.2270	9.9346	8	20	
	50	9.7007	21	9.7759	29	0.2241	9.9338	7	10	
31	0	9.7118	21	9.7788	28	0.2212	9.0331	8	0	**59**
	10	9.7139	21	9.7816	29	0.2184	9.9323	8	50	
	20	9.7160	21	9.7845	28	0.2155	9.9315	7	40	
	30	9.7181	20	9.7873	29	0.2127	9.9308	8	30	
	40	9.7201	21	9.7902	28	0.2098	9.9300	8	20	
	50	9.7222	20	9.7930	28	0.2070	9.9292	8	10	
32	0	9.7242	20	9.7958	28	0.2042	9.9284	8	0	**58**
	10	9.7262	20	9.7986	28	0.2014	9.9276	8	50	
	20	9.7282	20	9.8014	28	0.1986	9.9268	8	40	
	30	9.7302	20	9.8042	28	0.1958	9.9260	8	30	
	40	9.7322	20	9.8070	27	0.1930	9.9252	8	20	
	50	9.7342	19	9.8097	28	0.1903	9.9244	8	10	
33	0	9.7361	19	9.8125	28	0.1875	9.9236	8	0	**57**
	10	9.7380	20	9.8153	27	0.1847	9.9228	9	50	
	20	9.7400	19	9.8180	28	0.1820	9.9219	8	40	
	30	9.7419	19	9.8208	27	0.1792	9.9211	8	30	
	40	9.7438	19	9.8235	28	0.1765	9.9203	9	20	
	50	9.7457	19	9.8263	27	0.1737	9.9194	8	10	
34	0	9.7476	18	9.8290	27	0.1710	9.9186	9	0	**56**
	10	9.7494	19	9.8317	27	0.1683	9.9177	8	50	
	20	9.7513	18	9.8344	27	0.1656	9.9169	9	40	
	30	9.7531	19	9.8371	27	0.1629	9.9160	9	30	
	40	9.7550	18	9.8398	27	0.1602	9.9151	9	20	
	50	9.7568	18	9.8425	27	0.1575	9.9142	8	10	
35	0	9.7586	18	9.8452	27	0.1548	9.9134	9	0	**55**
	10	9.7604	18	9.8479	27	0.1521	9.9125	9	50	
	20	9.7622	18	9.8506	27	0.1494	9.9116	9	40	
	30	9.7640	17	9.8533	26	0.1467	9.9107	9	30	
	40	9.7657	18	9.8559	27	0.1441	9.9098	9	20	
	50	9.7675	17	9.8586	27	0.1414	9.9089	9	10	
36	0	9.7692	18	9.8613	26	0.1387	9.9080	10	0	**54**
	10	9.7710	17	9.8639	27	0.1361	9.9070	9	50	
	20	9.7727	17	9.8666	26	0.1334	9.9061	9	40	
	30	9.7744	17	9.8692	26	0.1308	9.9052	10	30	
	40	9.7761	17	9.8718	27	0.1282	9.9042	9	20	
	50	9.7778	17	9.8745	26	0.1255	9.9033	10	10	
37	0	9.7795	16	9.8771	26	0.1229	9.9023	9	0	**53**
	10	9.7811	17	9.8797	27	0.1203	9.9014	10	50	
	20	9.7828	16	9.8824	26	0.1176	9.9004	9	40	
	30	9.7844	17	9.8850	26	0.1150	9.8995	10	30	
	40	9.7861	16	9.8876	26	0.1124	9.8985	10	20	
	50	9.7877	16	9.8902	26	0.1098	9.8975	10	10	
38	0	9.7893	17	9.8928	26	0.1072	9.8965	10	0	**52**
	10	9.7910	16	9.8954	26	0.1046	9.8955	10	50	
	20	9.7926	15	9.8980	26	0.1020	9.8945	10	40	
	30	9.7941	16	9.9006	26	0.0994	9.8935	10	30	
	40	9.7957	16	9.9032	26	0.0968	9.8925	10	20	
	50	9.7973	16	9.9058	26	0.0942	9.8915	10	10	
39	0	9.7989	15	9.9084	26	0.0916	9.8905	10	0	**51**
	10	9.8004	16	9.9110	25	0.0890	9.8895	11	50	
	20	9.8020	15	9.9135	26	0.0865	9.8884	10	40	
	30	9.8035	15	9.9161	26	0.0839	9.8874	10	30	
	40	9.8050	16	9.9187	25	0.0813	9.8864	11	20	
	50	9.8066	15	9.9212	26	0.0788	9.8853	10	10	
40	0	9.8081		9.9238		0.0762	9.8843		0	**50**
		L. Cos.	d.	L.Cotg.	d. c.	L.Tang	L. Sin.	d.	′	°

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	22	**21**	**30**
1	2.2	2.1	3.0
2	4.4	4.2	6.0
3	6.6	6.3	9.0
4	8.8	8.4	12.0
5	11.0	10.5	15.0
6	13.2	12.6	18.0
7	15.4	14.7	21.0
8	17.6	16.8	24.0
9	19.8	18.9	27.0

	20	**29**	**7**
1	2.0	2.9	0.7
2	4.0	5.8	1.4
3	6.0	8.7	2.1
4	8.0	11.6	2.8
5	10.0	14.5	3.5
6	12.0	17.4	4.2
7	14.0	20.3	4.9
8	16.0	23.2	5.6
9	18.0	26.1	6.3

	19	**28**	**8**
1	1.9	2.8	0.8
2	3.8	5.6	1.6
3	5.7	8.4	2.4
4	7.6	11.2	3.2
5	9.5	14.0	4.0
6	11.4	16.8	4.8
7	13.3	19.6	5.6
8	15.2	22.4	6.4
9	17.1	25.2	7.2

	18	**27**	**9**
1	1.8	2.7	0.9
2	3.6	5.4	1.8
3	5.4	8.1	2.7
4	7.2	10.8	3.6
5	9.0	13.5	4.5
6	10.8	16.2	5.4
7	12.6	18.9	6.3
8	14.4	21.6	7.2
9	16.2	24.3	8.1

	17	**26**	**10**
1	1.7	2.6	1.0
2	3.4	5.2	2.0
3	5.1	7.8	3.0
4	6.8	10.4	4.0
5	8.5	13.0	5.0
6	10.2	15.6	6.0
7	11.0	18.2	7.0
8	13.6	20.8	8.0
9	15.3	23.4	9.0

	16	**25**	**11**
1	1.6	2.5	1.1
2	3.2	5.0	2.2
3	4.8	7.5	3.3
4	6.4	10.0	4.4
5	8.0	12.5	5.5
6	9.6	15.0	6.6
7	11.2	17.5	7.7
8	12.8	20.0	8.8
9	14.4	22.5	9.9

P. P.

TABLE VI.
LOGARITHMS OF TRIGONOMETRIC FUNCTIONS.

° ′	L. Sin.	d.	L. Tang	d. c.	L. Cotg.	L. Cos.	d.	′ °
40 0	9.8081	15	9.9238	26	0.0762	9.8843	11	0 **50**
10	9.8096	15	9.9264	25	0.0736	9.8832	11	50
20	9.8111	14	9.9289	26	0.0711	9.8821	11	40
30	9.8125	15	9.931$\bar{5}$	26	0.0685	9.8810	10	30
40	9.8140	15	9.9341	25	0.0659	9.8800	11	20
50	9.815$\bar{5}$	14	0.0366	26	0.0634	9.8789	11	10
41 0	9.8169	15	9.9392	25	0.0608	9.8778	11	0 **49**
10	9.8184	14	9.9417	26	0.0583	9.8767	11	50
20	9.8198	15	9.9443	25	0.0557	9.8756	11	40
30	9.8213	14	9.9468	26	0.0532	9.874$\bar{5}$	12	30
40	9.8227	14	9.9494	25	0.0506	9.8733	11	20
50	9.8241	14	9.9519	25	0.0481	9.8722	11	10
42 0	9.8255	14	9.9544	26	0.0456	9.8711	12	0 **48**
10	9.8269	14	9.9570	25	0.0430	9.8699	11	50
20	9.8283	14	9.9595	26	0.040$\bar{5}$	9.8688	12	40
30	9.8297	14	9.9621	25	0.0379	9.8676	11	30
40	9.8311	13	9.9646	25	0.0354	9.866$\bar{5}$	12	20
50	9.8324	14	9.0671	26	0.0329	9.8653	12	10
43 0	9.8338	13	9.9697	25	0.0303	9.8641	12	0 **47**
10	9.8351	14	9.9722	25	0.0278	9.8628	11	50
20	9.836$\bar{5}$	13	9.9747	25	0.0253	9.8618	12	40
30	9.8378	13	9.9772	26	0.0228	9.8606	12	30
40	9.8391	14	9.9798	25	0.0202	9.8594	12	20
50	9.840$\bar{5}$	13	9.9823	25	0.0177	9.8582	13	10
44 0	9.8418	13	9.9848	26	0.0152	9.8569	12	0 **46**
10	9.8431	13	9.9874	25	0.0126	9.8557	12	50
20	9.8444	13	9.9899	25	0.0101	9.854$\bar{5}$	13	40
30	9.8457	12	9.9924	25	0.0076	9.8532	12	30
40	9.8469	13	9.9949	26	0.0051	9.8520	13	20
50	9.8482	13	9.997$\bar{5}$	25	0.0025	9.8507	12	10
45 0	9.849$\bar{5}$		0.0000		0.0000	9.849$\bar{5}$		0 **45**
	L. Cos.	d.	L. Cotg.	d. c.	L. Tang	L. Sin.	d.	′ °

P. P.

	15	**14**	**13**	**12**
1	1.5	1.4	1.3	1.2
2	3.0	2.8	2.6	2.4
3	4.5	4.2	3.9	3.6
4	6.0	5.6	5.2	4.8
5	7.5	7.0	6.5	6.0
6	9.0	8.4	7.8	7.2
7	10.5	9.8	9.1	8.4
8	12.0	11.2	10.4	9.6
9	13.5	12.6	11.7	10.8

P. P.

in the direction of the sight the graduation on the circle may be read.

In running a traverse with prismatic compass, distances are determined by pacing, timing an animal or a boat (Arts. 95 and 96), or by other exploratory method, and the record of the distance and of the angle read for the course run are entered on the same line in the note-book. These quantities can be platted with scale and protractor, and give a fair plan of the route traveled.

92. Magnetic Declination.—The compass-needle points to two magnetic poles, which coincide with the true north and south in but few places on the surface of the earth. The angle made with the true meridian by the magnetic meridian at any point is called the magnetic declination. Declination is subject in all places to changes which are diurnal, secular, annual, and lunar. The two latter are very small and may be neglected.

The *diurnal variation* is scarcely perceptible in any ordinary survey, being zero between 10 and 11 in the morning and at about 8 P.M. It is greatest, that is, the north end is farthest east at about 8 in the morning, and farthest west at about 1:30 in the afternoon. The limits of this diurnal variation are from five to fifteen minutes. The *secular variation* is quite important; it is fairly periodic in character and takes from 250 to 400 years to make a complete cycle.

Declination may be determined at any time by an observation on Polaris to ascertain the true north and its comparison with the magnetic north (Chap. XXXIII).

93. Secular Variation and Annual Change.—Owing to secular variation the declination determined at any date, say when some old survey was executed, has varied since. Therefore, compass-readings recorded on any date for a particular line will not agree with those observed for the same line or direction at another time. It is, accordingly, difficult to rerun old compass lines, and this can only be done with any degree of approximation by knowing the declination of the place for the date of the survey and reducing it to the present time. Numerous observations have been made by the various individual and government surveys, and from them there have been prepared diagrams and tables which aid in the determination of the declination at any known time. A line drawn on a map connecting points having the same magnetic declination is called an *isogonic line*, and the line joining points of no declination is called an *agonic line*.

Plate II, prepared from charts in the U. S. Coast and Geodetic Survey Report of 1896, shows the isogonic and agonic lines in the United States for the epoch of January, 1900. The isogonic curves, which are lines of equal magnetic declination—that is, compass variation from true meridian,—

are shown for each degree. The plus sign indicates west, and the minus sign east declination.

On the same plate are indicated in figures the amount of annual change of magnetic declination for the period 1895–1900. This change, or *secular variation*, indicates that the isogonic lines, as shown on the plate, are all moving westward, but not at the same rate; the movement being such that all western declinations are increasing and all eastern declinations are decreasing. Thus, to find the isogonic line for any year subsequent to 1900, the annual change which is indicated in minutes is to be applied, the plus sign signifying increasing west or decreasing east declination, and the minus sign the reverse.

94. **Local Attraction.**—In running any survey, be it traverse or otherwise, by means of the compass-needle, the indications of the same are apt to be misleading as a result of local magnetic attraction. This is due to the needle being drawn from its mean position in any locality by the attraction of masses of magnetic iron-ore or of iron. In fact, if the compass is set up alongside of the tracks of a railroad or near the wheels of an iron-tired conveyance, it may be attracted from its normal position. It not uncommonly occurs that a closed traverse circuit run with a compass-needle will fail to check by a large error due to some such cause.

Too much reliance cannot, therefore, be placed on compass traverses; and when there appears to be local attraction as shown by inaccurate closures of the surveys or of the lines run, the same must be allowed for in any subsequent work in the same locality. This is done by occupying every point or station in the traverse, or by reading backsights or bearings as well as foresights. It may be that local attraction will be so great in amount as to render it impossible to use the compass at all. During the running of any traverse with a compass, it is well to take the precaution of setting up and observing backsights and foresights on occasional lines, to determine whether local attraction exists.

CHAPTER XI

LINEAR MEASUREMENT OF DISTANCES.

95. Methods of Measuring Distances; Pacing.—The most difficult element in running rough traverse or route surveys is the determination of distances. For directions either a cavalry sketch-board, a traverse plane-table with tripod, or a prismatic compass (Arts. 64, 61, and 91) may be employed, while heights may be determined with the aneroid or by vertical angulation (Arts. 160 and 174).

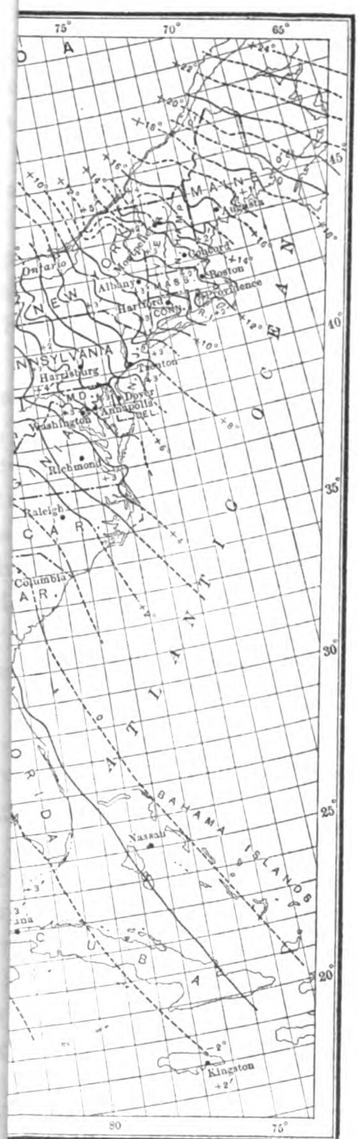
For the rough determination of distance, four methods may be employed, viz.:

1. Measurement by odometer;
2. By counting the paces of a man or animal;
3. By use of the range-finder; or
4. By time estimates.

Where distances are to be measured with greater accuracy some of the following methods may be used, viz.:

1. Tachymetric processes;
2. Chains or steel tape: or
3. Trigonometric processes.

Where walking is necessary in order to get over the ground, very satisfactory and economic measures of distance can be had by pacing. With a little practice a degree of accuracy may be attained quite equal to that had in the direction and elevation measurements with the crude instruments employed in reconnaissance work. It is desirable in *pacing* to adopt a stride shorter than the natural one; thus a man whose natural step in walking comfortably on level



ual change in declination given in
motion.

and Geodetic Survey Report for 1896.)

ground is a yard long should adopt in pacing a stride of 32 inches; and a man whose natural stride is 30 inches should adopt a 28-inch pace. The best way to ascertain the length of stride is to measure off a distance of, say, 200 feet and pace this several times, finding how many paces are required to measure the distance. If 70 strides are taken in this distance, for instance, the pacer should adopt a stride which will enable him to make the distance in 80 steps, and should practice it with sufficient frequency to enable him to make the distance in 80 steps every time. Such a stride is practically the normal one and is easy of calculation, since 40 paces equal 100 feet, 20 paces 50 feet, etc. Hence the number of paces multiplied by $2\frac{1}{2}$ gives the distance in feet. With such a shortened stride the pacer can lengthen out a little when going up-hill, and shorten his stride in going down-hill, and he therefore should practice pacing not only on level ground but on inclined ground, to determine how to alter his stride.

To further *simplify pacing* only every other step should be counted, as those of the left or right foot. In the foregoing case 20 double strides or the steps of one foot will equal 100 feet. Finally, as the length of this double stride is 5 feet, there will be nearly 1000 such steps to one mile; by lengthening the stride by practice to 5.5 feet, a thousand of these will almost exactly equal one mile of 5280 feet. Hence 100 such strides will measure one-tenth mile, etc.

Excellent results have been obtained in rough geographic surveys by using instrumental measurements over portions of the country, and running checked cross-lines between by pacing. Numbers of such lines which the writer has had run and plotted checked out in distances of 10 or 15 miles between fixed points within $\frac{1}{8}$ or $\frac{1}{10}$ of a mile, equivalent on a two-mile scale to $\frac{1}{16}$ or $\frac{1}{20}$ of an inch. Such results have not only been obtained once, but day after day for years, and by different men, in the course of rough surveys over rugged mountains and deep gorges, through brush and fallen timber.

Where it is possible to ride, fairly accurate results can be had by counting the *paces of a saddle-animal.* In determining the pace methods somewhat similar to those used in determining human pacing should be employed, though of course no attempt can be made to shorten or create an artificial pace for the animal. Distances should be measured not only on level country but on hilly land, and these should be between a thousand feet and a quarter of a mile in length, and over these stretches the animal should be paced both at a walk and at a trot, until a fair average has been ascertained of the number of steps at each gait in traveling the distance, when the length of stride can be determined. It is a remarkable fact that the same animal exhibits great uniformity in the length of its stride under similar conditions. This is especially true of mules, which are the most satisfactory animals for use in pacing, as they are slower, steadier, and more uniform in their stride than horses. The writer has run many miles of traverse in the rough regions of the West and under varying topographic conditions, where the distances were measured by the pacing of an animal and checked in at either end by fixed locations, and the results were frequently as accurate as those obtained by average human pacing: this not only at a walk, but at a mixed gait, generally a moderate trot. In such manner as many as 30 to 35 miles of cross-country traverse have been run in a day, which were plotted on a geographic map on a scale of four miles to an inch with very satisfactory closure checks. In pacing with animals the stride of one fore foot only should be counted.

96. Distances by Time.—Time estimates may be employed where uniform pacing is impracticable. With little practice the horseman learns the *rate of* his *animal,* that is, the number of miles per hour which it traverses at different gaits, and in rough reconnaissances and exploratory work he is thus enabled to estimate with fair accuracy the distance he has traveled, by noting the time consumed in passing from one point to another, providing he pays close attention to the gaits of his animal and notes the time consumed with each different gait.

In floating down a river a fairly satisfactory measure of the distance traveled can be obtained with currents of various velocities by *timing floats* over a measured distance in stretches of comparatively slow velocity, up to those in which the speediest rapids are encountered. The explorer may thus float down the stream, using a sketch-board or prismatic compass for direction, and by timing the boat from one course to another a fairly good survey may be made of the route traveled. Similar methods may be employed in ascertaining the time necessary to row or paddle a boat in still water or against streams of varying velocities, and by endeavoring to maintain a uniform rate in rowing or paddling it is possible by timing the courses to get a fair estimate of the distances.

In *platting paced and timed surveys* it will be found desirable to arrange a scale of pacing or timing. Thus, instead of transposing the number of paces into distance paced, a scale should be prepared on which should be graduated paces instead of distances (Fig. 55). For example, for a man who paces a yard at each stride, if the scale of plotting is to be one mile to the inch, there will be 10 paces to every $\frac{1}{176}$ of an inch, and 100 paces to every $\frac{1}{17.6}$ of an inch, so that by dividing an inch into 17.6 parts it will be equal to 100 paces, and lesser fractions can be interpolated. In the same manner, if a horse strides with the same foot a distance of 6 feet at each step, the inch may be divided into 88 parts, and each one of these will be equivalent to 10 strides. In similar manner a scale of time may be prepared, or, better still, in each case several scales for different strides or for different times. Thus, for a scale of one mile to one inch, 15 minutes' travel at the rate of 3 miles an hour will be represented by $\frac{3}{4}$ of an inch, and the same time at the rate of 4 miles an hour will be represented by one inch.

97. Measuring Distances with Linen Tape.—Various methods have been adopted for measuring distances on secondary and tertiary traverses in dense woods where the underbrush is so thick as to preclude the use of the stadia, and where the work required is such as to render unnecessary the accuracy attained by the use of steel tape or chain with two chainmen (Art. 99). Under such conditions two plans have generally been adopted: one, running of traverse lines by the topographer, directions being obtained by prismatic compass or plane-table (Arts. 91 and 61), and distances by the aid of an assistant who drags a chain; the other, by directions in the same manner, but distances by pacing (Art. 95). As the topographer can see but a few yards ahead of him, he rarely sights to a fixed object, but on small-scale work finds it sufficient to sight in the direction in which the assistant has preceded him, dragging the chain.

A more satisfactory and far more accurate mode of measurement under such circumstances has been found to consist in measuring distances with a long linen tape. This is made of tailor's linen binding-tape obtained at dry-goods stores in spools of five-hundred to one thousand feet in length, the best for this purpose being so finely woven and so smooth that it slips through the brush without catching, and is dragged ahead by one tapeman, the alignment of the tape giving the direction which the topographer is to sight for his azimuth. It is improved by immersion in boiling paraffin. The peculiarity of this apparatus consists in the fact that ordinarily the end of the tape will catch in brush and around trees, and tear and fray. To prevent this a narrow strip of celluloid, of the same dimensions as the tape, is sewed on its extreme end, the length of this celluloid appendage being from twelve to eighteen inches, and this causes it to slip between the bushes without becoming tangled or twisted. With such device numerous traverses have been run in the Adirondack woods and plotted on a scale of $1\frac{1}{8}$ inches to a mile, with average closure errors of $\frac{1}{50}$ to $\frac{1}{20}$ inch in circuits of 5 to 15 miles periphery.

98. **Odometer.**—The odometer is not a distance-measurer, but a *revolution-counter*; consequently a function of such distance-measuring is the circumference of a wheel, the number of revolutions of which are counted. This wheel may be one of a buggy or other light conveyance, preferably a front wheel, in order that the odometer which is attached to it may be clearly in view at all times; or the wheel may be attached to a light hand-barrow, so that it can be trundled along trails or other routes over which two- or four-wheeled conveyances could not be driven.

Distance-measuring by means of *rolling a wheel* over the surface and recording with the odometer the number of times the periphery of the wheel is applied to the surface may be done under the most favorable circumstances with nearly the accuracy of ordinary chain or stadia measuring. Such accuracy is not as great as that by the latter methods where they are carefully executed, but is sufficient for all purposes of distance-measuring where the results are to be plotted on a geographic map.

The *errors* inherent in this work are of four kinds:

1. Those due to the difficulty of reducing measures on an inclined surface to horizontal;

2. Failure of the odometer or counter to correctly record the number of the revolutions;

3. Slip or jolt in the wheel, due chiefly to striking stones, roots, and other obstacles; and

4. Errors resulting from failure to run the wheel in a direct line between two station points.

The first is perhaps the most serious, and as yet no satisfactory means have been devised whereby an instrument will record the changes in inclination passed over by an odometer wheel. The second may be partially guarded against only by using the best form of odometer and by the traverseman counting the revolutions of the wheel at the same time as a check. The third is not susceptible to correction, and errors due to this cause will occur unless the surface of the road be of exceptional quality. The errors due to the fourth cause may be practically eliminated by great care in driving or trundling the wheel in a straight line where the road surface will permit. These and like errors inherent in odometric surveys may be so greatly reduced by careful work as to render them of small moment when the survey is to be platted on a geographic map, and where there is sufficient

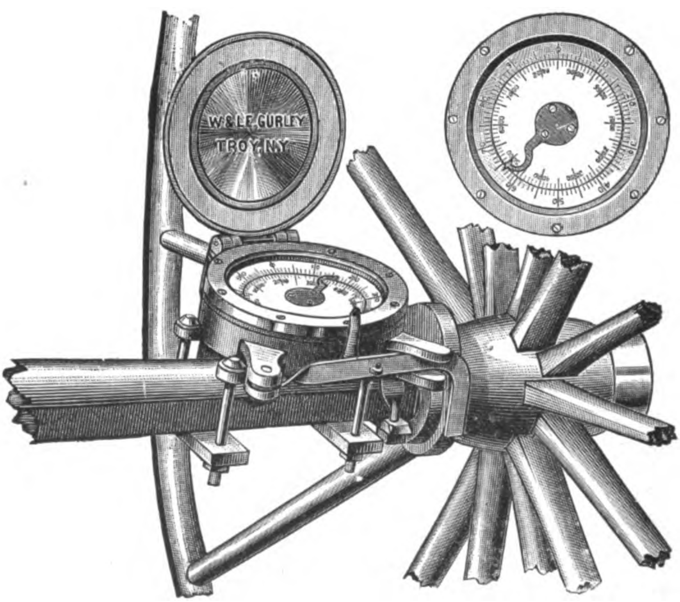


FIG. 73.—DOUGLAS ODOMETER ATTACHED TO WHEEL.

control by triangulation, stadia, or other equally good method to which to adjust the odometer traverses and thus eliminate their errors. As a general rule the errors due to odometer measurement for this class of work are no greater than those introduced in the measurement of directions and due to the difficulty of plotting short road tangents to a small scale.

There are several forms of odometer, among the best of which is the *Douglas odometer*, so named after its inventor, Mr. E. M. Douglas of the U. S. Geological Survey. (Fig. 73.) This is firmly fixed to the axle of the wheel, and a cam is welded around the hub, the lift of the cam being of such height that as it strikes the lever of the odometer it raises this by just the amount sufficient to turn the cog-wheels within the instrument and move the index forward one division for each lift of the cam, corresponding to each revolution of the wheel. This odometer records revolutions directly, and a similar result may be obtained by the use of the ordinary printing-press counter, which may be suitably rigged on the axle of the wheel. The old form of pendulum odometer is so unreliable as to be of practically no value at all for purposes of surveying.

Another form of odometer which has been found to be very satisfactory and accurate is the *bell odometer* (Fig. 74).

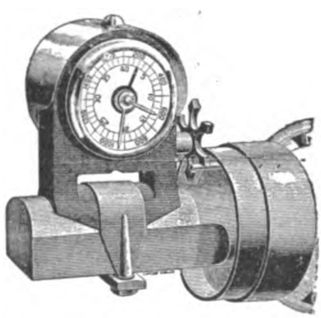


FIG. 74.—BELL ODOMETER.

The record of this is in miles, tenths, and hundredths, instead of in number of revolutions. As a consequence it is manufactured for different diameters of wheels. Knowing, therefore, the diameter of the wheel, the corresponding odometer must be ordered from the maker, and the record may be in miles or some scale unit, the latter being obtained by the surveyor making a false dial to be pasted over that which is furnished. This instrument is attached to the axle of the wheel and records by a small lug on the hub striking a star-shaped wheel connected with an endless screw within the odometer.

The mode of counting revolutions of a wheel most satisfactory to expert traversemen is by tying a rag to one of the spokes and *counting the revolutions* as it comes in view each time. The traverseman soon becomes so expert that he does this counting without any apparent effort, and he intuitively catches through the corner of his eye the flash of the white cloth. Others fasten gongs with heavy pendulum clappers to the spokes of the wheel, so that each time the wheel revolves the clapper falls and strikes the bell. Others rig gongs to the axle and cause them to be struck by clappers attached to the revolving wheel or hub. The simplest counter is, however, the most certain, and of these is the cloth tied to the spoke and counted mentally, or, best of all, on a *hand-recorder* which is pressed at each flash of the cloth as it

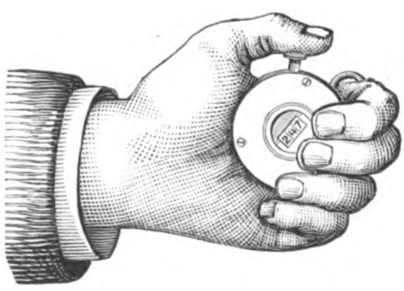


FIG. 75.—HAND RECORDER.

revolves, the recorder registering automatically the number of revolutions. (Fig. 75.)

In using any form of odometer measurement it becomes

TABLE VII.—FOR CONVERTING WHEEL REVOLUTIONS INTO DECIMALS OF A MILE.

Prepared by S. S. GANNETT.

Circumference of wheel.	Fractions of a Mile.																		
	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
	Revolutions.																		
Feet.																			
11.0	5	10	14	19	24	29	34	38	43	48	96	144	192	240	288	336	384	432	480
.1	5	10	14	19	24	29	33	38	43	48	95	143	190	238	286	333	381	428	476
.2	5	9	14	19	24	28	33	38	42	47	94	141	188	235	283	330	377	424	471
.3	5	9	14	19	23	28	33	37	42	47	93	140	187	233	280	327	374	420	467
.4	5	9	14	18	23	28	32	37	42	46	93	139	185	231	278	324	370	417	463
11.5	5	9	14	18	23	28	32	37	41	46	92	138	184	229	275	321	367	413	459
.6	5	9	14	18	23	27	32	37	41	46	91	136	182	227	273	318	364	410	455
.7	5	9	13	18	23	27	32	36	41	45	90	135	180	225	271	316	361	406	451
.8	4	9	13	18	22	27	32	36	40	45	89	134	179	223	268	313	358	402	447
.9	4	9	13	18	22	27	31	35	40	44	89	133	178	222	266	311	355	399	444
12.0	4	9	13	18	22	26	31	35	40	44	88	132	176	220	264	308	352	396	440
.1	4	9	13	17	22	26	31	35	39	44	87	131	174	218	262	305	349	392	436
.2	4	9	13	17	22	26	30	35	39	43	87	130	173	216	260	303	346	390	433
.3	4	9	13	17	21	25	30	34	39	43	86	129	172	214	257	300	343	386	429
.4	4	8	13	17	21	25	30	34	38	43	85	128	170	213	255	298	341	383	426
12.5	4	8	13	17	21	25	30	34	38	42	84	127	169	211	253	295	338	380	422
.6	4	8	13	17	21	25	29	34	38	42	84	126	168	209	251	293	335	377	419
.7	4	8	13	17	21	25	29	33	37	42	83	125	166	208	250	291	333	374	416
.8	4	8	12	16	21	25	29	33	37	41	82	124	165	206	247	288	330	371	412
.9	4	8	12	16	20	25	29	33	37	41	82	123	164	204	245	286	327	368	409
13.0	4	8	12	16	20	24	28	33	37	41	81	122	162	203	244	284	325	365	406
.1	4	8	12	16	20	24	28	32	36	40	81	121	161	201	242	282	322	363	403
.2	4	8	12	16	20	24	28	32	36	40	80	120	160	200	240	280	320	360	400
.3	4	8	12	16	20	24	28	32	36	40	79	119	159	198	238	278	318	357	397
.4	4	8	12	16	20	24	28	31	36	39	79	118	158	197	236	276	315	355	394
13 5	4	8	12	16	20	24	27	31	35	39	78	117	156	196	235	274	313	352	391

necessary to accurately measure the outer circumference of the wheel and then, for convenience in plotting, to arrange a small table in accordance with the scale of the map, so that the number of revolutions multiplied into the circumference will give decimals of the map scale. This table would therefore show for each tenth or hundredth of a mile or other unit the number of revolutions corresponding to such distance. (Table VII.)

99. **Chaining.**—The chain is little used by topographers excepting in dense woods and where the odometer cannot be run nor the stadia-rod seen. Chains are made of two lengths. The surveyor's or *Gunter's chain* is 66 feet long and consists of 100 links, each 17.92 inches long. It is only used in determining the areas of land where the acre is the unit of measure. It is universally used in all of the United States public-land surveys. In all deeds of conveyance of property, where the word "chain" is used it refers to the 66-foot chain.

The *engineer's chain*, that commonly used now, consists of 100 links of steel wire, each connected with the next by two or three rings. Each link with its rings is one foot in length, and the total length of the chain is 100 feet. At intervals of ten links brass tags are brazed on, having one to four points to indicate distances of 10, 20, 30 feet, etc. The chain is provided at either end with brass handles fastened with a swivel, and the length of the chain includes the handles.

The chain is done up from the middle, two links at a time being drawn into the hand. In measuring with the chain the front chainman carries a bundle of ten pins with pieces of cloth tied to them to attract the attention of the rear chainman, and one of these pins is pressed into the ground at each chain-length. They are picked up and tallied by the rear chainman as he progresses. It is very easy to make mistakes in chaining, because of the liability to drop a chain in counting or the tendency to measure on sloping ground without making the proper reductions to the horizontal. Moreover, the chain varies in length by tension, expansion, and contraction. In chaining on steep ground the endeavor should be to hold the chain level and to plumb down from one end, and where the slopes are very steep half-chains of fifty feet only should be measured at a time in order that the chain may be held horizontally.

The *chainmen* usually work ahead of the transitman towards the front flag, but they may be passed by him and follow him. The rear chainman is the more important of the two, as under ordinary methods of running he lines in the front chainman. The latter walks ahead, dragging the chain behind him, and moves to one side or the other according to the rear chainman's signaled directions. The rear chainman shakes the chain out to get rid of kinked links, holds the end of the handle against his pin, and when the front chainman is in line calls out "Down!" when the latter places the fore pin in the ground. The front chainman should be the more active of the two, as the speed of the party depends upon his movements.

CHAPTER XII.

STADIA TACHYMETRY.

100. Tachymetry.—Tachymetry, or, as it is sometimes called, tachyometry, is a method of rapid surveying which enables the operator, by a single observation upon a rod, to obtain the necessary horizontal and vertical data for the determination of the three elements of position of a point on the surface of the earth. Optical measurement of distances, azimuths, and heights by one observation is its essential principle, and is performed by means of stadia, telemeter, or special tachymetric attachment. Tachymetry furnishes at one operation all the controlling elements required in topographic surveying as distinguished from plane surveying, which for topographic requirements must be supplemented by hypsometric surveying.

The *instruments employed* in tachymetric measurement consist of a good transit or plane-table and alidade (Arts. 85, 56 and 59), and of a well-made rod variously divided according to the method employed (Art. 112). The requirements of this method are rapidity and comparative accuracy of measurement accomplished with the least cost, rather than with extreme precision. Where it is necessary to measure distances and elevations at the same time, tachymetry gives as nearly accurate results, at much less cost of time and money, as are possible with chain and spirit-level.

There are practically two systems of tachymetric measurement:

1. The angular or tangential system; and
2. The stadia, telemeter, or subtend system.

By *angular tachymetry* the horizontal distances are determined by measuring the vertical angle between two marks at a given distance apart on a rod. By *subtend or stadia tachymetry* the horizontal distance is determined by observing the number of divisions intercepted on a rod between two lines in the diaphragm of the instrument, the distance between which bears a fixed ratio to the distance intercepted.

The simplest form of tachymetry is with the plane-table, since on this is executed a graphic triangulation which attains the same end as does optical tachymetry or the range-finder (Art. 116). All forms of tachymetry are by means of triangulation, varying from a long base, as with plane-table or theodolite triangulation, to the short base of a Welden range-finder or of stadia-wires. Tachymeters may be divided into three classes:

1. Those in which the measured base forms an integral part of the instrument itself, as is the case with the Wagner-Fennel type of tachymeter and the large fixed range-finders employed at seacoast batteries and on board ships;

2. Those in which the measured base is on the point observed, as is the case with the stadia; and

3. Those in which the base is measured on the ground at the observer's station, as with the Welden range-finder and the plane-table used in range-finding.

101. Topography with Stadia.—In running a simple stadia traverse considerable topography may be obtained by the method of triangulation intersections (Art. 73) in conjunction with the stadia traverse, where it may be necessary to expand the area of topographic mapping. Thus signals may be established on commanding summits visible from the line, and directions and vertical angles (Arts. 54 and 160) be read to these, thus determining their positions. Then the stations marked by the signals can be occupied by the topographer; or else two or three assistants with stadia-rods may move about to the various positions which it is desired to determine from these stations; the topographer meantime observing on the rods and thus locating them, after which he sketches in the topography adjacent to their position.

The simplest method of *surveying a river* or narrow lake is with the stadia. The transit or plane-table should be carried in one boat and landings be made for stations; or, where banks will not permit of landing, the instrument may be firmly fastened in the boat. In a second boat a stadia-rod is carried and distances and directions are read to the rod by the topographer, thus locating its position. By moving along, the topographer alternately passing the stadia-man and the stadia-man the topographer, and repeating this process, it is possible to procure a fair map of such river at moderate cost and in a comparatively short time. Greater speed and accuracy are obtainable, where the conditions permit, by executing plane-table triangulation in conjunction with the stadia measurements, according as one or other method is more convenient.

102. Tachymetry with Stadia.—The stadia is a device for determining the distance of a point from the observer by means of a graduated rod and the distance subtended on it by auxiliary wires in the telescope of a transit or alidade. The principle upon which stadia measurements are based is the geometric one that the lengths of parallel lines subtending an angle are proportioned to their distances from its apex. This proportion is applied through the medium of two fine wires or cross-hairs, or a glass with lines etched on it at the positions of the cross-hairs, and equidistant from the central cross-hair or line. The space which any two of these lines subtends on a rod or other object of known length bears a direct ratio to the distance of that object from the cross-hairs of the instrument, and, accordingly, knowing the distance subtended on the rod, its distance from the instrument can be at once determined.

The term *stadia surveying* is used to include not only the measurement of the horizontal distance, but also the determination of heights by means of vertical angles observed to a fixed point on the rod. The stadia hairs may be horizontal and the rod held vertical, or *vice versa*, though the former method is usually preferred, for the rod can be more steadily and readily held in a vertical position than horizontally. The *stadia-rod* (Art. 112) may be held at right angles to the line of sight, which on a uniformly sloping hill would require it to be inclined at exactly right angles to the slope, or it may be held vertically, which is a much simpler operation, and the angle of inclination is then reduced by computation or tables (Arts. 104 and 105). The latter method is more safely and commonly employed than the former.

The *stadia-hairs* are usually three in number and are placed parallel to each other, the outer equally distant from the center one, and at an extreme distance from each other which bears a decimal ratio between the distance subtended and that measured horizontally on the ground. For very accurate work it is considered better practice to have the stadia-hairs fixed so that they are not adjustable, and to determine experimentlly the ratio between the distance which they subtend on the rod and that measured on the ground; or, in other words, the multiple of the distance subtended. The more common and convenient way, however, is to place the extreme hairs at such a distance apart that one foot subtended on the rod represents one hundred feet on the ground plus the focal length, and this ratio is obtained by having the hairs adjustable so that by testing the adjustment it can be ascertained at any time whether or not this ratio is correctly fixed. By this means it is possible to measure greater distances than would be observable on a rod of given length, by using the half-hairs, or the distance between one of the extreme hairs and the middle hair; in which case a given distance on the rod would correspond to double the distance on the ground measured by the extreme hairs.

Example: One foot subtended by extreme hairs equals

100 feet in distance; then, with level horizontal, if 5.68 feet are subtended on rod by hairs, the rod is distant from the telescope 568 feet plus the focal length *f*.

Example: If the sight be still horizontal but the half hairs set so that 1 foot on the rod equals 200 feet in distance, then for the above intercept, 5.68 feet, the distance from the center of the instrument to the rod will be 1136 feet plus the focal distance.

103. Accuracy and Speed of Stadia Tachymetry.—The accuracy and precision of well-conducted stadia-work is rarely fully appreciated. The stadia is essentially intended to secure rapidity rather than accuracy; nevertheless, with proper care to eliminate the chief sources of error, a high degree of accuracy may be attained. It is now generally believed by most of those who have employed the stadia in careful operations that where properly handled it will produce results as good as, and frequently better than, those with the chain, especially in rough country where the inclination of the ground affects chaining most seriously.

The degree of *precision is dependent* upon several quantities, chief among which are:

1. Length of sight;
2. Ratio of the space subtended on the rod to the distance on the ground;
3. Magnifying power of the telescope;
4. Fineness of the cross-hairs; and
5. Precautions taken to modify or eliminate the effects of refraction.

Numerous experiments have been made to ascertain the *effects of magnifying power*. By observing distances with telescopes magnifying 15 times and 25 times respectively, under essentially the same conditions, Prof. Ira O. Baker found the average error in the first case, that is, with the lower magnifying power, to be 1 in 282, and in the second case, with the higher magnifying power, 1 in 333. He similarly experimented with a view to determining the length of sight and corresponding error, with the result that the errors at distances of 100, 200 and 300 feet, respectively, were 1 in 282, 1 in 263, and 1 in 370. These results, however, are not as valuable in showing the effect of this form of error, because it is largely introduced by the quality of the instrument, its magnifying power, size of cross-hairs, atmospheric conditions, and similar modifying circumstances.

Experiments by Mr. R. E. Middleton showed the *limit of accuracy* of the stadia instrument with which he was experimenting to be about 800 feet. Between 100 and 800 feet the average error was minus .43 feet per thousand feet; beyond this distance it increased rapidly to minus .97 feet per thousand.

Perhaps some of the most interesting results obtained with stadia, as showing its *precision*, were those obtained by Mr. J. L. Van Ornum in taking topography on the international survey of the Mexican Boundary. The whole of the boundary line was measured with the stadia, and a large portion of it by the chain, and always tied in by a system of accurate primary triangulation. Corresponding distances were found by stadia and chain and compared with the known distances as obtained by triangulation, with the following results:

Of five different stretches measured by the three methods, the total distance shown by triangulation was 99,110 meters, by stadia 99,025 meters, by corrected chain 99,041 meters. The total ratio of error between triangulation and chaining was minus 1 in 1436, and between triangulation and stadia minus 1 in 1166. Other sections of the line were measured by stadia and triangulation, but not by chain. In all there were measured 182.5 miles by stadia which were triangulated and in which the total difference in length was plus 50 meters, or 1 in 5837. It may be noted that the chained distance was marked corrected chain, because in six measurements of the chained distance, dropping or omission of chain-lengths occurred which were detected in every instance by the stadia. The cause of this may be sought in the fact that the responsibility for correct measurement with stadia was placed on a trained instrumentman, thus reducing the danger of systematic error to a minimum. Moreover, there were some distances measured with the stadia which it would have been almost impossible to measure with the chain owing to the roughness of the country and the great error and confusion which would have resulted from breaking chain frequently on exceedingly rough ground.

In the surveys of the Indian Territory by the United States Geological Survey, a considerable number of section lines were run with stadia and chain with a view to determining in a general way whether the stadia measurements were as accurate as the chain measurements, or, in other words, if they could be kept within the limit of error allowed in the Manual of the U. S. Land Office. This work was not done with any great accuracy, but with sufficient care to ascertain the fact that in every case the stadia measurements were well within the closing limits allowed for chaining according to the Manual, namely, $\frac{5}{8}$ of a link per chain-length.

The degree of *accuracy* which may be attained by comparatively *crude stadia surveys* extending over a great area may be illustrated by a preliminary line run in Mexico by Mr. W. B. Landreth and the author from the west coast near Culiacan across the Sierra Madre to Durango and back by a different route. The method of running consisted in reading declination and distance with a transit to a stadia-rod held as foresight, the instrumentman leaving a small sapling with a piece of paper at his instrument position to serve as a rearsight. Vertical angles were read also in connection with this line. The total distance closed as a circuit 606 miles in length. It was computed by latitudes and departures which closed within 1100 feet, while the average *cost* of the survey was about $3.00 per linear mile.

The *speed* of stadia surveys is far greater than that of chaining where the surface of the ground is rough, since sights of one to two thousand feet length can be taken. Under similar circumstances the chain has to be laid down and stretched every hundred feet. Again, in a winding canyon or hilly road the instrument may have to be set up every hundred feet, perhaps ten times in 2000 feet. A single set-up and sight with the stadia may make the same measure and be not only speedier but far more accurate. The latter because one relatively crude measure will be less liable to error than ten separate measures of both angle and distance. Finally, the relative speed is further increased and the cost reduced correspondingly when elevations are to be obtained. For with stadia this is accomplished with the same men and instruments in conjunction with the plane survey. With chain survey, however, another party is necessary to determine elevations by spirit-level.

Over smooth country surveys may be made by stadia still more rapidly than by chaining or leveling if the rodman be mounted or ride a wheel, and the instrumentman ride or drive. On the Mexican survey above cited as many as 16 miles a day were often made, including the determination of height and the sketching of topography. Under favorable circumstances 10 to 20 miles a day can be made as easily as 5 to 8 miles walking with chain or level.

Surveys by stadia traverse and plane-table intersections combined have been made of the shore line of large lakes, as Raquette Lake in the Adirondacks, at remarkably low cost and with great accuracy. This lake has a most intricate shore line and contains many islands, all heavily wooded. Yet its 45 miles of outline were mapped on a scale of $1\frac{1}{3}$ inches to a mile by one topographer and one stadiaman, both in boats, in 12 days.

104. Stadia Formula with Perpendicular Sight. — In sighting the rod from the telescope, the stadia-wires appear to be projected upon the rod, thus intercepting a fixed distance upon it. In fact there is formed at the position of the stadia-wires an image of the rod which the wires intercept, and at points which are the respective foci of the two points subtended on the rod. If the object-glass be considered a simple bi-convex lens, then, by the principle of optics, the rays from any point of an object converge to a focus at a straight line which is a secondary axis connecting the point with its image and passing through the center of the lens. This point of intersection of the secondary axis is the optical center. It follows that the lines cC and bB, Fig. 76, drawn

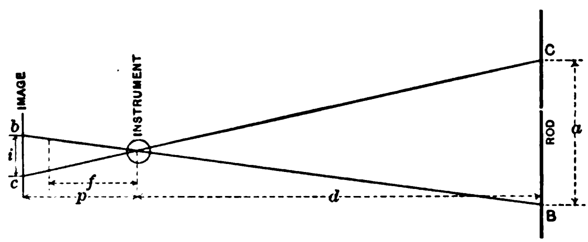


FIG. 76.—STADIA MEASUREMENT ON HORIZONTAL.

from the stadia-wires, will intersect the object at points corresponding to those which the wires cut on the image of the rod. From this follows the proportion

$$\frac{d}{p} = \frac{a}{i}; \quad \text{hence} \quad d = \frac{p}{i}a, \quad . \quad . \quad . \quad . \quad (2)$$

in which d = the distance of the rod from the center of the objective;

p = the distance of the stadia-wires from the center of the objective;

a = the distance intercepted on the rod by the stadia-hairs; and

i = the distance of the stadia-hairs apart.

If p remained the same for all lengths of sight, then $\frac{p}{i}$

would equal a constant, and d would be directly proportional to a. Unfortunately p varies with the length of the sight, and the relation between d and a is therefore variable. Representing the principal focal length by the letter f, and applying the general formula for bi-convex lenses, that the sum of the reciprocals of the conjugate focal distances of the convex lens is equal to the reciprocal of the focal length of the lens, we have

$$\frac{i}{p} + \frac{i}{d} = \frac{i}{f}, \qquad \ldots \ldots \qquad (3)$$

in which two different focal distances of image and object, p and p', are approximately the same as p and d respectively.

Substituting the above formula in (2) and transposing, we get

$$d = \frac{f}{i}a + f. \qquad \ldots \ldots \qquad (4)$$

Since in the above f and i remain constant for sights of all lengths, the factor by which a is multiplied is a constant, and d is equal to a constant multiplied by the length of $a + f$. The constant corresponding to $\frac{f}{i}$ is usually designated by k, and accordingly the distance from the rod to the objective of the telescope is equal to a constant times the reading on the rod plus the focal length of the objective.

To obtain the exact distance to the center of the instrument, it is further necessary to add the distance of the objective from the center of the instrument to f, which may be called c; the final expression for the distance of the horizontal sight is then

$$d = ka + f + c. \qquad \ldots \ldots \qquad (5)$$

The approximate value of f, the focal length, may be obtained by focusing the telescope on a distant object and measuring the distance from the center of the object-glass to the cross-hairs. The value of c is not constant in most instruments, since the objective is moved in focusing for the different distances. It may be determined by focusing on an object a few hundred feet away and measuring the distance from the objective to the center of the instrument.

The value of $\frac{f}{i} = k$ may be determined by driving a tackhead in a stake and setting up the instrument over this. From this point measure two distances—one, say, of 100 feet, and the other of 300 feet—and holding a rod at each, note the space intercepted on the rod at each point. Now, from the formulas,

$$d = \frac{f}{i}a + (f + c) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and

$$d' = \frac{f}{i}a' + (f + c), \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in each of which the values of d and d' and a and a' are known, the values of $\frac{f}{i} = k$ and $f + c$ may be deduced.

105. Stadia Formula with Inclined Sight.—Formula (5) is based on the assumption that the visual ray is horizontal and the rod held vertical; that is, the line of sight is assumed to be perpendicular to the rod. This formula is inaccurate, however, for most stadia-work, because the sights are not taken on a level, but usually on a slope or inclination. Accordingly, d is not the horizontal distance from the instrument to the rod, but the inclined distance from the horizontal axis of the telescope to the point on the rod covered by the central visual hair. Formula (5) may be used with an inclined sight, provided the rod is held perpendicular to the central visual ray, and such perpendicularity may be obtained by a telescope, or, more simply, by means of a pair of sights attached to the rod at right angles, or by a plumb-bob.

The effort to procure perpendicularity of the rod involves several serious difficulties, among which are: (1) the difficulty of holding the rod steadily in this position; (2) the fact that it is not always possible for the rodman to see the telescope at a long distance or great angle; and (3) because the formulas for computing the horizontal and vertical co-ordinates are more simple when the rod is held vertical than when it is held perpendicular to the line of sight.

The same effect as is obtained by perpendicularity is obtained by *holding* the *rod horizontally* and having the cross-wires of the telescope placed vertically. There are some advantages in this method, because there is no likelihood of confusing either of the stadia-hairs with the central horizontal leveling-hair used in obtaining elevations. Though in some detailed work this method has been employed, the chief objection to it is the difficulty of holding the rod horizontal, it being usually necessary to support it upon trestles or some similar device. Neither of the above methods is generally accepted, as they are less simple of accomplishment and produce no better results than the more usual method of holding the rod vertical. Hence they will not be further discussed.

The *rod* may be *held vertical* with as great ease as may a leveling-rod by balancing it between the fingers or by having attached to it plumbing-levels. The formula for reduction to verticality is comparatively simple. Let $\alpha =$ angle of central visual ray with the horizon. This angle is measured by the central stadia-hair, either as a process in determining trigonometric levels or merely with the object of reducing the stadia distances. It is generally small and should be kept as small as possible to produce the best results. Let $2\beta =$ the visual angle subtended by the extreme cross-hairs on the rod, an angle which is always small, rarely exceeding one half of a degree.

As CD (Fig. 77) is the actual distance subtended on the rod, and AB the distance which would be subtended if the

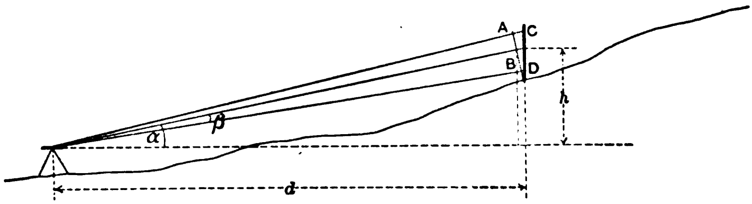


FIG. 77.—STADIA MEASUREMENT ON SLOPE.

rod were held perpendicular to the line of sight, the relation between AB and CD is required, and by simple mathematical deduction we obtain as an expression for this

$$AB = CD \cos \alpha. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Since AB is the distance subtended with the rod held perpendicular to the line of sight, it is the value of a corresponding to the distance d in Formula (5), and it therefore becomes

$$d = ka \cos \alpha + (f + c). \quad . \quad . \quad . \quad . \quad (9)$$

Let $d =$ the horizontal distance from the center of the instrument to the vertical foot of the rod, the actual distance which it is desired to measure. Then $d' = d \cos \alpha =$ actual distance from center of instrument to center of rod multiplied by $\cos \alpha$. Substituting in this the value of d from formula (9), we have

$$d' = ka \cos^2 \alpha + (f + c) \cos \alpha. \quad . \quad . \quad . \quad (10)$$

We also have

$$h = (f + c) \sin \alpha + ak\frac{\sin 2\alpha}{2}, \quad . \quad . \quad . \quad (11)$$

in which $h =$ the vertical distance or height of the object above the instrument.

With the aid of these two formulæ the horizontal and vertical distances can be immediately calculated when reading on a vertical rod and when the angle of elevation is observed. From them numerous stadia tables have been calculated, the earlier and more important of which were those published by Messrs. J. A. Ockerson and Jared Teeple and those published by Mr. Arthur Winslow.

106. Determining Horizontal Distances from Inclined Stadia Measures.—The following table (VIII), derived from the U. S. Coast and Geodetic Survey reports, is one of the most compact for use in reducing short stadia sights observed on slopes to their horizontal projections.

TABLE VIII.

REDUCTION OF INCLINED STADIA MEASURES TO HORIZONTAL DISTANCES.

Inclination in degrees.	Horizontal projection of—								
	10 feet.	20 feet.	30 feet.	40 feet.	50 feet.	60 feet.	70 feet.	80 feet.	90 feet.
Deg.									
1	9.997	19.995	29.993	39.991	49.988	59.986	69.984	79.981	89.979
2	9.99	19.98	29.97	39.96	49.95	59.04	69.94	79.92	89.91
3	9.98	19.96	29.93	39.91	49.88	59.86	69.84	79.82	89.80
4	9.96	19.92	29.88	39.84	49.80	59.76	69.72	79.68	89.64
5	9.94	19.88	29.81	39.75	49.69	59.63	69.57	79.50	89.44
6	9.91	19.82	29.73	39.64	49.56	59.46	69.37	79.27	89.20
7	9.88	19.76	29.64	39.52	49.40	59.28	69.16	79.04	88.91
8	9.84	19.68	29.53	39.37	49.21	59.06	68.90	78.74	88.58
9	9.80	19.60	29.40	39.21	49.01	58.80	68.61	78.41	88.21
10	9.75	19.51	29.27	39.02	48.78	58.54	68.29	78.05	87.79
11	9.70	19.41	29.11	38.82	48.52	58.22	67.93	77.64	87.34
12	9.65	19.30	28.95	38.60	48.24	57.90	67.55	77.20	86.84
13	9.60	19.20	28.80	38.40	48.00	57.60	67.20	76.80	86.40
14	9.55	19.10	28.65	38.20	47.75	57.30	66.85	76.40	85.95
15	9.50	19.00	28.50	38.00	47.50	57.00	66.50	76.00	85.50

Example: $d = 160.20$ and $\alpha = 7° 00'$.

$$160.2 \begin{cases} 100 \dotfill 98.80 \\ 60 \dotfill 59.28 \\ 0.2 \dotfill 0.1976 \end{cases}$$

$d' = 158.2776$

$d = 98.70$ and $\alpha = 4° 00$.

$$98.7 \begin{cases} 90 \dotfill 89.64 \\ 8 \dotfill 7.968 \\ 0.7 \dotfill 0.6972 \end{cases}$$

$d' = 98.3052$

107. Horizontal Distances and Elevations from Stadia Readings.—Table IX was computed by Mr. Arthur Winslow and is reproduced here from J. B. Johnson's "Surveying."

TABLE IX.*

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS.

Minutes.	0°		1°		2°		3°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23
2 . .	"	0.06	"	1.80	99.87	3.55	99.72	5.28
4 . .	"	0.12	"	1.86	"	3.60	99.71	5.34
6 . .	"	0.17	99.96	1.92	"	3.66	"	5.40
8 . .	"	0.23	"	1.98	99.86	3.72	99.70	5.46
10 . .	"	0.29	"	2.04	"	3.78	99.69	5.52
12 . .	"	0.35	"	2.09	99.85	3.84	"	5.57
14 . .	"	0.41	99.95	2.15	"	3.90	99.68	5.63
16 . .	"	0.47	"	2.21	99.84	3.95	"	5.69
18 . .	"	0.52	"	2.27	"	4.01	99.67	5.75
20 . .	"	0.58	"	2.33	99.83	4.07	99.66	5.80
22 . .	"	0.64	99.94	2.38	"	4.13	"	5.86
24 . .	"	0.70	"	2.44	99.82	4.18	99.65	5.92
26 . .	99.99	0.76	"	2.50	"	4.24	99.64	5.98
28 . .	"	0.81	99.93	2.56	99.81	4.30	99.63	6.04
30 . .	"	0.87	"	2.62	"	4.36	"	6.09
32 . .	"	0.93	"	2.67	99.80	4.42	99.62	6.15
34 . .	"	0.99	"	2.73	"	4.48	"	6.21
36 . .	"	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38 . .	"	1.11	"	2.85	"	4.59	99.60	6.33
40 . .	"	1.16	"	2.91	99.78	4.65	99.59	6.38
42 . .	"	1.22	99.91	2.97	"	4.71	"	6.44
44 . .	99.98	1.28	"	3.02	99.77	4.76	99.58	6.50
46 . .	"	1.34	99.90	3.08	"	4.82	99.57	6.56
48 . .	"	1.40	"	3.14	99.76	4.88	99.56	6.61
50 . .	"	1.45	"	3.20	"	4.94	"	6.67
52 . .	"	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54 . .	"	1.57	"	3.31	99.74	5.05	99.54	6.78
56 . .	99.97	1.63	"	3.37	"	5.11	99.53	6.84
58 . .	"	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60 . .	"	1.74	"	3.49	"	5.23	99.51	6.96
$c = 0.75$	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
$c = 1.00$	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
$c = 1.25$	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

* From "Theory and Practice of Surveying," by Prof. J. B. Johnson. New York: John Wiley & Sons.

TABLE IX.

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS.

Minutes.	4°		5°		6°		7°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2 . .	"	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4 . .	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21
6 . .	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26
8 . .	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10 . .	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12 . .	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14 . .	"	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16 . .	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18 . .	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20 . .	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22 . .	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24 . .	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26 . .	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28 . .	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30 . .	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94
32 . .	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34 . .	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36 . .	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38 . .	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40 . .	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42 . .	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44 . .	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46 . .	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48 . .	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50 . .	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52 . .	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54 . .	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56 . .	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58 . .	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60 . .	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
$c = 0.75$	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10
$c = 1.00$	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13
$c = 1.25$	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16

Table IX.

Horizontal Distances and Elevations from Stadia Readings.

Minutes.	8°		9°		10°		11°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2 . .	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4 . .	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6 . .	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8 . .	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10 . .	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12 . .	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14 . .	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16 . .	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18 . .	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20 . .	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22 . .	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24 . .	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26 . .	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28 . .	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30 . .	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32 . .	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34 . .	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36 . .	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38 . .	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40 . .	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42 . .	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44 . .	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46 . .	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48 . .	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50 . .	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52 . .	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54 . .	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56 . .	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58 . .	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60 . .	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
$c = 0.75$	0.74	0.11	0.74	0.12	0.74	0.14	0.73	0.15
$c = 1.00$	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
$c = 1.25$	1.23	0.18	1.23	0.21	1.23	0.23	1.22	0.25

TABLE IX.

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS.

Minutes.	12°		13°		14°		15°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2 . .	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4 . .	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6 . .	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8 . .	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10 . .	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12 . .	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14 . .	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16 . .	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18 . .	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20 . .	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22 . .	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24 . .	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26 . .	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28 . .	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30 . .	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32 . .	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34 . .	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36 . .	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38 . .	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40 . .	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42 . .	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44 . .	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46 . .	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48 . .	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50 . .	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52 . .	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54 . .	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56 . .	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58 . .	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60 . .	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
$c = 0.75$	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20
$c = 1.00$	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27
$c = 1.25$	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

TABLE IX.

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS.

Minutes.	16°		17°		18°		19°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2 . .	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4 . .	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6 . .	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8 . .	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10 . .	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12 . .	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14 . .	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16 . .	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18 . .	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20 . .	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22 . .	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24 . .	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33
26 . .	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28 . .	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30 . .	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32 . .	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34 . .	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36 . .	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38 . .	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40 . .	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42 . .	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44 . .	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46 . .	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48 . .	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50 . .	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52 . .	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54 . .	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56 . .	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58 . .	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60 . .	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
$c = 0.75$	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
$c = 1.00$	0.86	0.28	0.95	0.30	0.95	0.32	0.94	0.33
$c = 1.25$	1.20	0.35	1.19	0.38	1.19	0.40	1.18	0.42

TABLE IX.

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS.

Minutes.	20°		21°		22°		23°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2 . .	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4 . .	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6 . .	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8 . .	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10 . .	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12 . .	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14 . .	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16 . .	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18 . .	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20 . .	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22 . .	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24 . .	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26 . .	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28 . .	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30 . .	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32 . .	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34 . .	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36 . .	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38 . .	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40 . .	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42 . .	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44 . .	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46 . .	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48 . .	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50 . .	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52 . .	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54 . .	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56 . .	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58 . .	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60 . .	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
$c = 0.75$	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30
$c = 1.00$	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40
$c = 1.25$	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50

TABLE IX.

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS.

Minutes.	24°		25°		26°		27°	
	Hor. Dist	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2 . .	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4 . .	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6 . .	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8 . .	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10 . .	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12 . .	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14 . .	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16 . .	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18 . .	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20 . .	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22 . .	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24 . .	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26 . .	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28 . .	82.85	37.70	81.51	38.62	80.14	39.90	78.73	40.92
30 . .	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32 . .	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34 . .	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36 . .	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38 . .	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40 . .	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42 . .	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44 . .	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46 . .	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48 . .	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50 . .	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52 . .	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54 . .	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56 . .	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58 . .	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60 . .	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
$c = 0.75$	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35
$c = 1.00$	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46
$c = 1.25$	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58

TABLE IX.

HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS.

Minutes.	28°		29°		30°	
	Hor. Dist.	Diff. Elev.	Hor. Dist	Diff. Elev.	Hor. Dist.	Diff. Elev.
0 . .	77.96	41.45	76.50	42.40	75.00	43.30
2 . .	77.91	41.48	76.45	42.43	74.95	43.33
4 . .	77.86	41.52	76.40	42.46	74.90	43.36
6 . .	77.81	41.55	76.35	42.49	74.85	43.39
8 . .	77.77	41.58	76.30	42.53	74.80	43.42
10 . .	77.72	41.61	76.25	42.56	74.75	43.45
12 . .	77.67	41.65	76.20	42.59	74.70	43.47
14 . .	77.62	41.68	76.15	42.62	74.65	43.50
16 . .	77.57	41.71	76.10	42.65	74.60	43.53
18 . .	77.52	41.74	76.05	42.68	74.55	43.56
20 . .	77.48	41.77	76.00	42.71	74.49	43.59
22 . .	77.42	41.81	75.95	42.74	74.44	43.62
24 . .	77.38	41.84	75.90	42.77	74.39	43.65
26 . .	77.33	41.87	75.85	42.80	74.34	43.67
28 . .	77.28	41.90	75.80	42.83	74.29	43.70
30 . .	77.23	41.93	75.75	42.86	74.24	43.73
32 . .	77.18	41.97	75.70	42.89	74.19	43.76
34 . .	77.13	42.00	75.65	42.92	74.14	43.79
36 . .	77.09	42.03	75.60	42.95	74.09	43.82
38 . .	77.04	42.06	75.55	42.98	74.04	43.84
40 . .	76.99	42.09	75.50	43.01	73.99	43.87
42 . .	76.94	42.12	75.45	43.04	73.93	43.90
44 . .	76.89	42.15	75.40	43.07	73.88	43.93
46 . .	76.84	42.19	75.35	43.10	73.83	43.95
48 . .	76.79	42.22	75.30	43.13	73.78	43.98
50 . .	76.74	42.25	75.25	43.16	73.73	44.01
52 . .	76.69	42.28	75.20	43.18	73.68	44.04
54 . .	76.64	42.31	75.15	43.21	73.63	44.07
56 . .	76.59	42.34	75.10	43.24	73.58	44.09
58 . .	76.55	42.37	75.05	43.27	73.52	44.12
60 . .	76.50	42.40	75.00	43.30	73.47	44.15
$c = 0.75$	0.66	0.36	0.65	0.37	0.65	0.38
$c = 1.00$	0.88	0.48	0.87	0.49	0.86	0.51
$c = 1.25$	1.10	0.60	1.09	0.62	1.08	0.64

This is a most generally useful stadia table for rods reading 100 feet to the foot and with angles up to 30°. The values of other measures than those given in the table are obtained by multiplying the quantities under the proper vertical angle by stadia readings in hundreds of units. The quantity representing the focal distance is very small and is given at the bottom of each page for focal lengths between $\frac{3}{4}$ and $1\frac{1}{4}$ feet and is represented as a constant equal to c, which corresponds with the second term in the right side of equations (6) and (7) (Art. 104). The direct use of the table involves a multiplication for each result obtained.

Example: Let rod intercept be 3.25 feet, and the angle of inclination be 5° 35′. Then the distance on the horizontal would be

$$d = 325 \text{ feet.}$$

If we accept the focal distance $f + c$ as 1.25 feet, we have from the tables and by substituting in formulas (10) and (11)

$$d' = 3.25 \text{ ft.} \times 99.05 + 1.24 = 313.15 \text{ ft.},$$

and

$$h = 3.25 \text{ ft.} \times \ \ 9.68 + 0.11 = 31.57 \text{ ft.}$$

108. Determining Elevations by Stadia.—Table X, computed by Prof. R. S. Woodward, is one of the most convenient for determining differences of elevation from measures made with stadia.

This table is computed from the formula

$$h = d \sin \alpha \operatorname{cosin} \alpha; \quad . \quad . \quad . \quad . \quad (12)$$

in which d is the observed distance of the rod, α is the angle of elevation or depression, and h is the difference of elevation. To use the table, look for the observed angle in the first column, and the distance in the upper line under "D";

the differences of elevation will be found at the intersection of the two columns.

Example: Assuming the observed distance read directly from the stadia-rod as 360 feet and the angle 2° 40′, we have from the table directly the result

$$h = 16.7 \text{ feet.}$$

109. Diagram for Reducing Stadia Measures.—Stadia measures may be reduced by a diagram more simply, though not with the same degree of accuracy, as by tables. The following diagram (Fig. 78), from Ira O. Baker's "Engineer's

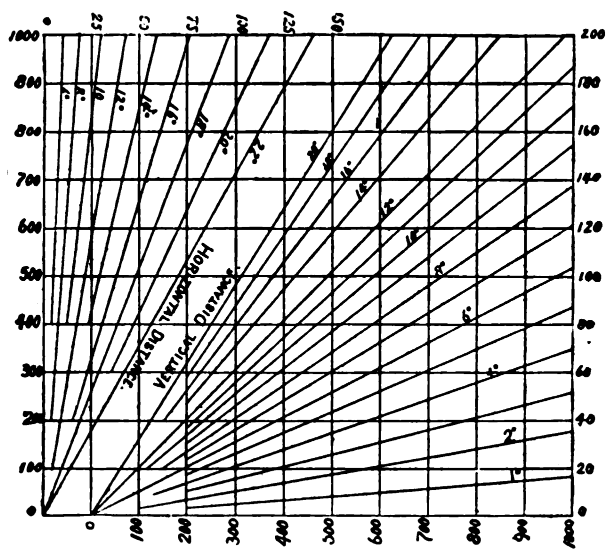


FIG. 78.—STADIA REDUCTION DIAGRAM.

Surveying Instruments," gives directly the corrections to horizontal distances in the upper horizontal line corresponding to the observed distances on the left-hand vertical line, and the angles indicated by diagonal intersecting lines. The right-hand vertical column of figures are differences of elevation. These are found by the intersection of the lines of

TABLE

DIFFERENCES OF ELEVATION

a	D 100	D 110	D 120	D 130	D 140	D 150	D 160	D 170	D 180	D 190	D 200	D 220	D 240	D 260
° ′														
0 01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1
0 02	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2
0 03	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2
0 04	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3
0 05	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.4
0 06	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.5
0 07	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4	0.5	0.5
0 08	0.2	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4	0.5	0.5	0.6	0.6
0 09	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.7
0 10	0.3	0.3	0.3	0.4	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.6	0.7	0.8
0 11	0.3	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.6	0.7	0.8	0.8
0 12	0.3	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.6	0.7	0.7	0.8	0.8	0.9
0 13	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.6	0.7	0.7	0.8	0.8	0.9	1.0
0 14	0.4	0.4	0.5	0.5	0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.9	1.0	1.1
0 15	0.4	0.5	0.5	0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1
0 16	0.5	0.5	0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.9	0.9	1.0	1.1	1.2
0 17	0.5	0.5	0.6	0.6	0.7	0.7	0.8	0.8	0.9	0.9	1.0	1.1	1.2	1.3
0 18	0.5	0.6	0.6	0.7	0.7	0.8	0.8	0.9	0.9	1.0	1.0	1.2	1.3	1.4
0 19	0.6	0.6	0.7	0.7	0.8	0.8	0.9	0.9	1.0	1.1	1.1	1.2	1.3	1.4
0 20	0.6	0.6	0.7	0.8	0.8	0.9	0.9	1.0	1.0	1.1	1.2	1.3	1.4	1.5
0 21	0.6	0.7	0.7	0.8	0.9	0.9	1.0	1.0	1.1	1.2	1.2	1.3	1.5	1.6
0 22	0.6	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.7
0 23	0.7	0.7	0.8	0.9	0.9	1.0	1.1	1.1	1.2	1.3	1.3	1.5	1.6	1.7
0 24	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.3	1.3	1.4	1.5	1.7	1.8
0 25	0.7	0.8	0.9	0.9	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.9
0 26	0.8	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.4	1.5	1.7	1.8	2.0
0 27	0.8	0.9	0.9	1.0	1.1	1.2	1.3	1.3	1.4	1.5	1.6	1.7	1.9	2.0
0 28	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.5	1.6	1.8	2.0	2.1
0 29	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.4	1.5	1.6	1.7	1.9	2.0	2.2
0 30	0.9	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.7	1.9	2.1	2.3
0 35	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6
0 40	1.2	1.3	1.4	1.5	1.6	1.7	1.9	2.0	2.1	2.2	2.3	2.6	2.8	3.0
0 45	1.3	1.4	1.6	1.7	1.8	2.0	2.1	2.2	2.4	2.5	2.6	2.9	3.1	3.4
0 50	1.5	1.6	1.7	1.9	2.0	2.2	2.3	2.5	2.6	2.8	2.9	3.2	3.5	3.8
0 55	1.6	1.8	1.9	2.1	2.2	2.4	2.6	2.7	2.9	3.0	3.2	3.5	3.8	4.2
1 00	1.7	1.9	201	2.3	2.4	2.6	2.8	3.0	3.1	3.3	3.5	3.8	4.2	4.5
1 10	2.0	2.2	2.4	2.6	2.9	3.1	3.3	3.5	3.7	3.9	4.1	4.5	4.9	5.3
1 20	2.3	2.6	2.8	3.0	3.3	3.5	3.7	4.0	4.2	4.4	4.7	5.1	5.6	6.0
1 30	2.6	2.9	3.1	3.4	3.7	3.9	4.2	4.4	4.7	5.0	5.2	5.8	6.3	6.8
1 40	2.9	3.2	3.5	3.8	4.1	4.4	4.7	4.9	5.2	5.5	5.8	6.4	7.0	7.6
1 50	3.2	3.5	3.8	4.2	4.5	4.8	5.1	5.4	5.8	6.1	6.4	7.0	7.7	8.3
2 00	3.5	3.8	4.2	4.5	4.9	5.2	5.6	5.9	6.3	6.6	7.0	7.7	8.4	9.1
2 10	3.8	4.2	4.5	4.9	5.3	5.7	6.0	6.4	6.8	7.2	7.6	8.3	9.1	9.8
2 20	4.1	4.5	4.9	5.3	5.7	6.1	6.5	6.9	7.3	7.7	8.1	8.9	9.8	10.6
2 30	4.4	4.8	5.2	5.7	6.1	6.5	7.0	7.4	7.8	8.3	8.7	9.6	10.5	11.3
2 40	4.6	5.1	5.6	6.0	6.5	7.0	7.4	7.9	8.4	8.8	9.3	10.2	11.2	12.1
2 50	4.9	5.4	5.9	6.4	6.9	7.4	7.9	8.4	8.9	9.4	9.9	10.9	11.8	12.8
3 00	5.2	5.7	6.3	6.8	7.3	7.8	8.4	8.9	9.4	9.9	10.5	11.5	12.5	13.6
4 00	7.0	7.7	8.4	9.0	9.7	10.4	11.1	11.8	12.5	13.2	13.9	15.3	16.7	18.1
5 00	8.7	9.6	10.4	11.3	12.2	13.0	13.9	14.8	15.6	16.5	17.4	19.1	20.8	22.6
a	D 100	D 110	D 120	D 130	D 140	D 150	D 160	D 170	D 180	D 190	D 200	D 220	D 240	D 260

X.

FROM STADIA MEASURES.

D 280	*D* 300	*D* 320	*D* 340	*D* 360	*D* 380	*D* 400	*D* 420	*D* 440	*D* 460	*D* 480	*D* 500	*D* 520	*D* 540
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.2
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.3
0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.4	0.4	0.5	0.5
0.3	0.3	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.5	0.6	0.6	0.6	0.6
0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.6	0.6	0.7	0.7	0.7	0.8	0.8
0.5	0.5	0.6	0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.8	0.9	0.9	0.9
0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.9	0.9	0.9	1.0	1.0	1.1	1.1
0.7	0.7	0.7	0.8	0.8	0.9	0.9	1.0	1.0	1.1	1.1	1.2	1.2	1.3
0.7	0.8	0.8	0.9	0.9	1.0	1.0	1.1	1.2	1.2	1.3	1.3	1.4	1.4
0.8	0.9	0.9	1.0	1.0	1.1	1.2	1.2	1.3	1.3	1.4	1.5	1.5	1.6
0.9	1.0	1.0	1.1	1.2	1.2	1.3	1.3	1.4	1.5	1.5	1.6	1.7	1.7
1.0	1.0	1.1	1.2	1.3	1.3	1.4	1.5	1.5	1.6	1.7	1.7	1.8	1.9
1.1	1.1	1.2	1.3	1.4	1.4	1.5	1.6	1.7	1.7	1.8	1.9	2.0	2.0
1.1	1.2	1.3	1.4	1.5	1.5	1.6	1.7	1.8	1.9	2.0	2.0	2.1	2.2
1.2	1.3	1.4	1.5	1.6	1.7	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4
1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.0	2.1	2.2	2.3	2.4	2.5
1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7
1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8
1.5	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.7	2.8	2.9	3.0
1.6	1.7	1.9	2.0	2.1	2.2	2.3	2.4	2.6	2.7	2.8	2.9	3.0	3.1
1.7	1.8	2.0	2.1	2.2	2.3	2.4	2.6	2.7	2.8	2.9	3.1	3.2	3.3
1.8	1.9	2.0	2.2	2.3	2.4	2.6	2.7	2.8	2.9	3.1	3.2	3.3	3.5
1.9	2.0	2.1	2.3	2.4	2.5	2.7	2.8	2.9	3.1	3.2	3.3	3.5	3.6
2.0	2.1	2.2	2.4	2.5	2.7	2.8	2.9	3.1	3.2	3.4	3.5	3.6	3.8
2.0	2.2	2.3	2.5	2.6	2.8	2.9	3.1	3.2	3.3	3.5	3.6	3.8	3.9
2.1	2.3	2.4	2.6	2.7	2.9	3.0	3.2	3.3	3.5	3.6	3.8	3.9	4.1
2.2	2.4	2.5	2.7	2.8	3.0	3.1	3.3	3.5	3.6	3.8	3.9	4.1	4.2
2.3	2.4	2.6	2.8	2.9	3.1	3.3	3.4	3.6	3.7	3.9	4.1	4.2	4.4
2.4	2.5	2.7	2.9	3.0	3.2	3.4	3.5	3.7	3.9	4.1	4.2	4.4	4.6
2.4	2.6	2.8	3.0	3.1	3.3	3.5	3.7	3.8	4.0	4.2	4.4	4.5	4.7
2.9	3.1	3.3	3.5	3.7	3.9	4.1	4.3	4.5	4.7	4.9	5.1	5.3	5.5
3.3	3.5	3.7	4.0	4.2	4.4	4.7	4.9	5.1	5.3	5.6	5.8	6.0	6.3
3.7	3.9	4.2	4.5	4.7	5.0	5.2	5.5	5.8	6.0	6.3	6.5	6.8	7.1
4.1	4.4	4.7	4.9	5.2	5.5	5.8	6.1	6.4	6.7	7.0	7.3	7.6	7.9
4.5	4.8	5.1	5.4	5.8	6.1	6.4	6.7	7.0	7.4	7.7	8.0	8.3	8.6
4.9	5.2	5.6	5.9	6.3	6.6	7.0	7.3	7.7	8.0	8.4	8.7	9.1	9.4
5.7	6.1	6.5	6.9	7.3	7.7	8.1	8.6	9.0	9.4	9.8	10.2	10.6	11.0
6.5	7.0	7.4	7.9	8.4	8.8	9.3	9.8	10.2	10.7	11.2	11.6	12.1	12.6
7.3	7.9	8.4	8.9	9.4	9.9	10.5	11.0	11.5	12.0	12.6	13.1	13.6	14.1
8.1	8.7	9.3	9.9	10.5	11.0	11.6	12.2	12.8	13.4	14.0	14.5	15.1	15.7
9.0	9.6	10.2	10.9	11.5	12.2	12.8	13.4	14.1	14.7	15.4	16.0	16.6	17.3
9.8	10.5	11.2	11.9	12.6	13.3	14.0	14.6	15.3	16.0	16.7	17.4	18.1	18.8
10.6	11.3	12.1	12.8	13.6	14.4	15.1	15.9	16.6	17.4	18.1	18.9	19.6	20.4
11.4	12.2	13.0	13.8	14.6	15.5	16.3	17.1	17.9	18.7	19.5	20.3	21.2	22.0
12.2	13.1	13.9	14.8	15.7	16.6	17.4	18.3	19.2	20.0	20.9	21.8	22.7	23.5
13.0	13.9	14.8	15.8	16.7	17.7	18.6	19.5	20.5	21.4	22.3	23.2	24.2	25.1
13.8	14.8	15.8	16.8	17.8	18.8	19.7	20.7	21.7	22.7	23.7	24.7	25.7	26.7
14.6	15.7	16.7	17.8	18.8	19.9	20.9	21.9	23.0	24.0	25.1	26.1	27.2	28.2
19.5	20.9	22.3	23.7	25.1	26.4	27.8	29.2	30.6	32.0	33.4	34.8	36.2	37.6
24.3	26.0	27.8	29.5	31.3	33.0	34.7	36.5	38.2	39.9	41.7	43.4	45.1	46.9
D 280	*D* 300	*D* 320	*D* 340	*D* 360	*D* 380	*D* 400	*D* 420	*D* 440	*D* 460	*D* 480	*D* 500	*D* 520	*D* 540

TABLE

DIFFERENCES OF ELEVATION

α	D 560	D 580	D 600	D 620	D 640	D 660	D 680	D 700	D 720	D 740	D 760	D 780	D 800	D 820
° ′														
0 01	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0 02	0.3	0.3	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.5	0.5	0.5
0 03	0.5	0.5	0.5	0.5	0.6	0.6	0.6	0.6	0.6	0.6	0.7	0.7	0.7	0.7
0 04	0.6	0.7	0.7	0.7	0.7	0.8	0.8	0.8	0.8	0.9	0.9	0.9	0.9	1.0
0 05	0.8	0.8	0.9	0.9	0.9	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.2	1.2
0 06	1.0	1.0	1.1	1.1	1.1	1.2	1.2	1.2	1.3	1.3	1.3	1.4	1.4	1.4
0 07	1.1	1.2	1.2	1.3	1.3	1.3	1.4	1.4	1.5	1.5	1.6	1.6	1.6	1.7
0 08	1.3	1.4	1.4	1.4	1.5	1.5	1.6	1.6	1.7	1.7	1.8	1.8	1.9	1.9
0 09	1.5	1.5	1.6	1.6	1.7	1.7	1.8	1.8	1.9	1.9	2.0	2.0	2.1	2.1
0 10	1.6	1.7	1.7	1.8	1.9	1.9	2.0	2.0	2.1	2.2	2.2	2.3	2.3	2.4
0 11	1.8	1.9	1.9	2.0	2.0	2.1	2.2	2.2	2.3	2.4	2.4	2.5	2.6	2.6
0 12	2.0	2.0	2.1	2.2	2.2	2.3	2.4	2.4	2.5	2.6	2.7	2.7	2.8	2.9
0 13	2.1	2.2	2.3	2.3	2.4	2.5	2.6	2.6	2.7	2.8	2.9	2.9	3.0	3.1
0 14	2.3	2.4	2.4	2.5	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.3
0 15	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.1	3.2	3.3	3.4	3.5	3.6
0 16	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.3	3.4	3.5	3.6	3.7	3.8
0 17	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1
0 18	2.9	3.0	3.1	3.2	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3
0 19	3.1	3.2	3.3	3.4	3.5	3.6	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5
0 20	3.3	3.4	3.5	3.6	3.7	3.8	4.0	4.1	4.2	4.3	4.4	4.5	4.7	4.8
0 21	3.4	3.5	3.7	3.8	3.9	4.0	4.2	4.3	4.4	4.5	4.6	4.8	4.9	5.0
0 22	3.6	3.7	3.8	4.0	4.1	4.2	4.4	4.5	4.6	4.7	4.9	5.0	5.1	5.2
0 23	3.7	3.9	4.0	4.1	4.3	4.4	4.5	4.7	4.8	5.0	5.1	5.2	5.4	5.5
0 24	3.9	4.0	4.2	4.3	4.5	4.6	4.7	4.9	5.0	5.2	5.3	5.4	5.6	5.7
0 25	4.1	4.2	4.4	4.5	4.7	4.8	4.9	5.1	5.2	5.4	5.5	5.7	5.8	6.0
0 26	4.2	4.4	4.5	4.7	4.8	5.0	5.1	5.3	5.4	5.6	5.7	5.9	6.0	6.2
0 27	4.4	4.6	4.7	4.9	5.0	5.2	5.3	5.5	5.7	5.8	6.0	6.1	6.3	6.4
0 28	4.6	4.7	4.9	5.0	5.2	5.4	5.5	5.7	5.9	6.0	6.2	6.3	6.5	6.7
0 29	4.7	4.9	5.1	5.2	5.4	5.6	5.7	5.9	6.1	6.2	6.4	6.6	6.8	6.9
0 30	4.9	5.1	5.2	5.4	5.6	5.8	5.9	6.1	6.3	6.5	6.6	6.8	7.0	7.2
0 35	5.7	5.9	6.1	6.3	6.5	6.7	6.9	7.1	7.3	7.5	7.7	7.9	8.1	8.4
0 40	6.5	6.7	7.0	7.2	7.4	7.7	7.9	8.1	8.4	8.6	8.8	9.1	9.3	9.5
0 45	7.3	7.6	7.9	8.1	8.4	8.6	8.9	9.2	9.4	9.7	9.9	10.2	10.5	10.7
0 50	8.1	8.4	8.7	9.0	9.3	9.6	9.9	10.2	10.5	10.8	11.1	11.3	11.6	11.9
0 55	9.0	9.3	9.6	9.9	10.2	10.6	10.9	11.2	11.5	11.8	12.2	12.5	12.8	13.1
1 00	9.8	10.1	10.5	10.8	11.2	11.5	11.9	12.2	12.6	12.9	13.3	13.6	14.0	14.3
1 10	11.4	11.8	12.2	12.6	13.0	13.4	13.8	14.3	14.7	15.1	15.5	15.9	16.3	16.7
1 20	13.0	13.5	14.0	14.4	14.9	15.4	15.8	16.3	16.7	17.2	17.7	18.1	18.6	19.1
1 30	14.7	15.2	15.7	16.2	16.7	17.3	17.8	18.3	18.8	19.4	19.9	20.4	20.9	21.5
1 40	16.3	16.9	17.4	18.0	18.6	19.2	19.8	20.3	20.9	21.5	22.1	22.7	23.3	23.8
1 50	17.9	18.5	19.2	19.8	20.5	21.1	21.7	22.4	23.0	23.7	24.3	24.9	25.6	26.2
2 00	19.5	20.2	20.9	21.6	22.3	23.0	23.7	24.4	25.1	25.8	26.5	27.2	27.9	28.6
2 10	21.2	21.9	22.7	23.4	24.2	24.9	25.7	26.4	27.2	28.0	28.7	29.5	30.2	31.0
2 20	22.8	23.6	24.4	25.2	26.0	26.8	27.7	28.5	29.3	30.1	30.9	31.7	32.5	33.4
2 30	24.4	25.3	26.1	27.0	27.9	28.8	29.6	30.5	31.4	32.2	33.1	34.0	34.9	35.7
2 40	26.0	27.0	27.9	28.8	29.7	30.7	31.6	32.5	33.5	34.4	35.3	36.3	37.2	38.1
2 50	27.6	28.6	29.6	30.6	31.6	32.6	33.6	34.6	35.5	36.5	37.5	38.5	39.5	40.5
3 00	29.3	30.3	31.4	32.4	33.4	34.5	35.5	36.6	37.6	38.7	39.7	40.8	41.8	42.9
4 00	39.0	40.4	41.8	43.1	44.5	45.9	47.3	48.7	50.1	51.5	52.9	54.3	55.7	57.1
5 00	48.6	50.4	52.1	53.8	55.6	57.3	59.0	60.8	62.5	64.2	66.0	67.7	69.5	71.2
α	D 560	D 580	D 600	D 620	D 640	D 660	D 680	D 700	D 720	D 740	D 760	D 780	D 800	D 820

X.

FROM STADIA MEASURES.

D 840	*D* 860	*D* 880	*D* 900	*D* 920	*D* 940	*D* 960	*D* 980	*D* 1000	*D* 1100	*D* 1200	*D* 1300	*D* 1400	*D* 1500	*D* 2000
0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.6
0.5	0.5	0.5	0.5	0.5	0.5	0.6	0.6	0.6	0.6	0.7	0.8	0.8	0.9	1.2
0.7	0.7	0.8	0.8	0.8	0.8	0.8	0.9	0.9	1.0	1.0	1.1	1.2	1.3	1.7
1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.3
1.2	1.2	1.3	1.3	1.3	1.4	1.4	1.4	1.5	1.6	1.7	1.9	2.0	2.2	2.9
1.5	1.5	1.5	1.6	1.6	1.6	1.7	1.7	1.7	1.9	2.1	2.3	2.4	2.6	3.5
1.7	1.8	1.8	1.8	1.9	1.9	2.0	2.0	2.0	2.2	2.4	2.7	2.9	3.1	4.1
2.0	2.0	2.1	2.1	2.1	2.2	2.2	2.3	2.3	2.6	2.8	3.0	3.3	3.5	4.7
2.2	2.3	2.3	2.4	2.4	2.5	2.5	2.6	2.6	2.9	3.1	3.4	3.7	3.9	5.2
2.4	2.5	2.6	2.6	2.7	2.7	2.8	2.9	2.9	3.2	3.5	3.8	4.1	4.4	5.8
2.7	2.8	2.8	2.9	2.9	3.0	3.1	3.1	3.2	3.5	3.8	4.2	4.5	4.8	6.4
2.9	3.0	3.1	3.1	3.2	3.3	5.4	3.4	3.5	3.8	4.2	4.5	4.9	5.2	7.0
3.2	3.3	3.3	3.4	3.5	3.6	3.6	3.7	3.8	4.2	4.5	4.9	5.3	5.7	7.6
3.4	3.5	3.6	3.7	3.7	3.8	3.9	4.0	4.1	4.5	4.9	5.3	5.7	6.1	8.1
3.7	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.8	5.2	5.7	6.1	6.5	8.7
3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	5.1	5.6	6.0	6.5	7.0	9.3
4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.4	5.9	6.4	6.9	7.4	9.9
4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.8	6.3	6.8	7.3	7.9	10.5
4.6	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	6.1	6.6	7.2	7.7	8.3	11.1
4.9	5.0	5.1	5.2	5.4	5.5	5.6	5.7	5.8	6.4	7.0	7.5	8.1	8.7	11.6
5.1	5.3	5.4	5.5	5.6	5.7	5.9	6.0	6.1	6.7	7.3	7.9	8.6	9.2	12.2
5.4	5.5	5.6	5.8	5.9	6.0	6.1	6.3	6.4	7.0	7.7	8.3	9.0	9.6	12.8
5.6	5.8	5.9	6.0	6.2	6.3	6.4	6.6	6.7	7.4	8.0	8.7	9.4	10.0	13.4
5.9	6.0	6.1	6.3	6.4	6.6	6.7	6.8	7.0	7.7	8.4	9.1	9.8	10.5	14.0
6.1	6.3	6.4	6.5	6.7	6.8	7.0	7.1	7.3	8.0	8.7	9.5	10.2	10.9	14.5
6.4	6.5	6.7	6.8	7.0	7.1	7.3	7.4	7.6	8.3	9.1	9.8	10.5	11.3	15.1
6.6	6.8	6.9	7.1	7.2	7.4	7.5	7.7	7.9	8.6	9.4	10.2	11.0	11.8	15.7
6.8	7.0	7.2	7.3	7.5	7.7	7.8	8.0	8.1	9.0	9.7	10.6	11.4	12.2	16.3
7.1	7.3	7.4	7.6	7.8	7.9	8.1	8.3	8.4	9.3	10.1	11.0	11.8	12.7	16.9
7.3	7.5	7.7	7.9	8.0	8.2	8.4	8.6	8.7	9.6	10.5	11.3	12.2	13.1	17.5
8.6	8.8	9.0	9.2	9.4	9.6	9.8	10.0	10.2	11.2	12.2	13.2	14.3	15.3	20.4
9.8	10.0	10.2	10.5	10.6	10.9	11.2	11.4	11.6	12.8	14.0	15.1	16.3	17.4	23.3
11.0	11.3	11.5	11.8	12.0	12.3	12.6	12.8	13.1	14.4	15.7	17.0	18.3	19.6	26.2
12.2	12.5	12.8	13.1	13.4	13.7	14.0	14.2	14.5	16.0	17.4	18.9	20.3	21.8	29.1
13.4	13.8	14.1	14.4	14.7	15.0	15.4	15.7	16.0	17.6	19.2	20.8	22.4	24.0	32.0
14.7	15.0	15.4	15.7	16.1	16.4	16.8	17.1	17.5	19.2	20.9	22.7	24.4	26.2	34.9
17.1	17.5	17.9	18.3	18.7	19.1	19.5	20.0	20.4	22.4	24.4	26.5	28.5	30.5	40.7
19.5	20.0	20.5	20.9	21.4	21.9	22.3	22.8	23.3	25.6	27.9	30.2	32.6	34.9	46.5
22.0	22.5	23.0	23.6	24.1	24.6	25.1	25.6	26.2	28.8	31.4	34.0	36.6	39.3	52.3
24.4	25.0	25.6	26.2	26.7	27.3	27.9	28.5	29.1	32.0	34.9	37.8	40.7	43.6	58.1
26.9	27.5	28.1	28.8	29.4	30.1	30.7	31.3	32.0	35.2	38.4	41.6	44.8	48.0	64.0
29.3	30.0	30.7	31.4	32.1	32.8	33.5	34.2	34.9	38.4	41.9	45.3	48.8	52.3	69.8
31.7	32.5	33.2	34.0	34.8	35.5	36.3	37.0	37.8	41.6	45.3	49.1	52.9	56.7	75.6
34.2	35.0	35.8	36.6	37.4	38.2	39.1	39.9	40.7	44.7	48.8	52.9	57.0	61.0	81.4
36.6	37.5	38.4	39.2	40.1	41.0	41.8	42.7	43.6	47.9	52.3	56.7	61.0	65.4	87.2
39.0	40.0	40.9	41.8	42.8	43.7	44.6	45.6	46.5	51.1	55.8	60.4	65.1	69.7	93.0
41.5	42.5	43.4	44.4	45.4	46.4	47.4	48.4	49.4	54.3	59.2	64.2	69.1	74.1	98.7
43.9	44.9	46.0	47.0	48.1	49.1	50.2	51.2	52.3	57.5	62.7	67.9	73.2	78.4	104.5
58.5	59.8	61.2	62.6	64.0	65.4	66.8	68.2	69.6	76.5	83.5	90.5	97.4	104.4	139.2
72.9	74.7	76.4	78.1	79.9	81.6	83.3	85.1	86.8	95.5	104.2	112.9	121.5	130.2	173.6
D 840	*D* 860	*D* 880	*D* 900	*D* 920	*D* 940	*D* 960	*D* 980	*D* 1000	*D* 1100	*D* 1200	*D* 1300	*D* 1400	*D* 1500	*D* 2000

horizontal distances indicated in the lower margin, with the diagonal lines corresponding to angles of elevation.

Example: Let rod intercept be 3.60 feet, and the angle 2° 40′; then it will be seen that the correction to the horizontal distance is too small to note on the diagram. The difference of elevation comes from the intersection of a vertical line between 300 and 400, and the right diagonal between 2° and 3°, and is approximately 16 to 18 feet.

110. Diagram for Reducing Inclined Stadia Distances to Horizontal.—The following diagram (Fig. 79), prepared by Prof. Ira O. Baker, gives with some accuracy the correction to the horizontal distance corresponding with any observed angle of inclination in the stadia-rod. The observed distance is indicated on the lower and right-hand marginal lines, the angle of inclination by the intersecting diagonal lines, and the correction to the horizontal distance, always minus, is given on the upper and left-hand marginal lines.

Another though more complicated diagram for the same purpose is that illustrated in Fig. 80, designed by Mr. E. McCulloch and published in *Engineering News*. This diagram has a wide range and is well suited to the most detailed work. On the lower and left-hand outer margins are figured stadia readings in feet or meters, decimals of the same being interpolated on the inner margin on all four sides, where the angles of inclination are also indicated. Corrections to observed distances are found at the intersections of the vertical rod readings with the horizontal angle lines, or *vice versa*, the horizontal rod lines with the vertical angle lines, and by following out to the margins the diagonals at which these intersections occur; opposite the ends of the diagonals will be found the corrections in feet to the distances observed, such corrections being on all four outer margins.

This correction is always to be subtracted from the distance and is applied in the following manner: If the angle appears on the left margin, multiply the correction by 0.01;

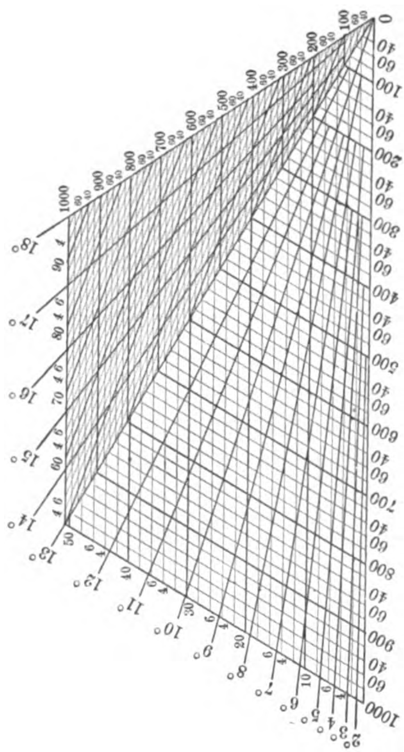


FIG. 79.—STADIA REDUCTION DIAGRAM TO HORIZONTAL DISTANCES.

if on the top margin, multiply the correction by 0.1; if on the bottom margin, multiply the correction by 1.0; and if on the right-hand margin, multiply the correction by 10.

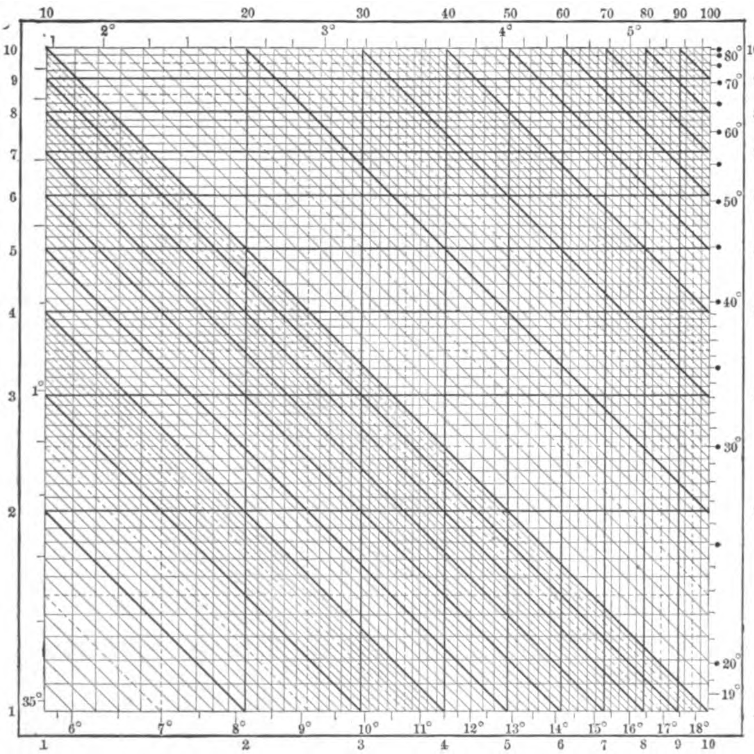


FIG. 80.—DIAGRAM FOR REDUCING INCLINED STADIA DISTANCES TO HORIZONTAL.

111. Effects of Refraction on Stadia Measurements.—Experiments by Mr. J. L. Van Ornum in his stadia-work showed that the disturbing effect of refraction increased enormously toward the ground, even if the foot of the rod were unobstructed. When no error in observation is made and the refraction remains constant from the time of the foresight to the time of the backsight, the elevation of any point can be computed by the formula

$$H' = H + \tfrac{1}{2}(h + m + n - h'), \quad . \quad . \quad . \quad (13)$$

in which H = elevation of known station;
h = height of instrument at that station;
m = total vertical component of the foresight;
H' = elevation of the unknown station;
h' = height of the instrument at the unknown station; and
n = the total vertical component of the backsight.

The average error of closing in Mr. Van Ornum's work before the adoption of this formula was more than half greater than after its use.

The effect of differential refraction on the determination of stadia distances is well set forth in a paper on stadia measurements by Mr. Leonard S. Smith of the University of Wisconsin. In these experiments Mr. Smith ascertained that refraction is a variable quantity, dependent on variable temperature of air and ground, and that it is much greater near the ground than 3 feet above it; also greater at noon than before or after it; that the effects vary for different distances and also for different observers. He called *differential refraction* that due to the different amount of refraction on the line of sight of the upper stadia-hair and that on the lower stadia-hair. He further found that the effects of refraction accumulated as distance increased. Twelve miles of stadia measurements with centers averaging 600 feet, in the morning and evening hours, showed an accuracy of plus 1 in 2685. The same distances measured by centers at midday showed an accuracy of minus 1 in 655. Again, 30 miles of measurements made in the morning and evening hours with centers between one and two thousand feet showed an accumulated error of but 1 in 1741, while the same distance, in midday, developed an accumulative error of 1 in 289.

The practical results of these experiments may be taken chiefly as suggestions, and the most interesting deductions to be obtained therefrom are:

1. To obtain accuracy in stadia-work it is best to obtain an interval error of the rod for the effect of refraction at different hours of the day; and

2. This correction for refraction may be made to readings for the hours corresponding to the refraction ascertained, or the stadia-rod may be graduated in proportion to the distances of the various intercepts above the ground; this latter method is not recommended for its simplicity, however.

It is evident that at midday long readings which require the lower stadia-wire to be lower than 3 feet from the bottom of the rod should not be taken. Where a rod interval is determined for correction of refraction, that interval which is determined for summer months or midday should not be used in colder months or winter, without testing by an independent interval. Moreover, such interval should not be determined for ordinary soil when the work is to be conducted over curbstones, in cities.

To sum Mr. Smith's results it may be stated that the time of greatest atmospheric vibration is about the middle of the forenoon, when the maximum difference in temperature occurs between atmosphere and ground; the stadia interval should be determined, not at any particular time of day, but at many hours during the day; and such interval for the hours should be selected as will approximate as closely as possible the average field conditions. Perhaps one of the simplest ways of applying the interval is not by dividing the rod unequally by incorporating the interval in the rod divisions, but by using a rod divided to standard lengths and computing distances by means of an interval factor.

112. Stadia-rods.—Several methods of dividing stadia-rods for very accurate work have been used, such as making special subdivisions of the rod to correspond with distances subtended between fixed hairs, or of dividing the rod irregularly so as to incorporate within the divisions a stadia interval which will correct the effects of refraction. The most approved practice is, however, to use *rods of standard division* and to prepare tables, or, better still, a rod interval factor to be applied to the observed distance as a correction. The following are some of the disadvantages of using other than standard rods:

1. Subsequent tests for interval cannot be made without the expense of repainting and regraduating the rods;

2. Rods specially divided cannot be interchanged among instruments; such rods cannot be used in leveling without computing the necessary correction;

3. Leveling-rods cannot be used in stadia-work; and

4. Observers with different personal equations cannot use the same rods without causing systematic errors in the work.

Stadia-rods are of two general forms:

1. Target; and

2. Self-reading.

The former is the only one which can be satisfactorily used in taking very long sights; the latter is most satisfactorily employed on short sights. *Target-rods* are similar to the simpler forms of leveling-rods, as the Philadelphia rod (Fig. 96). *Self-reading* or *speaking* rods may be of the type of the Philadelphia rod, for short distances, but are more satisfactorily made of flat boards 10 to 15 feet in length, 4 inches wide, and $\frac{3}{4}$ to $\frac{7}{8}$ inch thick, of well-seasoned pine. These can be graduated by the topographer or by some painter after one of the numerous patterns which have been found satisfactory under various circumstances. Such forms of graduation are better than the ordinary self-reading leveling-rods, because as a rule divisions on the latter are small and the figures small and they are therefore difficult to read at long distances. As visibility is the first requisite in a good stadia-rod, the graduations should consist of a number of divisions so large and yet of such varying shapes as to make them readily distinguishable at long distances, the pattern being either painted or stenciled on the wood or else on canvas or paper which may be fastened to the rod with glue, varnish, or both.

In Fig. 81 are shown two of the best forms of *rod*

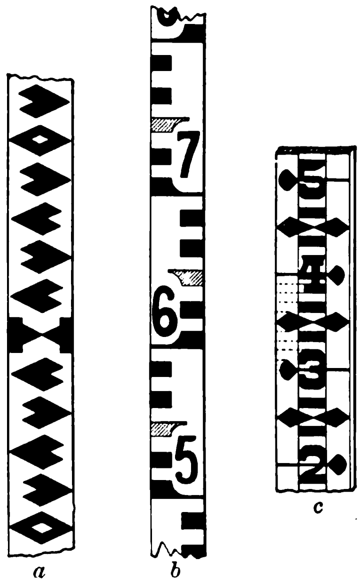


FIG. 81.—SPEAKING STADIA AND LEVEL-RODS.

graduation for long sights. Rod (*a*) is well adapted to either metric or foot graduation, but in its use care must be exercised to accurately count the units, as these are not numbered. Rod (*b*) has the feet only numbered, but the figures are so large as to be readily legible at very considerable distances. The forms of the figures on rod (*c*) are such as to clearly define the tenths.

In Fig. 97 are shown patterns of *speaking level-rods*, and these are also well adapted to stadia-work at short distances. All are suited to either metric or foot graduation. Rod (*a*) is perhaps best suited to close, detailed work. Rod (*b*) has the clearest figures and is therefore best suited to longer distances. Rod (*c*) has the advantage of having odd and even tenths figured on opposite sides of the center of the rod, with the order changed for units so as to bring their numberings nearer the rod center; moreover, the graduations are less liable to injury because they are in the center of the rod. For rough work at long distances a pole may be cut and white string or cloth tied around it at every foot to serve as graduation.

CHAPTER XIII.

ANGULAR TACHYMETRY.

113. Angular Tachymetry with Transit or Theodolite. —The angular system of tachymetry has the advantages—

1. Of enabling the survey to be made by the ordinary transit without the addition of stadia-wires;
2. That there are no divisions of the rod to be read; and
3. That, because of the ease and simplicity of the observations, very long sights can be taken with a small telescope with great accuracy.

The *field-work* by this method is nearly as rapid as that by any other form of tachymetry, as there are but two angles to be read, as against one angle and a rod reading with stadia method. The reduction of the work is less simple and more laborious than that by means of stadia measurements. That this method is as accurate as the stadia method appears from experiments made by Messrs. Airy and Middleton in England. The former used a five-inch theodolite and an ordinary leveling-rod 16 feet in length. He ran a circuit $1\frac{1}{2}$ miles in length, involving differences in elevation of 118 feet: the average length of stations was 341 feet, and the resulting probable error of distance was 3.64 feet in a mile, and of leveling 0.033 feet. The average closure error of four series of measurements was 2.84 feet. According to Mr. Middleton the limit of accuracy in this method is reached when the rod is held at a distance of 1000 feet, as determined from his measurements, and the average error in one mile was found to be 6.15 feet.

The accompanying diagram (Fig. 82) illustrates the method

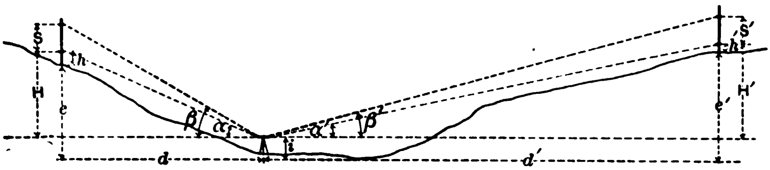


FIG. 82.—ANGULAR TACHYMETRY.

of angular tachymetry with a transit or theodolite. For simplicity the rod should be unmarked excepting for two or three broad lines painted at known distances apart, one near the bottom, one near the top, and one near the middle. These lines, if properly proportioned, can be seen at distances as great as a mile, and thus permit of tachymetric measurements of moderate accuracy for that distance.

Let s = distance on the rod between two well-defined marks;

h and h' = height of the lower marks above the ground;

H and H' = height of the lower marks above line of collimation of instrument;

e and e' = height of ground surface at foot of rod above base or ground surface at instrument;

i = height of collimation of instrument above ground surface;

α = angle at instrument between horizon and lower mark;

β = angle at instrument between horizon and upper mark;

d and d' = horizontal distance from instrument to rods;—then

$$s = d(\tan \beta - \tan \alpha). \quad . \quad . \quad . \quad . \quad (14)$$

Transposing, we have

$$d = \frac{s}{\tan\beta - \tan\alpha}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

also

$$.H = d \tan\alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Substituting, we have

$$H = \frac{s \tan\alpha}{\tan\beta - \tan\alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

It is also seen from an inspection of the figure that approximately

$$e = H - h + i. \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

From these the height of the ground at observed stations can be reduced to sea-level by addition of the height of ground at instrument. The above computations are easily made with the aid of a table of natural functions (Tables XI and XII). H and H' must have their proper signs, and will result in giving the difference of level according as the new stations are above or below the position of the instrument. From the expression dH it may be shown that a small angular error will affect the horizontal distance by nearly the same amount, whether the ground be level or steep, but will affect the vertical height very much more on steep than on level ground. It may be further shown that it is necessary to keep d tolerably small and s as large as possible, especially on steep ground.

114. Measuring Distances with Gradienter.—The gradienter is used as a telemeter in measuring horizontal distances in two ways: first, by measuring the space on the rod passed over by the horizontal cross-hair for a given number of revolutions of the gradienter screw; second, by noticing the number of revolutions of the screw required to carry the horizontal cross-hair over a fixed space on the rod. Gradienter screws are so made that a single revolution may carry the

TABLE XI.—NATURAL SINES AND COSINES.

(From the Smithsonian Tables.)

NATURAL SINES.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′
0°	.0000 00	.0029 09	.0058 18	.0087 27	.0116 35	.0145 44	.0174 52	**89°**	2.9
1	.0174 52	.0203 6	.0232 7	.0261 8	.0290 8	.0319 9	.0349 0	88	2.9
2	.0349 0	.0378 1	.0407 1	.0436 2	.0465 3	.0494 3	.0523 4	87	2.9
3	.0523 4	.0552 4	.0581 4	.0610 5	.0639 5	.0668 5	.0697 6	86	2.9
4	.0697 6	.0726 6	.0755 6	.0784 6	.0813 6	.0842 6	.0871 6	85	2.9
5	.0871 6	.0900 5	.0929 5	.0958 5	.0987 4	.1016 4	.1045 3	**84**	2.9
6	.1045 3	.1074 2	.1103 1	.1132 0	.1160 9	.1189 8	.1218 7	83	2.9
7	.1218 7	.1247 6	.1276 4	.1305 3	.1334	.1363	.1392	82	2.9
8	.1392	.1421	.1449	.1478	.1507	.1536	.1564	81	2.9
9	.1564	.1593	.1622	.1650	.1679	.1708	.1736	80	2.9
10	.1736	.1765	.1794	.1822	.1851	.1880	.1908	**79**	2.9
11	.1908	.1937	.1965	.1994	.2022	.2051	.2079	78	2.9
12	.2079	.2108	.2136	.2164	.2193	.2221	.2250	77	2.8
13	.2250	.2278	.2306	.2334	.2363	.2391	.2419	76	2.8
14	.2419	.2447	.2476	.2504	.2532	.2560	.2588	75	2.8
15	.2588	.2616	.2644	.2672	.2700	.2728	.2756	**74**	2.8
16	.2756	.2784	.2812	.2840	.2868	.2896	.2924	73	2.8
17	.2924	.2952	.2979	.3007	.3035	.3062	.3090	72	2.8
18	.3090	.3118	.3145	.3173	.3201	.3228	.3256	71	2.8
19	.3256	.3283	.3311	.3338	.3365	.3393	.3420	70	2.7
20	.3420	.3448	.3475	.3502	.3529	.3557	.3584	**69**	2.7
21	.3584	.3611	.3638	.3665	.3692	.3719	.3746	68	2.7
22	.3746	.3773	.3800	.3827	.3854	.3881	.3907	67	2.7
23	.3907	.3934	.3961	.3987	.4014	.4041	.4067	66	2.7
24	.4067	.4094	.4120	.4147	.4173	.4200	.4226	65	2.7
25	.4226	.4253	.4279	.4305	.4331	.4358	.4384	**64**	2.6
26	.4384	.4410	.4436	.4462	.4488	.4514	.4540	63	2.6
27	.4540	.4566	.4592	.4617	.4643	.4669	.4695	62	2.6
28	.4695	.4720	.4746	.4772	.4797	.4823	.4848	61	2.6
29	.4848	.4874	.4899	.4924	.4950	.4975	.5000	60	2.5
30	.5000	.5025	.5050	.5075	.5100	.5125	.5150	**59**	2.5
31	.5150	.5175	.5200	.5225	.5250	.5275	.5299	58	2.5
32	.5299	.5324	.5348	.5373	.5398	.5422	.5446	57	2.5
33	.5446	.5471	.5495	.5519	.5544	.5568	.5592	56	2.4
34	.5592	.5616	.5640	.5664	.5688	.5712	.5736	55	2.4
35	.5736	.5760	.5783	.5807	.5831	.5854	.5878	**54**	2.4
36	.5878	.5901	.5925	.5948	.5972	.5995	.6018	53	2.3
37	.6018	.6041	.6065	.6088	.6111	.6134	.6157	52	2.3
38	.6157	.6180	.6202	.6225	.6248	.6271	.6293	51	2.3
39	.6293	.6316	.6338	.6361	.6383	.6406	.6428	50	2.3
40	.6428	.6450	.6472	.6494	.6517	.6539	.6561	**49**	2.2
41	.6561	.6583	.6604	.6626	.6648	.6670	.6691	48	2.2
42	.6691	.6713	.6734	.6756	.6777	.6799	.6820	47	2.2
43	.6820	.6841	.6862	.6884	.6905	.6926	.6947	46	2.1
44	.6947	.6967	.6988	.7009	.7030	.7050	.7071	45	2.1
	60′	50′	40′	30′	20′	10′	0′	Angle	

NATURAL COSINES.

TABLE XI.—NATURAL SINES AND COSINES.

NATURAL SINES.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′
45°	.7071	.7092	.7112	.7133	.7153	.7173	.7193	44°	2.0
46	.7193	.7214	.7234	.7254	.7274	.7294	.7314	43	2.0
47	.7314	.7333	.7353	.7373	.7392	.7412	.7431	42	2.0
48	.7431	.7451	.7479	.7490	.7509	.7528	.7547	41	1.9
49	.7547	.7566	.7585	.7604	.7623	.7642	.7660	40	1.9
50	.7660	.7679	.7698	.7716	.7735	.7753	.7771	39	1.9
51	.7771	.7790	.7808	.7826	.7844	.7862	.7880	38	1.8
52	.7880	.7898	.7916	.7934	.7951	.7969	.7986	37	1.8
53	.7986	.8004	.8021	.8039	.8056	.8073	.8090	36	1.7
54	.8090	.8107	.8124	.8141	.8158	.8175	.8192	35	1.7
55	.8192	.8208	.8225	.8241	.8258	.8274	.8290	34	1.6
56	.8290	.8307	.8323	.8339	.8355	.8371	.8387	33	1.6
57	.8387	.8403	.8418	.8434	.8450	.8465	.8480	32	1.6
58	.8480	.8496	.8511	.8526	.8542	.8557	.8572	31	1.5
59	.8572	.8587	.8601	.8616	.8631	.8646	.8660	30	1.5
60	.8660	.8675	.8689	.8704	.8718	.8732	.8746	29	1.4
61	.8746	.8760	.8774	.8788	.8802	.8816	.8829	28	1.4
62	.8829	.8843	.8857	.8870	.8884	.8897	.8910	27	1.4
63	.8910	.8923	.8936	.8949	.8962	.8975	.8988	26	1.3
64	.8988	.9001	.9013	.9026	.9038	.9051	.9063	25	1.3
65	.9063	.9075	.9088	.9100	.9112	.9124	.9135	24	1.2
66	.9135	.9147	.9159	.9171	.9182	.9194	.9205	23	1.2
67	.9205	.9216	.9228	.9239	.9250	.9261	.9272	22	1.1
68	.9272	.9283	.9293	.9304	.9315	.9325	.9336	21	1.1
69	.9336	.9346	.9356	.9367	.9377	.9387	.9397	20	1.0
70	.9397	.9407	.9417	.9426	.9436	.9446	.9455	19	1.0
71	.9455	.9465	.9474	.9483	.9492	.9502	.9511	18	0.9
72	.9511	.9520	.9528	.9537	.9546	.9555	.9563	17	0.9
73	.9563	.9572	.9580	.9588	.9596	.9605	.9613	16	0.8
74	.9613	.9621	.9628	.9636	.9644	.9652	.9659	15	0.8
75	.9659	.9667	.9674	.9681	.9689	.9696	.9703	14	0.7
76	.9703	.9710	.9717	.9724	.9730	.9737	.9744	13	0.7
77	.9744	.9750	.9757	.9763	.9769	.9775	.9781	12	0.6
78	.9781	.9787	.9793	.9799	.9805	.9811	.9816	11	0.6
79	.9816	.9822	.9827	.9833	.9838	.9843	.9848	10	0.5
80	.9848	.9853	.9858	.9863	.9868	.9872	.9877	9	0.5
81	.9877	.9881	.9886	.9890	.9894	.9899	.9903	8	0.4
82	.9903	.9907	.9911	.9914	.9918	.9922	.9925	7	0.4
83	.9925	.9929	.9932	.9936	.9939	.9942	.9945	6	0.3
84	.9945	.9948	.9951	.9954	.9957	.9959	.9962	5	0.3
85	.9962	.9964	.9967	.9969	.9971	.9974	.9976	4	0.2
86	.9976	.9978	.9980	.9981	.9983	.9985	.9986	3	0.2
87	.9986	.9988	.9989	.9990	.9992	.9993	.9994	2	0.1
88	.9994	.9995	.9996	.9997	.9997	.9998	.9998	1	0.1
89	.9998	.9999	.9999	1.0000	1.0000	1.0000	1.0000	0	0.0
	60′	50′	40′	30′	20′	10′	0′	Angle.	

NATURAL COSINES.

TABLE XII.—NATURAL TANGENTS AND COTANGENTS.

(From the Smithsonion Tables.)

NATURAL TANGENTS.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′
0°	.0000 0	.0029 1	.0058 2	.0087 3	.0116 4	.0145 5	.0174 6	89°	2.9
1	.0174 6	.0203 6	.0232 8	.0261 9	.0291 0	.0320 1	.0349 2	88	2.9
2	.0349 2	.0378 3	.0407 5	.0436 6	.0465 8	.0494 9	.0524 1	87	2.9
3	.0524 1	.0553 3	.0582 4	.0611 6	.0640 8	.0670 0	.0699 3	86	2.9
4	.0699 3	.0728 5	.0757 8	.0787 0	.0816 3	.0845 6	.0874 9	85	2.9
5	.0874 9	.0904 2	.0933 5	.0962 9	.0992 3	.1021 6	.1051 0	84	2.9
6	.1051 0	.1080 5	.1109 9	.1139 4	.1168 8	.1198 3	.1227 8	83	2.9
7	.1227 8	.1257 4	.1286 9	.1316 5	.1346	.1376	.1405	82	3.0
8	.1405	.1435	.1465	.1495	.1524	.1554	.1584	81	3.0
9	.1584	.1614	.1644	.1673	.1703	.1733	.1763	80	3.0
10	.1763	.1793	.1823	.1853	.1883	.1914	.1944	79	3.0
11	.1944	.1974	.2004	.2035	.2065	.2095	.2126	78	3.0
12	.2126	.2156	.2186	.2217	.2247	.2278	.2309	77	3.1
13	.2309	.2339	.2370	.2401	.2432	.2462	.2493	76	3.1
14	.2493	.2524	.2555	.2586	.2617	.2648	.2679	75	3.1
15	.2679	.2711	.2742	.2773	.2805	.2836	.2867	74	3.1
16	.2867	.2899	.2931	.2962	.2994	.3026	.3057	73	3.2
17	.3057	.3089	.3121	.3153	.3185	.3217	.3249	72	3.2
18	.3249	.3281	.3314	.3346	.3378	.3411	.3443	71	3.2
19	.3443	.3476	.3508	.3541	.3574	.3607	.3640	70	3.3
20	.3640	.3673	.3706	.3739	.3772	.3805	.3839	69	3.3
21	.3839	.3872	.3906	.3939	.3973	.4006	.4040	68	3.4
22	.4040	.4074	.4108	.4142	.4176	.4210	.4245	67	3.4
23	.4245	.4279	.4314	.4348	.4383	.4417	.4452	66	3.5
24	.4452	.4487	.4522	.4557	.4592	.4628	.4663	65	3.5
25	.4663	.4699	.4734	.4770	.4806	.4841	.4877	64	3.6
26	.4877	.4913	.4950	.4986	.5022	.5059	.5095	63	3.6
27	.5095	.5132	.5169	.5206	.5243	.5280	.5317	62	3.7
28	.5317	.5354	.5392	.5430	.5467	.5505	.5543	61	3.8
29	.5543	.5581	.5619	.5658	.5696	.5735	.5774	60	3.8
30	.5774	.5812	.5851	.5890	.5930	.5969	.6009	59	3.9
31	.6009	.6048	.6088	.6128	.6168	.6208	.6249	58	4.0
32	.6249	.6289	.6330	.6371	.6412	.6453	.6494	57	4.1
33	.6494	.6536	.6577	.6619	.6661	.6703	.6745	56	4.2
34	.6745	.6787	.6830	.6873	.6916	.6959	.7002	55	4.3
35	.7002	.7046	.7089	.7133	.7177	.7221	.7265	54	4.4
36	.7265	.7310	.7355	.7400	.7445	.7490	.7536	53	4.5
37	.7536	.7581	.7627	.7673	.7720	.7766	.7813	52	4.6
38	.7813	.7860	.7907	.7954	.8002	.8050	.8098	51	4.7
39	.8098	.8146	.8195	.8243	.8292	.8342	.8391	50	4.9
40	.8391	.8441	.8491	.8541	.8591	.8642	.8693	49	5.0
41	.8693	.8744	.8796	.8847	.8899	.8952	.9004	48	5.2
42	.9004	.9057	.9110	.9163	.9217	.9271	.9325	47	5.4
43	.9325	.9380	.9435	.9490	.9545	.9601	.9657	46	5.5
44	.9657	.9713	.9770	.9827	.9884	.9942	1.0000	45	5.7
	60′	50′	40′	30′	20′	10′	0′	Angle.	

NATURAL COTANGENTS.

TABLE XII.—NATURAL TANGENTS AND COTANGENTS.

NATURAL TANGENTS.

Angle.	0′	10′	20′	30′	40′	50′	60′	Angle.	Prop. Parts for 1′
45°	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44°	5.9
46	1.0355	1.0416	1.0477	1.0538	1.0599	1.0661	1.0724	43	6.1
47	1.0724	1.0786	1.0850	1.0913	1.0977	1.1041	1.1106	42	6.4
48	1.1106	1.1171	1.1237	1.1303	1.1369	1.1436	1.1504	41	6.6
49	1.1504	1.1571	1.1640	1.1708	1.1778	1.1847	1.1918	40	6.9
50	1.1918	1.1988	1.2059	1.2131	1.2203	1.2276	1.2349	39	7.2
51	1.2349	1.2423	1.2497	1.2572	1.2647	1.2723	1.2799	38	7.5
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37	7.9
53	1.3270	1.3351	1.3432	1.3514	1.3597	1.3680	1.3764	36	8.2
54	1.3764	1.3848	1.3934	1.4019	1.4106	1.4193	1.4281	35	8.6
55	1.4281	1.4370	1.4460	1.4550	1.4641	1.4733	1.4826	34	9.1
56	1.4826	1.4919	1.5013	1.5108	1.5204	1.5301	1.5399	33	9.6
57	1.5399	1.5497	1.5597	1.5697	1.5798	1.5900	1.6003	32	10.1
58	1.6003	1.6107	1.6212	1.6319	1.6426	1.6534	1.6643	31	10.7
59	1.6643	1.6753	1.6864	1.6977	1.7090	1.7205	1.7321	30	11.3
60	1.7321	1.7437	1.7556	1.7675	1.7796	1.7917	1.8040	29	12.0
61	1.8040	1.8165	1.8291	1.8418	1.8546	1.8676	1.8807	28	12.8
62	1.8807	1.8940	1.9074	1.9210	1.9347	1.9486	1.9626	27	13.6
63	1.9626	1.9768	1.9912	2.0057	2.0204	2.0353	2.0503	26	14.6
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25	15.7
65	2.1445	2.1609	2.1775	2.1943	2.2113	2.2286	2.2460	24	16.9
66	2.2460	2.2637	2.2817	2.2998	2.3183	2.3369	2.3559	23	18.3
67	2.3559	2.3750	2.3945	2.4142	2.4342	2.4545	2.4751	22	19.9
68	2.4751	2.4960	2.5172	2.5386	2.5605	2.5826	2.6051	21	21.7
69	2.6051	2.6279	2.6511	2.6746	2.6985	2.7228	2.7475	20	23.7
70	2.7475	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	19	
71	2.9042	2.9319	2.9600	2.9887	3.0178	3.0475	3.0777	18	
72	3.0777	3.1084	3.1397	3.1716	3.2041	3.2371	3.2709	17	
73	3.2709	3.3052	3.3402	3.3759	3.4124	3.4495	3.4874	16	
74	3.4874	3.5261	3.5656	3.6059	3.6470	3.6891	3.7321	15	
75	3.7321	3.7760	3.8208	3.8667	3.9136	3.9617	4.0108	14	
76	4.0108	4.0611	4.1126	4.1653	4.2193	4.2747	4.3315	13	
77	4.3315	4.3897	4.4494	4.5107	4.5736	4.6382	4.7046	12	
78	4.7046	4.7729	4.8430	4.9152	4.9894	5.0658	5.1446	11	
79	5.1446	5.2257	5.3093	5.3955	5.4845	5.5764	5.6713	10	
80	5.6713	5.7694	5.8708	5.9758	6.0844	6.1970	6.3138	9	
81	6.3138	6.4348	6.5606	6.6912	6.8269	6.9682	7.1154	8	
82	7.1154	7.2687	7.4287	7.5958	7.7704	7.9530	8.1443	7	
83	8.1443	8.3450	8.5555	8.7769	9.0098	9.2553	9.5144	6	
84	9.5144	9.7882	10.0780	10.3854	10.7119	11.0594	11.5301	5	
85	11.4301	11.8262	12.2505	12.7062	13.1969	13.7267	14.3007	4	
86	14.3007	14.9244	15.6048	16.3499	17.1693	18.0750	19.0811	3	
87	19.0811	20.2056	21.4704	22.9038	24.5418	26.4316	28.6363	2	
88	28.6363	31.2416	34.3678	38.1885	42.9641	49.1039	57.2900	1	
89	57.2900	68.7501	85.9398	114.5887	171.8854	343.7737	∞	0	
	60′	50′	40′	30′	20′	10′	0′	Angle	

NATURAL COTANGENTS.

hair over either one foot or two feet on the stadia-rod at a distance of 100 feet. Prof. Ira O. Baker gives the following formula for measuring the distance by means of gradienter used in either of the above ways:

$$D = 100i, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

in which D = horizontal distance in feet between instrument and rod, and i = intercept on the rod for one revolution of the gradienter screw.

This *fundamental equation* corresponds closely with that for the stadia and applies to a rod held perpendicular to the line of sight. In working on slopes, however, the rod will for convenience be held vertically, when the line of sight will be inclined to the rod, and the formula for this case then becomes

$$D = i(100 \cos \alpha - \sin \alpha), \quad . \quad . \quad . \quad . \quad (20)$$

in which α is the angle between the lower visual ray and the horizontal. The above gives the distance directly observed on the lower visual ray, and from it we derive

$$D = i(100 \cos^2 \alpha - \tfrac{1}{2} \sin^2 \alpha). \quad . \quad . \quad . \quad (21)$$

For a constant intercept on the rod, the formula deduced by Prof. Baker is

$$D = \frac{100}{n} S, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

in which S is the distance between the fixed targets and n = number of revolutions required to move the line of sight over the constant intercept at the distance D. The horizontal distance then becomes

$$d = \frac{100 \cos^2 \alpha}{n} S. \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

115. Wagner-Fennel Tachymeter.—By means of an instrument made in Germany and known by the above name, surveys to be completed on large scale and with great detail may be conducted more rapidly than with ordinary transit and stadia. There are *two forms of* this *instrument*, both of which are so arranged that the horizontal distance and absolute height of the point to be determined are read direct from the instrument after the simple pointing and some intermediate manipulation, without moving the telescope or making any computations. The first of these instruments (Fig. 83) corresponds to a transit, and the second to an alidade. The latter called a tachygraphometer, for use with the plane-table, will probably be of service on large-scale surveys in which the elevations of numerous points are to be determined. With this instrument the positions and elevations of the points can be plotted on the plane-table in the field with great precision and facility and without the danger of omitting details of form. Both instruments are fully described and figured in Appendix 16, Report of U. S. Coast and Geodetic Survey for 1891.

The *field observations* with this class of instrument consist in determining, first, the distance on the slope, and then the horizontal and vertical angle without taking account as a separate observation of the space subtended on the rod from which the inclined distance is determined. From these data the azimuth of the desired point is determined, then the reduced distance and relative height from the point of observation. These operations are all mechanical and graphical, calling for no computations whatever. The *manipulation of* the *instrument* is simple and, with practice, rapid. It is adjusted to the occupied station, and the height of the latter set upon the scale of heights. The determination of separate points is then proceeded with in the following order:

The rodman sets his rod at the desired point; the instrumentman brings the middle wire upon the zero point of the rod, reads the space on the rod intercepted by the distance-wires, and records in his note-book the corresponding inclined

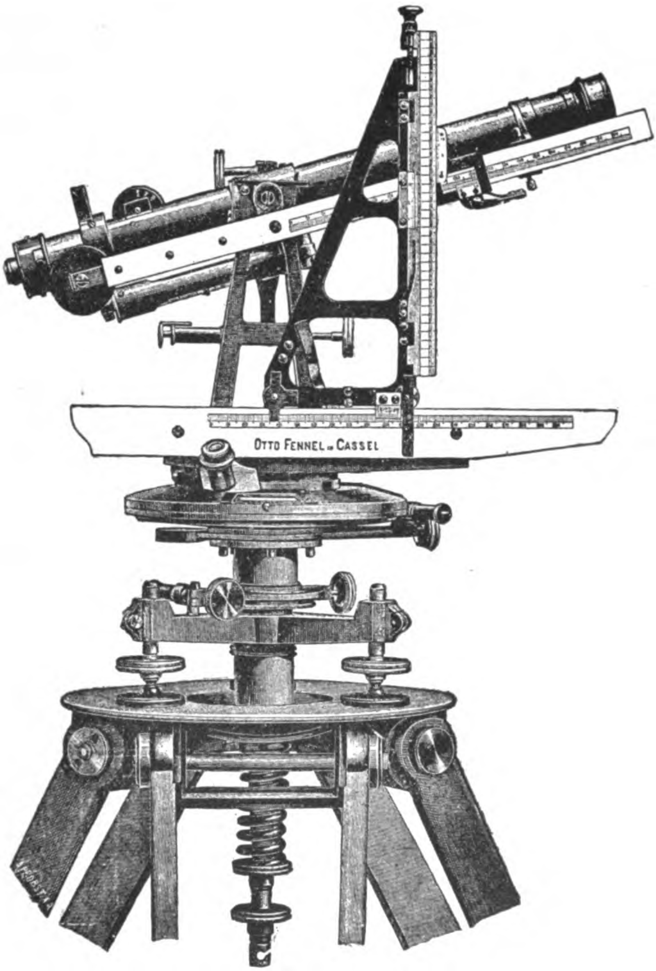


FIG. 83.—WAGNER-FENNEL THEODOLITE TACHYMETER.

distance. These he sets off upon the rule parallel to the line of sight, pressing the projection angle against the vernier of heights, and from the latter reads the required height, and from the horizontal rule the reduced distance of the observed point. Lastly, the horizontal angle is read with the tachymeter or the tachygeometer and plane-table. The first portion of the operations with the two instruments are the same, excepting that with the latter the horizontal projection of the observed point is pricked upon the drawing-paper in its correct position by pressing a needle suitably arranged against the horizontal scale instead of recording the same in a notebook. These various operations take from one and a half to two minutes. This instrument can be used satisfactorily for distances of a thousand feet, making the central base of the instrument two thousand feet.

116. **Range-finding.**—Range-finders are instruments which are primarily intended for military use in determining the distances or ranges of objects as an aid to directing artillery fire. There are many forms of this instrument, some of which are exceedingly large and elaborate and are fixed permanently, as those employed on seacoast fortifications. Of the smaller and more portable forms used with light horse-batteries, that known as the Weldon range-finder, from its inventor, has given the most satisfactory results in actual practice.

Range-finding may also be done with the plane-table, on the same scale when this is sufficiently large; on a provisionally large scale when the scale of the map is small. The light plane-table as a traverse or triangulation instrument, in connection with its use as a range-finder for distances, and with a vertical-angle sight-alidade for elevations, furnishes a most satisfactory tachymeter, both for filling in details on large-scale maps, and for carrying on rough geographic or exploratory surveys.

The range-finder furnishes a satisfactory rough telemetric method of obtaining a fairly accurate measure of inaccessible distances. Pacing or time-sketching may be depended upon where the surveyor may travel, but over rough country or for determining the positions of points on either side of a traversed route the range-finder is unequalled except by intersection methods, and the latter can only be employed where the instrument may be set up and angles taken by which to obtain intersections. The range-finder is most useful in military sketching and in route surveying. The more important advantages which it has are in enabling the surveyor to fix the position of a number of points which lie within the limits of his vision from one point. In ordinary surveying, to measure the distance of an object 5000 feet or more away with any degree of accuracy by intersections would require a base of at least 2000 feet in length or, better still, according to theoretical methods, 5000 feet in length; yet with the range-finder the same distance can be measured with comparative accuracy from a base but 100 feet in length.

117. Surveying with Range-finder.—The range-finder can be most successfully used in three ways. First, in geographic or topographic surveying, while occupying a plane-table or triangulation station the position of which is known, and which is surrounded by a few locations. The remainder of the country can be sketched by *locating with the range-finder* numerous minor points which will so control the sketching as to permit of a greater amount being done from one station than could be accomplished by other methods. Even in comparatively detailed topographic work the range-finder may be thus used in place of the stadia, for with the latter rodmen would have to be sent to the points the positions of which are to be determined, while with the range-finder it is but necessary to obtain a definite object to which to sight.

The second method of using the *range-finder* is *in traverse or route surveying*, where the positions of points on either side of the route can be determined by the range-finder more rapidly than by setting up an intersection instrument, and the country thus controlled by points ranged on either side can be rapidly sketched in.

The third method of using the *range-finder* is by employing it to *measure distances* along the route traversed when the latter is especially irregular or winding. Thus the traveled route may be measured by ordinary means only by going over the ground along which the line of sight is taken; but with the range-finder, as with the stadia or other telemetric instrument, though the road twist and turn and wind about in a ravine, canyon, or over tortuous country, it is unnecessary to measure the route traveled. It suffices to range-in some object in the line of travel and plat the same, when the surveyor may pursue any route he chooses to reach that object without the necessity of measuring the distance as he progresses, the same having already been obtained by the range-finder.

The extreme *adaptability of the range-finder* may be realized when it is known that a base can be accurately measured between two points selected as convenient stations for its use without taking into consideration the irregularities of the ground between them. In other words, it is not necessary to directly measure by pacing or taping the base used with the range-finder, but it is perfectly feasible to take the platted distance between two inaccessible points, as determined by a good range-finder, from a third point whence the two in question are both visible.

118. Traversing with Range-finder.—The most satisfactory method of using the range-finder in traversing or route-surveying is that described by Captain Willoughby Verner, in which he used a combination of range-finder, compass, and intersection which enabled him to sketch a considerable distance to either side of the route traversed. The directions were taken with a cavalry sketch-board (Art. 64) mounted on a tripod, and distances were observed with the range-finder. At the starting-point (Fig. 84) a round of directions were

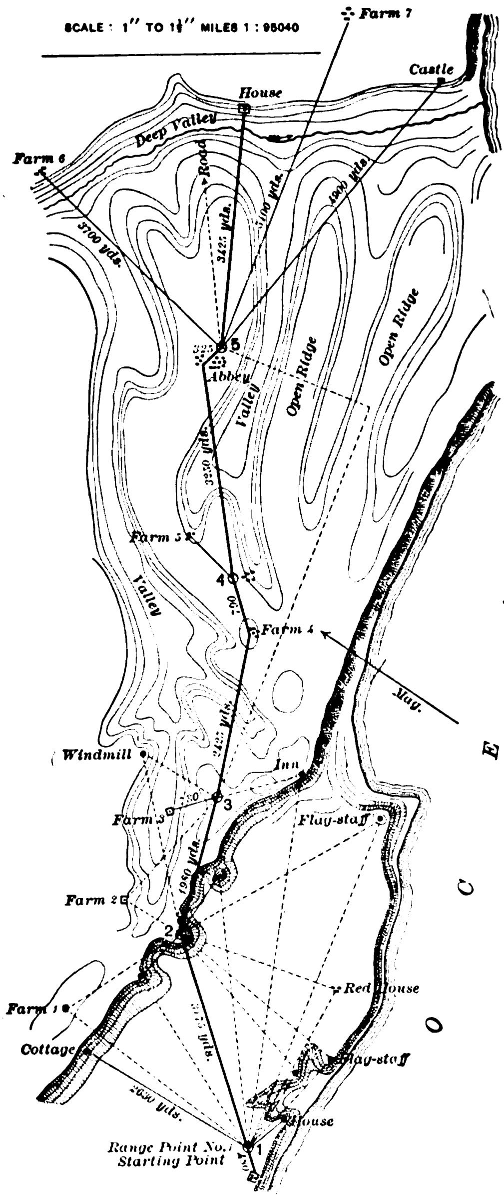


FIG. 84.—RECONNAISSANCE SKETCH-MAP WITH CAVALRY-BOARD AND RANGE-FINDER. After Capt. Willoughby Verner.

drawn on the board, and the ranges of a number of the more prominent objects were taken with the range-finder and their positions marked on the sketch. He then rode rapidly to high ground 3175 yards distant, the direction and range of which had been plotted from the first point. Arrived there, the first thing was to find a conspicuous point in the direction to be traveled, which was again ranged in and plotted on the board, its distance being 1980 yards.

The board being mounted on a tripod and oriented by the needle, *intersections* were *taken* on a number of points previously indicated by direction lines, while new direction lines were plotted to various objects, a few of which were again ranged-in, and this process was continued. Its chief advantages were that the surveyor was able to ride rapidly over the ground, along the most accessible route, from one point to another, and to locate a number of points in every direction, some by intersection, others by ranging. Sometimes the range in the direction of the route of travel is obstructed by trees or other objects; when it is possible to sight in that direction on the sketch-board, measure the distance by pacing or otherwise until the obstacle is passed, and then resume the range-finding.

119. Weldon Range-finder.—This instrument *consists of three prisms* accurately ground to the following angles: first, 90 degrees; second, 88 degrees 51 minutes 15 seconds; third, 74 degrees 15 minutes 53 seconds. The distance or *range of an object*, O (Fig. 85), from an observer is obtained by observing the angles OAD and OBA at the base of a right-angled triangle, ABO, the measured base, AB, of which bears the ratio 1 to 50 of the distance or range AO when the first or 90-degree prism and the second or 88-degree prism are used at either end of the base. A more accurate determination of the range may be obtained by use of the second prism only when the measured base is 1 : 25 of the distance or range AO (Fig. 86), in which case the angles of an isosceles triangle at ABO and ACO at either end of the fixed base are measured. Finally, the Weldon range-finder may be used for measuring rapidly a base AB or BC by using the third prism

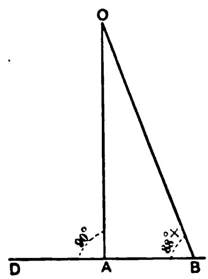


FIG. 85.—RANGE-FINDING WITH A DIRECTION-POINT *D*.

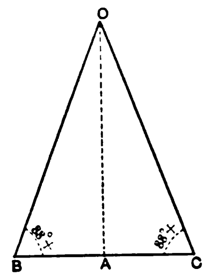


FIG. 86.—RANGE-FINDING WITHOUT DIRECTION-POINT.

of 74+ degrees (Fig. 87), but this is merely as a convenience and not as a necessity except under very unusual circumstances.

In *taking a range* choose a good direction-point, D, (Fig. 85), or else put in a ranging rod at D, making its reflec-

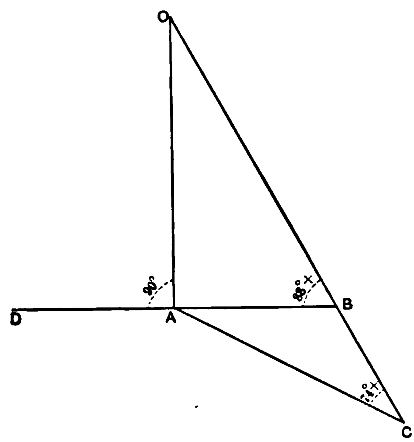


FIG. 87.—MEASURING LONG BASE WITH RANGE-FINDER.

tion coincide with the object by means of the 90° prism; then, using the 38°+ prism, retire along the line AB, leaving a mark at A to keep yourself in line, and when the $88° +$ prism shows a coincidence of D with O, B is reached. AB is then measured either by pacing or, more accurately, by the tape, and multiplied by 50, the product being the range of O from A.

The Weldon range-finder is manufactured in two forms: 1, as a small watch-shaped affair about 2 inches in diameter; and, 2, in a semi-cylindrical case about $1\frac{1}{2}$ by $2\frac{1}{8}$ inches. (Fig. 88.) The latter, which is the most serviceable of the two, has the three prisms arranged one above the other, and is used by holding it directly in front of the eye, grasping the projecting case as a handle, and with the first or 90° prism uppermost. The apex of this prism is then held in a direct line or range with one of the objects sighted, and is superimposed over the edge of the metal back. This object is then looked at by direct vision through the open space above or below the prism, and is viewed simultaneously with the second object reflected at right angles in the prism.

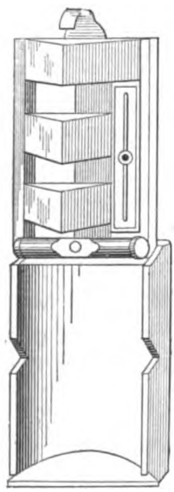


FIG. 88.—WELDON RANGE-FINDER.

The reflected object appears directly above or below that seen by direct vision, and forms with the eye of the observer the angle to which the prism is cut. Considerable practice is required to readily determine in the prism the reflected object, but by holding the instrument quite close to the eye a large field is obtained and a slight movement of the head to either side is all that is required to bring a fresh field into view. When the reflected image has been obtained it can be made to move up or down by slightly tilting the instrument so as to make the reflection coincide with the object selected in front of the observer for direct viewing. In order *to get a correct angle* the object should be kept upright and the reflected horizon as level as it is in nature, since any inclination affects the angle.

If in range-finding a good natural *direction-point* cannot be found, a flag or other mark may be placed to get a direction-point, and its distance from the observer is dependent on the distance of the object the range of which is required. Thus for a range of 3000 feet the direction-point may be 150 feet away, but for a mile to two miles the marker should certainly not be nearer than 200 to 300 feet. In fact, the further the direction-point from the observer the more accurate is the measurement of the range, other conditions being equal. If an assistant accompanies the observer, he may be used as a direction-point, when very little time will be lost in finding one for a long range.

120. Accuracy and Difficulties of Range-finding.—The accuracy obtained with the Weldon range-finder is remarkable considering its crudity as a surveying instrument. In tests for *accuracy* made at the Infantry and Cavalry School at Fort Leavenworth, Kansas, distances of 2000 and 3500 feet were determined within 2.5% error in every case, and an average for a large number of observations and for distances of 2000 to 12,000 feet, measured by enlisted soldiers unaccustomed to the use of the instrument, was 2.43%.

The chief *difficulty in* the *use of* the *instrument* is one inherent in any prismatic instrument; namely, that the object the range of which is desired is often hidden from the further end of the base by an intervening tree, knoll, or other obstacle, so that, except under very favorable circumstances, several trials are necessary in order to get a range, whereas this even is sometimes impossible. Other objections to this apparently simple process are the difficulty of obtaining a definite mark at right angles to the object of reflection when employing the base of 1 : 50, and the difficulty of always finding ground suitable for the measurement of a base as regards view, general configuration, and space. Again, *considerable practice is necessary* in order to obtain reliable results every time, and to attain facility in the selection of suitable range-points. And finally there is the difficulty, soon overcome with practice, of learning to recognize the reflected image, and of producing the coincidence of this and the direct view of the range-point.

121. Range-finding with Plane-table.—Range-finding may be performed with a plane-table as satisfactorily as with the prismatic range-finder for all the purposes of ordinary surveys. The plane-table would, of course, not serve as a satisfactory range-finder for military purposes, because it offers too large a mark and is not sufficiently portable for ordinary military reconnaissance.

While the plane-table may be satisfactorily employed as a range-finder in cases of map-making similar to that described in Article 118, its especial adaptability appears to be in connection with the determination of positions and elevations of unimportant points near the route of travel of the topographer who is sketching small-scale maps—assuming the topographer to be traveling over a road previously traversed and adjusted, and sketching in the topography on either side (Arts. 13 and 17), and that he finds a point C (Fig. 89), either a house in a field a mile or less distant, or a summit which has not been previously located, and the position and elevation of which are essential in order that he may properly sketch his surroundings.

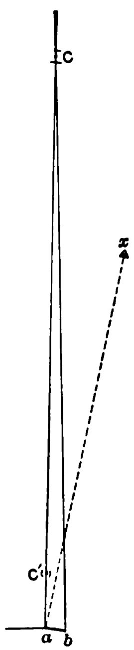


FIG. 89. RANGE-FINDING WITH PLANE-TABLE.

Let him set up his plane-table at a; then orienting by a backsight down the road, if a sufficiently long and straight one can be had, or by some point x already located and visible from the position a, draw a line in the direction of C. Now sighting in the direction b, whence C can also be seen, let him draw the line ab and measure off the base with a tape, or by carefully pacing ab say 100 feet. This should then be platted on the plane-table on a scale ten times as great as the scale of his map. Now removing his plane-table to *b* and orienting on *a*, let him draw a direction line to *C* which will approximately locate it by its intersection with the line from *a*. The angle is necessarily so acute that the actual position of *C* is indefinite, but the distance *aC* may be scaled off, and this divided by 10 will reduce it to the scale of the map. Platted on the line *aC*, it will give the location *C'* so closely, because of the great reduction in scale, as to fix the position of the point well within the map scale.

The chief *precautions to be taken* in this mode of location are that the base *ab* shall not be too small, a ratio of 1 to 25 being a very good one and 1 to 50 less satisfactory. Accordingly, with a measured distance *ab* of 100 feet, a point 2500 feet distant could be quite accurately paltted to a scale 10 times as great as that of the map. The topographer must take especial care in range-finding by this method to set his instrument exactly over the points *a* and *b* in order that his orientation may be accurate. Point *a* on the plane-table board must be plumbed over a stake or other mark, and *b* likewise must be plumbed over the mark sighted at; moreover the backsight from *b* to *a* must be exactly at the mark *a*.

CHAPTER XIV.

PHOTOGRAPHIC SURVEYING.

122. **Photo-surveying.**—The camera has recently come into limited favor as a topographic surveying instrument. Its first extended use was in Italy, where it was employed chiefly in making perspective views of buildings for the purpose of constructing therefrom their elevations and ground-plans, for architectural and military purposes, and this form of photo-surveying has been styled photogrammetry. As a result the word photo-topography has been recently adopted as applying to the survey of the terrane by means of the camera. Photo-surveying methods have been employed to a minor extent in India, France and Italy, and almost exclusively in the Dominion of Canada, in the making of topographic surveys.

123. **Photo-surveying and Plane-table Surveying Compared.**—A careful study of the method and results of photographic surveying leads to the following conclusions:

Photo-surveying consists ultimately in constructing a topographic map in office from photographs of the terrane in conjunction with angular measures taken by the camera. Necessarily the draftsman who does not see the country cannot make as detailed and accurate a map of it from photographs as the topographer could make while viewing the country itself from which the photograph had been made. It seems, therefore, fair to assert that a map made from photographs and constructed in the office on a drawing-board, much on the same principle as a map is made on a

Fig. 90.—Photograph by Canadian Survey and used in Map Construction.

plane-table board in the field, is less accurate and less satisfactory than the latter.

On the other hand, the use of the *plane-table* requires the expenditure of some time in the field in constructing the map, while the expenses of a large party organization are running on. Considerable outlay is saved in photo-surveying by drafting the map in office at the expense of only the individual draftsman; moreover, under advantageous conditions of light, photo-surveying field operations can be conducted more rapidly than plane-table surveys.

Finally, photo-surveying methods can be employed only in mapping a limited class of topographic forms, such as bold and open mountainous country, and then only on generalized geographic scales. For in highly eroded and detailed topography it would be necessary to occupy a multitude of camera stations that all the forms might be recorded in photographs. In wooded regions, and on plains or plateaus it is impossible to use photo-surveying methods. With the plane-table it is possible to supplement the facts mapped from the occupied stations by any amount of traverse surveying.

The ultimate *conclusion* is that a fair map can be made by photo-topographic methods, under favorable conditions, more rapidly in the field and at less cost than a good map can be made on the same scale by plane-table methods. On the other hand, where it is desirable to make a first-class topographic map on a given scale, the best results will be obtained with the plane-table under most conditions of atmosphere. For it must be borne in mind that when surveying by trigonometric methods, where the topographer leaves camp and ascends a mountain to make a plane-table station or photographic station, he will under ordinary circumstances succeed in making but one or two stations at most in a day, where the scale is of geographic proportions.

In the average atmospheric conditions met with in the United States the topographer will therefore accomplish as much in a day with the plane-table as with the camera, while the resulting map will be decidedly superior. Again, under such atmospheric conditions as exist in western British America and in Alaska, where the higher summits are covered with cloud and mist during the greater portion of the day or for several days, and when the occasional glimpses that may be had of surrounding country are accompanied by a clear and bright sunshine, the topography can be procured by photo-topographic methods, completing in an hour of clear weather the work necessary to be done at one station, which would require the better part of a day by plane-table methods. Therefore it is probable that photo-topographic methods are cheaper and more rapid than plane-table methods and furnish a much more practicable and economic mode of making geographic surveys under such conditions. Mr. E. Deville, Surveyor-General of Canada, estimates the cost of plane-table surveying in western British America, as compared with photo-topographic surveying, as 3 to 1.

124. Principles of Photo-topography.—The practice of photo-topography requires a thorough knowledge of descriptive geometry and perspective. The *camera* is specially prepared, resting on a horizontal plate divided like the circle of a transit instrument and read with verniers, and having attached to its side or on top a small telescope, with vertical arc for the measurement of angles. There are a vertical and a horizontal cross-hair in the focal plane of the camera, and it is fitted with a magnetic needle inside of the box, and a scale, so placed that, when the exposure is made, the magnetic declination, the scale, as well as the intersection of the cross-hairs, are all photographed on the plate containing the view (Fig. 91). If the instrument has been carefully leveled, the horizontal cross-hair becomes the horizon line, and the vertical cross-hair the center zero line, to which angular measures are referred in the office computations. A group of views are taken at each station, abutting one against the other, and the angular distance between each is noted by the reading of the horizontal plate of the camera, horizontal angles being also read by a small theodolite or by the camera to the more prominent peaks.

The objects represented in perspective are of an irregular shape and at various distances from the camera. If the

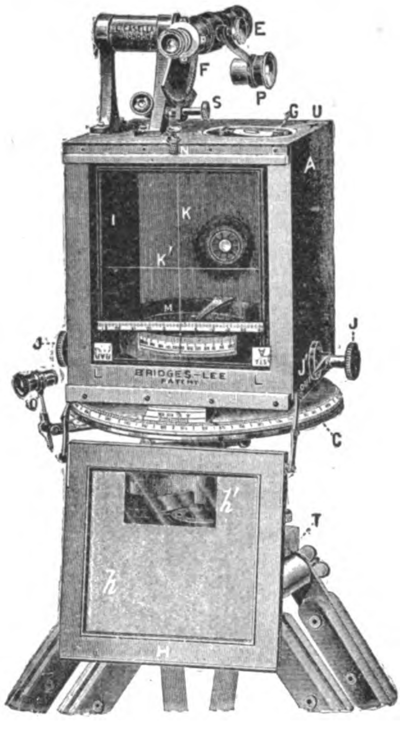


FIG. 91.—BRIDGES-LEE PHOTO-THEODOLITE.

picture or image of the object is a true perspective in a plane, it is possible to construct therefrom a geometric projection of the object in a plane at right angles to the picture plane. This, providing the distance and the relative position of the point of view be known with reference to the picture plane, and providing views have been taken from a sufficient number of stations to surround the irregularly formed objects viewed. *Photo-topography is,* therefore, *the art* of reconstructing geometrically horizontal projections from perspective views. The process of this reconstruction consists in platting the skeleton triangulation as obtained by angular measures with theodolite or the horizontal circle of the camera. The photo-topographic survey should be preferably preceded by a primary triangulation. Then, with several stations platted, the view from each of them of a given portion of the terrane may be projected on the plane of the map, and intersections be platted for each salient point seen in perspective.

125. **Camera and Plates.**—There are a number of forms of photo-topographic cameras, among the more complete and satisfactory of which are those employed by the Canadian Topographic Survey, the Italian instruments, and the Bridges-Lee (Fig. 91) instruments. The apparatus is packed in several small cases for easy transportation in the most inaccessible country, the tripod, camera, and plates making separate packages. The *equipment* of a photo-topographic party in the Canadian surveys consists of a transit theodolite and two cameras. These cameras are rectangular boxes of metal, open at one end and provided in addition to the lens with two sets of cross-levels, read through openings in the outer mahogany box; the plate-holders are made for single plates and are inserted in a frame which can be moved forwards and backwards by means of adjusting-screws. The camera rests on a triangular base with leveling foot-screws, similar to those of the transit instrument, so that both may be used on the same tripod.

The surveyor first adjusts the transit and measures the azimuths and vertical angles to triangulation points and to the camera stations, recording the same. The camera is then mounted on the tripod, leveled, and the plate-holder inserted, and its number is noted, as is also the approximate direction of the view by means of lines drawn on the outer case of the camera-box; the topographer then revolves the box until these lines show that the camera is properly pointed. Then, by looking at the lines on the side of the camera-box, he notes whether the view is in the correct vertical plane. Exposure is then made, and the camera sighted for the next view.

126. **Field-work of a Photo-topographic Survey.** — In the field-work of a photo-topographic survey the *primary triangulation* is *first executed* by ordinary methods, and secondary triangulation is executed during the progress of the photo-topographic survey. The object of the secondary triangulation being to fix the camera stations, the summits located in the secondary triangulation are selected for this purpose only, all topographic details of the plat being drawn from the photographs made at the camera stations. The positions of the camera stations may be fixed either by angles from them or by angles from primary triangulation points or both, and as it is easier and more accurate to plat the camera stations by means of angles taken from the primary triangulation points, the camera stations should, if possible, be occupied before the triangulation summits.

In *selecting camera stations* it must be borne in mind that views taken from a great altitude and overlooking a large expanse of country are desirable chiefly as aids in the expansion of the triangulation, while those taken from low altitudes are of the greatest service in drawing in details of topography, especially in valleys and lowlands. Difficulty is frequently experienced in obtaining two views which will furnish intersections over a certain portion of the terrane, in which case in very rugged country the method of vertical intersection may be employed, views being taken from different altitudes. Such a process can of necessity be employed only when the differences in elevation are great and the points to be determined not distant.

The greatest *difficulties* in *photo-topography* are encountered in bad lights, which must necessarily be met in making panoramic views; for while the camera will have the lights in the right direction for viewing one way, in taking views in the opposite direction the lights will be unfavorable (Art. 389). Moreover, views taken of the same object or portions of the terrane at different times of the day have the shades cast in different manners, so that it becomes difficult to identify the topographic detail or even salient points. If the number of photographs taken is large enough to cover the ground completely, the identificaion of points even under different lighting offers no serious obstacle.

In *making exposures* two or three points in each view must be observed with the altazimuth on the camera or with a theodolite, so as to obtain horizontal and vertical angles between them, and this aids in the orientation of the view and in platting and computing the details of the map. It is desirable in conducting such surveys to establish a small field laboratory at a central point to which the camera and plates may be taken for the purpose of development, changes of plates, etc. (Chap. XLI.) In making field surveys an outline sketch of the terrane should be made in a note-book, on which memoranda must be made of names, roads, paths, buildings, and other information essential to the map.

127. **Projecting the Photographic Map.**—Two drawing-boards are covered with paper, one of which is used as a constructing board, on which the graphical determination of the points is made, and the other is used for the final drawing of the topographic map. On both are projected the trigonometric points which are platted by means of their coordinates. The camera stations are platted on the board either by coordinates or by means of the protractor. The intermediate points are then projected by searching for well-defined points coming on two or more negatives, selecting such as seem most useful as guides for the drawing of the contours, and tracing the trend of mountain ranges, streams, etc.

Assuming that ten views have been taken panoramically from one station, then the *horizontal projection* of the ten plates exposed from such a station forms a decagon (Fig. 92), with a radius of inscribed circle equal to the principal focal length of the camera. After the position of one of these panoramic views has been found on the map by platting the angle from the occupied station to some located point, the orientation of the other point is accomplished by adding one tenth of 360° to this angle, and thus the entire decagon can be platted with reference to the occupied station and the orienting triangulation point. When this orientation of the horizontal plan is accomplished, the direction lines are drawn from the platted camera station to points photographed in the camera. The following example taken from the U. S. Coast

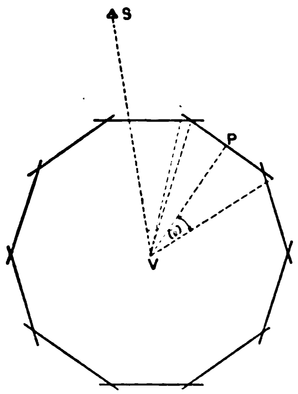


FIG. 92.—PROJECTION OF CAMERA-PLATES FROM A STATION.

Survey report for 1893, by A. J. Flemer, further describes the process:

Let *mm'nn'* (Fig. 93) represent a vertical and oriented perspective view, and OO' be the line of the horizon of the plate, V the point of view, and ω the angle of orientation of the plate in reference to a secondary point A. Now, $VP = f$ is the principal focal length, and if a is the representation on the plate of a point A in nature, and a vertical aa' has been drawn on the plate through it to the horizon line, then Pa' will be the abscissa of the point a. From the rectangular triangle VPa' we have then

$$x = f \tan \omega. \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

In order to draw the *horizontal position* of the ray from V to A, the distance $p'a'$, equal to x, is laid off upon the horizon line OO' from P'. This distance x is taken from the picture by means of a pair of dividers. The position of the point A' will be in the intersection of two or more lines of direction obtained in a similar manner from other pictures containing a and taken from other stations, and the same applies to all other points of the terrane if they can be identified upon plates taken from different panoramic stations.

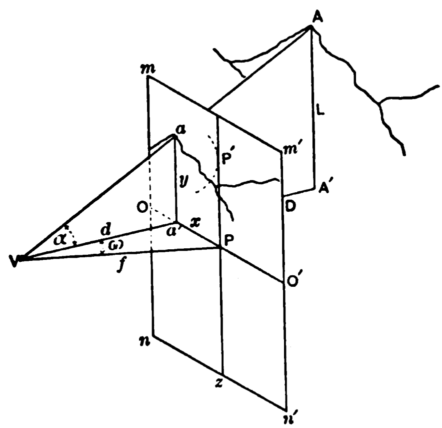


FIG. 93.—PROJECTION OF PHOTOGRAPH.

The elevations of points on the terrane are determined, after the selected points have been platted in horizontal plan as above, in the following manner:

If the elevation of the camera station V is known, the elevation of the horizon line on the plate, mn, can be obtained by adding the height of the instrument to the elevation of

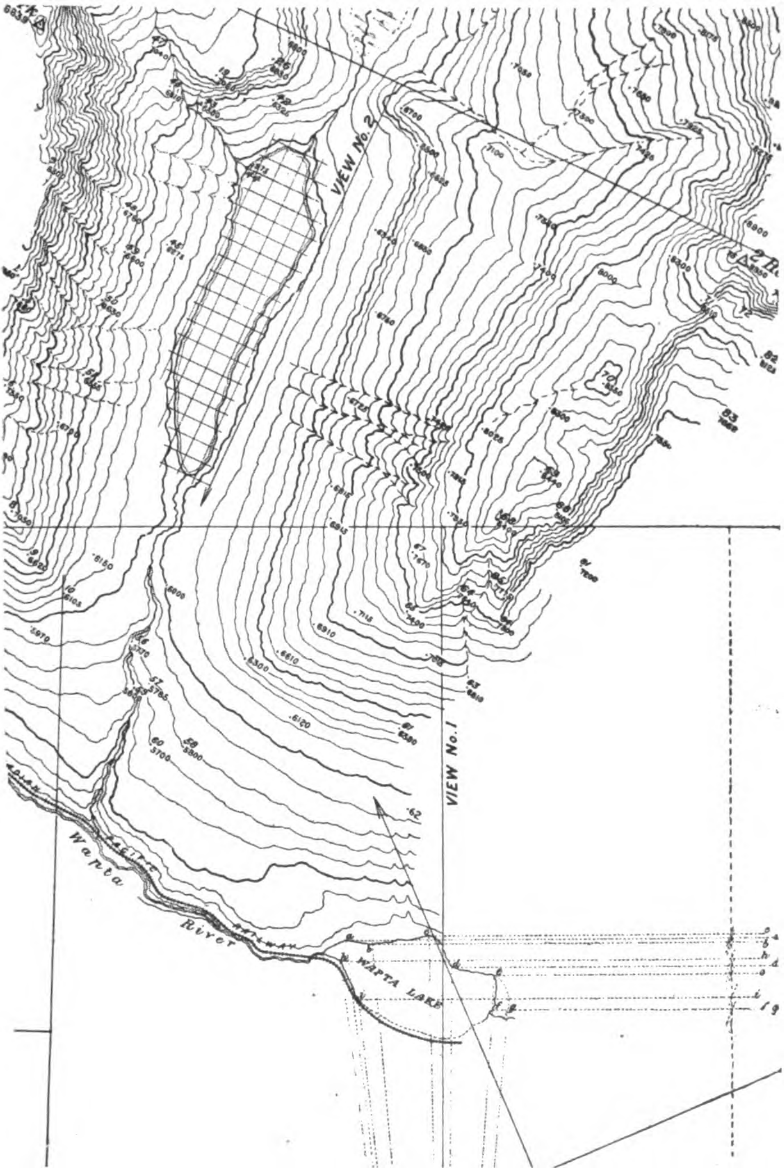


FIG. 94.—CONSTRUCTION OF MAP FROM FOUR PHOTOGRAPHIC STATIONS.

the station V. The *elevations of* all *points* on the plates which are bisected by the horizontal line have the same elevation as the horizontal axis of the instrument at the station, disregarding curvature and refraction. The elevations of other secondary points selected from the plates are obtained by determining their elevations above or below the horizon line. From the relations of similar triangles we have

$$h = \frac{Dy}{d}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

in which h is the difference of elevation between the occupied station and the point observed, D the horizontal distance to the observed station A from the occupied station V, and d the horizontal distance of the same on the picture, y being the ordinate of points. From the rectangular triangle VPa' we find $d = f \sec \omega$, when

$$h = \frac{Dy}{f \sec \omega}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The differences of elevation taken from the perspective are positive or negative according to the relative positions of their points in respect to the horizon.

The computations and office platting connected with photo-topographic surveying are long and tedious operations, one day's work in the field frequently requiring from four to eight days' office work for the accomplishment of the platting of the map. In Fig. 94 is shown the mode of constructing of a portion of the map of the Canadian Survey. This was made from four camera stations, the view from one of which is shown in Fig. 90.

PART III.

HYPSOMETRY, OR DETERMINATION OF HEIGHTS.

CHAPTER XV.

SPIRIT-LEVELING.

128. Hypsometry.—Hypsometry is that branch of surveying which treats of the determination of absolute heights or relative elevations. Mean sea level is the usual plane of reference from which such heights are determined, though not infrequently other arbitrary base levels are assumed for special purposes. There are three principal hypsometric methods, noted here in their order of accuracy, viz.:

1. By spirit-level;
2. By trigonometric or angular measurement; and
3. By barometer or atmospheric pressure.

Barometric leveling may be performed whenever the station the height of which is to be determined can be occupied; trigonometric leveling can be prosecuted when one or both of the stations is inaccessible; and spirit-leveling, only when both stations are accessible and visible one from the other.

Hypsometry, or *leveling*, is the determination of the relative elevations or heights above sea level of points upon the earth's surface, and may be further classed as direct and indirect. *Direct leveling* is performed by the spirit-level and consists of the prolongation of a level line and the determination by actual measurement, on a vertical rod, of heights above or below this line. *Indirect leveling* is the determination of heights by calculation from measured angles and distances or by barometric methods.

Two points are said to be upon the same level when they are equidistant from the earth's center. A *level line* is at a uniform distance from the equal potential surface, and, owing to the figure of the earth, the difference between the polar and equatorial levels is 13 miles vertical. A level line is not a *horizontal line*, for the latter is a straight line parallel to a tangent of the earth's circumference, whereas a level line is a curved line, because it is parallel to the curvature of the sea. But for all ordinary purposes a level line and a horizontal line are synonymous even for leveling operations conducted over such great distances as to be affected by the curvature of the earth. A level surface may also be defined as one which is everywhere perpendicular to the direction of gravity as indicated by a plumb-line; and the spirit-level, like the plummet, is a device for utilizing the law of gravity to establish a horizontal or perpendicular line.

129. Spirit-leveling.—The operation of spirit-leveling is the most accurate of hypsometric methods, because it is the simplest and most direct and is subject to the fewest sources of error in measurement or instrument. It is not dependent upon the exact measurement of horizontal distances nor of angles, nor is it affected by atmospheric changes. It is practically subject only to errors of instrument and level-bubble and of the staff or rod by which the vertical heights are measured.

Spirit-leveling may for convenience be divided into three general classes:

1. Ordinary or engineering spirit-leveling;

2. Precise spirit-leveling; and

3. Trigonometric or, as it is sometimes called, geodesic spirit leveling.

The first two of these methods of spirit-leveling are essentially similar, differing chiefly in the care taken in the conduct of the work, and in the elimination or correction of instrument errors. In *engineering spirit-leveling* it is assumed that the adjustments of the instrument eliminate instrument errors, and no attempt is therefore made to correct these: moreover, the work is conducted with but moderate care, both in the quality of the instrument and rods employed, the turning-points upon which these rest, and in the various other phases of the operation of leveling.

Precise spirit-leveling is conducted with finer instruments and rods and with all the care which it is possible to exercise in every detail of the work, especially in the elimination of the errors of instrument in the process of leveling. Account may or may not be taken of instrumental errors, and correction may or may not be made for them, though with proper precautions to eliminate these more accurate results can be obtained than by attempting their correction, since the method of determining and compensating for such corrections involves other operations which may introduce counterbalancing errors.

Geodesic spirit-leveling accepts the instruments as inaccurate, and corrections are made for the instrumental inaccuracies by determining the instrument constants and applying them. Moreover, the operation is a combination of direct and indirect leveling, because, in addition to prolonging the horizontal line as determined from the level-bubble, a slight angular measurement, calling for a correction to height dependent upon the distance, is introduced in each sight. This is done by making the instrument approximately level and reading the rods, then by making it truly level by a milled-head micrometer leveling-screw; the angular distance through which the telescope is moved in the performance of this operation, as recorded on the micrometer, is multiplied into the distance between the instrument and rod, and the resulting difference in height is a correction to the height directly measured by the instrument used as a spirit-level.

130. Engineering Spirit-levels. — There are several methods of leveling according to the sequence of rod and instrument. In ordinary spirit-leveling the practice is to use one rod, to read a backsight upon it, then have the rod moved forward and observe a foresight upon it. The same methods may be employed, but with greater rapidity, by having two rodmen, so that immediately after the backsight is read on the rear rod this may be moved sufficiently in advance for the second foresight while the instrumentman is reading a foresight on the front rod (Art. 144). In addition to increasing the speed, this method gives a slight increase in the accuracy because of the rapidity with which the backsight and foresight can be read, thus avoiding a possible settlement in the instrument between the two, but for ordinary purposes this method is too expensive. In addition to these two single leveling methods, duplicate leveling may be done with one rod or with two rods and one instrument (Arts. 143 and 144), and these methods are those commonly employed in precise leveling.

A description of the *engineer's spirit-level* (Fig. 95) is superfluous in a work of this sort. The best results are procured by using any good make of instrument of 18 to 20 inches length. It must have a stout tripod, good glasses, and bubble graduated preferably to 10 seconds of arc. The ordinary bubble graduated to 20 seconds (Art. 147) increases the speed but slightly and is not nearly so accurate.

131. Adjustments of the Level. — Before any of the adjustments of the level can be properly undertaken, the cross-wires must be focused by pointing them on an object and moving the diaphragm until a strong definition of them is obtained. The ordinary adjustments of the Y level are:

1. The adjustment of the line of collimation, by which

the cross-hairs are brought into optical axis, so that their point of intersection remains on a fixed point during an entire revolution of the telescope on its wyes;

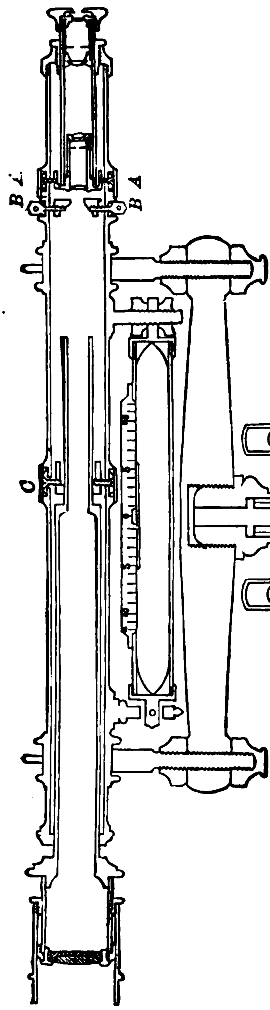


FIG. 95.—ENGINEERS' WYE LEVEL.

2. The level-bubble must be brought parallel to the bearings of the wyes, that is, to the longitudinal axis of the telescope; and

3. The wyes must be adjusted, that is, the bubble brought into position at right angles with the vertical axis of the instrument.

To *adjust* the *line* of *collimation*, make the vertical cross-hair tangent to any vertical profile, as a wall, and then turn the telescope half-way round in its wyes. If the vertical cross-hair is still tangent to the edge selected, it is collimated. Select some horizontal line, and cause the horizontal cross-hair to be brought tangent to it. Again rotate the telescope half-way round in its wyes, and if the horizontal cross-hair is still tangent to the edge selected, it is collimated.

Having adjusted the two wires separately in this manner, select some well-defined point which the cross-hairs are made to bisect. Now rotate the telescope half-way round in its wyes. If the point is still bisected, the telescope is collimated. A very excellent mark to use is the intersection of the cross-hairs of a transit instrument.

To *adjust* the *level-bubble*, bring the level-bar over two of the leveling-screws, focus the telescope upon some object about 300 feet distant, and put on the sunshade. Clamp the spindle, throw open the two arms which hold the telescope down in its wyes, and carefully level the instrument over the two level-screws parallel to the telescope. Lift the telescope out of its wyes, turn it end for end, and carefully replace it. If the level-tube is adjusted, the level will indicate the same reading as before. If it does not, correct half the deviation by the two leveling-screws and the remainder by moving the level-tube vertically by means of the two cylinder-nuts which secure the level-tube to the telescope-tube at its eye-piece end. Loosen the upper nut with an adjusting-pin, and then raise or lower the lower nut as the case requires, and finally clamp that end of the level-tube by bringing home the upper nut. Repeat until the adjustment is perfect.

To make the *level-tube parallel* to the *axis* of the *telescope*, rotate the telescope about 20° in its wyes, and note whether the level-bubble has the same reading as when the bubble was under the telescope. If it has, this adjustment is made. If it has not the same reading, move the end of the level-tube nearest the object-glass in a horizontal direction, when the telescope is in its proper position, by means of the two small capstan-headed screws which secure that end of the level to the telescope-tube.

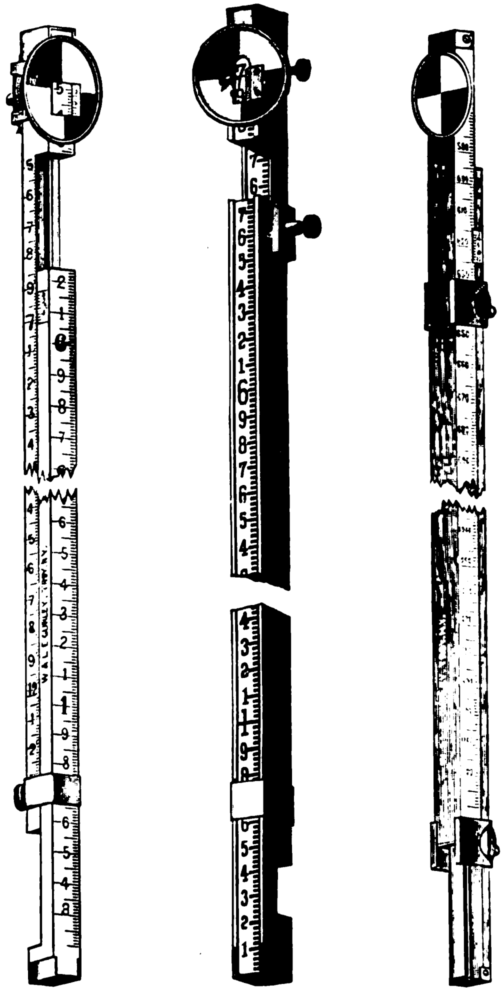
To make the *level-bar parallel* to the *axis* of the *level-tube*, level the instrument carefully over two of its leveling-screws, the other two being set as nearly level as may be; turn the instrument 180° in azimuth, and if the level indicates the same inclination, the level-bar is adjusted. If the level-bubble indicates a change of inclination of the telescope in turning 180°, correct half the amount of the change by the two level-screws, and the remainder by the two capstan-headed nuts at the end of the level-bar which is to the engineer's left hand when he can read the maker's name. Turn both nuts in the same direction an equal part of a revolution, starting that nut first which is in the direction of the desired movement of the level-bar.

132. Target Leveling-rods.—Leveling-rods are of two general types:

1. Target-rods; and
2. Speaking or self-reading rods.

These, again, may be extensible or of one piece. The three more usual types of target-rods are made in two pieces, one of which slides on the other so as to extend their length when in use, yet when not in use the length is reduced to one-half its possible limit for convenience in transportation. These three forms of rods are known respectively as the New York, Philadelphia, and Boston rods. Each of the two pieces of which these rods are constructed is about 7 feet in length, and the graduations are so arranged that the total extension possible with them is 12 feet.

The *New York rod* (Fig. 96, *a*) is the best constructed and the most accurate of the three and is divided to hundredths of a foot, reading with the vernier on the target to thousandths.



a b c

FIG. 96.—TARGET-RODS.

The divisions are so arranged, however, that only those below $6\frac{1}{2}$ feet, that is, only those visible when the rod is not extended, can be read from the instrument. On extension the rod is read by a vernier on the rear side. The *Philadelphia rod* (Fig. 96, *b*) is divided to hundredths and is so graduated as to be easily read by the levelman at all distances at which it is visible. There is no vernier on the target of the Philadelphia rod, so that the least reading practicable with it is one-half a hundredth, and by estimation perhaps to two-thousandths, of a foot.

Unlike the rods just described, the *Boston rod* (Fig. 96, *c*) has a fixed target, and all readings upon it are obtained by extending the rod. It is held with the target down for readings less than 5½ feet, and is inverted for greater readings. The vernier and the scales by which the rod is read are on the sides, and the divisions are such as to permit of its being read to one-thousandth of a foot. This rod is lighter and more compact than the others, but is not so commonly used.

For very *accurate work with a New York* rod, the foot-plate, instead of being the full width of the rod and of brass, should be a small truncated pyramid of phosphor-bronze or steel, the least dimensions of which at the bottom should be about one-half inch, in order that the rod when rested on the turning-point shall surely be balanced over its center and that the same point of the foot-plate should always be in contact with the turning-point. Great care should be taken to keep this foot-plate wiped clean, and in making extensions of the rod care should be taken that the vernier of the target is exactly set on the 6.5-foot mark when clamped. Also, after extension, care should be taken that no grit or dirt gets into either of the abutting joints, else readings taken between 6 feet and 6.5 feet might be in error. Plumbing-levels should also be used where careful work is attempted.

133. Speaking-rods.—The greater part of the leveling ordinarily done is of the more hasty and rougher kind, readings being taken on intermediate stakes to one-tenth foot only, and on turning points rarely nearer than one one-hundredth foot. For this reason most levelmen prefer to use speaking-rods, and, as a consequence, of the extensible rod the Philadelphia is the more commonly used because it is also a speaking-rod.

The *non-extensible speaking-rods* are, however, more easily and safely employed than extensible rods. They are more popular with the more experienced levelmen, as with them better work can be performed than with extensible speaking-rods. They are, moreover, frequently used in precise leveling, as preferable to target and vernier rods. There are many modes of *graduating speaking-rods* so as to make the divisions legible at the greatest distance at which the rod is sighted. Few such rods can be purchased of instrument-makers, the easiest way to obtain them being for the levelman to divide and paint them himself. They consist usually of well-seasoned pine ½ to 1 inch in thickness and from 3 to 5 inches in width. The figures are made as large as possible, so as to be legible, and various markings are introduced between these or in the shapes of the figures themselves, so that the eye shall have a guide whereby to divide the spaces (Figs. 81 and 97).

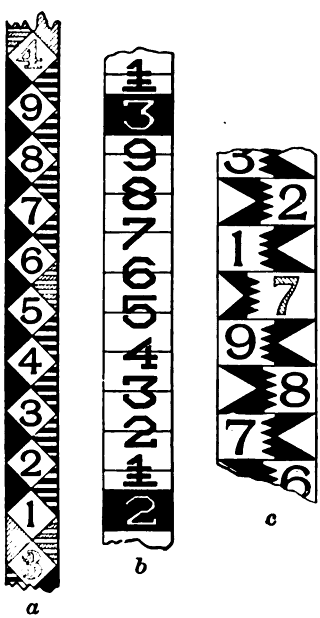


FIG. 97.—SPEAKING LEVEL-RODS.

In order to get more accurate work from a speaking-rod the level should have *three horizontal cross-hairs* in the diaphragm, and the levelman should tell the rodman where to place his finger or pencil, and the latter should record this as an approximate check on the reading. The levelman should then read each of the three cross-hairs and record its reading separately, so that by taking a mean of these he has a greater check on the reading observed and gets a more accurate determination of the height than by reading one cross-hair only.

134. Turning-points.—In rough leveling it is of little consequence what manner of turning-point be used where the readings on each are attempted no closer than .1 or .01 of a foot. The turning-point may be on a pebble or other hard object on the ground, or on a short stake driven into the ground, or a hatchet laid on the ground. Where, however, more accurate work is attempted a better form of turning-point must be employed. Several such have been commonly used, the more usual being the head of a hatchet the blade of which is driven firmly into the ground, or a spike-shaped hammer, or a stone which is well embedded in the ground.

For *precise* work, however, these forms of *turning-point* are not sufficiently stable, and two general forms have been employed, one consisting of a hemispherical disk of iron, about 6 inches in diameter, with short spikes on the under side which are pressed into the ground by the foot. (Fig. 98, *a*.) This form is approximately that employed by the British Ordnance Survey, but it is not believed to be as satisfactory as a long steel peg well driven into the ground. (Fig. 98, *b*.) Such pegs should be $\frac{3}{4}$ to 1 inch in diameter at top and

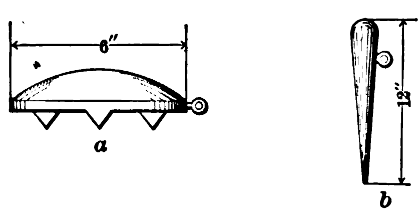


FIG. 98.—TURNING-POINTS.

from 12 to 18 inches in length, according to the consistency of the soil into which they are set. These should be firmly driven into the ground with a heavy hammer, a sufficient number of blows being struck to assure that the last few blows cause it to subside but little, and that friction is sufficient to prevent its further subsidence by the weight of the leveling-rod. The turning-point should be made of hardened steel, and the top rounded and kept so by frequent dressing at a smithy in order that there shall be but one point of contact, and that the highest.

135. Bench-marks.—In the course of any line of levels, be it short or long, accurate or approximate, marks should be left, the heights of which are determined by the leveling-rod, and these should be of such permanent character as not to be liable to mutilation or injury either accidentally or maliciously. This is in order that any future leveling which may be done in the neighborhood may start from or connect with the previous level line; and in order that the point of connection may be fully identified, such marks must be left and be fully described in the notes.

These bench-marks, as they are called, should be left preferably not farther apart than one mile, but may be farther than this or nearer together according, 1, to the character of the work; 2, the opportunity for description; 3, the purpose for which it is done; and 4, the chances of connection with other lines of levels. On a line of railway they should be at such distance from the right of way as to assure their not being destroyed during construction of the road. Along highways or across country they should be so placed that they can be easily identified by descriptions which state their relation to some well-known object (Fig. 99), and they should not be placed upon rocks, etc., which are liable to disturbance either by repairs to the highway or by work in the adjacent fields.

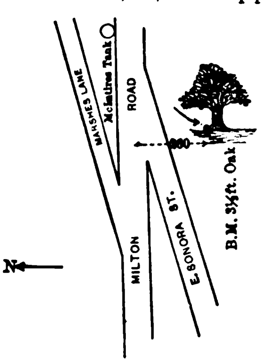


FIG. 99.
ILLUSTRATED DESCRIPTION OF BENCH-MARK.

One of the more common forms of bench-mark is a nail driven into the root of a tree. The nail should not be driven into the trunk above the ground because of the difficulty of placing the rod upon it. The nail placed in the root should be as near to the trunk as possible, in order to guard against its being accidentally struck, and a notch should be so cut in the root as to leave one part of it a little higher than any of the surrounding wood, and into the highest point of the notched root the nail should be driven flush to its surface. The best nail for such purposes is one of copper, as it can always be surely identified as distinct from nails which may accidentally or maliciously be driven in its neighborhood. Next to copper nails, wire nails are most satisfactory as bench-marks.

The corner-stone or water-table of a building, a door-sill, abutment of a bridge, or massive rock pier, all furnish desirable sites for bench-marks. The exact spot should be marked by a chisel-cut. For more permanent bench-marks, such as are left in precise leveling, it is customary to drill a hole in solid rock or the foundation-stone of some stable structure and to place a copper bolt in this. For greatest security from subsidence a building had better not be used, but a stone or iron post should be planted deep into the earth and the top of this be used as a bench-mark.

Fig. 100 shows a form of iron post used by the U. S. Geological Survey. Under this, in the bottom of the hole, is placed a large flat stone. This post is cheap and light, as it is of wrought-iron pipe. The same organization uses bronze tablets of similar design for cementing in masonry walls. In Germany small wrought-iron pins with round heads are cemented into walls or posts for bench-marks.

136. Method of Running Single Lines of Levels.—In general the details of the most approved methods of running spirit levels, as practiced by the more successful levelmen, may be stated as follows:

The rodman, after examining and wiping the bottom of the level-rod, standing behind it, balances it vertically on a bench-mark or a steel turning-point firmly driven into the ground. He waves it back and forth gently as he balances it, so that the levelman may see that it is plumb in the direction of the line of sight, and the latter calls to him, not by signaling with the hand, but by word of mouth, the exact

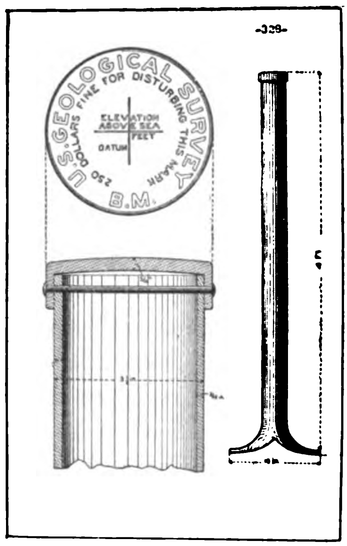


FIG. 100.—BRONZE TABLET AND WROUGHT-IRON BENCH-MARK POST.

figures on which to set the target. The rodman then takes down the rod, sets the target, clamps it and again holds it on the turning-point, when the levelmen may call to him to raise or lowér it one or more thousandths. Reclamping the target as directed, he now levels the rod carefully by watching the fore and back plumbing-level, the levelman waving him to level it across the line of sight as indicated by the vertical cross-hair.

The levelman, having his instrument well planted, and sighting first at the rod and then examining the level-bubble, if he finds the target exactly set at the same time that the instrument, as shown by the bubble, is exactly level, calls out "plumb," which expression, or some equivalent thereto, is instantly repeated by the rodman if he finds his rod plumb, and if the target is then perfectly set, the levelman gives the signal "all right"; if not, he calls again to the rodman the amount by which the target is to be raised or lowered, and the same operation is repeated until the rod is found to be precisely plumb at the same instant that the instrument is level and the horizontal cross-hair bisects the clamped target.

The rodman reads the rod and records his reading before he removes his turning-point, then he shows the rod to the levelman as they pass, the latter reading and recording the same. Both at once compute the height of instrument and compare results without having made any remarks one to the other concerning the rod reading, and if the results differ, as stated in the instructions, they then both reread the rod, recompute, and if the difference still exists, they must go back to the nearest bench-mark and rerun that much of the line. When a target setting falls above the 6.5-ft. mark, at which point is the break in the jointed rod, the closure of the rod at this point must be examined by both to make sure that it is perfect, otherwise the joint must be cleaned or a correction made for the failure of the rod to properly close.

In running a long single-rodded line of levels, the following additional precaution may be taken. Instead of setting and reading the target once, it is set and read twice; that is, it is *double-targeted* by the levelman first signaling the target to a setting up or down, when it is clamped, read and recorded by the rodman, who then loosens the target, continues to move it in the same direction in which it was going for, say, a tenth of a foot, when he is then signaled to a setting in the opposite direction. This gives a double target reading on each turning-point, and the method of making these tends to eliminate parallax in target-clamping. If the two readings differ by more than two thousandths of a foot, additional settings are made. As the rodman and the levelman pass, the latter reads the target, which is left clamped at the last setting, and the rodman, though he records all readings, uses in his computations only the first of the pair adopted, while the levelman uses the last.

137. Instructions for Leveling.—The following are the instructions for levelmen issued by the Director of the U. S. Geological Survey:

1. Primary level lines should be run with one or two rodmen and one levelmen, and when necessary a bubble-tender. Where such lines are run in circuits which will check back upon themselves or other lines, one rodman will suffice. Where long, unchecked lines are run, two rodmen must be employed.

2. SINGLE-RODDED LINES.—Levelman and rodman must keep separate notes and compute differences of elevation immediately. As levelman and rodman pass, the former must read the rod himself, record and compare readings, then compute the H. I., and after computations are made compare results with the rodman. No comparisons should be made until the record is complete. If the results differ, each must read the rod before comparing anything but results.

3. Work on primary lines should not be carried on during high winds or when the air is "boiling" badly. During very hot weather an effort should be made to get to work early and remain out late, rather than to work during midday.

4. Foresights and backsights should be of equal length, and no sight over 300 feet should be taken excepting under unavoidable circumstances, as in crossing rivers at fords or ferries or in crossing ravines. In such cases extraordinary precautions must be taken, as repeated readings at changed positions of rod and level, etc.

5. If it is impracticable to take equal foresights and backsights, as soon as the steep slope is passed take enough unequal sights to make each set balance. In this case extra care must be taken to insure correct adjustment of the level.

6. Distances along a railroad can be obtained by counting rails; at other times stadia or pacing may be used, according to the quality of the work. The distances in feet of both the foresights and backsights must be recorded in both note-books in the proper columns.

7. Always level the instrument exactly before setting the target. After setting it and before giving the signal "all right" examine the level-bubble. If found to be away from center, correct it and reset target.

8. The level must be adjusted daily, or oftener if necessary. The adjustment of the line of collimation and of the level-tube is especially important.

9. Provide rodmen with conical steel pegs, 6 to 12 inches long, with round heads, to be used as turning-points. Never take turning-points on rails, ties, or between them. Always drive the pegs firmly into the ground.

10. When the rod is lengthened beyond 6.5 feet, both the rodman and the levelman must examine the setting of the target as well as the reading of the rod vernier. When the rod is closed see that the rod vernier indicates 6.5 feet, not depending upon the abutting end to bring it back to place. Keep the lower end of the rod and the top of the turning-point free from mud and dirt.

11. Plumbing-levels must always be used and kept in adjustment, and long extensions of the rod avoided.

12. Leave temporary bench-marks at frequent intervals, marked so that they can be easily identified. These may be on a solid rock well marked, a nail driven in the root of a tree or post, or on any place where the mark will not be disturbed for a few weeks. One such bench-mark should be left for every mile run, in order to give sufficient points to which to tie future levels. Mark in large figures, in a conspicuous place when possible, the elevation to the nearest foot. Make notes opposite all elevations at crossings of roads, railroads, streams, bridges, and in front of railway stations and public buildings, and of such other facts as may aid the topographer in his work.

13. All permanent bench-marks must be on copper bolts or bronze tablets let in drill-holes in masonry structures or in solid rock, or be on the iron posts adopted by this Survey. The figures of elevation must be stamped well into the metal, to the nearest foot only, also name or initial letter of the central datum point.

14. A complete description, accompanied by a large-scale sketch, must be made of each bench-mark, giving its exact elevation as computed from the mean of the two sets of notes. After bench-marks are stamped both levelman and rodman must examine them, and record in note-books the figures stamped thereon.

15. The limit of error in feet should not exceed $.05 \sqrt{\text{distance in miles}}$.

16. Use the regular Survey level-books; keep full descriptive notes on title-page of every book, giving names, dates, etc. Each man should be responsible for his own note-book; and under no circumstances should erasures be made, a single pencil-line being drawn through erroneous records.

17. When errors are discovered as the work progresses, report the same at once to the topographer in charge.

18. Keep each set of notes separately and independently as taken, paying no attention whatever to other notes except to compare results. If on comparison errors are discovered, correct them only by new observations or computations. All notes must be recorded directly in note-book. Separate pieces of paper for figuring or temporary records must not, under any circumstances, be used.

19. In long, single-rodded lines make two target-settings on each turning-point, by first signaling "up" or "down" to a setting, which is recorded by the rodman, then unclamping and signaling in the opposite direction to a setting. If the two differ more than .002 of a foot, additional readings must be made. The rodman should record all readings, using in his computations only the first of the pair adopted, and the levelman the last.

20. DOUBLE-RODDED LINES.—In running unchecked or single primary lines with two rodmen, they should set on turning-points 10 to 20 feet apart, but each at equal distances for foresights and backsights; otherwise the above instructions are to be followed with the following modifications:

21. The tripod clamping-screws should be loosened when the instrument is set, and tightened only after the legs are firmly planted, and the instrument must be shaded at all times by the bubble-tender.

22. The laborer should place the steel turning-points for foresights and then return and not remove the backsight points until the levelman has set targets on the new foresight, so that there shall be in the ground at all times two turning-points the elevations of which are known.

23. Bench-marks left at termination of work at night, or for rain or other cause, should be practically turning-points in a continuous line. They should consist of large wooden pegs driven below the surface of the ground, with a copper nail firmly embedded in the top. One of these pegs is to be used as the final turning-point for each rodman. They are to be covered with dirt or otherwise hidden, their location being marked by sketches in note-books showing relation to railroad ties, telegraph-poles, etc.

24. An index-book or list of bench-marks must be kept posted in the field, in ink, for all classes of leveling done. In these, location sketches of permanent bench-marks may be made, and descriptions should in every case refer, with distance, to some village, section corner, or other place of local importance. All circuit-closure errors should be distinctly noted, with cross-reference by page to the connecting lines.

138. Note-books.—There are several methods commonly used in keeping notes of ordinary levels. Where the leveling is for a line of railway or canal, elevations are taken at every one hundred feet and at intermediate points to note sudden changes in slope or at stream crossings and similar features. The more usual ruling in a level note-book is to have one page divided into five columns, the opposite page being left free for remarks and for a plot of the level line, showing position of turning-points, road and stream crossings, etc.

LEVEL NOTES.

					Date, Sept. 26, 1898.	
Dist. B. S.	Dist. F. S.	Backsight.	H. I., Feet.	Foresight.	Elevation, Feet.	Sta.
		Morehouse	ville to Pis	eco, N. Y.		
					1910.429	1
55	55	10.721	1921.150	7.938	1913.212	
20	45	0.786	1913.998	11.984	1902.014	
23	55	0.801	1902.815	10.587	1892.228	2

The five columns into which the note-book is ruled are generally marked at their heads respectively "Station," "B. S." for backsight, "H. I." for height of instrument, "F. S." for foresight, and "E." for elevation. In the first or station column are placed the letters "B. M." with number, for bench-marks, and "T. P." with number, to indicate the position of turning-points. In the backsight column is placed the reading observed in backsighting on any bench-mark or turning-point. In the height of instrument column is placed the height of the line of collimation of the instrument as obtained by adding to the last recorded elevation in the fifth column the reading of the rod recorded in the backsight column. In the foresight column is placed the reading of the rod recorded at each of the intermediate stations, and next to it in the elevation column the elevation is obtained by subtracting the foresight from the height of instrument; also the reading of the rod at the foresight on the next turning-point or bench-mark is obtained by subtracting the foresight from the height of instrument. Not uncommonly the notes in the book are kept by having the foresight and elevation of the next turning-point recorded on the line below that on which the backsight and height of instrument and the last turning-point are recorded.

139. Platting Profiles.—For purposes of construction and in order that levels may be more readily interpreted, the notes are platted on what is called cross-section or profile paper so as to show graphically the undulations of the surface passed over. There are numerous forms of ruling for profile papers which are kept in stock by various dealers in mathematical instruments, the more common being a vertical ruling which divides the paper horizontally into spaces about ¼ inch apart, while the horizontal divisions or elevations are shone by vertical spaces of like size, but heavily ruled and divided into five smaller spaces by finer ruled horizontal lines.

In platting the profile a convenient elevation is assumed for the bottom horizontal line, perhaps sea-level or some datum which will be the lowest point on the line of the route leveled, and opposite it may be marked zero as datum or its elevation above sea-level, if this is known. For railway or canal work where construction is to follow, it is usual to assume one foot as the smallest vertical interval of the profile-paper, and 10 feet as the smallest horizontal interval, the proportion then being 5 feet of vertical to 1 of horizontal, or 5 to 1. Various other proportions may be used, a greater disproportion of vertical to horizontal being employed to accentuate the irregularities of very rough country, each horizontal division being assumed as 10 feet, or 100 feet, or a fraction of a mile, as the case may be. The distance to each turning-point or station at which the elevation is determined is that ascertained by counting the vertical lines from left to right, and above it the corresponding elevation is platted by counting from the datum or base line the proper number of horizontal lines.

CHAPTER XVI.

LEVELING OF PRECISION.

140. **Precise Leveling.**—When for any reason it is necessary to determine elevations with the greatest precision attainable, as in government work along the Mississippi and Missouri rivers, and where elevations have to be carried great distances from the ocean, in order to give datums on which to base other levels, as the primary level of the U. S. Geological Survey or the geodetic investigations of the U. S. Coast Survey, spirit-leveling is executed by methods which differ materially from those just described.

In the United States three methods of precise leveling have been practiced by three different government organizations. One of the oldest and most satisfactory is that employed by the *U. S. Engineers* on the Mississippi River, and is an adaptation of the European modes of leveling, in which a Swiss instrument, the Kern level, is used and a speaking-rod is employed. The *U. S. Coast and Geodetic Survey* have devised a peculiar instrument, called a "geodesic" level, which has been exclusively used by them in connection with the target-rod. The *U. S. Geological Survey* uses a modification of an instrument originally designed by Mr. Van Orden of the Coast Survey, and employs purely spirit-leveling methods, using either target- or speaking-rods. Each method has its own advocates, but that of the Coast Survey is so cumbersome, involves such lengthy and expensive computations, and is so influenced by instrumental errors which must be corrected, that it is not one which is likely to find favor elsewhere, nor is its use likely to continue much longer. It is fully described in various reports of the Coast and Geodetic Survey, as well as in Johnson's "Surveying" and Baker's "Engineer's Surveying Instruments," and will, therefore, not be described in detail here.

The method of leveling employed by the U. S. Engineers is free from the cumbersome computations and the troublesome corrections for instrumental constants that occur in the method employed by the Coast Survey. At the same time it is hampered by a few necessary adjustments and corrections which render it more complex and less expeditious than the method of the Geological Survey. As this mode of precise leveling is fully described in the Reports of the U. S. Engineers, as well as in the surveying text-books just referred to, it will not be described in detail.

141. Geodetic Leveling.—There are numerous serious disadvantages to this variety of leveling, among the more prominent of which are:

1. There is no check on the work of the instrumentman, as the rodmen are not able to make or keep duplicate notes;

2. A high order of skill is required in the levelman, as the instrument is delicate and complicated and there are to be made many corrections and tests to determine its constants;

3. The results are not immediately available, the office reductions and computations consuming even more time than the actual field-work. As a result this mode of leveling has found but limited favor in the past and has now been practically abandoned even by the U. S. Coast Survey. It is therefore but briefly described here, and more as a matter of interest than information.

A prime requisite in geodesic leveling is that the distance between the rod and the instrument must be exactly known, since there is added to the ordinary spirit-leveling features the operation of reading small vertical angles of a few seconds and computing and reducing them to elevations. For, in geodesic leveling as practiced by the United States Coast and Geodetic Survey, after getting the instrument practically level, it is assumed that it is impossible to watch the level-bubble and see that it is absolutely level at the same instant that bisection of the target is obtained, and therefore the observer, after leveling his instrument approximately, pays no further attention to the bubble while sighting. The rodman is first signaled to move the target until nearly bisected by the cross-hair, when it is clamped, and thereafter the rodman devotes his attention wholly to keeping the rod steady and plumb. Watching the level-bubble, the levelman then brings it to exact center by turning the micrometer-screw, and he notes and records the micrometer reading. Then, without further watching the bubble, he exactly bisects the target with the cross-hair by turning the micrometer-screw, and again records the reading of the latter, and the designating number on the rod. This operation is then repeated with the level and telescope reversed, the mean of the four readings taken, and the difference in elevation between the telescope pointed at the target and the instrument level, as shown by the micrometer readings, is added to or subtracted from the rod reading. It must be noted, however, that an important factor in this operation is the difference of elevation between rod reading and horizon reading as obtained from a trigonometric computation, depending upon a minute gradienter angle and the distance of the rod from the instrument.

142. **Precise Spirit-level.**—The adjustments of precise levels do not differ essentially from those of ordinary Y levels. In the latter the less important adjustments are neglected as being less than the degree of accuracy aimed at; but as extreme accuracy is desired in precise leveling, every adjustment must be carefully made, even though the instrument is used in such manner as to eliminate errors of adjustment. Accordingly, the instrument is adjusted as nearly as prac-

ticable, and then the errors of instrument are determined and each single observation corrected for these errors. As the inequality of the diameters of the collars cannot be eliminated by a system of double observations, since the line of vertical axis is invariable, it is practically eliminated from the final result by reading equal foresights and backsights. Although the inequality of the diameters of the collars cannot be eliminated by double readings, it can be determined by observations with a striding-level, as in the case of the astronomic transit, and can be applied as a correction to the rod readings where a system of double-rodding is employed.

The *precise level* used by the U. S. Geological Survey was designed and is made by Messrs. Buff and Berger of Boston. Like other precise levels, one of its essentials is a

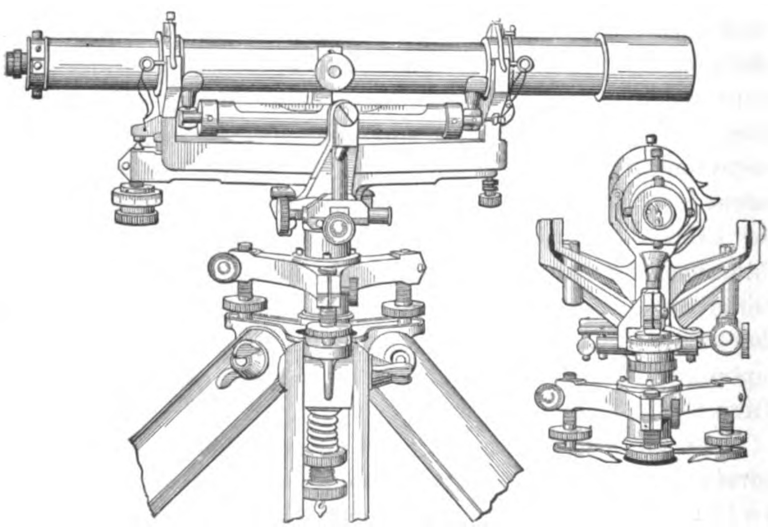


FIG. 101.—PRECISE SPIRIT-LEVEL.

very firm tripod with split legs, so as to give a broad head and correspondingly firm base for the support of the instrument. (Fig. 101.) On the head the level is supported freely by three leveling-screws, and it is clamped to the tripod-head by a stout center screw when not in use. The telescope has an aperture of $1\frac{1}{2}$ inches and magnifying power of 40 diameters, and is inverting. It likewise rotates in vertical plane by means of a milled-head screw nearly under the eyepiece; but this rotation is about a horizontal axis, not under the object-glass as in the other precise instruments, but placed opposite the center of the instrument by means of a cradle the axes of which are within a fraction of an inch of the line of collimation, thus securing the telescope a motion in altitude free from any change in the height of the line of collimation, as must occur in the other instruments.

It is leveled by a long spirit-bubble hanging from the telescope, as in ordinary spirit-levels, and in addition is supplied with an auxiliary striding-level. The bubble is so graduated that one division $\frac{1}{10}$ inch in length is equivalent to 4 seconds in arc. The author does not approve the use of the striding-level nor the micrometer leveling-screw as such, but merely as a milled-head screw for final leveling. In place of the chambered bubble as furnished by the makers, two bubbles of different sensitiveness should be carried in the field, one in which a division is equivalent to 4 seconds of arc, and the other in which a division is equivalent to 8 seconds of arc, and both should be carried in separate metal frames so that they can be changed without much delay.

143. Sequence in Simultaneous Double-rodded Leveling.—The method of leveling approved by the author as most satisfactory is not to run a single-rodded line forward and a similar line backward over the same series of turning-points as do the U. S. Engineers, but to check the work by running a simultaneous line of levels with one instrument and one instrumentman, but two rods and rodmen turning on separate turning-points (Art. 134). The idea of any form of duplicate or simultaneous rodded leveling is that checks shall be had on various bench-marks, the result of observing in opposite or reverse directions, so as to correct errors introduced by refraction and to get a mean elevation on the lines run in opposite directions. A notable peculiarity in all precise leveling is a constant *divergence between the duplicate lines*, which is to be largely ascribed to settlement in instrument between the time of observing foresight and backsight on each line (Art. 149). To reduce divergence the effort should be to allow the least lapse of time between the back- and foresights. To procure this result the *sequence in running* should be to backsight on rodman A at a_1 (Fig. 102), immediately

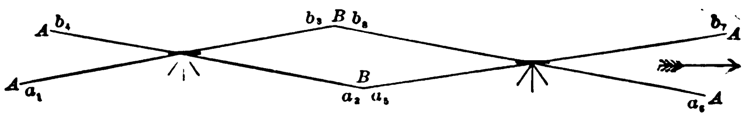


FIG. 102.—DUPLICATE DIRECT AND REVERSE LEVELING WITH SINGLE RODS.

reversing the instrument foresight on rodman B at a_2, then foresight on rodman B at b_3, and backsight on rodman A at b_4. In this method it will be observed that the level notes are complicated or divided between the two rodmen, because one rodman acts as backsight on both rear turning-points, and the other rodman as foresight on both fore turning-points. The levelman, however, keeps a clear set of notes of both rod readings, and the rodmen exchange foresight and backsight notes at the end of a day's work by summation between bench-marks.

The advantage of this method is in the quick observing between foresights and backsights on each line, practically no time elapsing between the making of these sights other than that required in reversing the instrument and watching the bubble. It was believed that by this method practically no subsidence occurrs between these sights, a belief borne out by the fact that the greater length of time elapsing between the sighting of the two lines results in a greater divergence than might even be anticipated. The order of sequence in sighting is such as to practically run one line in a direct and the other in the reverse or opposite direction.

The method of *exchanging notes* requires each man to practically walk the distance leveled twice, for after rod a_1 is sighted, then the rodman moves towards the levelman for his inspection; meantime the latter sights rod a_2, then the levelman moves towards A to meet him, and they exchange notes, A returning to point b_4, and the levelman going to meet B, whose rod he reads, B then returning to point b_3. The second set of sights having been made, the notes are exchanged as the men pass each other, the rear rodman and levelman moving forward each time.

The above method requires a great amount of extra walking by the rodman and levelman when exchanging notes. This may be obviated by the employment of the *double-faced rods* described in Art. 145, and the most satisfactory results are obtained by the use of this double rod. The procedure in using it is as follows:

Backsight on rodman A (Fig. 103) to turning-point a_1, red rectangular target, immediately foresight to rodman B

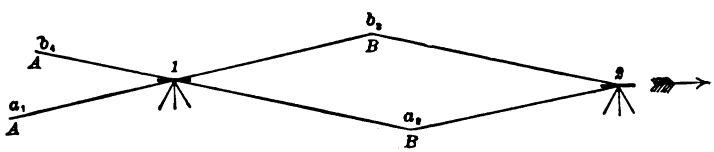


FIG. 103.—DUPLICATE DIRECT AND REVERSE LEVELING WITH DOUBLE ROD.

at turning-point a_2, red rectangular target (Art. 145); then, without any lapse of time for exchange of notes, the two rodmen move to the adjacent turning-points, and the levelman foresights on rodman B at turning-point b_3, black oval target, and backsights on rodman A at b_4, black oval target. The rodmen clamp their targets after each setting, and the rear rodman and the levelman move forward together, so that, as the levelman passes the rodmen, he is able to read and record the clamped targets, and the whole is accomplished with such a considerable reduction of elapsed time between the two lines, since the targets have not to be read until all the observing is completed, as to materially increase the speed, reduce the cost, and reduce the divergence.

144. Methods of Running.—In precise leveling a double line is invariably run for the purpose of check on every bench-mark. The *U. S. Engineers* adopt a method of sequence which is that already described for double rod for ordinary spirit-levels (Art. 130). For speed they use two rodmen, and the levelman backsights on rodman A at a_1 (Fig. 104)

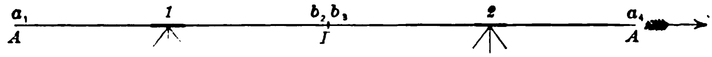


FIG. 104.—SINGLE-RODDING WITH TWO RODMEN.

and foresights on rodman B at b_2. Then the levelman and A move forward, and the former backsights on B at b_3 and foresights on A at a_4. This is a single line of levels, and the party duplicate their own work by rerunning over the same line in an opposite direction.

In the *U. S. Coast Survey* the levelman backsights on rodman A (Fig. 105) at the turning-point a_1, and then back-

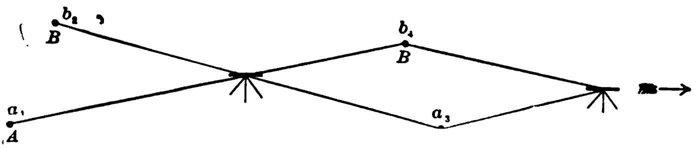


FIG. 105.—DUPLICATE RODDING, BOTH LINES DIRECT ONLY.

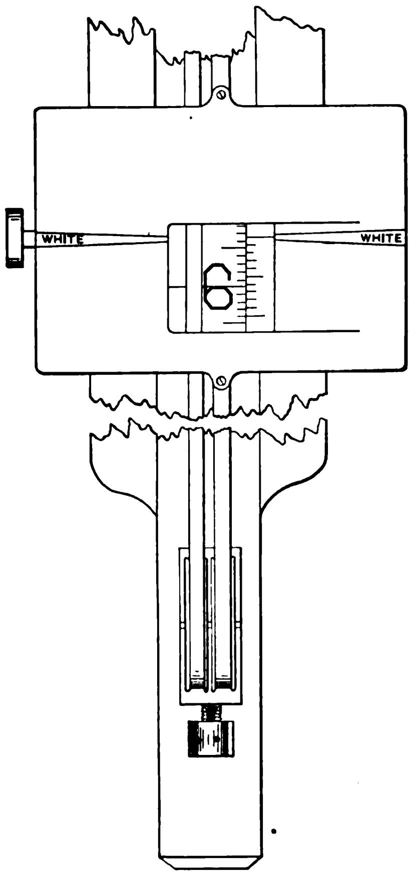
sights on rodman B at the turning-point b_2. Both A and B then pass him, and he then foresights on rodman A at turning-point a_3 and on rodman B at b_4, the rear turning-points a_1 and b_2 being left in the ground until the turning-points a_3 and b_4 are set.

145. Precise Rods. — The Coast Survey rod is of thoroughly paraffined wood, and the bottom, which is hemispherical, is set in saucer-shaped turning-points, the curvature of which is greater than that of the rod foot. This rod is single and non-extensible, 12 feet long, and divided into fractions of a meter by large, easily legible markings. At short intervals on its face are inserted in the pine wood metal plugs on each of which is engraved a fine line, and these are the zero marks on which the vernier is read; it being believed that these lines are finer than divisions can possibly be made upon wood. The rod can be read directly to thousandths of a meter, and by estimation to one ten-thousandth of a meter, by means of a target which is moved up and down by an endless chain passing over pulleys at either end of the rod, while the target can be clamped by means of another chain which is convenient to the hand of the rodman.

The U. S. Engineers use a rod made of one piece of wood 12 feet in length. It has a T-shaped cross-section, a foot-plate, and a turning-point similar to the above. The rod is self-reading, that is, without targets, and graduated to centimeters. Closer records are made by estimation by the levelman, since there are three horizontal cross-wires in the instrument, on each of which readings are made, and the mean of these is the value used.

The precise rods used by the U. S. Geological Survey are of two kinds, target-rods and speaking-rods. The *double-target rods* are made by Messrs. W. & L. E. Gurley of the best selected white pine, well seasoned and heated to a high temperature, when they are impregnated with boiling paraffine to a depth of one-eighth inch. The rods are a little over 10 feet long, and the graduations are commenced about a foot from the bottom of the rod to prevent readings being taken too near the bottom of the rod because of refraction. They are made of three pieces of wood bolted together, the cross-section forming a + (Fig. 106). These rods are graduated on both sides, and each is supplied with two targets, which are, one oval and red, the other rectangular and black, with verniers on the edge of a square hole in the face. These verniers can be moved by means of a spring in a direction at right angles to the line of sight, so as to bring them to a close bearing against the graduations and thus prevent parallax in reading.

Elevation.



Section.

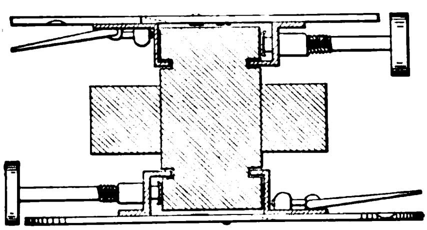


Fig. 106.—U. S. Geological Survey Double-target Level-rod.
One-third size.

The zero of the targets consists of a white stripe, wider at the outer edges than in the middle and of such width that at the nearest possible setting of the level the cross-hair of the latter will easily bisect the narrower part of the stripe, it being found preferable to bisect a stripe rather than cover a line with the cross-hair. The targets are handled by endless tapes running over pulleys at either end of the rods. The bottoms of the rods are protected by steel plates narrowed down to one half inch in area by giving them the shape of truncated pyramids. The rods are graduated in feet and hundredths, and read by vernier to thousandths.

These rods greatly increase the speed of leveling and reduce the amount of walking. The difference in shape and color of the two targets reduces to a minimum the possibility of error in the record of the two faces; much of the time expended in comparison of notes and check-reading of rod by instrumentman and rodmen is saved, because the instrumentman can set the target on the face of the rear rod and then on the corresponding face of the front rod and, without the necessity of reading or exchange of notes, both rodmen set on the next turning-points, clamping the targets on the other face.

The *single-target rods* used are similar in all essential respects to the double rods just described, but have only one face divided and one target. They lack, therefore, the advantages gained by speed in manipulation with the double rods. They are, however, superior in speed and accuracy to other forms of target-rods (Fig. 96).

The *precise speaking-rods* used by the U. S. Geological Survey are an adaptation of the non-extensible speaking-rods used in European geodetic surveys. They were designed by the author after suggestions received from Mr. Horace Andrews. They are a little over 10 feet in length and are graduated for 10 feet. The divisions of this rod are peculiar and are illustrated to half-scale in Fig. 107. They are so arranged as to divide the spaces into five parts, on the theory that the eye can estimate the position of the cross-hair on the rod to five parts more readily than by attempting to divide the same space by estimation into ten parts. In order to get the desired result, and as a foot is too small a space to be divided in the manner required, this rod is divided into units of 2 feet each. Accordingly, each actual foot is but a half of a unit, and so on for tenths and hundredths, the result being that the 10-foot rod is divided into five units and each of these into ten others and each of these again into ten spaces. Thus one-fifth of the smallest unit space, the hundredths, can be easily estimated by eye with the aid of the cross-hair at the greatest distance permitted in precise leveling whereas tenths could not be estimated. This space being .01 of a unit, a fifth of it is .002 of a unit, actually equivalent to .004 of a foot. The portion of the rod hatched in the illustration is painted red and the remainder is painted black on white enamel, the ruling of the black lines being very fine, as shown.

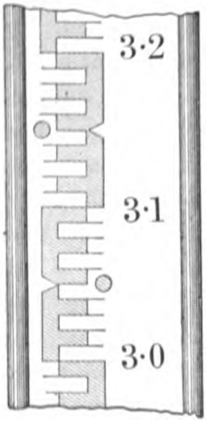


FIG. 107.—U. S. GEOLOGICAL SURVEY PRECISE SPEAKING-ROD. One-half size.

The mode of keeping the notes with this rod is unique. Whatever the initial elevation may be, say 100 feet, it is put in the column of elevations as being one half of this, or 50 feet. Then the backsights and foresights are recorded, and the computations made as with any other rod, the actual figures read being used. Whenever a bench-mark is reached and it is desired to know its elevation, that given in the book is doubled. This introduces no complications in note-keeping, simplifies the rod reading, and permits of the estimation of differences of heights on a speaking-rod to .001 of a foot.

146. Manipulation of Instrument.—In precise leveling several important details of manipulation, although apparently trivial, add greatly to the accuracy of the result. In addition to the necessity of exactly equalizing sights and of taking care not to refocus the instrument without adjustment, care should be taken to loosen the instrument from the tripod by freeing the central holding-screw after the tripod has been firmly planted in the ground. The instrument then rests on the tripod merely by its own weight and is not subject to the torsional strain which may be brought upon it by the tension of the center holding-screw. The three screws which bind the wooden tripod legs to the metal tripod head should be loosened after the tripod has been firmly planted, and then retightened before the observations are made, so as to obviate strain in the tripod and its head due to any twist brought against these screws in planting the tripod. After giving the final signal to clamp the target, the instrumentman should have the rod replaced on the turning-point, should again notice the level-bubble, and take a last look at the target bisection, calling out to the rodman, "plumb," or some similar word, at the moment the same is repeated by the rodman, so as to make sure that the rod is plumb at the moment of target bisection and after the target has been clamped.

147. **Length of Sight.**—There is a limit of distance at which the rod should be placed from the instrument, which is variable and is dependent chiefly upon—

1. Magnifying power of the telescope;
2. Quality of work being done;
3. Atmospheric conditions; and
4. Sensitiveness of the level-bubble.

The first condition affects both the *nearness and* the *extreme distance* at which sights should be taken. If the rod is too close to the instrument, difficulty will be experienced in properly setting the target or bisecting the divisions of the rod if the latter is self-reading, and the levelman may waste much time in an effort to find too close a reading. There is also sometimes difficulty in focusing on a very near rod, but above all is the slowness caused by the short sights. Effort should therefore be made to take as long sights as are permissible. Accordingly, the four classes of limitations above specified may be all taken to limit the greatest length of sight rather than the least. The distance of the rod from the instrument should not be so far that the magnifying power of the telescope will not permit of reading the rod or setting the target to the smallest division of the rod.

The second limitation to distance, the *quality of* the *work*, gives the greatest latitude in distance of sight. If rough or flying levels are being run and only turning-points taken, and these as far apart as the power of the instrument will permit, or if the rod is being read to the .01 or even .1 of a foot for the obtaining of approximate elevations only, the rod may be placed at as great a distance as the target or the divisions upon the rod are clearly visible, providing, of course, that the greater the distance of the rod from the instrument the more nearly the foresights and backsights should be equalized, otherwise errors will be introduced into the work owing to the errors in the bubble and the instrument adjustments.

The third limitation to distance, *atmospheric conditions*, is one of the most important, since, when the atmosphere is vibrating rapidly because of heat, the difficulties of accurately reading the rod or bisecting the target become so great as to render it impossible to make the observations within the limit of a rod division, the cross-hairs of the instrument frequently dancing over several thousandths or even hundredths of a foot on the rod if it is placed at a considerable distance. Accordingly, as heat vibrations increase, the lengths of the sights must be diminished; and it is not uncommon, in very accurate work, to have to reduce sights to as low as 100 feet, and even then the results of a rod setting may be in doubt. Precise leveling should not be carried on in very hot weather or when the atmosphere is vibrating violently from heat or other causes.

Atmospheric conditions, the magnifying power of the glasses, and other elements being satisfactory, the true limit of distance is fixed by the *sensitiveness of the bubble.* For instance, with an 8-second bubble the target can be set with comparative certainty to within .001 of a foot at a distance of a little less than 300 feet. Likewise, with a 4-second bubble on the same instrument the target can be set to .001 of a foot with comparative accuracy at a distance of about 400 feet. Accordingly, these distances for the instrument under consideration practically fix the limits of distance at which the rod may be placed under favorable atmospheric and other conditions. The ordinary engineer's level has a 20-second bubble, one which therefore for accurate work would limit the distance even more greatly; that is, with such an instrument rod readings of less than .01 of a foot are rarely possible with accuracy. The accuracy of the same instrument is greatly increased by use of a 10-second bubble. It may be stated that, in ordinary engineering levels, sights as long as 300 to 500 feet may be regularly taken. In precise levels, however, 350 feet should not be exceeded even with an instrument having a 2-second bubble, for though the sensitiveness of the bubble is increased, the other functions of the instrumental error, atmosphere, magnifying power, etc., do not increase in equal ratio.

148. Sources of Error.—The operation of spirit-leveling involves perhaps more varieties of errors than occur in the use of any other engineering instrument. Moreover, these are of such peculiar kinds as to involve a fine distinction between such as are compensating and such as are cumulative. The sources of error may be divided into—

1. Instrumental errors;
2. Atmospheric errors;
3. Rod errors, including turning-point and record; and
4. Errors of manipulation.

Among *instrumental errors* the most important is perhaps that due to the line of sight not being parallel to the level-bubble, and may be caused by imperfect adjustment or unequal size of the rings or both. If the telescope-slide is not straight or does not fit well, it will introduce an error. All of these errors may be eliminated by placing the instrument midway between the turning-points, and wherever accurate results, as in precise leveling, are desired, the lengths of foresights and backsights should be exactly equalized. In precise work the error of the telescope-slide is practically eliminated by not changing the focus after adjustment of the instrument. This would necessitate readjusting the instrument if for any reason the lengths of sights should be changed in any part of the day's run. Another source of error arising from the instrument is produced by the adhesion of the fluid inside the glass tube, which prevents the bubble from coming precisely to its true point of equilibrium. This frequently occurs owing to the crystallization of something which is contained in the ether, little granules or crystals forming on the inside of the glass which catch the bubble and keep it from running smoothly. Careful microscopic examination of the bubble tube may show these crystals, and if discovered it should be discarded.

The most important of *atmospheric errors* is the effect of the *heat of the sun* on one end of the telescope raising it by unequal expansion. This error may be partially eliminated in ordinary leveling by rapid manipulation of the instrument, so as to leave the least interval in which the sun may act. The error is greatest in work towards or from the sun and is cumulative; for if on the backsight the Y nearer the object glass is expanded, thus elevating the line of sight, then the other Y is expanded in the foresight, thus depressing the line of sight. This is a much greater source of error than is ordinarily recognized, for the error in the case above cited is further increased on the foresight by the cooling of the Y, which is expanded on the backsight. The sources of error due to this cause may be largely eliminated by shading the instrument from the sun, and this should be done in careful engineering as well as in precise leveling.

Another class of atmospheric error is due to the jarring or shaking both of the instrument and of the rod by *high winds*. When the wind has become so high that in looking through the telescope the cross-hairs dance to such an extent as to prevent accurately sighting the target; or when it is evident that the jarring of the instrument interferes with the exact leveling of the bubble; or when the rod itself vibrates to such an extent as to make it impracticable to exactly sight it by the instrument, precise leveling observations should be discontinued. The effect of high winds may be partially obviated by using fine wires or cords held by men to guy the top of the rod, and they may be obviated in the instrument by screening it either with an umbrella, wind-break, or a tent. In precise leveling by the Coast Survey on the plains of Nebraska, the wind has been so high continuously for weeks at a time as to render it necessary even to work in a high wind, and the harmful effect of the latter has been neutralized by guying the rods and by erecting a shelter-tent at every sighting. In running along the line of the Union Pacific Railroad a shelter-tent was carried on a frame on a hand-car in such manner that the instrument could be set up on the ground under the tent, and thus scarcely any time was lost in the operation.

A most serious atmospheric error is that due to *frost*, or especially a frost following rain or melting snow. The writer has observed instances where tripod legs, firmly inserted in the frozen ground in the morning, when the sun was causing rapid thawing, have in the course of a few minutes—in fact, during the time the instrument was being sighted after leveling—sunk so quickly as to keep the bubble continuously in motion, thus rendering it impossible to get a stationary position of the bubble. This was due to the heat of the metal tips of the tripod, warmed while the instrument was carried in the air, thawing the surrounding frozen ground, the water from which acted as a lubricant and permitted the tripod to sink. Precise leveling should not be conducted under such circumstances; for not only is the instrument affected, but also the turning-points on which the rod rests are liable to some movement, however carefully made and placed. The effects of *dancing of the air* and of *refraction* are referred to in Articles 111 and 152.

Rod and turning-point errors are of the same kind. Among the latter is error due to settlement or jarring of the turning-point or to its inferior quality for the object sought. The first of these is to be guarded against only by using steel turning-points and driving them into the ground as firmly as possible with a heavy hand-sledge; and by care in placing the rod on the point so as not to produce any impact; and by carefully wiping the bottom of the rod and top of the turning-point prior to each setting. Errors of rod reading are to be guarded against by the levelman reading the rod and recording it himself when he and the rodman pass, so as to get a check on the reading of the rod by the latter, also in duplicate rodding by the two rods being read by the two rodmen as well as by the levelman.

Lack of *verticality of rod* is to be remedied by waving it slowly backward and forward that the instrumentman may see that the cross-hair is tangent to a rod graduation at its highest point; or, better, by the use of rod levels, two of which are attached at right angles to the side of the rod, though a single circular level may be employed. In the use of these levels, that which determines the verticality of the rod laterally scarcely need be noted by the rodman, as the vertical cross-hair of the spirit-level determines it in that direction. Another source of error in rods is due to inaccurate graduation. When done by a first-class instrument-maker and tested by the standards which he has in his possession, this source of error is generally found to be very small, yet for precise leveling the graduation should be tested by means of an official standard, and the error, however small, recorded and applied to each rod reading. Changes in rod length due to variation in temperature and moisture are so small that they may be disregarded in rods made of the best quality of well-seasoned white pine treated with paraffine as described in Article 145.

149. Divergence of Duplicate Level Lines.—A curious fact, probably first noted in the United States in the report of the Chief of Engineers of the Army for 1884, but since frequently observed by the U. S. Coast and Geodetic Survey, the U. S. Geological Survey, and others doing precise leveling, is the fact that when duplicate lines are run, either in opposite directions by two sets of levelers or by the use of a single instrument reading on two rods, the discrepancies between the two lines have an average tendency in one direction or to one sign, and increase with the distance. In other words, the *two lines separate* as they progress, the distance between the heights of any fixed bench-mark as determined by them increasing with the length of the line. Many reasons have been assigned for this, as settlement of instrument or of turning-points, effect of sun, illumination of target, frost, etc., but scarcely any are quite satisfactory. Remedies have been suggested, such as leveling alternate sections in opposite directions, or reading the backsight first at each alternate setting of the instrument, but no complete remedy has been yet discovered.

The writer's experience with such work on the Geological Survey indicates that the best results are obtained by a *duplicate rodded line* (Art. 143), and not by running two lines in opposite directions or in alternate sections. He believes that these errors are largely due to the settlement of the instrument between the time of taking backsights and foresights and between the time of observing on the two separate lines or rods. With the aid of Mr. W. Carvel Hall of the Geological Survey he has reduced this form of error to a minimum by quick manipulation; by the employment of the method of rod succession, whereby immediately after the backsight the foresight can be at once read (Art. 143), and by using double-faced rods (Art. 145), thus reducing the time consumed in reading the rod between the various sights. The reversal of the direction of the sights on the two lines tends to give the notes the effect of two lines run in opposite directions, since the computations go on in opposite directions and the instrument is manipulated in opposite directions. To sum up, the adoption of the method of rod succession, and the use of double-faced rods to produce quick backsighting and foresighting, and care to neutralize instrument subsidence in frosty ground have had the effect of greatly diminishing the divergence of duplicate lines.

150. Limit of Precision.—The final *error of a series of observations will*, according to the theory of probabilities, *vary as the square root of the number of observations* when affected only by accidental errors. Accordingly, when the instrument is set up the same number of times per mile, the error of leveling a given distance is assumed to be in proportion to the square root of the distance, and not to vary directly as the distance. In fact, a limit of error based on this presumption, while found to be very satisfactory for short distances, say those under one hundred miles, proves too severe for greater distances, and it is almost impossible to maintain it for such great distances as are leveled over by lines of precision. This is probably true because accidental errors are not the only ones made, and the number of observations are not solely proportional to the distance leveled, that is, the lengths of sight are not constant. While a fixed limit of precision may be maintained for a number of short pieces of leveling, it will generally be exceeded if the sums of errors be added together as the total discrepancy.

Various limits of precision have been fixed in accordance with the theory of probabilities by different precise-leveling surveys. If the probable error of leveling one mile be e', then that for leveling d miles is $e = e'\sqrt{d}$. Levels of precision executed in Europe of late years show that the probable error of level lines of precision should not exceed 5 mm. $\sqrt{\text{distance in kilometers}}$, equivalent to about .021 ft. $\sqrt{\text{distance in miles}}$, the result being in feet. The U. S. Coast and Geodetic Survey calls for a precision in feet equivalent to .02 ft. $\sqrt{\text{distance in miles}}$; the British Ordnance Survey endeavors to place a high limit in fixing a constant error of 0.01 foot per mile, and yet this same limit applied to any of the long lines of precision run in the United States is very much easier to attain than any of the limits fixed above, because it varies directly as the distance.

The U. S. Geological Survey has fixed as its limits of precision in its precise leveling that of the Coast Survey, namely, a result in feet = 0.02 ft. $\sqrt{\text{distance in miles}}$, or = .02 ft. $\sqrt{2d}$ miles for duplicate lines. The U. S. Mississippi and Missouri River Commissions aim at a limit represented by the formula 0.0126 ft. $\sqrt{2 \times \text{distance in miles}}$ for direct lines.

151. Adjustment of Group of Level Circuits.—Where a line of levels has been run in such manner as to connect back on itself, thus forming a *polygonal figure or circuit*, there will occur some error of closure. If the instrument be set up the same number of times in one mile, the probable error of the result increases as the square root of the distance. In attempting to distribute the error in such a closed circuit it must be remembered that the *weights to be applied are inversely proportional to the squares of the probable errors*, or, in other words, to the distance over which the leveling is carried. If the leveling be run over three routes, E, C, and D between the points A and B (Fig. 108) and the lengths of these be respectively 5, 7, and 8 miles, the weights to be applied to them will be respectively $\frac{1}{5}$, $\frac{1}{7}$, and $\frac{1}{8}$.

If a *closed circuit* of levels is run from A via C, B, and D back to A, and bench-marks are set at each of those points, the adjusted elevations of these benches should be in direct proportion to the distances between the benches. If the distance from A to C is 4 miles, from C to B 3 miles, from B to D 3 miles, and from the latter to A again 5 miles, then the total distance is 15 miles. Therefore $\frac{4}{15}$ of the total discrepancy

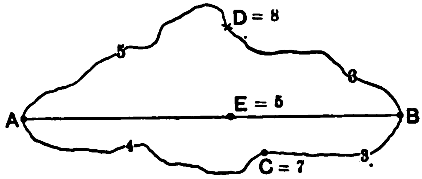


FIG. 108.—LEVEL CIRCUIT.

is to be subtracted from the elevation of the first bench, C; $\frac{7}{15}$ of the total discrepancy is to be subtracted from the second bench, B; $\frac{10}{15}$ from the third bench, D, etc.,—account of course to be taken of signs.

A *group* or *net of levels* such as that shown in Fig. 109 permits of the computation of the elevations of the various bench-marks by several different routes. If now the elevation of any one bench be given, the elevations of the other junction-points are to be obtained. The number of independent quantities in any such group of level circuits is one less than the number of connecting benches. If this group of levels be adjusted by the method of least squares, there will be introduced as many conditional equations as there are separate geometric figures and one less independent quantity than there are connecting bench-points.

A simpler method of adjustment, however, that recommended by Prof. J. B. Johnson and preferred by the author, is to consider the errors in proportion to the square roots of the distances or lengths of the sides of the polygonal figures. This is because the errors are compensating in their nature and increase with the square roots of the lengths of the lines.

Instead, therefore, of solving the group by least squares as one system, that polygonal figure having the largest error of closure should be first adjusted by distributing its error among its sides in proportion to the square roots of their length. Then the circuit or polygon having the next largest error

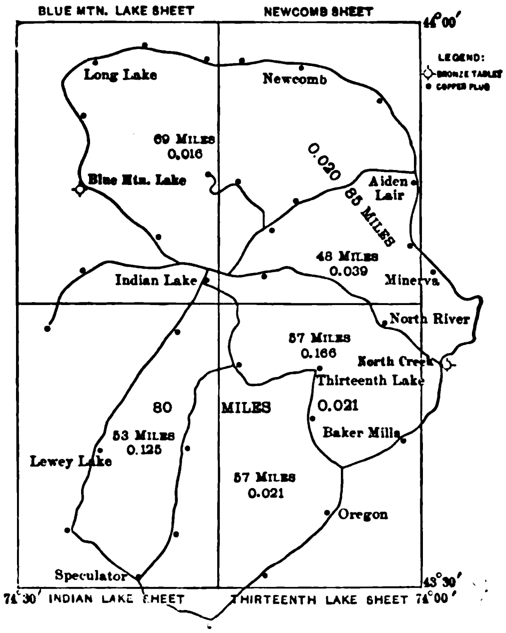


FIG. 109.—GROUP OF CONNECTED LEVEL CIRCUITS.

should be similarly adjusted, using the new values for the adjusted side if contiguous to the former, and distributing the remaining error among the remaining sides of the figure without distributing the side already adjusted.

152. Refraction and Curvature.—The line of sight of a telescope when the bubble is level is theoretically parallel to that of the surface of the ocean at rest. In fact, however, it is depressed below that plane by the action of *refraction*, and it lies between the level or curved surface of the ocean and a tangent plane to the same, but is nearer the latter. The deviation of the tangent plane from the level surface is about $\frac{2}{3}$ of a foot per mile, and for n miles it is $\frac{2}{3}n^2$ feet.

In all spirit-leveling and trigonometric operations *curvature and refraction* are rarely considered separately, but are usually treated in combination (Art. 166). Their combined effect is to cause the line of sight to be elevated above the level of the surface by an amount equal to about 0.57 foot in one mile, or for n miles by $0.57n^2$ feet. The above facts, however, have little bearing on the ordinary operations of spirit-leveling, as the lengths of the sights taken are too short to be affected appreciably by them. Moreover, so long as the rule is strictly adhered to that the lengths of backsights and foresights shall be equal, all effects due to curvature and refraction will be eliminated.

In long-distance leveling (Art. 155) the effects of curvature and refraction become immediately appreciable in amount and must be taken into consideration if sights are not equalized. Ordinarily, however, they are eliminated in this form of leveling by simultaneous reciprocal readings with two instruments, or ordinarily less accurately by frequently repeated reciprocal readings from either end, thus equalizing the lengths of the sights.

One of the most abundant causes of error in leveling is the refraction encountered by the line of sight passing near to the surface of the earth, and also another phenomenon nearly related to it—the dancing of the air due to heat-waves near the ground surface. This latter can only be eliminated satisfactorily by reducing the length of the sight when the *air* is *boiling* badly. This reduction must be of such amount that the space on the rod danced over by the cross-hair will not be of appreciable amount. Refraction may be reduced to a minimum by exercising the precaution of never sighting too short a rod—that is, never allowing the line of sight to come nearer the ground than $1\frac{1}{2}$ to 2 feet (Art. 111).

This precaution should be especially observed at that time of day at which refraction is greatest.

153. Speed in Leveling.—The speed with which levels can be run varies greatly with the accuracy desired, the character of the country, the atmospheric conditions, the method of running employed, and the levelmen and rodmen. In *ordinary* or *flying levels*, in which merely turning-points are taken and no great accuracy is aimed at and a self-reading rod employed, speeds of from 3 to 15 miles a working day are attainable, the lowest in very hilly country, the highest on comparatively flat plains. *Engineering levels* of considerable accuracy, such as the primary spirit-levels of the Geological Survey, are run at speeds varying under average conditions from 50 miles to 90 miles per month of about twenty working days.

Strange as it may seem, *precise levels* are run with a generally higher average speed than are the ordinary levels above cited. One reason is because they are invariably run over the best and most favorable grades, generally following the lines of railways. The chief reason is because they are run with two rodmen, so that no time is lost by the levelman or rodmen waiting for one another to move to the next position. The geodesic levels of the Coast Survey have been run in recent years with speeds of from 3 to 5 miles a day, the greater speed being made under favorable atmospheric conditions. The precise levels of the Geological Survey are run with greater speed, the average for the seasons 1896 to 1899 varying between 4 and 8 miles per day as limits.

154. Cost of Leveling.—Necessarily the cost of leveling varies according to the character of the work. A party which is organized for a long season of work will operate less expensively than one which is placed in the field for but a short period of time. The following estimates are based on seasons of at least several months' duration.

Ordinary or *flying levels* run by the Geological Survey along good roads in New England with a party consisting of levelman and rodman only, living on the country, average a cost of $2.50 per linear mile. The primary or engineering levels of the same organization run by a levelman and rodman only, but over all sorts of routes, since they are compelled to place a bench-mark once in every thirty-six square miles, and where subsistence is had either in hotels or farm-houses or in camp, vary in cost from $6.50 per linear mile in rough mountain country like the Adirondacks, West Virginia mountains, or Oregon, as one extreme, to $3.50 per linear mile in flat country like Alabama, western New York, and the Mississippi valley.

Where the work is executed in the best manner, as above described, and the rod is set only on turning-points and not on intermediate stations, a fair estimate of the cost can be had from an inspection of Table XIII giving the result of the work done by the various leveling parties working in different States and under different climatic and topographic conditions for the U. S. Geological Survey during the field season of 1896. The bench-marks enumerated were

TABLE XIII.

COST OF LEVELING PER MILE IN VARIOUS STATES.

State.	Miles of Levels.	Number of Bench-marks.	Cost per Linear Mile.	State.	Miles of Levels.	Number of Bench-marks.	Cost per Linear Mile.
Alabama	65	10	$4.30	Nebraska	365	100	$2.85
Arkansas	179	15	3.75	New York	925	105	3.66
California	338	72	11.27	North Carolina	597	108	4.15
Colorado	404	77	5.80	North Dakota	76	16	6.53
Delaware	40	14	2.80	Oregon	130	24	3.26
Georgia	278	38	4.30	South Dakota	320	42	2.79
Idaho	140	25	7.53	Texas	1,008	222	4.44
Illinois	129	7	....	Vermont	40	8	3.80
Indian Territory	4,174	700	....	Washington	186	40	8.44
Iowa	236	43	3.98	West Virginia	180	35	4.44
Kansas	43	15	....	Wyoming	304	58	8.17
Maryland	120	20	2.80				
Michigan	90	6	4.50	Totals and average	10,968	1,924	$4.78
Missouri	316	35	3.90				
Montana	200	29	4.49				

of the permanent metal forms (Fig. 100), and these added somewhat to the cost. Where less careful work is attempted, the cost may be reduced as much as one-half for each kind of country, and where intermediate stakes are set, say for every one hundred feet for railway leveling, the cost will be increased by at least one-half.

Precise leveling executed in connection with *city surveys* is necessarily more expensive and scarcely as accurate as that carried on elsewhere, because of the annoyance and jarring from passing vehicles, rapid alternation of sunshine and shadow about buildings, etc. In the precise leveling done in connection with the survey of the city of Baltimore, there were run 141 miles of double line, in the course of which there were established 606 permanent bench-marks, or one to every 1228 linear feet. As the area of the city survey was 30 square miles, there were established 20 bench-marks per square mile. The computed probable error of the work was about 0.003 of a foot per mile, about the same being the probable error of the precise leveling in the city of St. Louis. The cost of precise leveling in the city of Baltimore for field

TABLE XIV.

COST AND SPEED OF GOVERNMENT PRECISE LEVELING.

Organization.	Year.	Locality.	Days of Actual Field-work.	Miles of Duplicate Line.	Total Cost.	Speed. Miles per Day.	Cost per Mile.	Cost per Day.
Engineer Corps......	1882	Carrollton, La., to Biloxi, Miss......	35	87	$2778	2.5	$31.93	$79.37
"	1882	Keokuk, Ia., to Fulton, Ill	50	170	3252	3.4	19.08	65.04
" ...	1893	Blair, Neb., to Dewitt, Mo.........	22	32	736	1.5	23.00	33.50
Coast Survey........	1895	Richmond, Va., to Washington, D.C.	55	115	3900	2.0	10.94	3[illegible].20
"	1895	Lamar, Mo., to Chester, Ark.....	70	150				
Geological Survey ..	1896	Morehead City, N. C., to Paint Rock, N. C..............	105	457	2280	4.3	5.00	21.70
"	1897	Paint Rock, N. C., to Atlanta, Ga...	48	308	1172	6.4	3.78	24.40

and office work averaged $23.56 per mile, that for the city of St. Louis averaging $45.38 per mile.

155. Long-distance Precise Leveling.—In running precise levels it may occur that, owing to unusual physical conditions, the line cannot be carried forward by short and equal foresights and backsights, as in crossing an expanse of water. Under such circumstances, long sights, involving special methods of observation and reduction, become necessary. In long-distance leveling, in order to attain the accuracy of precise leveling, instrumental and atmospheric errors are eliminated by taking simultaneous reciprocal observations.

The *instrument* employed should be a good precise level, and the *rods* should be provided with large targets up to 12 inches square for distances of two miles. The target should be painted one color, preferably red, with a white band across its center, one to two inches wide at the outer edge of the target and narrowing to $\frac{1}{8}$ inch wide at the opening in the target center, this white streak to be bisected by the cross-hairs, and provided with a cross-wire opposite its center for convenience in target reading. The instruments and rods should rest on *solid foundations;* and in leveling across water, the more usual case in which such work is done, the telescope should be 10 to 15 feet above the water surface to avoid extreme refraction. The instrument should rest on a platform independent from any surrounding platform on which the observer may stand.

In such a piece of work conducted by Mr. Gerald Bagnall for the U. S. Engineer Corps at Galveston, Texas, platforms had to be erected in the water, and owing to the unstable character of the bottom an apron of rubble was placed around them. The corners and supports for the instruments were heavy piles driven 16 feet into the bottom, well braced horizontally and diagonally. Rocks were placed around the outside piles, and rows of sheet-piling were driven along them. A reference bench-mark was placed near each instrument and nearly at right angles to the directions of the line joining the two instruments, so that the long sight of both observers might be equal. Each leveling party consisted of an observer, a recorder, rodman, umbrellaman with the instrument, and an assistant to signal and watch signals with the glasses. *Simultaneous reciprocal observations* were taken with the two instruments, one at each end of the line, in order to eliminate the error due to refraction, and this was effected by signaling between the two so that the targets were set at the same moment.

The *errors* which have to be eliminated by this system are:

1. Those due to the inclination of the bubble and to collimation, which are eliminated by each instrument independently.

2. Those due to curvature and refraction, which are eliminated by the simultaneous reciprocal observations.

3. Those due to the inequalities of pivot-rings of both telescopes, which are eliminated by the observers changing stations and repeating the observations.

A *set of observations* at each station should consist of at least four rod readings taken with telescope and level direct and reversed, thus: 1st, telescope and level direct; 2d, telescope direct and level reversed; 3d, telescope inverted and level reversed; 4th, telescope inverted and level direct. When a sufficient number of sets of observations have been taken the observers change stations and repeat the operation, determining the true difference of elevation of the reference bench-marks, from which the heights of instruments for the long sights are determined. The maximum distance at which satisfactory results may be obtained depends on the instrument used and the conditions surrounding the work. With an instrument having a powerful object-glass, and high magnifying power being used, fair results may be obtained at distances up to two miles.

Another example of long-distance leveling is given here from the observations from one of three days in which the precise levels of the U. S. Geological Survey were carried across the Tennessee River by Mr. W. Carvel Hall, the greatest length of sight being 1810 ft. (Fig. 110.) In this work a 4-second bubble was used and a 40-diameter magnifying power. Lozenge-shaped pieces of paper, 0.07 ft. in width, were placed on the targets as markers. The sights were 85 ft. clear above the river surface. Two reference points, *A* and *D*, were placed on the near bank at distances of 15 and 20 ft. respectively, as backsights in the forward crossing, these being terminal points in the regular line. On the far bank of the river two other reference points were

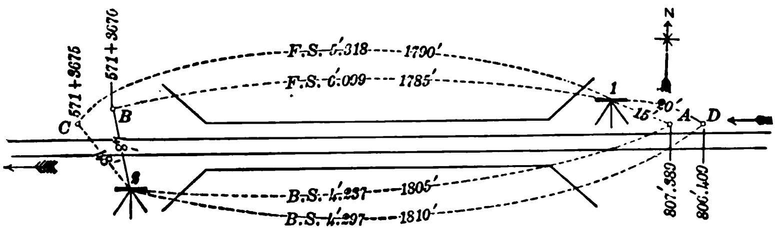


Fig. 110.—Long-distance Leveling across Tennessee River.

placed, both at a distance of 48 ft. beyond the instrument, but at some little distance apart one from the other, and these became rear turning-points in the continuation of the regular line. The foresights taken from the near bank on the two lines, after backsighting on the reference points, were respectively 1785 and 1790 ft. in length, and the backsights taken from the far bank to the rods on the near bank, which were placed on the reference marks as turning-points, were respectively 1805 and 1810 ft., or sufficiently close to the length of the foresights to practically eliminate errors due to curvature. The errors due to refraction were eliminated as far as possible by observing at such times in the day as refraction was least, namely, late in the morning, and when the day was slightly cloudy, the atmosphere still, and there was no perceptible "boiling" of the air; also by observing at three different times on another day under different atmospheric conditions.

From the instrument position 1 on the near bank a reading was first made on rear reference point A, and then ten readings were made on fore reference point B on the other side of the river; then a reading was made on reference point D on the second line, and ten readings were made on the distant point C across the river. Likewise, from instrument position 2 on the far bank one reading was made on reference point B, and ten on the distant back turning-point A on the rear bank; also one on the near reference point C, and ten on the distant reference point D on the rear bank. The following are the results of the four sets of observations:

	5.301	6.076	.....	4.295
	5.319	6.096	4.219	4.293
	5.312	6.109	4.233	4.288
	5.329	6.108	4.272	4.299
	5.324	6.094	4.242	4.307
	5.318	6.103	4.249	4.300
	5.338	6.109	4.225	4.304
	5.301	6.105	4.247	4.292
	5.317	6.091	4.245	4.282
	5.319	6.098	4.224	4.311
Means:	5.318	6.099	4.237	4.297

The resulting elevations of the two turning-points on the far bank, as obtained from the above observations, were, in feet:

Turning-point 571 + 3675: from east bank, 807.211; from west bank, 807.203; mean, 807.207; extreme difference of elevation, 0.008.

Turning-point 571 + 3670: from east bank, 805.523; from west bank, 805.514; mean, 805.518; extreme difference of elevation, 0.009.

The divergence of the lines for this day's work was: at the east bank, 0.911 ft.; at the west bank, 0.927 ft.

156. Hand-levels.—A very useful little instrument for the topographer is the hand-level, by which approximate level lines can be determined for some distance from the position of the observer and thus aid him in following the course of level or contour lines. This instrument consists of a brass tube six inches in length with a small level on top near the object end. (Fig. 111.) Beneath is an opening through which the bubble can be seen as reflected from a prism into the eye at one end. Both ends are covered by plain glass, while there is a small semi-convex lens in the eye end to magnify the level-bubble and the cross-wires beneath the bubble. The cross-

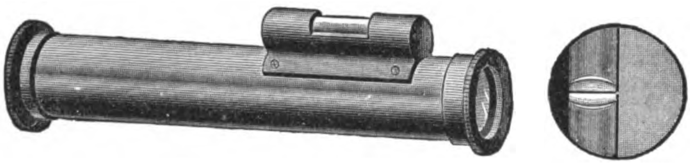


FIG. 111.—LOCKE HAND-LEVEL.

wires are fastened to a small frame moving under the bubble-tube, and are adjusted in place by a small screw at the end of the bubble-case. By standing erect and sighting any object and lowering or raising the object end of the level by hand until the reflection of the bubble is exactly bisected by the cross-wires, a horizontal line will then be sighted and the position of the horizontal cross-wire will indicate approximately the elevation of any object which is at the same height as the eye of the observer.

157. **Using the Locke Hand-level.**—There are two ways of *leveling with the Locke hand-level.* One is for the observer to stand erect, measure the height of his eye against a pole and note this height—say five feet. Then he directs the hand-level at the side of a hill or of a tree-trunk and notes where the horizontal wire intersects this. Then this object is at exactly the height of his eye above the ground, or five feet. Moving forward to it and standing with his feet on a level with this object, he is raised five feet, and, continuing the process, he levels along differences of five feet in elevation at a time.

In *sketching contours* the hand-level is used differently. Standing on the ground and knowing his elevation, he adds to that the height of his eye. Then sighting along the slopes of the land with the Locke level, he observes where the horizontal line strikes the hillsides, and knows that such points are on a level with his eye, or five feet above the contour on which he stands, and he is thus able to sketch that contour with a considerable degree of accuracy.

The topographer can with the Locke level *determine the elevations of points* about him which are but a little above or below his height, by sighting them and estimating the distance above or below the level line as indicated by the cross-hair. If the points are at any considerable distance, he must make allowance for curvature and refraction. Great reliance must not be placed, however, on the accuracy of this instrument, as its results are but approximate.

158. Abney Clinometer Level.—This is but an English modification of the Locke level, and is most useful in estimating the angles of slope, or grades, and thus in sketching contours.

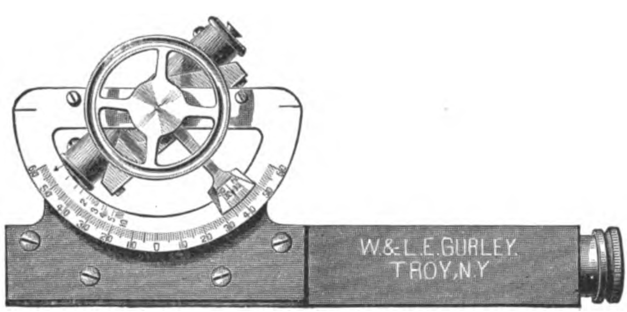


FIG. 112.—ABNEY CLINOMETER HAND-LEVEL.

It is also useful in reading rough vertical angles. Where a traverse plane-table (Art. 61) is used, however, it is more accurately replaced by a vertical angle sight-alidade (Art. 62). Attached to a hand-level is a small telescope revolving about a vertical arc graduated to 60 degrees on either side of zero when the instrument is held level (Fig. 112). It can be used as the Locke level, and also with considerable accuracy by resting the tube, which is square, on a plane-table board or other surface which can be leveled. Having leveled the tube by holding it in the hand or resting it on a plane-table and noting that it is level by bringing the bubble against the horizontal cross-hair, the small telescope is then directed up the slope and the angle of the slope read; or it is directed at some object the distance of which is known, and with the angle read the difference in height can be computed (Art. 160).

CHAPTER XVII.

TRIGONOMETRIC LEVELING.

159. Trigonometric Leveling.—Trigonometric leveling is the process of determining the difference in elevation between two points by means of the angle measured at one of them between the horizontal or level line and the other; or by measuring the zenith distance of the other. This method of leveling is especially suited to finding the heights of stations in a triangulation survey, and in connection with stadia traverse. In triangulation the vertical angles are measured with the same instrument as are the horizontal angles. In stadia and odometer traverse the vertical angles are measured with the same instrument and at the same time as is the distance or the deflection angle.

Trigonometric leveling is primary or secondary in quality, depending upon the instruments and methods employed. In either a vertical angle is observed to the point the height of which is to be determined, and this, with the distance between the occupied and the observed points, gives the quantities necessary to determine their difference in elevation. *Primary trigonometric leveling* is performed by measuring at one station, with the vertical circle of a large theodolite (Art. 241), the double zenith distance (Art. 297) of the signal at the other station; or by the measurement, by means of a micrometer inserted in the eyepiece of the telescope (Art. 242), of the differences in altitude between different stations, in connection with a reference mark the absolute height of which, or its zenith distance, has been previously obtained. *Secondary trigonometrical leveling*, or, as commonly called, *vertical angulation*, is performed with a small theodolite or with a telescopic alidade (Art. 59), and consists of direct measurement of the angle between stations observed and the horizon, as the latter is determined by the level-bubble on the instrument. A similar series of observations is taken at each successive station, and if the elevation of one of these is known the elevations of the others can be computed.

In the process of trigonometric leveling, the height of the telescope above ground and the height of the signal must be carefully measured and made a part of the record, also the hour of making the observation, as in accurate work this has a bearing upon the correction for refraction. In trigonometric leveling of primary order the state of the level at the commencement and end of the observation, and observations made to determine value and sequence of arc corresponding to a turn of the micrometer-screw, become a part of the record, as does also the object sighted.

The best results are obtained by measuring *reciprocal zenith distances* at two stations at the same moment of time, in which case the conditions of atmosphere are practically the same and the effects of refraction are eliminated. When reciprocal zenith distances are measured, not simultaneously but by the same observer on different dates, these should be made on various days from each station in order to obtain as far as possible a mean value of the angle and an average value of the refraction. The *relative refraction* (Art. 166) may be so different between various stations at distances greater than 15 or 20 miles apart as to seriously affect the results unless a very large number of measures are taken on numerous and favorable days. The higher the elevation at which observations are made the more reliable the results; also, the larger the number of stations included in a scheme of vertical triangulation the better the results, owing to the possibility of the adjustment of the whole.

The results obtained by trigonometric leveling are of far greater accuracy than ordinarily supposed. The best work of this kind is that executed by the U. S. Coast and Geodetic Survey in connection with its transcontinental belt of primary triangulation. Checks on these levels have been obtained by means of precise spirit-levels to some of the triangulation stations. At St. Albans base near Charleston, W. Va., the elevation by triangulation brought from the Atlantic coast is 594.78 feet. The elevation of the same point by precise spirit-levels from Sandy Hook via Chillicothe is 595.616 feet, a difference of only 0.836 feet, which is much better than could be expected from spirit-levels of less accuracy than precise quality would produce.

160. Vertical Angulation.—This term is used to designate the process of obtaining elevations by angular methods of ordinary quality, as by telescopic alidade used with plane-table or by the vertical circle of a transit instrument. In this work the distances and angles are measured with only approximate accuracy because of the qualities of the instruments employed, the signal sights had are not clearly defined and accordingly corrections for curvature and refraction (Art. 166) are made but approximately. Instead of having a vertical arc which can be set at zero when the level-bubble attached to the telescope is leveled, it is better to record an index error and correct the angle for this. Thus the telescope is made level by the bubble, and the reading on the vernier is recorded under the title *index error*. Then the cross-hairs are directed to the object the elevations of which are to be determined, and the vernier is again read. The difference between the two readings gives the angle between the object sighted and the horizon, and is recorded in the notes as plus or minus. To apply the correction to vertical arc to the vertical angle attention must be paid to the signs;

for a plus error in vertical arc subtract the error from plus angles and add to minus angles.

An example of the mode of keeping such notes is as follows:

Station, XXIII. *Elevation*, 2960′.

Date, Nov. 16, 1898.

Description of Point Sighted.	No.	Point.	Level.	Angle.	Dist. Miles.	Diff. Elev.	Elev.	Adj. Elev.
		° ′	° ′	° ′				
Kitty Cobble; top	XV	14 09	13 10	+ 0 59	2.93	270′	3236	3239
Wolf Lake house; base......	15-3	10 02	14 03	− 4 01	1.02	377	2583	
Top of ledge over Brooktrout	17-9	13 07	14 12	− 1 05	1.43	138	2822	2824
Russia; cupola red barn.....	1	13 00	14 12	− 1 12	1.01	110	2850	

In vertical angulation corrections to the observed angles must be made for *curvature* and *refraction* (Art. 166), which may be taken from tables (Tables XVI and XXXI), also for the height of the instrument above ground surface and the height of signal. (See Table XV and example Art. 163, also Art. 239.) The correction for difference between the heights of signal and instrument above ground may be computed by the formula

$$\text{Cor.} = \frac{h}{d \sin 1''}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

in which d is the distance between stations, and h the difference in height (Art. 239.) Or, with fair approximation, a correction may be made by determining the differences in elevation observed, adding to the known height of the occupied station the height of the telescope above it before making the computations, and subtracting from the result or computed elevation of the station sighted at, the height of the target above ground.

To sight the telescope on visible *points of equal elevation* the correction for curvature and refraction must be applied to the vernier reading, that is, the vernier must not be set at zero, but at a minus angle the number of minutes of which is nearly three-eighths of the distance in miles.

161. Vertical Angulation, Computation.—The quantity entered in the *distance column* above is measured directly on the plane-table board or on the map. In the *number column* is the number of the station corresponding to the summit sighted if it has been occupied already; or if the point has been sighted from some other station, the number of the pointing which was given from that station; or if it has never been sighted for the other station, it is given a new number for the occupied station. Under the columns *point* and *level* are placed the angles read when the instrument is pointed at the object and when the telescope is leveled, providing it is an instrument which has not an adjustable vernier. In the column *difference of elevation* is placed a quantity either computed (Art. 164) or taken from a simple table.

Table XV is one which can be used for determining angles of elevation or depression up to any distance. For the first angle, for instance, take out 59′ in the first column of the table. In the second column, that headed 0°, the difference of height is found corresponding to the unit distance one mile, and this is 90.6. This quantity multiplied by the distance in miles, 2.93, gives a difference of elevation of 265.5 feet. The correction to *curvature and refraction* for 2.93 miles is 4.8 feet, which is always additive. As the angle in this case is positive the total difference of elevation is 270.3 feet.

For all distances less than 1.6 miles the correction to curvature and refraction may be taken as 5 feet, as the height of instrument, about 4.5 feet, has to be added.

Under the column *adjusted elevation*, in the above example, is given the final height of the point as obtained by averaging its elevation as determined from several stations.

162. Vertical Angulation in Sketching.—The elevations of positions occupied by the topographer while sketching (Arts. 13 and 17) may be checked in practically the same

TABLE XV.—DIFFERENCES OF ALTITUDE FROM

Difference of altitude = $\begin{cases} + Dh_1 + h_2 \text{ for angles of elevation.} \\ - Dh_1 + h_2 \text{ for angles of depression.} \end{cases}$

	0° h_1	1° h_1	2° h_1	3° h_1	4° h_1	5° h_1	6° h_1	7° h_1
′	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
0	0.0	92.2	184.4	276.7	369.2	461.9	555.0	648.3
1	1.5	93.7	185.9	278.2	370.7	463.5	556.5	649.9
2	3.1	95.2	187.4	279.8	372.3	465.0	558.0	651.4
3	4.6	96.8	189.0	281.3	373.8	466.6	559.6	653.0
4	6.1	98.3	190.5	282.9	375.4	468.1	561.2	654.5
5	7.7	99.8	192.1	284.4	376.9	469.7	562.7	656.1
6	9.2	101.4	193.6	286.0	378.5	471.2	564.3	657.7
7	10.7	102.9	195.1	287.5	380.0	472.8	565.8	659.2
8	12.3	104.4	196.7	289.0	381.6	474.3	567.4	660.8
9	13.8	106.0	198.2	290.6	383.1	475.9	568.9	662.3
10	15.4	107.5	199.8	292.1	384.6	477.4	570.5	663.9
11	16.9	109.1	201.3	293.7	386.2	479.0	572.0	665.5
12	18.4	110.6	202.8	295.2	387.7	480.5	573.6	667.0
13	20.0	112.1	204.4	296.7	389.3	482.1	575.1	668.6
14	21.5	113.7	205.9	298.3	390.8	483.6	576.7	670.1
15	23.0	115.2	207.5	299.8	392.4	485.2	578.2	671.7
16	24.6	116.7	209.0	301.3	393.9	486.7	579.8	673.3
17	26.1	118.3	210.5	302.9	395.5	488.3	581.3	674.8
18	27.6	119.8	212.1	304.4	397.0	489.8	582.9	676.4
19	29.2	121.4	213.6	306.0	398.6	491.3	584.4	677.9
20	30.7	122.9	215.1	307.5	400.1	492.9	586.0	679.5
21	32.3	124.4	216.7	309.1	401.6	494.5	587.6	681.1
22	33.8	126.0	218.2	310.6	403.2	496.0	589.1	682.6
23	35.3	127.5	219.8	312.1	404.7	497.6	590.7	684.2
24	36.9	129.0	221.3	313.7	406.3	499.1	592.2	685.7
25	38.4	130.6	222.8	315.2	407.8	500.7	593.8	687.3
26	39.9	132.1	224.4	316.8	409.4	502.2	595.4	688.9
27	41.5	133.6	225.9	318.3	410.9	503.8	596.9	690.4
28	43.0	135.2	227.4	319.9	412.5	505.3	598.5	692.0
29	44.5	136.7	229.0	321.4	414.0	506.9	600.0	693.6
30	46.1	138.3	230.5	322.9	415.5	508.4	601.6	695.1
31	47.6	139.8	232.1	324.5	417.1	510.0	603.1	696.7
32	49.2	141.3	233.6	326.0	418.6	511.5	604.7	698.2
33	50.7	142.9	235.1	327.6	420.2	513.0	606.2	699.8
34	52.2	144.4	236.7	329.1	421.7	514.6	607.8	701.4
35	53.8	146.0	238.2	330.6	423.3	516.2	609.3	702.9
36	55.3	147.5	239.8	332.2	424.8	517.7	610.9	704.5
37	56.8	149.0	241.3	333.7	426.4	519.3	612.5	706.1
38	58.4	150.6	242.8	335.3	427.9	520.8	614.0	707.6
39	59.9	152.1	244.4	336.8	429.5	522.4	615.6	709.2
40	61.4	153.6	245.9	338.4	431.0	523.9	617.1	710.7
41	63.0	155.2	247.5	339.8	432.6	525.5	618.7	712.3
42	64.5	156.7	249.0	341.4	434.1	527.0	620.2	713.9
43	66.0	158.2	250.5	343.0	435.6	528.6	621.8	715.4
44	67.6	159.8	252.1	344.5	437.2	530.1	623.3	717.0
45	69.1	161.3	253.6	346.1	438.7	531.7	624.9	718.6
46	70.6	162.9	255.1	347.6	440.3	533.2	626.4	720.1
47	72.2	164.4	256.7	349.1	441.8	534.8	628.0	721.7
48	73.7	165.9	258.2	350.7	443.4	536.3	629.6	723.3
49	75.3	167.5	259.8	352.2	444.9	537.9	631.1	724.8
50	76.8	169.0	261.3	353.8	446.5	539.4	632.7	726.4
51	78.3	170.6	262.8	355.3	448.0	541.0	634.2	728.0
52	79.9	172.1	264.4	356.9	449.6	542.5	635.8	729.5
53	81.4	173.6	265.9	358.4	451.1	544.1	637.3	731.1
54	82.9	175.2	267.5	360.0	452.7	545.1	638.9	732.7
55	84.5	176.7	269.0	361.5	454.2	547.2	640.4	734.2
56	86.0	178.2	270.5	363.0	455.8	548.7	642.0	735.8
57	87.5	179.8	272.1	364.6	457.3	550.3	643.6	737.4
58	89.1	181.3	273.6	366.1	458.9	551.8	645.1	738.9
59	90.6	182.9	275.2	367.7	460.4	553.4	646.7	740.5
60	92.2	184.4	276.7	369.2	461.9	555.0	648.3	742.0

ANGLES OF ELEVATION OR DEPRESSION.

D = distance in miles, a = angle of elevation or depression; h_1 = 5280 ft. × tan a; h_2 = correction for curvature and refraction. Argument for h_1 is a; argument for h_2 is D.

Corrections for curvature and refraction (always to be added algebraically).

8° h_1	9° h_1	10° h_1	11° h_1	12° h_1	13° h_1	14° h_1	15° h_1	D	h_2	D	h_2
feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	miles	feet.	miles	feet.
742.0	836.3	931.0	1026.3	1122.3	1219.0	1316.5	1414.8	1.0	0.6	5.5	17.3
743.6	837.8	932.6	1027.9	1123.9	1220.6	1318.1	1416.4	1.1	0.7	5.6	18.0
745.2	839.4	934.2	1029.5	1125.5	1222.2	1319.7	1418.0	1.2	0.8	5.7	18.6
746.7	841.0	935.8	1031.1	1127.1	1223.8	1321.3	1419.7	1.3	1.0	5.8	19.3
748.3	842.6	937.3	1032.7	1128.7	1225.5	1323.0	1421.3	1.4	1.1	5.9	20.0
749.9	844.1	938.9	1034.3	1130.3	1227.1	1324.6	1423.0	1.5	1.3	6.0	20.6
751.4	845.7	940.5	1035.9	1131.9	1228.7	1326.2	1424.6	1.6	1.5	6.1	21.3
753.0	847.3	942.1	1037.5	1133.5	1230.3	1327.9	1423.9	1.7	1.7	6.2	22.0
754.6	848.9	943.6	1039.1	1135.2	1231.9	1329.5	1427.9	1.8	1.9	6.3	22.8
756.1	850.4	945.3	1040.7	1136.8	1233.6	1331.1	1429.6	1.9	2.1	6.4	23.5
757.7	852.0	946.8	1042.3	1138.4	1235.2	1332.8	1431.2	2.0	2.3	6.5	24.2
759.3	853.6	948.4	1043.8	1140.0	1236.8	1334.4	1432.9	2.1	2.5	6.6	25.0
760.9	855.2	950.0	1045.4	1141.6	1238.4	1335.0	1434.5	2.2	2.8	6.7	25.7
762.4	856.8	951.6	1047.0	1143.2	1240.0	1337.7	1436.2	2.3	3.0	6.8	26.5
765.0	858.3	953.2	1048.6	1144.8	1241.7	1339.3	1437.8	2.4	3.3	6.9	27.3
765.6	859.9	954.7	1050.2	1146.4	1243.3	1340.9	1439.5	2.5	3.6	7.0	28.1
767.1	861.5	956.3	1051.8	1148.0	1244.9	1342.6	1441.1	2.6	3.9	7.1	28.9
768.7	863.0	957.9	1053.4	1149.6	1246.5	1344.2	1442.8	2.7	4.2	7.2	29.7
770.3	864.6	959.5	1055.0	1151.2	1248.1	1345.8	1444.4	2.8	4.5	7.3	30.5
771.8	866.2	961.1	1056.6	1152.8	1249.8	1347.5	1446.1	2.9	4.8	7.4	31.4
773.4	867.8	962.7	1058.2	1154.4	1251.4	1349.1	1447.7	3.0	5.2	7.5	32.2
775.0	869.4	964.3	1059.8	1156.1	1253.0	1350.8	1449.4	3.1	5.5	7.6	33.1
776.5	870.9	965.9	1061.4	1157.7	1254.6	1352.4	1451.0	3.2	5.9	7.7	34.0
778.1	872.5	967.5	1063.0	1159.3	1256.2	1354.0	1452.7	3.3	6.2	7.8	34.9
779.7	874.1	969.0	1064.6	1160.9	1257.9	1355.7	1454.4	3.4	6.6	7.9	35.8
781.3	875.7	970.6	1066.2	1162.5	1259.5	1357.3	1456.0	3.5	7.0	8.0	36.7
782.8	877.3	972.2	1067.8	1164.1	1261.1	1358.9	1457.7	3.6	7.4	8.1	37.6
784.4	878.8	973.8	1069.4	1165.7	1262.7	1360.6	1459.3	3.7	7.8	8.2	38.6
786.0	880.4	975.4	1071.0	1167.3	1264.4	1362.2	1461.0	3.8	8.3	8.3	39.5
787.5	882.0	977.0	1072.6	1168.9	1266.0	1363.9	1462.6	3.9	8.7	8.4	40.5
789.1	883.6	978.6	1074.2	1170.6	1267.6	1365.5	1464.3	4.0	9.2	8.5	41.4
790.7	885.1	980.1	1075.8	1172.2	1269.3	1367.1	1465.9	4.1	9.6	8.6	42.4
792.2	886.7	981.7	1077.4	1173.8	1270.9	1368.8	1467.6	4.2	10.1	8.7	43.4
793.8	888.3	983.3	1079.0	1175.4	1272.5	1370.4	1469.2	4.3	10.6	8.8	44.4
795.4	889.9	984.9	1080.6	1177.0	1274.1	1372.1	1470.9	4.4	11.1	8.9	45.4
797.0	891.5	986.5	1082.2	1178.6	1275.7	1373.7	1472.5	4.5	11.6	9.0	46.4
798.5	893.0	988.1	1083.0	1180.2	1277.4	1375.3	1474.2	4.6	12.1	9.1	47.5
800.1	894.6	989.7	1085.4	1181.8	1279.0	1377.0	1475.9	4.7	12.7	9.2	48.5
801.7	896.2	991.3	1087.0	1183.4	1280.6	1378.6	1477.5	4.8	13.2	9.3	49.6
803.2	897.8	992.9	1088.6	1185.0	1282.2	1380.3	1479.2	4.9	13.8	9.4	50.7
804.8	899.4	994.5	1090.2	1186.7	1283.9	1381.9	1480.8	5.0	14.3	9.5	51.7
806.4	900.9	996.0	1091.8	1188.3	1285.5	1383.5	1482.5	5.1	14.9	9.6	52.8
807.9	902.5	997.6	1093.4	1189.9	1287.1	1385.2	1484.1	5.2	15.5	9.7	53.9
809.5	904.1	999.2	1095.0	1191.5	1288.8	1386.8	1485.8	5.3	16.1	9.8	55.1
811.1	905.7	1000.8	1096.6	1193.1	1290.4	1388.5	1487.5	5.4	16.7	9.9	56.2
812.7	907.3	1002.4	1098.2	1194.7	1292.0	1390.1	1489.1	5.5	17.3	10.0	57.3
814.2	908.8	1004.0	1099.8	1196.3	1293.7	1391.8	1490.8				
815.8	910.4	1005.6	1101.4	1197.9	1295.3	1393.4	1492.4				
817.4	912.0	1007.2	1103.0	1199.6	1296.9	1395.0	1494.1				
819.0	913.6	1008.8	1104.6	1201.2	1298.5	1396.7	1495.8				
820.5	915.2	1010.4	1106.3	1202.8	1300.2	1398.3	1497.4				
822.1	916.7	1012.0	1107.9	1204.4	1301.8	1400.0	1409.1				
823.7	918.3	1013.6	1109.5	1206.0	1303.4	1401.6	1500.7				
825.2	919.9	1015.2	1111.1	1207.7	1305.0	1403.3	1502.4				
826.8	921.5	1016.8	1112.7	1209.3	1306.7	1404.9	1504.1				
828.4	923.1	1018.4	1114.3	1210.9	1308.3	1406.5	1505.7				
830.0	924.7	1020.0	1115.9	1212.5	1309.9	1408.2	1507.4				
831.5	926.2	1021.5	1117.5	1214.1	1311.6	1409.8	1509.0				
833.1	927.8	1023.1	1119.1	1215.8	1313.2	1410.5	1510.7				
834.7	929.4	1024.7	1120.7	1217.4	1314.8	1413.1	1512.4				
836.3	931.0	1026.3	1122.3	1219.0	1316.5	1414.8	1514.0				

manner as vertical angulation is conducted in the course of traverse-work (Art. 163). While sketching, the topographer has before him on his plane-table board all of the plotted control, including positions of triangulation stations, of adjusted traverse lines, and of points intersected from the traverses (Art. 84). Assuming now that he has been sketching for some little time by means of an aneroid adjusted at some fixed elevation along the route of his traverse (Art. 176), and it becomes desirable either to check the aneroid or to determine the elevation of some nearby point which he is sketching.

Setting up the *plane-table* at a known position he reads an angle with the telescopic alidade or vertical-angle sight-alidade (Arts. 59 and 62) to some house on a neighboring road or hillside, or to some near-by summit which is plotted on the map, and this angle, with the distance measured on the plane-table sheet, furnishes the data from which to compute his height (Art. 161). Or, *vice versa*, knowing his elevation by an aneroid which has been recently checked, or being at some point the height of which has been determined by spirit-level or previous vertical angulation, he may determine the elevation of other located points which are within view, as a house on a neighboring road or hillside, or a summit, by reading an angle to them with the alidade and measuring the plotted distance on the plane-table. In this way he may keep elevations placed ahead of him on adjacent roads or hills over which he expects to travel, or he may bring those elevations to him after he has reached such positions.

In all such vertical angulation, either performed in the course of traverse-work or of sketching, the topographer must bear in mind clearly the fact that the *accuracy* of the determination is dependent on the distance and on the difference of elevation or degree of the angle read. The smaller the distance the steeper may be the angle, and yet produce no great error; the greater the distance the smaller must be the angle. (Verify by Table XV.) Reliance should not be placed where the scale is about one mile to one inch and the contour interval about 20 feet on angles exceeding 2 degrees at distances of 2 or 3 miles. The same proportion holds true for different contour intervals and scales, and the degree of accuracy with which the base elevation is determined and the platted positions are fixed.

163. Vertical Angulation from Traverse.—In traversing with the plane-table opportunity frequently arises to obtain the elevation of near-by points as referred to the known heights of the traverse stations; or, *vice versa*, the heights of the traverse stations may be obtained by vertical angulation from points the elevation of which is already known. This is done by reading angles with the telescopic alidade or with the vertical-angle sight-alidade (Arts. 59 and 62) from some traverse station to the object the difference in elevation of which is to be determined. The angle read, together with the distance measured on the plane-table board, furnish data from which to compute or to obtain from tables the differences in elevation. By this means the heights of traverse stations may be frequently obtained and the aneroid checked thereby, and then the heights of minor surrounding points may be obtained at intermediate stations on the traverse from the adjusted aneroid elevations.

An example of traverse notes accompanied by vertical angulation is as follows:

Date, Nov. 16, 1898. Traverse from Jonesboro, N. C., to Walnut Grove, N. C.

Remarks.	Station.	Aneroid.	Cor. Curv. and Refr.	Height of Signal.	Point.	Level.	Angle.	Dist., Miles.	Diff. Elevation.	Elevation.
										3069′
House..	+ 25.1		+ 2′	0′	15 00	15° 15′	− 0° 13′	1.86	− 38′	3033
Flag ...	− 26.25		+ 1	0	14 56	14 50	+ 0 06	1.46	+ 15	3049
Flag ...	− 27.26			0	15 00	14 53	+ 0 07	.69	+ 7	3056

The *angle used in the computation* is the difference between the angle read when pointing at the station sighted and that when the telescope is horizontal, as shown by the striding-level (Art. 161). The distance is measured directly by stadia, odometer, chain, or upon the plane-table sheet (Arts. 102, 98, and 99). The difference of elevation is obtained by computation (Arts. 161 and 164). The correction to curvature and refraction (Art. 166) is applied to the difference of elevation and gives a resulting elevation in the last column. Account must be taken of the height of instrument, about 4½ feet. Such an example would apply either to observations taken from stations to points about the traverse or, as in this case, to backsights and foresights on the traverse line. The fact of the sight being a backsight or a foresight is indicated by a + or − sign in the index column, which affects the application of the correction for curvature and refraction, as the latter is always algebraically positive.

164. Trigonometric Leveling, Computation.—To determine the difference of elevation by zenith distances, let

Z and Z' = the measured zenith distances at the two stations;

D = the distance between stations, in meters;

R = radius of curvature of the arc joining the two, in meters;

C = angle at the center of the earth subtended by this arc; and

H and H' = the heights of the two stations observed;—then

$$C = \frac{D}{R \sin 1''} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

and

$$H - H' = \frac{D \sin \frac{1}{2}(Z - Z')}{\operatorname{cosin} \frac{1}{8}(Z - Z' + C)}. \quad . \quad . \quad (29)$$

The value of R or of $\dfrac{1}{R \sin 1''}$ may be computed for different latitudes and for varying angles from Table XVI, based on Clarke's Constants and taken from the report of the U. S. Coast and Geodetic Survey for 1877.

TABLE XVI.

LOGARITHMS OF RADIUS OF CURVATURE, R, IN METERS.

	Azimuth.	Latitude.						
	°	24°	26°	28°	30°	32°	34°	36°
Meridian.........	0	6.802479	6.802597	6.802722	6.802852	6.802988	6.803129	6.803274
	5	2498	2615	2739	2869	3004	3145	3289
	10	2553	2669	2791	2919	3052	3190	3332
	15	2644	2756	2875	3000	3130	3265	3404
	20	2766	2875	2990	3111	3236	3366	3500
	30	3093	3192	3296	3405	3518	3636	3757
	40	3496	3580	3671	3766	3864	3967	4072
	50	3923	3994	4070	4150	4233	4319	4407
	60	4325	4384	4446	4512	4580	4650	4723
	70	4653	4702	4753	4807	4863	4921	4980
	75	4776	4822	4869	4918	4969	5022	5076
	80	4867	4909	4953	4999	5047	5097	5148
	85	4923	4963	5006	5049	5096	5143	5192
Perpendicular ...	90	6.804942	6.804981	6.805023	6.805066	6.805112	6.805159	6.805207
		38°	40°	42°	44°	46°	48°	50°
Meridian	0	6.803422	6.803573	6.803726	6.803880	6.804035	6.804189	6.804342
	5	3436	3586	3739	3892	4045	4199	4351
	10	3478	3626	3775	3926	4077	4228	4378
	15	3546	3690	3835	3982	4130	4277	4423
	20	3637	3776	3917	4059	4201	4343	4484
	30	3880	4006	4133	4262	4391	4519	4647
	40	4179	4289	4400	4511	4623	4735	4846
	50	4498	4590	4683	4777	4871	4965	5058
	60	4797	4873	4949	5025	5104	5181	5257
	70	5041	5104	5166	5229	5293	5357	5420
	75	5133	5190	5248	5307	5364	5423	5481
	80	5201	5254	5308	5363	5417	5472	5526
	85	5242	5294	5345	5397	5450	5502	5554
Perpendicular...	90	6.805256	6.805307	6.805358	6.805409	6.805460	6.805512	6.805563

EXAMPLE.

$K = 23931^{m}.6$......distance between two stations, Santa Cruz and Mount Bache, California.

$Z = 87° \; 35' \; 01''.06$ observed at Santa Cruz station, reduced to ground at Mount Bache.

$Z' = 92° \; 35' \; 34''.20$ observed at Mount Bache, reduced to ground at Santa Cruz station.

$L = 37° \; 02'$........mean latitude of the two stations

Angle $= 51° \; 55'$........angle made by line with the meridian.

COMPUTATION OF $h - h'$.

log K.......	4.3790	$Z' - Z$....	$5^\circ\ 00'\ 33''.14$	log K..............	4.3790
colog R sin 1″	8.5101	$\frac{1}{2}(Z' - Z)$..	2 30 16.57	log sin $\frac{1}{2}(Z' - Z)$....	8.6405
		$Z' - Z + C$	5 13 28.06	colog cos $\frac{1}{2}(Z' - Z + C)$	0.0004
log C.......	2.8891	$\frac{1}{2}(Z' - Z + C)$	2 36 44.03		
$C =$	774.56				3.0199

	m
Difference in height..	1046.90
Santa Cruz station above mean tide—by spirit-level..................	108.87
Mount Bache above mean tide....................................	1155.77

165. Errors in Vertical Triangulation.—In this class of leveling there are several sources of error, the most important of which, perhaps, is the refraction of the atmosphere. In vertical angulation (Art. 161) this may be compensated by applying approximate or mean corrections. In more precise trigonometric leveling the amount of *refraction* should be determined by direct observation in order that the correction may be most accurately applied. The correction of largest amount is that for *curvature*, but this is accurately known. Other sources of error are due to—

1. Errors of measurement of the distance between the objects;

2. Errors of the instrument, both of graduation and of level-bubble; and

3. Errors of pointing on the signal, or its height or definition.

Most of the *errors of instrument* excepting those of graduation may be eliminated by taking direct instrumental observations on the object sighted and reading the level and vertical circle, then reversing the instrument in its wyes and again reading the angle. Half the difference of the reading would thus be corrected for the difference of level. Shifting the vertical circle and repeating the reading would aid slightly in further reducing the errors of graduation and observation. These errors are small, however, compared with the errors arising from refraction, which can only be partially eliminated by observing on different days in order to get different atmospheric conditions. The best results in trigonometric leveling are to be obtained at such times of the day as refraction is least.

166. Refraction and Curvature.—The coefficient of refraction or the proportion of intercepted arc is determined from the observed zenith distances to two stations, the relative altitudes of which have been determined by the spirit-level; or from reciprocal zenith distances, simultaneous or not, under the assumption that the mean of a number of observations taken under favorable conditions will eliminate the differences of refraction found to exist even at the same moment at two stations a few miles apart. The difference of height from trigonometric leveling being dependent upon the coefficient of refraction multiplied by the square of the distance, it is therefore evident that the longer the line the greater will be the error caused by any uncertainty in the coefficient, and that there is therefore a limit to the distances for which any assumed mean values of refraction can be depended upon for accurate results.

The *coefficient of refraction* is the angle of refraction divided by the arc of the earth's circumference intercepted between the observer and the station observed.

Let c = angle at the earth's center, subtended by two stations, s and s';
f = angle of refraction; and
r = coefficient of refraction;—then

$$f = \frac{c}{2} - \frac{1}{2}(Z' + Z - 180°),$$

and
$$r = \frac{f}{c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

The value of c in seconds can be found from the expression

$$c'' = \frac{d}{y \sin 1''}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

in which d is the distance between the two stations, and y is the radius of the earth.

Refraction is least and is comparatively stationary between 9 A.M. and 3 P.M. It is greatest early in the morning, and after 3 P.M. it increases in amount and variation to a maximum during the night. The value of the coefficient of refraction r differs according to various observations from 0.06, observed by the U. S. Lake Survey in central Illinois, to 0.08, observed by the U. S. Coast and Geodetic Survey in New England near the sea-level, and in the interior of the country or at considerable altitudes between 0.065 and 0.07.

The amount and method of application of the correction for the *curvature of the earth* have been briefly indicated in Articles 160 and 161. The amount of this correction for various distances is more fully shown in Article 239, which gives also in tabular form (Table XXXI) the amount of refraction and the combined amount of the two.

167. Leveling with Gradienter.—The gradienter screw may be used as an adjunct to a tachymetric instrument, 1st, for the purpose of measuring vertical angles and thus determining differences of elevation; and, 2d, as a telemeter for the measurement of horizontal distances (Art. 114). The gradienter is a tangent screw with micrometer head attached to the horizontal axis of the telescope. Originally, as its name implies, the gradienter was employed in locating grades on railway and canal surveys. It has also been satisfactorily employed by the writer in interpolating contours on uniform slopes especially in the survey of reservoir sites.

To *locate a grade* of 2½ per cent, for example, which is a grade of 2½ feet per hundred, the telescope is leveled and the head of the gradienter screw read. Then, for a screw graduated so that one revolution corresponds to one foot in 100, the same must be revolved 2½ turns, when the line of sight of the telescope will be on the grade desired. The gradienter may be employed in *measuring elevations* by means of vertical angles in terms of the tangent. For, with a knowledge of the horizontal distance obtained by the gradienter (Art. 114) or otherwise, a small vertical angle may be read by the micrometer screw, or large ones read with the vertical arc of the instrument supplemented by the micrometer screw, and this vertical angle in connection with the distance gives the data from which to compute the difference of height (Art. 161).

CHAPTER XVIII.

BAROMETRIC LEVELING.

168. Barometric Leveling.—Barometric leveling is especially adapted to finding the difference between two points at considerable horizontal or vertical distances apart and which are unconnected by any system of plane survey. As a result it is the most speedy though least accurate of the methods of leveling. It is, however, very useful in making exploratory or geographic surveys over extensive areas or for making reconnaissance surveys for railroads or similar engineering works. Barometric hypsometry is frequently the only means by which approximate elevations may be determined in the progress of rough or reconnaissance surveys.

Two general classes of instruments are employed in the making of hypsometric observations in such surveys, namely:

1. The cistern or mercurial barometer; and
2. The aneroid.

Both of these instruments are dependent upon the *differences of atmospheric pressure* at two different elevations. The higher we rise above sea level the less the depth of atmosphere above us, and consequently the less its weight and the height to which it will raise or counterbalance a column of mercury. Thus, if the barometer records 30 inches of pressure, that is, sustains a column of mercury thirty inches in height, at the level of the sea, it will, at an elevation of 1000 feet, sustain a column of approximately 28.9 inches. The aneroid is a much more compact instrument than the mercurial barometer, more portable, and is carried in a metal case similar to that of a large watch.

169. Methods and Accuracy of Barometric Leveling.—The differences of atmospheric pressure as recorded by barometers is affected by the temperature, and compensation for temperature must be made in order to obtain the best results from barometric measurements. Ordinarily the aneroid or mercurial barometer is adjusted at the elevation of the starting-point, and readings are taken at various points the heights of which are to be determined and the elevations to which they correspond are computed therefrom. More accurate results can be obtained by the synchronous readings of two barometers, one of which remains stationary at a known elevation, while the other is read at points the heights of which are to be determined, and the difference between the two gives the data from which to compute the differences in height.

As the *weight of the atmosphere* and the consequent record of the barometer are affected by humidity far more than by temperature, the readings of two instruments which are affected by approximately the same atmospheric conditions give a better relative difference in height than could be obtained by the reading of one. Forty or fifty determinations of elevations by mercurial barometer were obtained ten or fifteen years ago in widely separated regions in the course of the early hyposometric surveys of the U. S. Geological Survey at points the elevations of which were known from spirit-leveling. It is interesting to note that the average error in these determinations was but a little over 8 feet, and the extreme error 17 feet. It is thus seen that under the most varying conditions where a barometer is carefully and well used fairly satisfactory results may be looked for, though unaccountable atmospheric disturbances may give results in error over 100 feet under apparently favorable conditions.

170. Mercurial Barometer.—The mercurial barometer consists of two parts, the cistern and the tube. The *cistern* is made up of a glass cylinder, *E*, through which the surface of the mercury can be seen; an upper inclosing plate, *G*, through

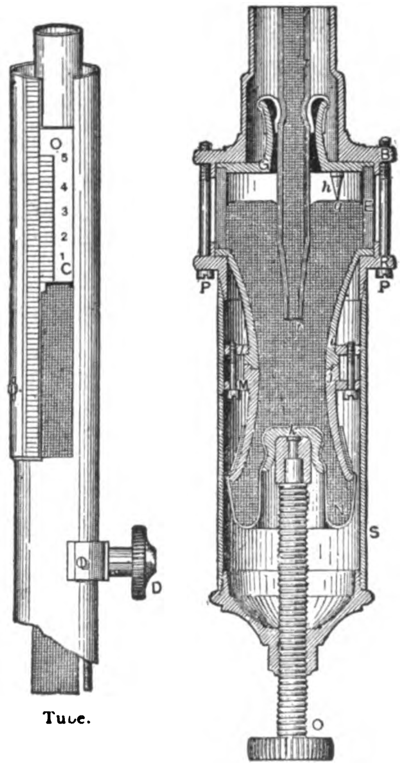


FIG. 113.—SECTION THROUGH CISTERN AND TUBE OF MERCURIAL BAROMETER.

which the lower end of the barometer tube, *t*, passes and to which it is fastened by a piece of kid leather, so as to make a strong but flexible joint. (Fig. 113.) Below these and to this plate is attached by long screws, *P*, a lower metal cup from which is suspended a wooden reservoir or cistern, *M*, the bottom portion of which is formed by a kid or chamois-leather bag, *N*. This is so contrived that it may be raised or lowered by means of an adjusting-screw, *O*, and the surface of the mercury, as seen through the glass cylinder, can be brought to exact contact with an ivory pointer, *q*, when the instrument is to be read. When being *transported* the adjusting-screw is turned up tightly until the mercury completely fills the tube, when the latter can be inverted and carried with the cistern end uppermost, so as not to be liable to breakage by the jar or shock of the mercury splashing against the upper end of the glass tube. When being read, and after the index-point has been brought to exact contact with the mercury surface, a sliding scale, on which is a vernier, is brought to corresponding contact with the upper surface of the mercury in the tube by turning a screw, *D*, and the reading on the vernier is recorded.

Though the barometer may be received filled from the maker, one who uses it should understand how to fill it in case of the not improbable breakage of the tube. The mercury used in *filling a barometer* should be mechanically pure, and is best transported in short iron tubes made of sections of gas-pipe. It is rarely necessary to boil the mercury in the tube to expel moisture or air, as is the general practice, since barometers can with a little practice be filled in a sufficiently satisfactory manner with cold mercury.

If a new *glass tube* is to be inserted, this should be transported in a box for safe packing, and the ends should be sealed until required for insertion, when the lower end is to be cut off with a sharp file and the edges filed straight and smooth, or, better, heated over a flame until they are rounded by fusion. The mercury should then be dropped in the open end of the tube slowly through a clean, rough paper funnel with a hole so small as only to let the mercury through a drop at a time, thus filtering it. When the tube is filled within a quarter of an inch of the top, the open end must be closed with a piece of chamois placed over the thumb, and the bubble of air which remains is to be run back and forth in the tube by inclining it so as to gather together all small air-bubbles adhering to the inside of the glass.

When all the bubbles have been collected turn the tube up again so that the large bubble shall pass to the open end. This should then be completely filled with mercury, and a little of the mercury may be again let out and the same operation repeated with an expanded or vacuum air-bubble until all the air has been removed. This can be distinguished by letting the column of mercury run sharply against the closed end of the tube, when it will give a clear metallic click if there is no free air in the tube. The tube is then placed open end upward, again filled to overflowing with mercury, top plate and glass plate on upper half of wooden cistern screwed tightly on. The cistern is then filled with mercury to overflowing, and the lower half, carrying the kid bag is placed on it and the two halves of the cistern joined together. Now screwing on the outer metal case and having the adjusting-screw tightly fastened up, the instrument may be reversed or placed in its upright position, when it is ready for use. A similar operation has to be repeated in case the mercury in the cistern has become dirty or the ivory point dirty from oxidation, it being necessary to first tighten up the adjusting-screw, invert the instrument, and remove the lower half of the cistern.

171. Barometric Notes and Computation.—There are several modes of keeping barometric notes as well as of computing them, according to the formulæ employed. The general *theory on which barometric work is computed* depends upon the fact that at sea-level the weight of the column of atmosphere above any given point is approximately 15 pounds to the square inch, which is sufficient to raise a column of mercury in a vacuum tube to the height of 30 inches. As one ascends the pressure diminishes because of the diminution in the height of the column of air above. But this diminution is not in a simple ratio depending on altitude because there are varying densities in the strata of air produced largely by retained moisture and wind-pressure. Moreover, each succeeding layer of air is less dense than that which underlies it by the weight of the stratum beneath it. The difference in heights of any two places is equal to the difference between the logarithms of the air-pressures at those two places multiplied by a certain constant distance. It is this relation which gives the first and principal term in the various tables for reducing barometric work. Numerous determinations of the pressure constants have been made, and these produce the principal differences in the various tables.

The more important *barometric tables* are dependent originally on Laplace's formula and the use of his coefficients. One of the tables first and most extensively used in this country is known as Williamson's Table, having been first expounded in a treatise by Lieutenant-Colonel Williamson on the "Use of the Barometer." The tables generally accepted now as giving the best results are A. Guyot's.

Laplace's formula reduced to English measures is as follows:

$$Z = \log \frac{h}{H} \times 60158.6 \text{ Eng. ft.} \left\{ \begin{array}{l} \left(1 + \dfrac{t + t' - 64}{900}\right) \\ (1 + 0.0026 \cos 2L) \\ \left(1 + \dfrac{Z + 52252}{20886860} + \dfrac{h}{10443430}\right), \end{array} \right.$$

in which

$$\left.\begin{array}{l} h = \text{the observed height of the barometer,} \\ \tau = \text{the temperature of the barometer,} \\ t = \text{the temperature of the air,} \end{array}\right\} \text{at the lower station;}$$

$$\left.\begin{array}{l} h' = \text{the observed height of the barometer,} \\ \tau' = \text{the temperature of the barometer,} \\ t' = \text{the temperature of the air,} \end{array}\right\} \text{at the upper station.}$$

Z = the difference of level between the two barometers;

L = the mean latitude between the two stations;

$H =$ the height of the barometer at the upper station reduced to the temperature of the barometer at the lower station, or

$H = h' \left\{ 1 + 0.00008967(\tau - \tau') \right\}$.

The expansion of the mercurial column for 1° Fahrenheit $= 0.00008967$;

The increase of gravity from the equator to the poles $=$ 0.00520048 or 0.0026 to the 45th degree of latitude;

The earth's mean radius $= 20,886,860$ Eng. ft.

An extremely interesting method of computing differences of elevations barometrically was devised by Mr. G. K. Gilbert of the U. S. Geological Survey. Mr. Gilbert made an entirely new departure in barometric measuring. He abandoned Laplace's formula, substituting a new formula involving none of his constants and having but a single element in common. The old method, that based on Laplace's, and by which Guyot's and Williamson's Tables were prepared, was dependent on the thermometer and the difference of temperature as recorded by it. The new method abandons the thermometer and employs the barometer alone.

Gilbert decided that there was an *atmospheric gradient;* that is, that the difference of atmospheric pressure between two points at different altitudes differed in some proportion to these altitudes. Thus a plane passing through the summits of verticals erected above the two points is inclined in some direction because the pressures are on unequally different altitudes. Gilbert determined that there were diurnal and annual variations in this gradient, and that in order to properly determine difference of altitude by the barometer the gradient must be considered, and his mode of so doing is to establish two-base barometer stations, one as high as the highest of the points the elevations of which are to be determined, the other as low as the lowest. These should be read synchronously at intervals, say of one hour, and the moving barometer is corrected by reduction, not to one-base barometer but to two, so that it can be placed in its gradient somewhere between the two barometers which are at known altitudes.

172. Example of Barometric Computation.—Below is given an example of an *observation* made by a moving barometer at McKenzie Mountain, N. M., while at the same hour a station barometer was observed at Fort Wingate, N. M., the altitude of which is known. The station or base barometer was assumed to be without an instrumental error. The moving barometer was compared with it at the beginning of the season, May 1, and was reduced to it by first reducing the readings to 32 degrees Fahrenheit and then subtracting the readings of the moving from the base barometer. The five comparative readings ranged between + .002 and — .005, with a mean of — .003 inches as the error of the moving barometer.

BAROMETRIC COMPUTATION.

Observations at Fort Wingate, N. M.

Date.	Hour.	Barom. No.	Upper Vernier.	Lower Vernier.	Att. Ther.	Temp. Cor.	Inst. Error.	Total Cor.	Reduced Readings.	Thermom.	
										Dry B.	Wet B.
May 31, 1883	9 A.M.	2606	23.512	14.789	75°.5	— .099	0	— .099	23.413	76°	46°
" " "	10 A.M.	2606	23 502	14.780	79°	— .106	0	— .106	23.396	78	44
Means......	....	....	.....	......	......	....	....	......	13.404	77	45

Observations at McKenzie Mountain, N. M.

Date.	Hour.	Barom. No.	Upper Vernier.	Lower Vernier.	Att. Ther.	Temp. Cor.	Inst. Error.	Total Cor.	Reduced Readings.	Dry B.	Wet B.
May 31, 1883........	9 A.M.	2679	30.823	22.124	58	— .058	— .003	—.061	22.063	56	40
" " "	10 A.M.	2679	30.819	22.119	61	— .064	— .003	— .067	22.052	59	41
Means..............	....	.....	......	......	......	......	... ..	......	22.057	57.5	40.5

The *computation* by the Guyot method is illustrated in the following example side by side with a computation by the Williamson method in order that the difference between the two may be noted. The terms $D_{\prime}(h)$ and $D_{\prime}(H)$ are obtained from Table XVII, the argument for $D_{\prime}(h)$ being the height of the barometer at the base station, and the

BAROMETRIC DETERMINATION OF HEIGHTS.

FIELD SEASON, 1883.

Party No. 1. *Division of Fort Wingate, N. M.*

H. M. Wilson, Computer.

Observations recorded in books No. 306 *and No.* 309.

Names of Tables, etc.	Williamson's Computation.	Guyot's Computation.
Date	May 31, 9 and 10 A.M.	
No. of synchronous obs.	2	2
Lower station	Wingate	Wingate
Upper station	McKenzie	McKenzie
Bar. at 32° h =	23.404	23.404
Bar. at 32° H =	22.057	22.057
Temperature $t_{,}$ =	77	77
Temperature t' =	57.5	57.5
Temperature $t + t'$ =	134.5	134.5
Humidity a =	.130	
Humidity a' =	.284	
Humidity $a + a'$ =	.414	
Latitude =	35° 30′	35° 30′
$D_{i}(h)$ =	22299	22216
$D_{i}(H)$ =	20745	20667
1st approx. =	1554	1549
D_{ii} =	112	122
2d approx. =	1666	1671
D_{iii} =	2	2
D_{iv} =	4	4
D_{v} =	1	1
3d approx. =	1673	1678
D_{vi} =	22′	
D_{vii} =	10′.2	
Correct for $(a + a')$ =	4	
Diff. of altitude =	1677	1678
Altitude of reference station =	6978	6978
Altitude of new station, feet =	8655	8656
Remarks:		

argument for $D_{,}(H)$ the height of the moving barometer. If the new station be lower than the base, the difference between $D'(h)$ and $D_{,}(H)$ is given a negative sign. The corrections $D_{,,}$, $D_{,,,}$, etc., are added to the first approximate result regardless of its signs, attention being paid to the signs of the corrections, which are generally positive.

The correction $D_{,,}$ is the product of the first approximation into the factor found in Table XVIII, the argument for which is $t - t'$ or the sum of temperatures of the detached thermometers of the two stations. When the humidity correction is used the relative humidities are first found from Lee's Tables, the arguments being the difference between the wet and dry bulbs and the reading of the wet bulb, though this correction scarce affects the result appreciably and may be omitted.

173. **Guyot's Barometric Tables.**—Table XVII gives in English feet the value of log H or $h \times 60158.6$ for each hundredth of an inch from 12 to 31 inches of barometric pressure. The additional thousandths are obtained in a separate column.

Table XVIII gives the correction 2.343 feet $\times (\tau - \tau')$ for the different temperatures of the barometers at the two stations; and as that at the upper station is generally lower, $\tau - \tau'$ is generally positive and the correction negative. This correction becomes positive only when the temperature of the upper barometer is higher.

Table XIX shows the correction $D'\dfrac{Z + 52252}{20886860}$ to be applied to the approximate altitude for the decrease of gravity on a vertical acting on the density of the mercurial column. It is always added.

Table XX furnishes the small correction $\dfrac{h}{10443430}$ for the decrease of gravity on a vertical acting on the density of the air. This correction is always added.

TABLE XVII.

REDUCTION OF BAROMETRIC READINGS TO FEET.

$D = 60158.58 \times \log H$ or h. Argument: The observed height of the barometer at either station.

(Extracted from Smithsonian Miscellaneous Contributions.)

Barometer in Eng. inches.	Hundredths of an inch. .00	.01	.02	.03	.04	.05	.06	.07	.08	.09	Thousandths of an inch.		Barometer in Eng. inches.
	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.		Feet.	
12.0	4763.4	4785.2	4806.9	4828.7	4850.4	4872.1	4893.7	4915.4	4937.0	4958.6			12.0
12.1	4980.2	5001.8	5023.4	5044.9	5066.4	5087.9	5109.4	5130.9	5152.4	5173.8			12.1
12.2	5195.2	5216.6	5238.0	5259.4	5280.7	5302.1	5323.4	5344.7	5367.0	5387.2			12.2
12.3	5408.5	5429.8	5452.0	5472.2	5493.4	5514.5	5535.7	5556.8	5578.9	5599.0	1	2.1	12.3
12.4	5620.1	5641.2	5662.2	5683.2	5704.3	5725.3	5746.2	5767.2	5788.1	5809.0	2	4.2	12.4
12.5	5829.9	5850.8	5871.7	5892.6	5913.4	5934.2	5955.0	5975.8	5996.6	6017.4	3	6.2	12.5
12.6	6038.1	6058.8	6079.6	6100.2	6120.9	6141 6	6162.2	6182.8	6203.5	6224.0	4	8.3	12.6
12.7	6244.6	6265.2	6285.8	6306.3	6326.8	6347.3	6367.8	6388.3	6408.8	6429.2	5	10.4	12.7
12.8	6449.6	6470.0	6490.4	6510.8	6531.1	6551.5	6571.8	6592.1	6612.4	6632.7	6	12.5	12.8
12.9	6652.9	6673.2	6693.4	6713.6	6733.8	6754.0	6774.1	6794.3	6814.4	6834.5	7	14.6	12.9
13.0	6854.7	6874.7	6894.8	6914.9	6934.9	6955.0	6975.0	6995.0	7014.9	7034.9	8	16.6	13.0
13.1	7054.9	7074.8	7094.7	7114.6	7134.5	7154.4	7174.3	7194.1	7213.9	7233.8	9	18.7	13.1
13.2	7253.6	7273.3	7293.1	7312.9	7332.6	7352.3	7372.1	7391.8	7411.4	7431.1			13.2
13.3	7450.8	7470.4	7490.0	7509.6	7529.2	7548.8	7568.4	7587.9	7607.4	7627.0			13.3
13.4	7646.5	7666.0	7685.4	7704.9	7724.4	7743.8	7763.2	7782.6	7802.0	7821.4			13.4
13.5	7840.8	7860.1	7879.4	7898.7	7918.0	7937.3	7956.6	7975.8	7995.1	8014.3			13.5
13.6	8033.6	8052.8	8071.9	8091.1	8110.3	8129.4	8148.6	8167.7	8186.8	8205.9			13.6
13.7	8225.0	8244.0	8263.1	8282.1	8301.1	8320.1	8339.1	8358.1	8377.1	8396.0	1	1.9	13.7
13.8	8415.0	8433.9	8452.8	8471.7	8490.6	8509.1	8528.3	8547.1	8565.9	8574.8	2	3.8	13.8
13.9	8603.6	8622.3	8641.1	8659.9	8678.6	8697.4	8716 1	8734.8	8753.5	8772.2	3	5.6	13.9

TABLE XVII.—REDUCTION OF BAROMETRIC READINGS TO FEET.

Barometer in Eng. inches.	Hundredths of an inch.										Thousandths of an inch.		Barometer in Eng. inches.
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09			
	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.		Feet.	
14.0	8790.8	8809.5	8828.2	8846.8	8865.4	8884.0	8902.6	8921.2	8939.7	8958.3	4	7.5	14.0
14.1	8976.8	8995.4	9013.9	9032.4	9050.8	9069.3	9087.8	9106.2	9124.6	9143.0	5	9.4	14.1
14.2	9161.4	9179.8	9198.2	9216.6	9234.9	9253.3	9271.6	9289.9	9308.2	9326.5	6	11.3	14.2
14.3	9344.7	9363.0	9381.3	9399.5	9417.7	9436.0	9454.2	9472.3	9490.5	9508.7	7	13.2	14.3
14.4	9526.8	9545.0	9563.1	9581.2	9599.3	9617.4	9635.5	9653.5	9671.6	9689.6	8	15.0	14.4
14.5	9707.6	9725.7	9743.7	9761.7	9779.6	9797.6	9815.6	9833.5	9851.4	9869.3	9	17.0	14.5
14.6	9887.2	9905.1	9923.0	9940.9	9958.7	9976.5	9994.4	10012.2	10030.0	10047.8			14.6
14.7	10065.5	10083.3	10101.1	10118.8	10136.6	10154.3	10172.0	10189.7	10207.4	10225.1			14.7
14.8	10242.7	10260.4	10278.0	10295.7	10313.3	10330.9	10348.5	10366.1	10383.6	10401.2	1	1.7	14.8
14.9	10418.7	10436.3	10453.8	10471.3	10488.8	10506.3	10523.7	10541.2	10558.6	10576.0	2	3.4	14.9
15.0	10593.4	10610.8	10628.2	10645.6	10662.9	10680.3	10697.6	10715.0	10732.3	10749.6	3	5.1	15.0
15.1	10766.9	10784.1	10801.5	10818.7	10836.0	10853.2	10870.5	10887.7	10904.9	10922.1	4	6.8	15.1
15.2	10939.3	10956.5	10973.6	10990.8	11008.0	11025.1	11042.2	11059.3	11076.4	11093.5	5	8.5	15.2
15.3	11110.6	11127.7	11144.7	11161.8	11178.8	11195.8	11212.8	11229.8	11246.8	11263.8	6	10.2	15.3
15.4	11280.8	11297.8	11314.7	11331.6	11348.6	11365.5	11382.4	11399.3	11416.2	11433.0	7	11.9	15.4
15.5	11449.9	11466.7	11483.6	11500.4	11517.2	11534.0	11550.8	11567.6	11584.4	11601.1	8	13.6	15.5
15.6	11617.9	11634.6	11651.4	11668.1	11684.8	11701.5	11718.2	11734.9	11751.6	11768.2	9	15.3	15.6
15.7	11784.9	11801.5	11818.2	11834.8	11851.4	11868.0	11884.6	11901.1	11917.7	11934.3			15.7
15.8	11950.8	11967.3	11983.0	12000.4	12016.9	12033.3	12049.8	12066.3	12082.7	12099.2			15.8
15.9	12115.6	12132.0	12148.4	12164.8	12181.2	12197.6	12214.0	12230.4	12246.7	12263.1			15.9
16.0	12279.6	12295.9	12312.2	12328.5	12344.8	12361.1	12377.4	12393.6	12409.9	12426.1			16.0
16.1	12442.4	12458.6	12474.8	12491.0	12507.2	12523.4	12539.6	12555.7	12571.9	12588.0			16.1
16.2	12604.2	12620.3	12636.4	12652.5	12668.6	12684.7	12700.8	12716.8	12732.9	12748.9	1	1.6	16.2
16.3	12765.0	12781.0	12797.0	12813.0	12829.0	12845.0	12861.0	12876.9	12892.9	12908.8	2	3.1	16.3
16.4	12924.8	12940.7	12956.6	12972.5	12988.4	13004.3	13020.2	13036.0	13051.9	13067.7	3	4.7	16.4

TABLE XVII.—REDUCTION OF BAROMETRIC READINGS TO FEET.

Barometer in Eng. inches.	Hundredths of an inch. .00	.01	.02	.03	.04	.05	.06	.07	.08	.09	Thousandths of an inch.		Barometer in Eng. inches.
	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.		Feet.	
16.5	13083.6	13099.4	13115.2	13131.0	13146.8	13162.6	13178.0	13194.2	13210.0	13225.7	4	6.3	16.5
16.6	13241.5	13257.2	13272.9	13288.6	13304.3	13320.0	13335.7	13351.5	13367.1	13382.7	5	7.8	16.6
16.7	13398.4	13414.0	13429.6	13445.2	13460.8	13476.4	13492.0	13507.6	13523.2	13538.7	6	9.4	16.7
16.8	13554.3	13569.8	13585.4	13600.9	13616.4	13631.9	13647.4	13662.9	13678.4	13693.9	7	11.0	16.8
16.9	13709.4	13724.8	13740.3	13755.7	13771.1	13786.5	13801.9	13817.3	13832.7	13848.1	8	12.5	16.9
17.0	13863.5	13878.8	13894.2	13909.6	13924.9	13940.2	13955.6	13970.9	13986.2	14001.5	9	14.1	17.0
17.1	14016.8	14032.0	14047.3	14062.6	14077.8	14093.0	14108.3	14123.5	14138.7	14153.9			17.1
17.2	14169.1	14184.3	14199.4	14214.6	14229.8	14244.9	14260.1	14275.2	14290.3	14305.5			17.2
17.3	14320.6	14335.7	14350.8	14365.8	14380.9	14396.0	14411.0	14426.1	14441.1	14456.2			17.3
17.4	14471.2	14486.2	14501.2	14516.2	14531.2	14546.1	14561.1	14576.1	14591.0	14605.9	1	1.5	17.4
17.5	14620.9	14635.8	14650.7	14664.6	14680.5	14695.4	14710.3	14725.2	14740.1	14754.9	2	2.9	17.5
17.6	14769.8	14784.6	14799.4	14814.3	14829.1	14843.9	14858.7	14873.5	14888.2	14903.0	3	4.4	17.6
17.7	14917.8	14932.5	14947.3	14962.0	14976.8	14991.5	15006.2	15020.9	15035.6	15050.3	4	5.8	17.7
17.8	15065.0	15079.6	15094.3	15109.0	15123.6	15138.2	15152.9	15167.5	15122.1	15196.7	5	7.3	17.8
17.9	15211.3	15225.9	15240.5	15255.0	15269.6	15284.2	15298.7	15313.3	15327.8	15342.4	6	8.8	17.9
18.0	15356.8	15371.3	15385.8	15400.3	15414.8	15429.3	15443.7	15458.2	15472.7	15487.1	7	10.2	18.0
18.1	15501.5	15516.0	15530.4	15544.8	15559.2	15573.6	15588.0	15602.4	15616.8	15631.2	8	11.7	18.1
18.2	15645.5	15659.9	15674.2	15688.5	15702.9	15717.2	15731.5	15745.8	15760.1	15774.4	9	13.1	18.2
18.3	15788.6	15802.9	15817.2	15831.4	15845.7	15859.9	15874.2	15888.4	15902.6	15916.8			18.3
18.4	15931.0	15945.2	15959.4	15973.6	15987.8	16001.9	16016.1	16030.2	16044.4	16058.5			18.4
18.5	16072.6	16086.8	16100.9	16115.0	16129.1	16143.2	16157.3	16171.3	16185.4	16199.5			18.5
18.6	16213.5	16227.6	16241.6	16255.6	16269.7	16283.7	16297.7	16311.7	16325.7	16339.6			18.6
18.7	16353.5	16367.5	16381.5	16395.4	16409.4	16423.3	16437.2	16451.2	16465.1	16479.0	1	1.4	18.7
18.8	16492.9	16506.8	16520.7	16534.6	16548.5	16562.3	16576.2	16590.0	16603.9	16617.8	2	2.7	18.8
18.9	16631.5	16645.4	16659.2	16673.0	16686.8	16700.6	16714.4	16728.1	16741.9	16755.7	3	4.1	18.9

TABLE XVII.—REDUCTION OF BAROMETRIC READINGS TO FEET.

Barometer in Eng. inches.	Hundredths of an inch. .00	.01	.02	.03	.04	.05	.06	.07	.08	.09	Thousandths of an inch.		Barometer in Eng. inches.
	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.		Feet.	
19.0	16769.4	16783.2	16796.9	16810.6	16824.3	16838.1	16851.8	16865.5	16879.2	16892.8	4	5.4	19.0
19.1	16906.5	16920.2	16933.9	16947.5	16961.2	16974.9	16988.5	17002.1	17015.8	17029.4	5	6.8	19.1
19.2	17043.0	17056.6	17070.2	17083.8	17097.4	17110.9	17124.5	17138.1	17151.6	17165.2	6	8.1	19.2
19.3	17178.7	17192.2	17205.8	17219.3	17232.8	17246.3	17259.8	17273.3	17286.8	17300.3	7	9.5	19.3
19.4	17313.7	17327.2	17340.6	17354.1	17367.5	17380.9	17394.4	17407.8	17421.2	17434.6	8	10.9	19.4
19.5	17448.0	17461.4	17474.8	17488.2	17501.6	17516.0	17528.3	17541.7	17555.0	17568.4	9	12.2	19.5
19.6	17581.7	17595.0	17608.3	17621.7	17635.0	17648.2	17661.5	17674.8	17688.1	17701.4			19.6
19.7	17714.6	17727.9	17741.1	17754.4	17767.6	17780.8	17794.1	17807.3	17820.5	17833.7			19.7
19.8	17846.9	17860.1	17873.3	17886.5	17899.6	17912.8	17926.0	17939.1	17952.2	17965.4			19.8
19.9	17978.5	17991.6	18004.8	18017.9	18031.0	18044.1	18057.2	18070.3	18083.4	18096.4	1	1.3	19.9
20.0	18109.5	18122.6	18135.6	18148.7	18161.7	18174.8	18187.8	18200.8	18213.8	18226.8	2	2.6	20.0
20.1	18239.8	18252.8	18265.8	18278.8	18291.8	18304.8	18317.7	18330.7	18343.6	18356.6	3	3.9	20.1
20.2	18369.5	18382.5	18395.4	18408.3	18421.2	18434.1	18447.0	18459.9	18472.3	18485.7	4	5.1	20.2
20.3	18498.5	18511.4	18524.3	18537.1	18550.0	18562.8	18575.7	18588.5	18601.3	18614.1	5	6.4	20.3
20.4	18626.9	18639.7	18652.5	18665.3	18678.1	18690.9	18703.6	18716.4	18729.1	18741.9	6	7.7	20.4
20.5	18754.6	18767.4	18780.1	18792.0	18805.6	18818.3	18831.0	18843.7	18856.4	18869.1	7	9.0	20.5
20.6	18881.8	18894.5	18907.2	18919.9	18932.5	18945.2	18957.8	18970.5	18983.1	18995.7	8	10.3	20.6
20.7	19008.3	19021.0	19033.6	19046.2	19058.8	19071.4	19083.9	19096.5	19109.1	19121.7	9	11.6	20.7
20.8	19134.2	19146.8	19159.3	19171.9	19184.4	19196.9	19209.5	19222.0	19234.5	19247.0			20.8
20.9	19259.5	19272.0	19284.5	19297.1	19309.5	19322.0	19334.4	19346.9	19359.3	19371.8			20.9
21.0	19384.3	19396.7	19409.1	19421.5	19434.0	19446.4	19458.8	19471.2	19483.6	19496.0	1	1.2	21.0
21.1	19508.4	19520.8	19533.1	19545.5	19557.9	19570.2	19589.6	19594.9	19607.3	19619.6	2	2.4	21.1
21.2	19632.0	19644.3	19656.6	19668.9	19681.2	19693.5	19705.8	19718.0	19730.3	19742.6	3	3.6	21.2
21.3	19754.9	19767.1	19779.4	19791.6	19803.9	19816.1	19828.4	19840.6	19852.8	19865.0	4	4.8	21.3
21.4	19877.3	19889.5	19901.7	19913.9	19926.0	19938.2	19950.4	19962.6	19974.7	19986.9			21.4

TABLE XVII.—REDUCTION OF BAROMETRIC READINGS TO FEET.

Barometer in Eng. inches.	Hundredths of an inch.										Thousandths of an inch.		Barometer in Eng. inches.
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09			
	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft		Feet.	
21.5	19999.1	20011.2	20023.3	20035.5	20047.6	20059.7	20071.8	20083.9	20096.1	20108.2	5	6.0	21.5
21.6	20120.3	20132.3	20144.4	20156.5	20168.6	20180.7	20192.7	20204.8	20216.9	20228.9	6	7.2	21.6
21.7	20241.0	20253.0	20265.0	20276.6	20289.1	20301.1	20313.1	20325.1	20337.1	20349.1	7	8.4	21.7
21.8	20361.1	20373.0	20385.0	20397.0	20409.0	20420.9	20432.9	20444.8	20456.8	20468.7	8	9.7	21.8
21.9	20480.7	20492.6	20504.5	20516.4	20528.3	20540.2	20552.1	20564.0	20575.9	20587.8	9	10.9	21.9
22.0	20599.7	20611.5	20623.4	20635.9	20647.1	20659.0	20670.8	20682.7	20694.5	20706.3			22.0
22.1	20718.2	20732.0	20741.8	20753.6	20765.4	20777.2	20789.0	20801.8	20812.6	20824.4			22.1
22.2	20836.2	20847.9	20859.7	20871.4	20883.2	20894.0	20906.7	20918.4	20930.1	20941.9			22.2
22.3	20953.6	20965.3	20977.0	20988.7	21000.4	21012.1	21023.8	21035.4	21047.1	21058.8	1	1.1	22.3
22.4	21070.5	21082.1	21093.8	21105.4	21117.1	21128.7	21140.4	21152.0	21163.6	21175.3	2	2.3	22.4
22.5	21186.9	21198.5	21210.1	21221.6	21233.2	21244.8	21256.4	21268.0	21279.5	21291.1	3	3.4	22.5
22.6	21302.6	21314.2	21325.8	21337.3	21348.9	21360.4	21371.9	21383.5	21395.0	21406.5	4	4.6	22.6
22.7	31418.1	21429.6	21441.1	21452.5	21464.0	21465.5	21487.0	21498.5	21509.9	21521.4	5	5.7	22.7
22.8	21532.9	21544.9	21555.8	21577.2	21578.7	21590.1	21601.6	21613.0	21624.4	21635.8	6	6.8	22.8
22.9	21647.3	21658.7	21670.1	21681.4	21692.8	21704.2	21715.6	21727.0	21738.3	21749.7	7	8.0	22.9
23.0	21761.0	21772.4	21783.7	21795.1	21806.4	21817.7	21829.1	21840.4	21851.7	21863.0	8	9.1	23.0
23.1	21874.3	21885.6	21897.0	21908.3	21919.6	21930.8	21942.1	21953.4	21964.7	21976.6	9	10.2	23.1
23.2	21987.2	21998.5	22009.8	22021.0	22032.3	22043.5	22054.7	22066.0	22077.2	22088.4			23.2
23.3	22099.6	22110.8	22122.1	22133.3	22144.5	22155.6	22166.8	22178.0	22189.2	22200.4			23.3
23.4	22211.5	22222.7	22233.9	22245.0	22256.2	22267.3	22278.4	22289.6	22300.7	22311.8			23.4
23.5	22322.9	22334.0	22345.2	22356.3	22367.4	22378.3	22389.5	22400.6	22411.7	22422.8			23.5
23.6	22433.8	22444.9	22456.0	22467.0	22478.1	22489.1	22500.2	22511.2	22522.3	22533.3			23.6
23.7	22544.3	22555.4	22566.4	22577.4	22588.4	22599.4	22610.4	22621.4	22632.4	22643.4			23.7
23.8	22654.3	22665.3	22676.3	22687.2	22698.2	22709.1	22720.1	22731.0	22742.0	22752.9	1	1.1	23.8
23.9	22763.8	22774.8	22785.7	22796.6	22807.5	22818.4	22829.4	22840.3	22851.2	22862.0	2	2.2	23.9

TABLE XVII.—REDUCTION OF BAROMETRIC READINGS TO FEET.

Barometer in Eng. inches.	Hundredths of an inch. .00	.01	.02	.03	.04	.05	.06	.07	.08	.09	Thousandths of an inch.		Barometer in Eng. inches.
	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.		Feet.	
24.0	22873.0	22883.9	22894.7	22905.6	22916.5	22927.4	22938.2	22949.1	22960.0	22970.8	3	3.2	24.0
24.1	22981.7	22992.5	23003.3	23014.2	23025.0	23035.8	23046.6	23057.5	23068.3	23079.1	4	4.3	24.1
24.2	23089.9	23100.7	23111.4	23122.2	23133.0	23143.8	23154.5	23165.3	23176.1	23186.8	5	5.4	24.2
24.3	23197.6	23208.3	23219.1	23229.8	23240.5	23251.3	23262.0	23272.7	23283.4	23294.2	6	6.5	24.3
24.4	23304.9	23315.6	23326.3	23337.0	23347.6	23358.3	23369.0	23379.7	23390.3	23401.0	7	7.5	24.4
24.5	23411.7	23422.3	23433.0	23443.7	23454.3	23464.9	23475.6	23486.2	23496.8	23507.4	8	8.6	24.5
24.6	23518.1	23528.7	23539.3	23549.9	23560.5	23571.1	23581.7	23592.3	23602.9	23613.5	9	9.7	24.6
24.7	23624.1	23634.6	23645.2	23655.8	33666.3	23676.9	23687.5	23698.0	23708.6	23719.1			24.7
24.8	23729.7	23740.2	23750.7	23761.2	23771.7	23782.3	23792.8	23803.3	23813.8	23824.3			24.8
24.9	23834.8	23845.3	23855.7	23866.2	23876.7	23887.2	23897.7	23908.2	23918.6	23929.1	1	1.0	24.9
25.0	23939.5	23949.9	23960.4	23970.8	23981.3	23991.7	24002.1	24012.5	24023.0	24033.4	2	2.1	25.0
25.1	24043.8	24054.2	24064.6	24075.0	24085.4	24095.7	24106.1	24116.5	24126.9	24137.2	3	3.1	25.1
25.2	24147.6	24158.0	24168.3	24178.7	24189.0	24199.4	24209.7	24220.1	24230.4	24240.8	4	4.1	25.2
25.3	24251.1	24261.4	24271.8	24282.1	24292.4	24302.7	24313.0	24323.3	24333.3	24343.9	5	5.1	25.3
25.4	24354.2	24364.5	24374.7	24385.0	24395.3	24405.5	24415.8	24436.1	24436.3	24446.6	6	6.2	25.4
25.5	24456.8	24467.0	24477.3	24487.5	24497.8	24508.0	24518.2	24528.4	24538.7	24548.9	7	7.2	25.5
25.6	24559.1	24569.3	24579.5	24589.7	24599.9	24610.0	24620.2	24630.4	24640.6	24650.7	8	8.2	25.6
25.7	24660.9	24671.1	24681.2	24691.4	24701.5	24711.7	24721.8	24732.0	24742.1	24752.3	9	9.2	25.7
25.8	24762.4	24772.5	24782.6	24792.8	24802.9	24813.0	24823.1	24833.2	24843.3	24853.4			25.8
25.9	24863.5	24873.6	24883.7	24893.7	24903.8	24913.9	24921.0	24934.0	24944.1	24944.1			25.9
26.0	24964.2	24974.2	24984.3	24994.3	25004.4	25014.4	25024.4	25034.4	25044.5	25054.5			26.0
26.1	25064.5	25074.5	25084.5	25094.5	25104.5	25114.5	25124.5	25134.5	25144.4	25154.4			26.1
26.2	25164.4	25174.4	25184.3	25194.3	25204.2	25214.2	25224.1	25234.1	25244.0	25254.0	1	1.0	26.2
26.3	25263.9	25273.8	25283.8	25293.7	25303.6	25313.5	25323.4	25333.3	25343.2	25353.1	2	2.0	26.3
26.4	25363.0	25372.9	25382.8	25392.7	25402.6	25412.4	25422.3	25432.2	25442.1	25451.9	3	2.9	26.4

TABLE XVII.—REDUCTION OF BAROMETRIC READINGS TO FEET.

Barometer in Eng. inches.	Hundredths of an inch. .00	.01	.02	.03	.04	.05	.06	.07	.08	.09	Thousandths of an inch.		Barometer in Eng. inches.
	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.		Feet.	
26.5	25461.8	25471.7	25481.5	25491.4	25501.2	25511.0	25520.9	25530.7	25540.5	25550.4	4	3.9	26.5
26.6	25560.2	25570.0	25579.8	25589.7	25599.5	25609.3	25619.1	25628.9	25638.7	25648.5	5	4.9	26.6
26.7	25658.3	25668.1	25677.8	25687.6	25697.4	25707.1	25716.9	25726.7	25736.4	25746.2	6	5.9	26.7
26.8	25755.0	25765.6	25775.4	25785.1	25794.8	25804.6	25814.3	25824.0	25833.8	25843.5	7	6.9	26.8
26.9	25853.2	25862.9	25872.6	25882.2	25892.0	25901.7	25911.4	25921.1	25930.8	25940.5	8	7.8	26.9
27.0	25950.2	25959.9	25969.6	25979.2	25988.9	25998.6	26008.2	26017.9	26027.5	26037.2	9	8.8	27.0
27.1	26046.8	26056 5	26066.1	26075.7	26085.3	26095.0	26104.6	26114.2	26123.8	26133.4			27.1
27.2	26143.0	26152.6	26162.2	26171.8	26181.4	26191.0	26200.6	26210.2	26219.8	26229.3			27.2
27.3	26238.9	26248.0	26258.0	26267.6	26277.2	26286.7	26296.3	26305.8	26315.3	26324.9			27.3
27.4	26334.4	26344.0	26353.5	26363.0	26372.4	26382.1	26391.6	26401.1	26410.6	26420.1	1	0.9	27.4
27.5	26429.6	26439.1	26448.6	26458.1	26467.6	26477.1	26486.5	26496.0	26505 5	26514.9	2	1.9	27.5
27.6	26524.4	26533.9	26543.3	26552.8	26562.3	26571.7	26581.2	26590.6	26600.0	26609.5	3	2.8	27.6
27.7	26618.9	26628.4	26637.8	26647.2	26656.7	26666.1	26675.5	26684.9	26694.3	26703.7	4	3.7	27.7
27.8	26713.1	26722.5	26731.9	26741.3	26750.7	26760.1	26769.5	26778.8	26788 2	26797.6	5	4.7	27.8
27.9	26806.9	26816.3	26825.6	26835.0	26844.3	26853.7	26863.0	26872.3	26881.7	26891.0	6	5.6	27.9
28.0	26900.4	26909.7	26919.0	26928.4	26937.7	26947.0	26956.3	26965.6	26975.0	26984.3	7	6 5	28.0
28.1	26993 6	27002.9	27012.2	27021.5	27030.7	27040.0	27049.3	27059.6	27067.8	27077.1	8	7.5	28.1
28.2	27086.4	27095.6	27104.9	27114.2	27123.4	27132.7	27141.9	27151.2	27160.4	27169.6	9	8.4	28.2
28.3	27178.9	27188.1	27197.3	27206.5	27215.7	27225.0	27234.2	27243.4	27252.6	27261.8			28.3
28.4	27271.0	27280.2	27289.4	27298.6	27307.8	27317.0	27326.2	27335.3	27344.5	27353.7			28.4
28.5	27362.0	27372.0	27381.2	27390.4	27399.5	27408.7	27417.8	27427.0	27436.1	27445.2			28.5
28.6	27454.4	27463.5	27472.6	27481.8	27490.9	27500.0	27509.1	27518.2	27527.4	27536.5			28.6
28.7	27545.4	27554.7	27563.8	27572.9	27582.0	27591.1	27600.2	27609.3	27618.3	27627.4	1	0.9	28.7
28.8	27636.5	27645.5	27654.6	27663.7	27672.7	27681.8	27690.8	27699.9	27708.9	27717.9	2	1.8	28.8
28.9	27727.0	27736.0	27745.1	27754.1	27763.1	27772.2	27781.2	27790.2	27799.2	27808.3	3	2.7	28.9

TABLE XVII.—REDUCTION OF BAROMETRIC READINGS TO FEET.

Barometer in Eng. inches.	Hundredths of an inch.										Thousandths of an inch.		Barometer in Eng. inches.
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09			
	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.	Eng. ft.		Feet.	
29.0	27817.2	27826.2	27835.2	27844.2	27853.2	27862.2	27871.2	27880.2	27889.1	27898.1	4	3.6	29.0
29.1	27907.1	27016.1	27925.0	27934.0	27943.0	27951.9	27960.9	27969.8	27978.8	27987.7	5	4.5	29.1
29.2	27996.7	28005.6	28014.6	28023.5	28032.4	28041.4	28050.3	28059.2	28068.2	28077.1	6	5.4	29.2
29.3	28086.0	28094.9	28103.8	28112.8	28121.7	28130.6	28139.5	28148.4	28157.3	28166.2	7	6.3	29.3
29.4	28175.1	28184.0	28192.9	28201.7	28210.6	28219.5	28228.4	28237.2	28246.1	28254.9	8	7.2	29.4
29.5	28263.8	28272.6	28281.5	28290.3	28299.2	28308.0	28316.9	28325.7	28334.5	28343.4	9	8.1	29.5
29.6	28352.2	28361.0	28369.8	28378.7	28387.5	28396.3	28405.1	28413.9	28422.7	28431.5			29.6
29.7	28440.3	28449.1	28457.9	28466.7	28475.4	28484.2	28493.0	28501.8	28510.6	28519.3			29.7
29.8	28528.1	28536.9	28545.6	28554.4	28563.2	28571.9	28580.7	28589.4	28598.2	28606.9			29.8
29.9	28615.7	28624.4	28633.2	28641.9	28650.6	28659.3	28668.1	28676.8	28685.2	28694.2	1	8.6	29.9
30.0	28702.9	28711.6	28720.3	28729.0	28737.7	28746.4	28755.1	28763.8	28772.5	28781.1	2	1.7	30.0
30.1	28789.8	28798.5	28807.2	28815.9	28824.5	28833.2	28841.9	28850.5	28859.2	28867.9	3	2.6	30.1
30.2	28876.5	28885.2	28893.8	28902.5	28911.1	28919.8	28928.4	28937.0	28945.7	28954.3	4	3.4	30.2
30.3	28962.9	28971.5	28980.1	28988.8	28997.4	29006.0	29014.6	29023.2	29031.7	29040.3	5	4.3	30.3
30.4	29048.9	29057.5	29066.1	29074.7	29083.3	29091.8	29100.4	29109.0	29117.6	29126.2	6	5.2	30.4
30.5	29134.7	29143.3	29151.9	29160.4	29169.0	29177.6	29186.1	29194.7	29203.2	29211.8	7	6.0	30.5
30.6	29220.3	29228.9	29237.4	29245.9	29254.4	29262.9	29271.5	29280.0	29288.5	29297.0	8	6.9	30.6
30.7	29305.5	29314.0	29322.5	29331.1	29339.6	29348.1	29356.6	29365.1	29373.5	29382.0	9	7.7	30.7
30.8	29390.5	29399.0	29407.5	29416.0	29424.4	29432.9	29441.4	29449.8	29458.3	29466.8			30.8
30.9	29475.2	29483.7	29492.1	29500.6	29509.0	29517.5	29525.9	29534.3	29542.8	29551.2			30.9

TABLE XVIII.

CORRECTION FOR $\tau - \tau'$, OR DIFFERENCE IN THE TEMPERATURE OF THE BAROMETERS AT THE TWO STATIONS.

This correction is *negative* when the attached thermometer at the upper station is lowest; *positive* when the attached thermometer at the upper station is highest.

(From Smithsonian Miscellaneous Contributions.)

$\tau - \tau'$ F.	Correction.	$\tau - \tau'$ F.	Correction.	$\tau - \tau'$ F.	Correction.	$\tau - \tau'$ F.	Correction.	$\tau - \tau'$ F.	Correction.
°	Eng. ft.	°	Eng. ft.	°	Eng. ft.	°	Eng. ft.	°	Eng. ft.
1.0	2.3	21.0	49.2	41.0	96.0	61.0	142.9	81.0	189.7
1.5	3.5	21.5	50.4	41.5	97.2	61.5	144.1	81.5	190.9
2.0	4.7	22.0	51.5	42.0	98.4	62.0	145.2	82.0	192.1
2.5	5.9	22.5	52.7	42.5	99.6	62.5	146.4	82.5	193.3
3.0	7.0	23.0	53.9	43.0	100.7	63.0	147.6	83.0	194.4
3.5	8.2	23.5	55.1	43.5	101.9	63.5	148.8	83.5	195.6
4.0	9.4	24.0	56.2	44.0	103.1	64.0	149.9	84.0	196.8
4.5	10.5	24.5	57.4	44.5	104.2	64.5	151.1	84.5	197.9
5.0	11.7	25.0	58.6	45.0	105.4	65.0	152.3	85.0	199.1
5.5	12.9	25.5	59.7	45.5	106.6	65.5	153.4	85.5	200.3
6.0	14.1	26.0	60.9	46.0	107.8	66.0	154.6	86.0	201.5
6.5	15.2	26.5	62.1	46.5	108.9	66.5	155.8	86.5	202.6
7.0	16.4	27.0	63.2	47.0	110.1	67.0	157.0	87.0	203.8
7.5	17.6	27.5	64.4	47.5	111.3	67.5	158.1	87.5	205.0
8.0	18.7	28.0	65.6	48.0	112.4	68.0	159.3	88.0	206.1
8.5	19.9	28.5	66.8	48.5	113.6	68.5	160.5	88.5	207.3
9.0	21.1	29.0	67.9	49.0	114.8	69.0	161.6	89.0	208.5
9.5	22.3	29.5	69.1	49.5	116.0	69.5	162.8	89.5	209.7
10.0	23.4	30.0	70.3	50.0	117.1	70.0	164.0	90.0	210.8
10.5	24.6	30.5	71.4	50.5	118.3	70.5	165.2	90.5	212.0
11.0	25.8	31.0	72.6	51.0	119.5	71.0	166.3	91.0	213.2
11.5	26.9	31.5	73.8	51.5	120.6	71.5	167.5	91.5	214.3
12.0	28.1	32.0	75.0	52.0	121.8	72.0	168.7	92.0	215.5
12.5	29.3	32.5	76.1	52.5	123.0	72.5	169.8	92.5	216.7
13.0	30.5	33.0	77.3	53.0	124.2	73.0	171.0	93.0	217.9
13.5	31.6	33.5	78.5	53.5	125.3	73.5	172.2	93.5	219.0
14.0	32.8	34.0	79.6	54.0	126.5	74.0	173.4	94.0	220.2
14.5	34.0	34.5	80.8	54.5	127.7	74.5	174.5	94.5	221.4
15.0	35.1	35.0	82.0	55.0	128.8	75.0	175.7	95.0	222.5
15.5	36.3	35.5	83.2	55.5	130.0	75.5	176.9	95.5	223.7
16.0	37.5	36.0	84.3	56.0	131.2	76.0	178.0	96.0	224.9
16.5	38.7	36.5	85.5	56.5	132.4	76.5	179.2	96.5	226.1
17.0	39.8	37.0	86.7	57.0	133.5	77.0	180.4	97.0	227.2
17.5	41.0	37.5	87.8	57.5	134.7	77.5	181.6	97.5	228.4
18.0	42.2	38.0	89.0	58.0	135.9	78.0	182.7	98.0	229.6
18.5	43.3	38.5	90.2	58.5	137.0	78.5	183.9	98.5	230.7
19.0	44.5	39.0	91.4	59.0	138.2	79.0	185.1	99.0	231.9
19.5	45.7	39.5	92.5	59.5	139.4	79.5	186.2	99.5	233.1
20.0	46.9	40.0	93.6	60.0	140.6	80.0	187.4	100.0	234.3
20.5	48.0	40.5	94.9	60.5	141.7	80.5	188.6	100.5	235.4

Table XIX.

CORRECTION FOR THE DIFFERENCE OF GRAVITY AT VARIOUS LATITUDES.

Correction *positive* from latitude 0° to 45°; *negative* from 45° to 90°.

(From Smithsonian Miscellaneous Contributions.)

Approximate Difference of Level.	Latitude.																								Approximate Difference of Level.
	0° 90°	2° 88°	4° 86°	6° 84°	8° 82°	10° 80°	12° 78°	14° 76°	16° 74°	18° 72°	20° 70°	22° 68°	24° 66°	26° 64°	28° 62°	30° 60°	32° 58°	34° 56°	36° 54°	38° 52°	40° 50°	42° 48°	44° 46°	45°	
Eng. ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Eng. ft.
1,000	2.6	2.6	2.6	2.5	2.5	2.4	2.4	2.3	2.2	2.1	2.0	1.9	1.7	1.6	1.5	1.3	1.1	1.0	0.8	0.6	0.5	0.3	0.1	0	1,000
2,000	5.2	5.2	5.1	5.1	5.0	4.9	4.7	4.6	4.4	4.2	4.0	3.7	3.5	3.2	2.9	2.6	2.3	1.9	1.6	1.3	0.9	0.5	0.2	0	2,000
3,000	7.8	7.8	7.7	7.6	7.5	7.3	7.1	6.9	6.6	6.3	6.0	5.6	5.2	4.8	4.4	3.9	3.4	2.9	2.4	1.9	1.4	0.8	0.3	0	3,000
4,000	10.4	10.4	10.3	10.2	10.0	9.8	9.5	9.2	8.8	8.4	8.0	7.5	7.0	6.4	5.8	5.2	4.6	3.9	3.2	2.5	1.8	1.1	0.4	0	4,000
5,000	13.0	13.0	12.9	12.7	12.5	12.2	11.9	11.5	11.0	10.5	10.0	9.4	8.7	8.0	7.3	6.5	5.7	4.9	4.0	3.1	2.3	1.4	0.5	0	5,000
6,000	15.6	15.6	15.4	15.3	15.0	14.7	14.3	13.8	13.2	12.6	11.9	11.2	10.4	9.6	8.7	7.8	6.8	5.8	4.8	3.8	2.7	1.6	0.5	0	6,000
7,000	18.2	18.2	18.0	17.8	17.5	17.1	16.6	16.1	15.4	14.7	13.9	13.1	12.2	11.2	10.2	9.1	8.0	6.8	5.6	4.4	3.2	1.9	0.6	0	7,000
8,000	20.8	20.7	20.6	20.3	20.0	19.5	19.0	18.4	17.6	16.8	15.9	15.0	13.9	12.8	11.6	10.4	9.1	7.8	6.4	5.0	3.6	2.2	0.7	0	8,000
9,000	23.4	23.3	23.2	22.9	22.5	22.0	21.4	20.7	19.8	18.9	17.9	16.8	15.7	14.4	13.1	11.7	10.3	8.8	7.2	5.7	4.1	2.4	0.8	0	9,000
10,000	26.0	25.9	25.7	25.4	25.0	24.4	23.8	23.0	22.0	21.0	19.9	18.7	17.4	16.0	14.5	13.0	11.4	9.7	8.0	6.3	4.5	2.7	0.9	0	10,000
11,000	28.6	28.5	28.3	28.0	27.5	26.9	26.1	25.3	24.3	23.1	21.9	20.6	19.1	17.6	16.0	14.3	12.5	10.7	8.8	6.9	5.0	3.0	1.0	0	11,000
12,000	31.2	31.1	30.9	30.5	30.0	29.3	28.5	27.5	26.5	25.2	23.9	22.4	20.9	19.2	17.4	15.6	13.7	11.7	9.6	7.4	5.4	3.3	1.1	0	12,000
13,000	33.8	33.7	33.5	33.1	32.5	31.8	30.9	29.8	28.7	27.3	25.9	24.3	22.6	20.8	18.9	16.9	14.8	12.7	10.4	8.2	5.9	3.5	1.2	0	13,000
14,000	36.4	36.3	36.0	35.6	35.0	34.2	33.3	32.1	30.9	29.4	27.9	26.2	24.4	22.4	20.4	18.2	16.0	13.6	11.2	8.8	6.3	3.8	1.3	0	14,000
15,000	39.0	38.9	38.6	38.1	37.5	36.6	35.6	34.4	33.1	31.6	29.9	28.1	26.1	24.0	21.8	19.5	17.1	14.6	12.1	9.4	6.8	4.1	1.4	0	15,000
16,000	41.6	41.5	41.2	40.7	40.0	39.1	38.0	36.7	35.3	33.7	31.9	29.9	27.8	25.6	23.3	20.8	18.2	15.6	12.9	10.1	7.2	4.3	1.5	0	16,000
17,000	44.2	44.1	43.8	43.2	42.5	41.5	40.4	39.0	37.5	35.8	33.9	31.8	29.6	27.2	24.7	22.1	19.4	16.6	13.7	10.7	7.7	4.6	1.5	0	17,000
18,000	46.8	46.7	46.3	45.8	45.0	44.0	42.8	41.3	39.7	37.9	35.8	33.7	31.3	28.8	26.2	23.4	20.5	17.5	14.5	11.3	8.1	4.9	1.6	0	18,000
19,000	49.4	49.3	48.9	48.3	47.5	46.4	45.1	43.6	41.9	40.0	37.8	35.5	33.1	30.4	27.6	24.7	21.7	18.5	15.3	12.0	8.6	5.2	1.7	0	19,000
20,000	52.0	51.9	51.5	50.4	50.0	48.9	47.5	45.9	44.1	42.1	39.8	37.4	34.8	32.0	29.1	26.0	22.8	19.5	16.1	12.6	9.0	5.4	1.8	0	20,000
21,000	54.6	54.5	54.1	53.4	52.5	51.3	49.9	48.2	46.3	44.2	41.8	39.3	36.5	33.6	30.5	27.3	23.9	20.5	16.9	13.2	9.5	5.7	1.9	0	21,000
22,000	57.2	57.1	56.6	55.9	55.0	53.7	52.3	50.5	48.5	46.3	43.8	41.1	38.3	35.2	32.0	28.6	25.1	21.4	17.7	13.8	9.9	6.0	2.0	0	22,000
23,000	59.8	59.7	59.2	58.5	57.5	56.2	54.6	52.8	50.7	48.4	45.8	43.0	40.0	36.8	33.4	29.9	26.2	22.4	18.5	14.5	10.4	6.2	2.1	0	23,000
24,000	62.4	62.2	61.8	61.0	60.0	58.6	57.0	55.1	52.9	50.5	47.8	44.9	41.8	38.4	34.9	31.2	27.4	23.4	19.3	15.1	10.8	6.5	2.2	0	24,000
25,000	65.0	64.8	64.4	63.6	62.5	61.1	59.4	57.4	55.1	52.6	49.8	46.8	43.4	40.0	36.3	32.5	28.5	24.3	20.1	15.7	11.3	6.8	2.3	0	25,000

TABLE XX.

CORRECTION FOR DECREASE OF GRAVITY ON A VERTICAL.

(From Smithsonian Miscellaneous Contributions.)

Approximate difference of level.	Decrease of gravity on a vertical. *Positive.* 0	+500	Approximate difference of level.	Decrease of gravity on a vertical. *Positive.* 0	+500	Approximate difference of level.	Decrease of gravity on a vertical. *Positive.* 0	+500
Eng. feet.	Feet.	Feet.	Eng. feet.	Feet.	Feet.	Eng. feet.	Feet.	Feet.
1,000	2.5	3.9	10,000	29.8	31.5	19,000	64.8	67.0
2,000	5.2	6.6	11,000	33.3	35.1	20,000	69.2	71.4
3,000	7.9	9.3	12,000	36.9	38.7	21,000	73.6	75.9
4,000	10.8	12.2	13,000	40.6	42.5	22,000	78.2	80.5
5,000	13.7	15.2	14,000	44.4	46.3	23,000	82.9	85.2
6,000	16.7	18.3	15,000	48.3	50.3	24,000	87.6	90.0
7,000	19.9	21.5	16,000	52.3	54.3	25,000	92.5	94.9
8,000	23.1	24.7	17,000	56.4	58.4			
9,000	26.4	28.1	18,000	60.5	62.6			

TABLE XXI.

CORRECTION FOR THE HEIGHT OF THE LOWER STATION.—POSITIVE.

(From Smithsonian Miscellaneous Contributions.)

Approximate difference of level.	Height of the barometer, in English inches, at lower station. 16	18	20	22	24	26	28
Eng feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
1,000	1.6	1.3	1.0	0.8	0.6	0.4	0.2
2,000	3.1	2.5	2.0	1.5	1.1	0.7	0.3
3,000	4.7	3.8	3.0	2.3	1.7	1.1	0.5
4,000	6.3	5.1	4.0	3.1	2.2	1.4	0.7
5,000	7.8	6.4	5.0	3.8	2.8	1.8	0.8
6,000	9.4	7.6	6.0	4.6	3.3	2.1	1.0
7,000	11.0	8.9	7.1	5.4	3.9	2.5	1.2
8,000	12.5	10.2	8.1	6.2	4.4	2.8	1.3
9,000	14.1	11.4	9.1	6.9	5.0	3.2	1.5
10,000	15.7	12.7	10.1	7.7	5.5	3.5	1.7
11,000	17.2	14.0	11.1	8.5	6.1	3.9	1.8
12,000	18.8	15.3	12.1	9.2	6.6	4.2	2.0
13,000	20.4	16.5	13.1	10.0	7.2	4.6	2.2
14,000	21.9	17.8	14.1	10.8	7.7	4.9	2.3
15,000	23.5	19.1	15.1	11.5	8.3	5.3	2.5
16,000	25.1	20.3	16.1	12.3	8.8	5.6	2.7
17,000	26.6	21.6	17.1	13.1	9.4	6.0	2.8
18,000	28.2	22.9	18.1	13.8	9.9	6.3	3.0
19,000	29.8	24.1	19.2	14.6	10.5	6.7	3.2
20,000	31.3	25.4	20.2	15.4	11.0	7.0	3.3
21,000	32.9	26.7	21.2	16.1	11.6	7.4	3.5
22,000	34.5	28.0	22.2	16.9	12.1	7.7	3.7
23,000	36.0	29.2	23.2	17.7	12.7	8.1	3.8
24,000	37.6	30.5	24.2	18.5	13.2	8.4	4.0
25,000	39.1	31.8	25.2	19.2	13.8	8.8	4.1

174. Aneroid Barometer.—This instrument depends for its operation on a shallow cylindrical metal box, the top of which is made of *corrugated metal* and is so elastic as to readily yield to changes in the pressure of the atmosphere. The interior of this *box* is *exhausted of air*, so that when the atmospheric pressure increases the top is pressed inwards, and when it decreases the elasticity of the corrugated top moves it outwards. These movements are transmitted by multiplying levers, chains, and springs to an index which moves over a scale. (Fig. 114.)

Aneroids as made by various instrument-makers differ in the mechanism employed to multiply the linear motion of the end of the vacuum index and in the arrangement of figures on the face of the scale. The instrument is graduated by comparing its indicator under different pressures with those of a mercurial barometer, and is tested in a vacuum pump, and a scale of correction is usually prepared with a view to making it independent of temperature changes. At the back of the instrument is a screw which presses against the end of the vacuum box so that it may be adjusted at any base elevation.

The *scales* on the face are usually two in number, one for inches of atmospheric pressure, and the other for altitude in feet. The scale of feet is frequently made movable so that it may be set at a known altitude opposite to the index pointer, after which changes in the index hand will indicate relative changes in altitudes as based upon the setting.

175. Errors of Aneroid.—The aneroid is very convenient as a movable instrument, requiring no time to place it in position for observing, as does the mercurial barometer, and being at all times in condition for immediate and direct reading, as is a watch. It is inferior, however, as a hypsometric instrument to the mercurial barometer, chiefly because it is subject to the following sources of error:

1. The elasticity of the corrugated top of the vacuum chamber is affected by rapid changes in pressure.

2. Its readings are affected by changes in temperature which it is impossible to readily compensate.

3. The different spaces on the scale are seldom correct relatively one to the other, but the scale of pressure or inches is more accurate than the scale of feet, since the latter contains the errors due to the formula by which it was graduated.

4. The weight of the instrument affects its indications, its readings differing in accordance with the position in which it is held.

5. It lacks in sensitiveness, frequently not responding quickly to changes of altitude.

6. The chain and levers sometimes fail to quickly respond to the movements required of them.

7. Because of its containing so many mechanical parts these are subject to shifting or jarring by movement made in transporting it, the only remedy for which is frequent comparison with known altitudes or a mercurial barometer.

The aneroid is *not an instrument of precision*, and the least reading which it is capable of is about 0.025 of an inch, corresponding to nearly 25 feet, and no system of verniers nor multiplying scales will increase the precision. The range of pressure of the aneroid is limited, and if used for a greater altitude, or a pressure lower than that within its range, the spring runs down; in other words, the spring ceases to act after the pressure has been lowered too far.

It frequently happens, as on the approach of a storm, or change from stormy to clear weather, that atmospheric pressures will change in a few hours by over an inch. This means an apparent change of elevation at the same place of 1000 feet or more. (Art. 168.)

176. Using the Aneroid.—The aneroid barometer is used very extensively in the topographic surveys executed by the U. S. Geological Survey. Excepting in country of very flat slopes, it has been used almost exclusively by that organization in sketching contours over an area of 800,000 square miles which have been already mapped. As previously stated, the aneroid has been found to be erratic and unreliable for exact work. It has also been found that where properly handled and attention is paid to its eccentricities it is a sufficiently accurate instrument to permit of sketching contours of intervals not less than 20 feet, in moderately rolling country, with all the accuracy necessary for a scale of one mile to the inch, and in very mountainous country for even larger scales.

The topographers of the U. S. Geological Survey carry *aneroids of the simplest form*, unencumbered by verniers and similar in size and general appearance to that shown in Fig. 114. These instruments are from 2 to 2½ inches in diameter, and aneroids of various ranges are employed according to the altitudes of the country under survey. In a region in which the heights do not exceed 2000 feet a 3000-foot aneroid is carried. When the altitudes exceed 3000 or 4000 feet a 5000-foot aneroid is carried. An aneroid cannot be used with any safety in determining heights which approach nearly to its range.

The instrument is *carried loosely* as a watch in the pocket. The slight jolting which it thus receives in riding or walking is just sufficient to keep the needle from sticking and aid it in responding to the changes of altitude. In reading it it should invariably be *held in* the *same position*. Some prefer to hold it horizontally, the better way, however, is to hold it vertically in front of the eye, suspended by the carrying-ring. In reading it the eye should always be held in the same position with relation to the needle, to avoid the effect of *parallax*, and the case of the aneroid should be *rapped gently* but sharply in order to loosen the spring or needle should either stick, such rapping being more effective if performed with a hard substance, as the finger-nail or lead-pencil, than with the fleshy part of the finger.

In setting out to work, the reading of the aneroid should be noted to see if it has changed materially from the reading noted in camp on the previous day. This gives some indications of the condition of the atmosphere. Before starting out

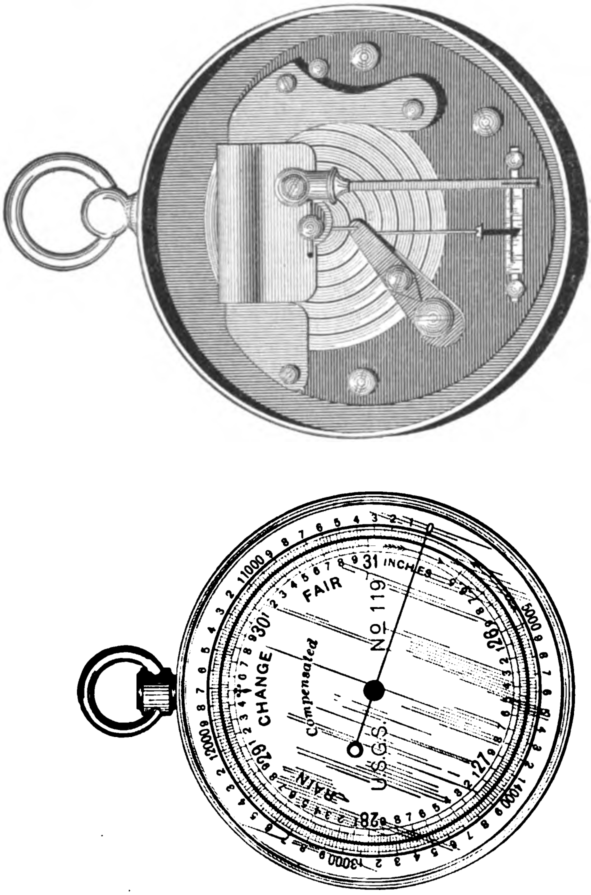


FIG. 114.—ANEROID BAROMETER.

the sliding foot-scale of the aneroid should be revolved so that the hand shall point to the altitude of the camp or other known elevation near the starting-point. On arrival at the field of work the aneroid should be again read at some point the elevation of which is known, and the effect of atmosphere or sudden change in height in preventing its recording this known elevation correctly, as compared with that at which it had been previously set in camp, should be noted. If the aneroid appears to have acted erratically, it should be used with great care at first and frequently checked, lest the atmospheric conditions be unsuited to its use. There often occur days on which it is impossible to use the aneroid (Art. 175), when results are desired which permit of sketching contours of intervals as small as 20 feet. On such days the topographer should either obtain numerous check elevations by vertical angulation, or should confine his route to sketching where elevations already obtained are numerous, or should devote himself to plane-table triangulation, office work, or some other phase of his duties.

If the aneroid seems in proper condition for use,—and this is best verified by carrying two aneroids, lest one for some reason be out of order,—the topographer may proceed to *sketch contours* by it (Arts. 13 and 17). In this work he should use only one of the aneroids, that which seems in best condition, making no attempt to check it by the other, or to take a mean of the readings of the two, but depending for such adjustment on checking it by elevations obtained by better methods. Setting the aneroid at his starting-point and at a known elevation, he drives over the roads, consulting it to determine the height at each contour crossing of the route traveled. He may rely upon it for sketching contours of small interval for distances not exceeding a couple of miles without rechecking it. Where the changes of slope are not great and the journey is made with considerable speed, as by driving in a vehicle, and where the time consumed in travel is comparatively short, the aneroid may be safely used for distances as great as three to five miles, though in such cases it may not check out within a contour interval on the next comparison, when a portion of the journey just made must be retraveled and the topography resketched. Where the contour interval is greater than 30 feet, as 50 or even 100 feet, longer journeys may be made and greater differences of altitude encountered without introducing errors in the aneroid reading which will equal a contour interval in amount.

In using the aneroid in the above manner its exact reading should frequently be marked on the map, especially at all junction points of roads and trails. Accordingly, as the topographer in driving or walking over the various roads or trails comes back to one of those points at which he has already noted the aneroid height, thus *closing a circuit*, he adjusts his aneroid by comparison with the recorded height as though he were adjusting it on an elevation obtained by better methods. In this manner he may be able to extend the range of use of the aneroid by throwing out closed circuits of aneroid elevations one from the other to distances as great as four or five miles to the next elevation of first quality without introducing errors beyond his contour interval. Such results can only be obtained under the most favorable atmospheric conditions.

In using the aneroid in geographic or exploratory surveys where frequent checks cannot be had on known elevations, or by closing back on aneroid heights already recorded, the instrument must be handled in a different manner. It must still be used with the same care, and the beginning of the journey must be made in the same manner. Immediately upon making a stop for a rest or overnight, or for an interval of time of even five minutes, the height indicated by the aneroid should be at once recorded. In starting out again the aneroid will be again read, and if the elevation which it records has changed, the scale must be reset to that noted when the stop was made. To get the best results the journey should be made as rapidly as possible from one stopping-point to the next.

As already stated (Art. 175), the *aneroid acts sluggishly* upon making any sudden change of elevation which is considerable in amount. Thus in ascending or descending a high and steep hill the aneroid will fail to record the full altitude passed over if read immediately upon arrival. It should be so read, however, but a stop of a few minutes should be made at the top or bottom of an inclination, and thereafter the aneroid be again read, in which case if not affected by unaccountable atmospheric conditions it will have responded gradually to the change of elevation and will note an increased difference of height. Frequent comparison with known elevations in the conduct of aneroid work has shown that the amount of change by which the record of the aneroid is too small varies from 2 to 5 per cent, according to the speed with which the journey has been made, the condition of the aneroid itself, and the difference of elevation. It is therefore safe to add this amount to or subtract it from the record of the aneroid, as noted upon immediate arrival at the top or bottom of a high, steep slope. The scale of the aneroid, however, should not be corrected for this difference, since the aneroid will gradually come back itself to the change of elevation which it should have originally noted.

In railway and other topographic surveys in Germany even more faith is placed in the results of aneroid readings than the most firm believers in the instrument in this country would advocate. Mr. F. A. Gelbcke states that a careful observer is able to reach an approximation of from three to six feet of elevation with certainty. Such a high degree of accuracy is obtained of course only where the aneroid is frequently checked by reference to spirit-level elevations, as in making a topographic survey for railroads, where a base line is leveled through and the aneroid is used at comparatively short distances and for small changes of elevation.

Calculations of heights from such observations are made graphically, the aneroid readings, after correction for temperature, being plotted on cross-section paper. On this, with the aid of the barographic notations and the readings at the bench-marks and other check stations, a horizontal curve is constructed. This is an *aneroid diagram*, from which it is only necessary to read the ordinates of the curve at the stations, with a scale varying to suit the observed changes of temperature, in order to obtain the elevations of the stations. Thus the desired heights are furnished without calculation and in the least time, and so that large errors in determination of the elevations are practically excluded.

177. Thermometric Leveling.—Differences in elevation may be ascertained with a certain degree of approximation by means of determining the *boiling-point of water*. This is because when water is heated the elastic force of the vapor produced as it is transformed into steam increases until it becomes equal to the incumbent weight of the atmosphere; this pressure then being overcome, the vapor bursts into steam. It is evident, therefore, that the temperature at which water boils in open air depends upon the weight of the column of atmosphere above it, and this fact is made use of in determining the differences of altitude.

The temperature at which water boils under different pressures has been determined by experiment. It is only necessary, therefore, to observe the temperature at which water boils at any place, and by referring to a table to find the corresponding height of the barometer or elevation above the sea. Account may be taken of the effect of variations in temperature, moisture, pressure, etc., but the errors inherent in the method itself are so great as to make such attempt at refinement of little value. Table XXII gives the approximate elevations above mean sea-level for different temperatures Fahrenheit between 190° and 213°, and is dependent upon the state of the atmosphere.

The thermometer should be a delicately graduated glass tube, made to show the largest possible fraction of a degree between those shown in the table. It may be immersed in a kettle of steam, but more advantageous results can be obtained by using some sort of steam-boiler which will bring the larger portion of its surface into immediate contact with a good current of steam. An apparatus of this sort may consist of a cylindrical boiler from the center of which rises a chimney about 2 inches in diameter by 4 inches high, open at the top, and covered by a similar inverted chimney, the whole being covered again by a still larger chimney; so that the current of steam rising through the inner chimney will circulate down through the middle one and up through the outer and off through a central vent, through which latter the thermometer will be inserted through the interior flue. Such a double passageway prevents the condensation of steam on the interior walls.

TABLE XXII.

ALTITUDE BY BOILING-POINT OF WATER.

Boiling-point. Degrees (Fahr.).	Altitude. Feet.	Boiling-point. Degrees (Fahr.).	Altitude. Feet.
190	11,720	208	2,050
195	8,950	209	1,545
200	6,250	210	1,020
202	5,185	211	510
204	4,130	212	0
206	3,085	213	— 505

The lack of delicacy in this instrument is evident when it is realized that an error of 0.1 degree in the temperature will cause an error of over 80 feet in the determination of elevations. In addition to being subject to all the errors of measurement by barometer, measurement by thermometer is also subject to errors in graduation of the thermometer, lack of precision in reading, the quality of glass, and the form of the vessel containing the water, as well as the purity of the latter, salts in solution materially affecting the boiling-point.

PART IV.

OFFICE WORK OF TOPOGRAPHIC MAPPING.

CHAPTER XIX.

MAP CONSTRUCTION.

178. Cartography.—Cartography is the art of constructing maps either (1) from existing material or (2) from original surveys. It includes not only the processes of copying, reducing or combining, platting or sketching maps, but also of incorporating into them such data as may be obtained from text notes or verbal descriptions of the territory represented.

The *expert cartographer* must therefore be not only a good draftsman, familiar with the methods of map construction and the conventional signs commonly employed, but he must be possessed of such actual knowledge of map-making as is only gained by practical experience in field surveying. Moreover, he must be able to distinguish between the quality and value of the various map materials which he is to utilize, discerning, by his knowledge of topographic forms, the good from the bad, and especially that which is based on original surveys from that which has been compiled from hearsay or existing map sources.

The *draftsman or topographer* who makes a map from original notes taken in the field is not a cartographer in the truest sense of the word. He should know some of those details of map construction with which the cartographer is familiar, as the projection of the map, conventional signs to be employed, and the values of scales, etc. He need not necessarily be familiar, however, with the relative value of existing map material, nor be possessed of especial discernment in the compilation and utilization of the same.

179. Map Projection.—Having executed the primary triangulation. (Chap. XXV) and computed the geodetic coordinates of the initial points (Chap. XXIX), these are platted on a plane-table sheet by the aid of a *projection*. This is a rectangular diagram on which unit meridians and parallels are platted to the scale of the map, and which thus serve as bases from which to measure the differential latitudes and longitudes of the points so that they may be platted by these co-ordinates, much as the points of a traverse are platted by latitudes and departures. (Art. 90.)

The only *absolutely true map* is a model of the terrestrial globe; but as globes are too awkward for general use, recourse is had for purposes of map publication to various forms of map projections, which are numerous in variety and are all artificial representations upon some plane surface of a spheroidal surface. For surveys extending over a large area it is necessary to adopt some method of projection by which the *convergence of* the *meridians* is shown as on a curved surface, and the distances are reduced to sea-level. Where areas which are to be mapped are small, the positions of points and the construction of the map may be fixed as upon a plane surface, by showing meridians of longitude and parallels of latitude as parallel straight lines at right angles to each other. It is practically impossible to fix limits within which the first or the second of these methods must be employed, as they are not only affected by the area covered, but by the scale of the map.

180. Kinds of Projection.—The varieties of map projections cannot be more clearly characterized than is done by

Prof. Dr. Friedrich Umlauff in his admirable little treatise on "The Understanding of Maps," published in Leipsic in 1889, from which the following is freely translated:

In drawing a small-scale map of a considerable area there must be considered: 1, the scale; 2, the projection by which it is made; and 3, the manner in which the spheroidal surface of the earth as a whole or in part is transferred to the plane of the surface.

A spherical surface cannot be spread on a plane without tearing, stretching, or folding; hence maps can never exhibit a perfectly true picture of the area represented. Thus there is simply a question of selecting a mode of representation which shall come as close as possible to the original. To solve this problem, various kinds of projections have been devised, aiming to plat the so-called degree-net of the globe, meridians and parallels, or a part of it, on a plane surface. There are distinguished, especially, (1) perspective projections, (2) non-perspective projections, (3) conical and (4) cylindrical projections.

181. Perspective Projections.—To project a figure from a spherical surface on a plane, nothing occurs to one more readily than to employ the same method that is used to depict any object in space, as a landscape; namely, by perspective drawing. The methods of platting based on the principles of the perspective are called *perspective projections.* The visual rays going from the eye to all points of the original are imagined to be cut by the plane of the drawing, and the point in the picture representing each point in the object is assumed to be the point where the visual ray in question cuts the plane of the drawing. The position of this plane is assumed to be perpendicular to the ray striking the middle of the area to be represented. A difference of the picture can only arise, in perspective projections, by a different position of the eye with relation to the surface of the sphere.

If the eye first of all is supposed to be placed at the center of the globe, we obtain the *gnomonic or central projection.*

As the visual rays pass from the eye through the various lines of the degree-net, they are inclined to the plane of drawing at smaller and smaller angles the farther they deviate from the center of the plane of drawing, and it is evident that this angle must finally dwindle to zero degrees—that the outermost visual rays run parallel with the plane of the picture and therefore do not intersect it. Thus the circles of the degree-net become farther and farther apart as we approach the periphery of the map. (Fig. 115, *a*.)

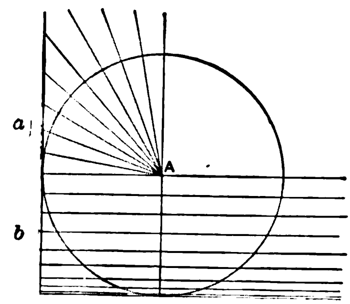


FIG. 115.—GNOMONIC (*a*) AND ORTHOGRAPHIC (*b*) PROJECTIONS.

If we imagine the eye placed at an infinite distance from the globe, we obtain the *orthographic or parallel projection*, so called because all the rays coming from the eye appear parallel and therefore strike the plane of drawing at right angles (Fig. 115, *b*). The parallel projection permits the representation of a complete hemisphere, which is impossible with the central projection.

The third perspective mode of platting is the *stereographic projection*, in which the eye is supposed to be placed at the surface of the sphere itself. Here, too, the visual rays diverge more and more toward the edges of the picture, but they intersect it at greater angles than in the central projection, and even the outermost ray still strikes the plane of the picture, so that this projection, too, permits the representation of a complete hemisphere. (Fig. 116, *a*.)

Finally, if the eye is placed outside of the sphere, but at a little distance, we obtain the *external projection* (Fig. 116, *b*), which, however, is but very rarely used.

To obtain an idea of the networks produced by these perspective projections one has to take other things into consideration. If the eye-point, aside from its distance from the terrestrial globe, lies in the axis of revolution of the earth, the projection is called *polar*; if the eye-point lies in the plane of the equator, the projection is called *equatorial*; if the eye-

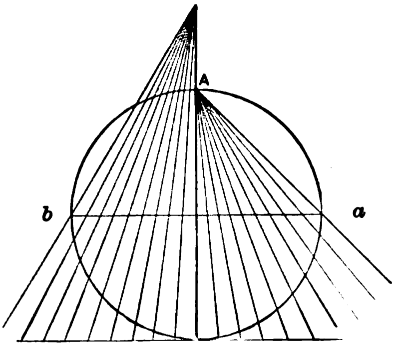


FIG. 116.—STEREOGRAPHIC (*a*) AND EXTERNAL (*b*) PROJECTIONS.

point lies outside the plane of the equator and outside the earth's axis of revolution, the projection is called *horizontal*. Thus, disregarding the external projection, we obtain the following nine kinds of perspective projections:

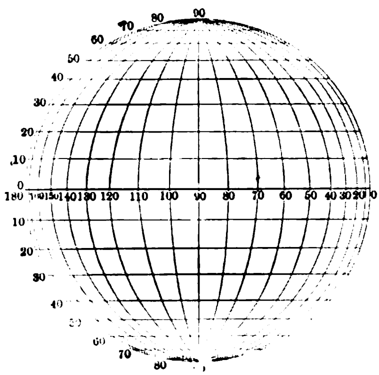


FIG. 117.—ORTHOGRAPHIC EQUATORIAL PROJECTION.

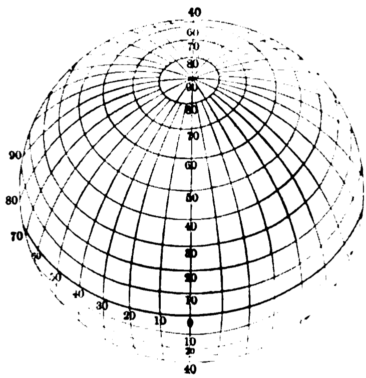


FIG. 118.—ORTHOGRAPHIC HORIZONTAL PROJECTION.

1. Orthographic polar, equatorial, and horizontal projections. (Figs. 115, 117, and 118.)

2. Central (gnomonic) polar, equatorial, and horizontal projections. (Fig. 115.)

3. Stereographic equatorial, polar, and horizontal projections. (Figs. 119, 120, and 121.)

Not all of these modes of map-platting find practical application.

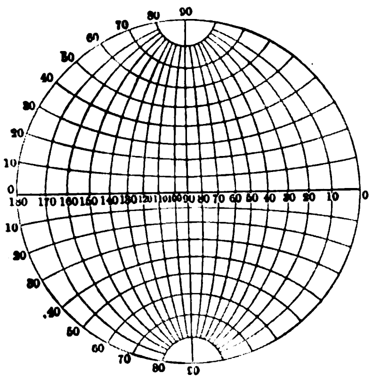


FIG. 119.—STEREOGRAPHIC EQUATORIAL PROJECTION.

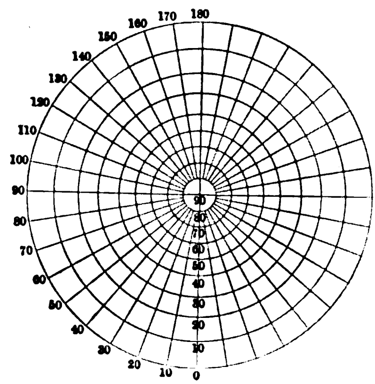


FIG. 120. — STEREOGRAPHIC POLAR PROJECTION.

Maps must comply with certain requirements:

1. They must be angle-true or *conformable;* that is to say, parallels and meridians must intersect on the map at the same angles as on the original.

2. They must be surface-true or *equivalent;* that is to say, the areas of given tracts on the original and on the map must agree.

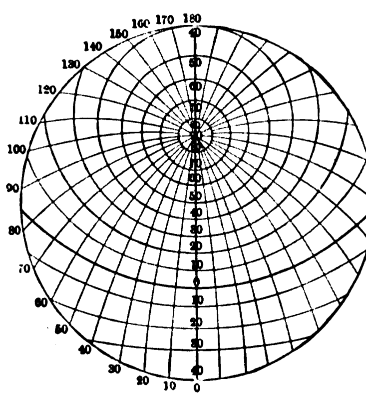


FIG. 121.—STEREOGRAPHIC HORIZONTAL PROJECTION.

From the standpoint of practical cartography surface equivalence is most important, because geographic comparisons relate mostly to phenomena manifesting their uniformity or diversity over areally extended regions. From this last-named requirement arose especially Lambert's *surface-true* *central projection*, which departs from the perspective modes of platting. It received its name from the fact that at all points of equal zenith distance from the middle of the area represented the distortions are the same. The equator and the central meridian appear as two straight lines perpendicular to each other; the other meridians appear as circles, the parallels as elliptic curves. (Fig. 122.) Lambert's surface-true central projection is not a perspective projection; neither is the so-called globular projection, invented by the Sicilian Nicolosi, the distinguishing feature of which is that all meridians and parallels are equally divided. This is used especially as an *equatorial projection.*

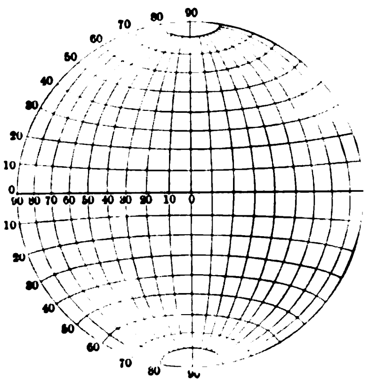


Fig. 122.—Lambert's Surface-true Central Projection.

Finally, there is a special mode of representation of the whole surface of the earth, related to the perspective projections, and the origin of which dates back to Ptolemy: the *star projection.* Every polar projection of the northern hemisphere may be extended into a representation of the whole surface of the earth, by appendages or wings; the southern half of the earth then divides into four or eight parts, to which is given the form of spherical triangles or star-like protuberances. The dividing meridians are so chosen as to avoid any cutting up of land masses as much as possible. For this reason such a star-polar projection is not suitable for representing the oceans. Dr. Jäger has devised an eight-rayed star projection, which was improved by Dr. Petermann; H. Berghaus has drawn a similar one with five appendages.

182. Cylinder Projections.—If we imagine the surface of the earth circumscribed by a cone tangent to it along a parallel, we obtain a *conical projection;* if the surface of the earth appears replaced by a cylinder tangent to it at the equator, we obtain the *cylindrical* or *Mercator projection.*

If we imagine the equator as the middle parallel, quite a broad zone of the globe north and south of the equator may be considered as coinciding with the surface of the cylinder. On this cylinder the meridians are represented as straight lines, and the equator and parallels as circles of equal length cutting the parallels at right angles (Fig. 123, *a*). To represent

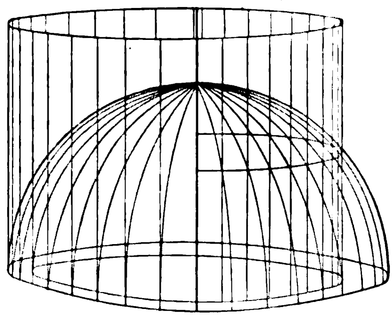


FIG. 123.—CYLINDER PROJECTIONS.

a zone at a higher latitude, we imagine, instead of the tangent cylinder, an intersecting one, also cutting the earth's surface in the middle of the area to be represented. (Fig. 123, *b.*) If thereupon we cut the cylinder along a meridian, we obtain two systems of straight lines intersecting at right angles, representing the parallels and meridians. Maps on such projections are in general called flat maps. If the distances of the various parallels from each other and also of the meridians are all equal, we obtain a network of square meshes, as shown in *equidistant flat maps.* (Fig. 124.) On such maps the distortion of the surfaces increases greatly as we approach the pole, because the parallels, instead of dwindling to zero, preserve the same length in all latitudes, while the meridians retain the natural length. This inconvenience is avoided in *Mercator's projection* by increasing the distances between the parallels toward the two poles at the same ratio that the parallels increase compared to the equator. (Fig. 125.) Mercator's projection is well adapted to maps representing the distribution of general, especially physical, conditions over the whole surface of the earth, and for sea-charts, and any direction may be represented upon it by a straight line.

A modification of the cylinder projection is found in the *Sanson-Flamsteed projection* (Fig. 126). According to this the parallels are drawn as parallel equidistant straight lines, and on these, to the

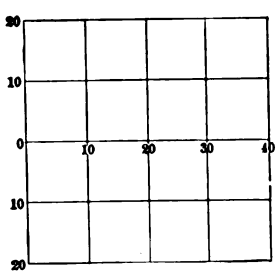


FIG. 124.—EQUIDISTANT FLAT PROJECTION.

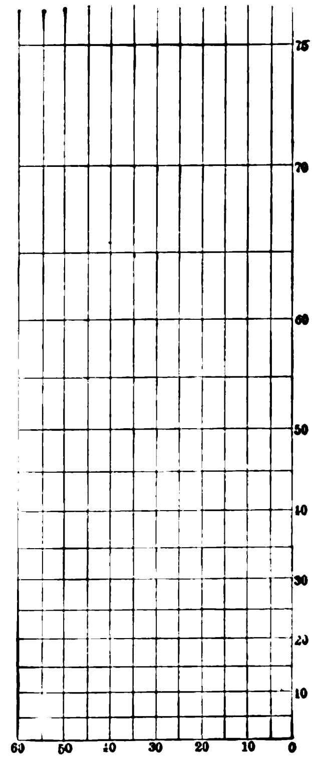


FIG. 125.—MERCATOR'S CYLINDER PROJECTION.

right and left of the middle meridian, the degrees of longitude are marked in their true size, and the corresponding points of intersection are connected by curves representing the meridians. If the equator be drawn as a straight line and the central meridian also as a straight line of half the length of the equator, we obtain an elliptic picture of the whole surface of the globe according to Mollweide's or Babinet's *homalographic projection* (Fig. 127).

183. Conical Projections.—Conical projections are quite analogous to cylinder projections. A certain zone of the globe which is to be represented is conceived to be replaced by a zone on the ***surface of a normal cone, either tangent to the sphere or intersecting it*** (Figs. 128 and 129). The parallels are drawn on the surface of the cone as parallel conical

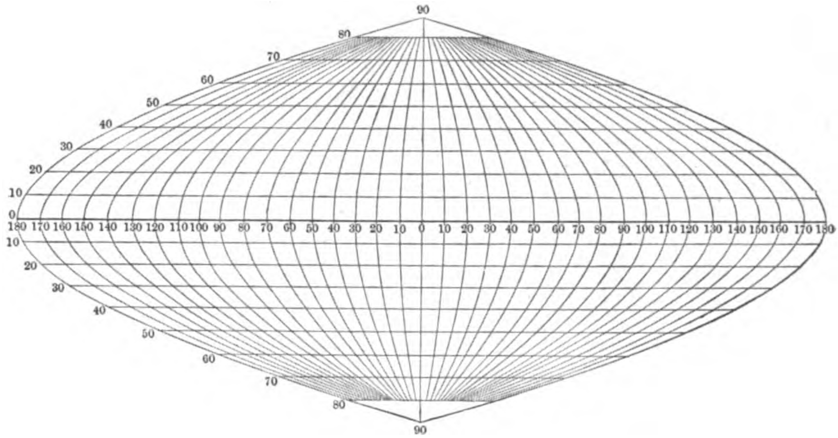


FIG. 126.—SANSON-FLAMSTEED PROJECTION.

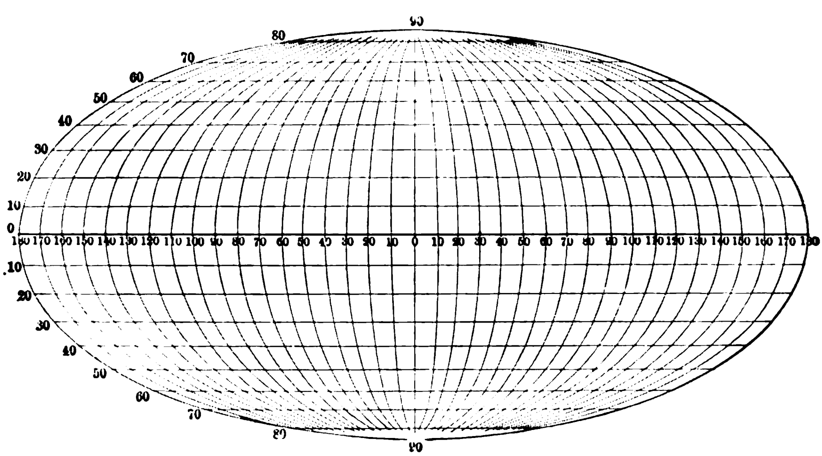


FIG. 127.—HOMALOGRAPHIC PROJECTION.

circles, while the meridians are drawn as straight lines on the conical surface. If the surface of the cone is developed, the parallel circles appear as arcs of concentric circles whose common center is the apex of the cone, while the meridians appear as straight lines converging to that center. The most important conical projections are those of Mercator, Lambert, and Bonne.

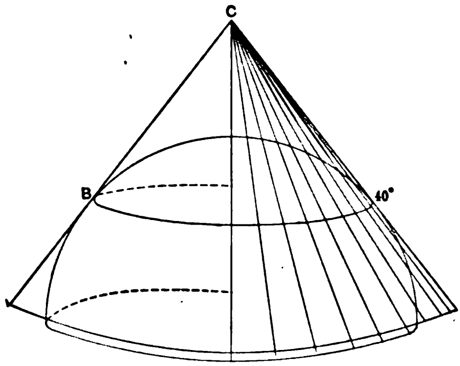


FIG. 128.—TANGENT CONE PROJECTION.

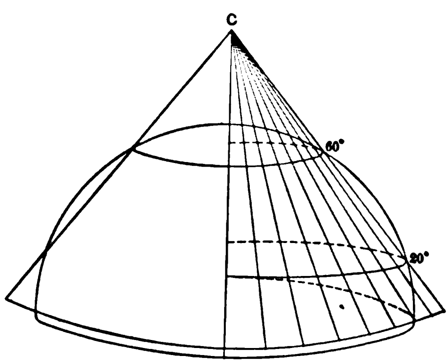


FIG. 129.—INTERSECTING CONE PROJECTION.

An ordinary or *equidistant conical projection* based on a tangent cone shows the meridians as straight lines proceeding from the apex of the cone at equal angles, while the parallel circles are equal-spaced circular arcs with the same apex as center. (Fig. 130.) In *Mercator's conical projection* the distortion is diminished by making the cone pass through two parallels of the area to be represented, so that two parallels of the sphere, instead of one, coincide with their pictures. (Fig. 131.) This is the projection on which the maps of our common atlases and geographies are drawn. Lambert's

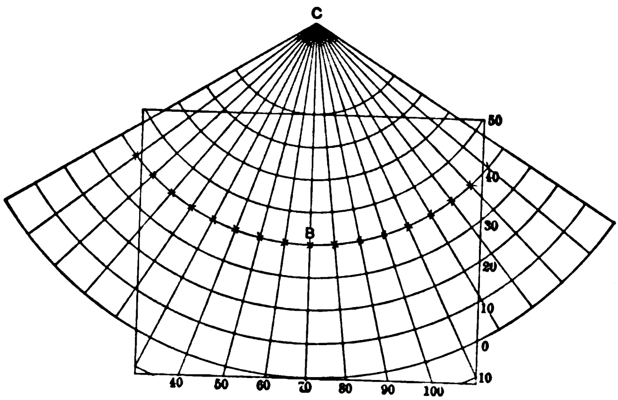


FIG. 130.—EQUAL-SPACED CONICAL PROJECTION.

equivalent conical projection is based on an intersecting cone, and the distances of the parallels increase with increase of

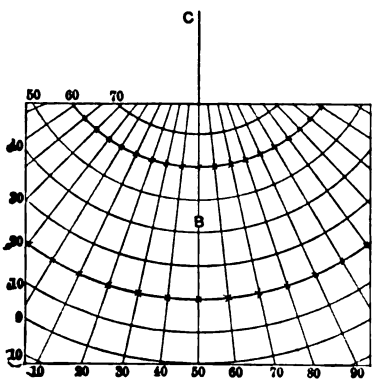


FIG. 131.—MERCATOR'S CONICAL PROJECTION.

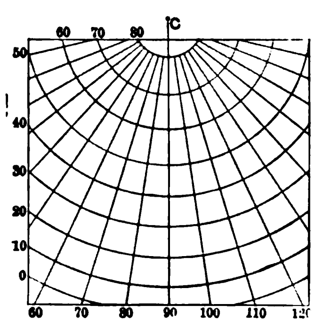


FIG. 132.—EQUIVALENT CONICAL PROJECTION.

latitude at such rate that the meshes included by them and the meridians show the same areas as on the sphere. (Fig. 132.)

Bonne's projection is a projection on the tangent cone in the center of the map, the parallel curves being drawn in the same way as in the ordinary conical projection. On these parallel curves, on both sides of the meridian, the parallel degrees are marked in their true size, and the points of intersection are joined by steady curves which give the meridians. (Fig. 133.)

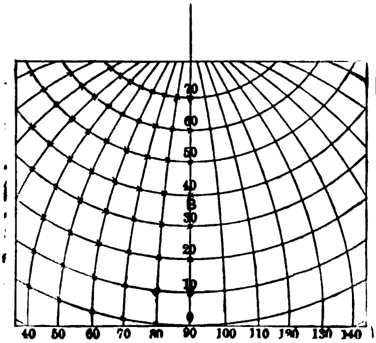


FIG. 133.—POLYCONIC PROJECTION.

184. Constructing a Polyconic Projection.—Of the various projections that are best suited to accurate topographic or geographic mapping the most suitable is the polyconic projection, as it corresponds most nearly on a plane surface with the spheroidal shape of the earth. It is the projection of a series of cones parallel to each parallel of latitude to be drawn on the map. Assume the scale of the map as one mile to one inch, or 1 : 63,360. For this scale it will be sufficient to draw the meridian and latitude lines at intervals of every five minutes or approximately five inches apart, though single minute lines may be drawn if desired. The construction of such a projection is a simple matter, requiring only the greatest care and accuracy in the use of the drafting and measuring instruments. The process is as follows:

Rule a fine vertical line down the center of the sheet. (Fig. 134.) Make it as straight as possible with an accurate straight-edge. On this lay off the lengths of the several five-minute spaces in latitude, these being the dl's as taken from Table XXIII for the scale 1 : 63,360. This fixes the points of intersection of the parallels at every five minutes with the central meridian. Erect on each of these points with the beam compass and straight-edge perpendiculars, and draw these across the map at right angles to the central meridian, as shown in dotted lines. On these approximate parallels lay off the quantities dm (Table XXIII) for half the distance of five minutes, that is, for 2′ 30″ and 7′ 30″ on either side of the central meridian and corresponding to the latitude as obtained from the table. On the points so obtained on each

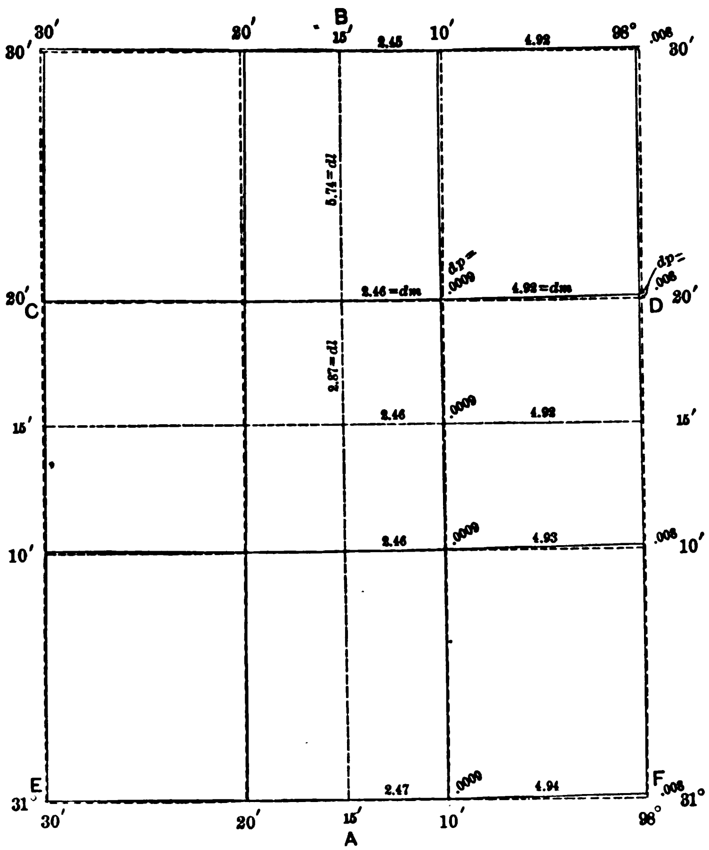


Fig. 134.—Construction of Polyconic Projection.
30′ of latitude and longitude. Scale 2 miles to 1 inch. Construction lines dotted. Final projection lines full.

approximate parallel erect short perpendiculars, and on these lay off the small quantity *dp* corresponding to the *dm*, and connect the various *dp*'s by straight lines in a horizontal and vertical direction. The result will be a projection similar to that shown in full lines in the figure.

185. Projection of Maps upon a Polyconic Development.—The following table (Table XXIII) is arranged for the projection of maps upon a polyconic development of the Clarke spheroid. It is on a scale of one mile to one inch, and is computed from the equator to the pole, the unit scale being one selected for presentation here, as that most generally useful, since the quantities shown in the table can be most readily reduced to those applicable to other scales which are even multiples of one mile to one inch. They are reproduced from the Smithsonian Miscellaneous Tables, for which they were prepared by Prof. R. S. Woodward.

The following formulas are those used in the preparation of this and similar tables, and are derived from the United States Coast and Geodetic Survey Report for 1884:

For lengths of degrees of the meridian (dm) and parallel (dp) we have

$$dm = 111\,132^{m}.09 - 566^{m}.05 \cos 2\phi + 1^{m}.20 \cos 4\phi - 0^{m}.003 \cos 6\phi;$$

$$dp = 111\,415^{m}.10 \cos \phi - 94^{m}.54 \cos 3\phi + 0^{m}.12 \cos 5\phi,$$ neglecting smaller terms,

where $\phi =$ the latitude.

We have also the square of the eccentricity,

$$e^2 = 0.006768658 = \frac{a^2 - b^2}{a^2}.$$

$N = \dfrac{a}{(1 - e^2 \sin^2 \phi)^{\frac{1}{2}}}$ = normal produced to minor axis; . . . . (33)

$Rm = N^3 \dfrac{1 - e^2}{a^2}$ = radius of curvature in the meridian; . . . (34)

$Rp = N \cos \phi$ = radius of the parallel; . . . . (35)

$r = N \cot \phi$ = radius of the developed parallel or side of the tangent cone; . . . . (36)

$\theta = n \sin \phi$,

in which n is any arc of the parallel to be developed, and θ the angle which it subtends at the vertex of the cone when developed. . . . . . (37)

For projecting from the middle meridian the points of intersection of the meridians and parallels we have, using rectangular coordinates X and Y,

$X = r \sin \theta$ . . . . (38)

and

$Y = 2r \sin^2 \frac{1}{2}\theta$. . . . . (39)

TABLE XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

(From Smithsonian Tables.)

Latitude of Parallel.	Meridional Distances from Even-degree Parallels. *dl*	ABSCISSAS OF DEVELOPED PARALLEL. *dm* 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches	inches.	inches.	inches.	inches.
0° 00′	......	5.764	11.529	17.293	23.058	28.822	34.586
10	11.451	5.764	11.528	17.293	23.057	28.821	34.585
20	22.901	5.764	11.528	17.292	23.056	28.821	34.585
30	34.352	5.764	11.528	17.292	23.056	28.820	34.583
40	45.803	5.764	11.528	17.291	23.055	28.819	34.583
50	57.254	5.764	11.527	17.291	23.054	28.818	34.582
1 00	68.704	5.764	11.527	17.291	23.054	28.818	34.581
10	11.451	5.763	11.526	17.289	23.052	28.816	34.579
20	22.901	5.763	11.525	17.288	23.050	28.813	34.576
30	34.352	5.762	11.524	17.287	23.049	28.811	34.573
40	45.803	5.762	11.524	17.285	23.047	28.809	34.571
50	57.254	5.761	11.523	17.284	23.045	28.807	34.568
2 00	68.704	5.761	11.522	17.283	23.044	28.805	34.565
10	11.451	5.760	11.520	17.281	23.041	28.801	34.561
20	22.902	5.759	11.519	17.278	23.038	28.797	34.556
30	34.353	5.759	11.517	17.276	23.035	28.794	34.552
40	45.804	5.758	11.516	17.274	23.032	28.790	34.548
50	57.254	5.757	11.514	17.272	23.029	28.786	34.543
3 00	68.705	5.756	11.513	17.270	23.026	28.783	34.539
10	11.451	5.756	11.511	17.267	23.022	28.778	34.533
20	22.902	5.754	11.509	17.264	23.018	28.773	34.527
30	34.353	5.753	11.507	17.260	23.014	28.767	34.520
40	45.804	5.752	11.505	17.257	23.010	28.762	34.514
50	57.255	5.751	11.503	17.254	23.006	28.757	34.508
4 00	68.706	5.750	11.501	17.251	23.002	28.752	34.502
10	11.451	5.749	11.498	17.247	22.996	28.746	34.495
20	22.903	5.748	11.496	17.243	22.991	28.739	34.487
30	34.354	5.746	11.493	17.240	22.986	28.733	34.479
40	45.805	5.745	11.490	17.236	22.981	28.726	34.471
50	57.256	5.744	11.488	17.232	22.976	28.720	34.463
5 00	63.708	5.743	11.485	17.228	22.970	28.713	34.456

ORDINATES OF DEVELOPED PARALLEL. *dp*

Longitude Interval.	0°	1°	2°	3°	4°	5°
	inches.	inches.				
5′	0.000	0.000	0.000	0.000	0.000	0.000
10	.000	.000	.001	.001	.001	.001
15	.000	.001	.001	.002	003	.003
20	.000	.001	.002	.003	.005	.006
25	.000	.002	.004	.005	.007	.009
30	.000	.003	.005	.008	.011	.013

TABLE XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even-degree Parallels. *dl*	ABSCISSAS OF DEVELOPED PARALLEL. *dm* 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
5°00′	68.708	5.743	11.485	17.228	22.970	28.713	34.456
10	11.452	5.741	11.482	17.223	22.964	28.705	34.446
20	22.903	5.739	11.479	17.218	22.958	28.697	34.436
30	34.355	5.738	11.476	17.213	22.951	28.689	34.427
40	45.806	5.736	11.472	17.209	22.945	28.681	34.417
50	57.258	5.735	11.469	17.204	22.938	28.673	34.408
6 00	68.710	5.733	11.466	17.199	22.932	28.665	34.398
10	11.452	5.731	11.462	17.193	22.924	28.656	34.387
20	22.904	5.729	11.458	17.188	22.917	28.646	34.375
30	34.356	5.727	11.455	17.182	22.910	28.637	34.364
40	45.808	5.726	11.451	17.177	22.902	28.628	34.353
50	57.260	5.724	11.447	17.171	22.894	28.618	34.342
7 00	68.712	5.722	11.443	17.165	22.887	28.609	34.330
10	11.452	5.720	11.439	17.159	22.878	28.598	34.317
20	22.905	5.717	11.435	17.152	22.869	28.587	34.304
30	34.358	5.715	11.430	17.146	22.861	28.576	34.291
40	45.810	5.713	11.426	17.139	22.852	28.565	34.278
50	57.262	5.711	11.422	17.132	22.843	28.554	34.265
8 00	68.715	5.709	11.417	17.126	22.834	28.543	34.252
10	11.453	5.706	11.412	17.119	22.825	28.531	34.237
20	22.906	5.704	11.407	17.111	22.815	28.519	34.222
30	34.359	5.701	11.403	17.104	22.805	28.507	34.208
40	45.812	5.699	11.398	17.096	22.795	28.494	34.193
50	57.265	5.696	11.393	17.089	22.786	28.482	34.178
9 00	68.718	5.694	11.388	17.082	22.776	28.470	34.163
10	11.454	5.691	11.382	17.073	22.764	28.456	34.147
20	22.907	5.688	11.377	17.065	22.754	28.442	34.130
30	33.361	5.686	11.371	17.057	22.742	28.428	34.114
40	45.814	5.683	11.366	17.049	22.732	28.415	34.097
50	57.268	5.680	11.360	17.040	22.720	28.401	34.081
10 00	68.722	5.677	11.355	17.032	22.710	28 387	34.064

ORDINATES OF DEVELOPED PARALLEL. *dp*

Longitude Interval.	5°	6°
	inches.	inches.
5′	0.000	0.000
10	.001	.002
15	.003	.004
20	.006	.007
25	.009	.011
30	.013	.016

	7°	8°
5	0.000	0.001
10	.002	.002
15	.005	.005
20	.008	.009
25	.013	.014
30	.018	.021

	9°	10°
5	0.001	0.001
10	.003	.003
15	.006	.006
20	.010	.011
25	.016	.018
30	.023	.026

TABLE XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even degree Parallels. *dl*	ABSCISSAS OF DEVELOPED PARALLEL. *dm* 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
10°00′	68.722	5.677	11.355	17.032	22.710	28.387	34.064
10	11.454	5.674	11.349	17.023	22.698	28.372	34.046
20	22.909	5.671	11.343	17.014	22.685	28.357	34.028
30	34.263	5.668	11.337	17.005	22.673	28.342	34.010
40	45.817	5.665	11.331	16.996	22.661	28.327	33.992
50	57.272	5.662	11.324	16.987	22.649	28.311	33.973
11 00	68.726	5.659	11.318	16.978	22.637	28.296	33.955
10	11.455	5.656	11.312	16.968	22.624	28.280	33.935
20	22.910	5.652	11.305	16.958	22.610	28.263	33.915
30	34.365	5.649	11.298	16.948	22.597	28.246	33.895
40	45.820	5.646	11.292	16.938	22.584	28.230	33.875
50	57.275	5.642	11.285	16.928	22.570	28.213	33.855
12 00	68.730	5.639	11.278	16.918	22.557	28.196	33.835
10	11.456	5.636	11.271	16.907	22.542	28.178	33.814
20	22.912	5.632	11.264	16.896	22.528	28.160	33.792
30	34.367	5.628	11.257	16.885	22.514	28.142	33.770
40	45.823	5.625	11.250	16.874	22.499	28.124	33.749
50	57.279	5.621	11.242	16.864	22.485	28.106	33.727
13 00	68.735	5.618	11.235	16.853	22.470	28.088	33.706
10	11.457	5.614	11.227	16.841	22.455	28.069	33.682
20	22.913	5.610	11.220	16.829	22.439	28.049	33.659
30	34.370	5.606	11.212	16.818	22.424	28.030	33.635
40	45.827	5.602	11.204	16.806	22.408	28.010	33.612
50	57.284	5.598	11.196	16.794	22.392	27.991	33.589
14 00	68.740	5.594	11.188	16.783	22.377	27.971	33.565
10	11.458	5.590	11.180	16.770	22.360	27.950	33.540
20	22.915	5.586	11.172	16.758	22.344	27.930	33.515
30	34.373	5.582	11.163	16.745	22.327	27.909	33.490
40	45.830	5.578	11.155	16.733	22.310	27.888	33.465
50	57.288	5.573	11.147	16.720	22.294	27.867	33.440
15 00	68.746	5.569	11.138	16.708	22.277	27.846	33.415

ORDINATES OF DEVELOPED PARALLEL. *dp*

Longitude Interval.	10°	11°
	inches.	inches.
5′	0.001	0.001
10	.003	.003
15	.006	.007
20	.011	.013
25	.018	.020
30	.026	.028

Longitude Interval.	12°	13°
5	0.001	0.001
10	.003	.004
15	.008	.008
20	.014	.015
25	.021	.023
30	.031	.033

Longitude Interval.	14°	15°
5	0.001	0.001
10	.004	.004
15	.009	.009
20	.016	.017
25	.025	.026
30	.035	.038

TABLE XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even-degree Parallels. dl	ABSCISSAS OF DEVELOPED PARALLEL. dm 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
15°00′	68.746	5.569	11.138	16.708	22.277	27.846	33.415
10	11.459	5.565	11.130	16.694	22.259	27.824	33.389
20	22.917	5.560	11.121	16.681	22.241	27.802	33.362
30	34.376	5.556	11.112	16.667	22.223	27.779	33.335
40	45.834	5.551	11.103	16.654	22.206	27.757	33.308
50	57.293	5.547	11.094	16.641	22.188	27.735	33.282
16 00	68.752	5.542	11.085	16.628	22.170	27.713	33.255
10	11.460	5.538	11.076	16.613	22.151	27.689	33.227
20	22.919	5.533	11.066	16.599	22.132	27.665	33.198
30	34.379	5.528	11.057	16.585	22.113	27.642	33.170
40	45.838	5.524	11.047	16.571	22.094	27.618	33.142
50	57.298	5.519	11.038	16.556	22.075	27.594	33.113
17 00	68.758	5.514	11.028	16.542	22.056	27.571	33.085
10	11.461	5.509	11.018	16.527	22.036	27.546	33.055
20	22.921	5.504	11.008	16.512	22.016	27.521	33.025
30	34.382	5.499	10.998	16.497	21.996	27.495	32.994
40	45.843	5.494	10.988	16.482	21.976	27.470	32.964
50	57.304	5.489	10.978	16.467	21.956	27.445	32.934
18 00	68.764	5.484	10.968	16.452	21.936	27.420	32.904
10	11.462	5.479	10.957	16.436	21.915	27.394	32.872
20	22.924	5.473	10.947	16.420	21.894	27.367	32.840
30	34.386	5.468	10.936	16.404	21.872	27.341	32.809
40	45.848	5.463	10.926	16.389	21.852	27.315	32.777
50	57.310	5.458	10.915	16.373	21.830	27.288	32.746
19 00	68.771	5.452	10.905	16.357	21.809	27.262	32.714
10	11.463	5.447	10.893	16.340	21.787	27.234	32.680
20	22.926	5.441	10.882	16.324	21.765	27.206	32.647
30	34.390	5.436	10.871	16.307	21.742	27.178	32.614
40	45.853	5.430	10.860	16.290	21.720	27.150	32.580
50	57.316	5.424	10.849	16.274	21.698	27.123	32.547
20 00	68.779	5.419	10.838	16.257	21.676	27.095	32.513

ORDINATES OF DEVELOPED PARALLEL. dp

Longitude Interval.	15°	16°
	inches.	inches.
5′	0.001	0.001
10	.004	.004
15	.009	.010
20	.017	.018
25	.026	.028
30	.038	.040

Longitude Interval.	17°	18°
5	0.001	0.001
10	.005	.005
15	.011	.011
20	.019	.020
25	.029	.031
30	.042	.044

Longitude Interval.	19°	20°
5	0.001	0.001
10	.005	.005
15	.012	.012
20	.021	.022
25	.032	.034
30	.046	.049

Table XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridinal Distances from Even-degree Parallel. *dl*	ABSCISSAS OF DEVELOPED PARALLEL. *dm* 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
20°00′	68.779	5.419	10.838	16.257	21.676	27.095	32.513
10	11.464	5.413	10.826	16.239	21.652	27.065	32.478
20	22.929	5.407	10.814	16.222	21.629	27.036	32.443
30	34.394	5.401	10.803	16.204	21.605	27.007	32.408
40	45.858	5.396	10.791	16.187	21.582	26.978	32.373
50	57.322	5.390	10.779	16.169	21.558	26.948	32.338
21 00	68.787	5.384	10.768	16.151	21.535	26.919	32.303
10	11.466	5.378	10.755	16.133	21.511	26.889	32.266
20	22.932	5.372	10.743	16.115	21.486	26.858	32.230
30	34.397	5.366	10.731	16.097	21.462	26.828	32.193
40	45.863	5.359	10.719	16.078	21.438	26.797	32.156
50	57.329	5.353	10.707	16.060	21.413	26.767	32.120
22 00	68.795	5.347	10.694	16.042	21.389	26.736	32.083
10	11.467	5.341	10.682	16.022	21.363	26.704	32.045
20	22.934	5.334	10.669	16.003	21.338	26.672	32.006
30	34.401	5.328	10.656	15.984	21.312	26.641	31.969
40	45.868	5.322	10.643	15.965	21.287	26.609	31.930
50	57.336	5.315	10.631	15.946	21.261	26.577	31.892
23 00	68.803	5.309	10.618	15.927	21.236	26.545	31.853
10	11.469	5.302	10.604	15.907	21.209	26.511	31.813
20	22.937	5.296	10.591	15.887	21.182	26.478	31.774
30	34.406	5.289	10.578	15.867	21.156	26.445	31.733
40	45.874	5.282	10.565	15.847	21.129	26.412	31.694
50	57.343	5.276	10.551	15.827	21.102	26.378	31.654
24 00	68.812	5.269	10.538	15.807	21.076	26.345	31.614
10	11.470	5.263	10.526	15.789	21.052	26.315	31.577
20	22.940	5.256	10.512	15.767	21.023	26.279	31.535
30	34.410	5.249	10.498	15.746	20.995	26.244	31.493
40	45.880	5.242	10.483	15.725	20.967	26.209	31.450
50	57.350	5.235	10.469	15.704	20.938	26.173	31.408
25 00	68.821	5.227	10.455	15.682	20.910	26.137	31.365

ORDINATES OF DEVELOPED PARALLEL. *dp*

Longitude Interval.	20°	21°
	inches.	inches.
5′	0.001	0.001
10	.005	.006
15	.012	.013
20	.022	.022
25	.034	.035
30	.049	.051

Longitude Interval.	22°	23°
5	0.001	0.001
10	.006	.006
15	.013	.014
20	.023	.024
25	.036	.038
30	.052	.054

Longitude Interval.	24°	25°
5	0.002	0.002
10	.006	.006
15	.014	.014
20	.025	.026
25	.039	.040
30	.056	.058

Table XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridinal Distances from Even-degree Parallel. dl	Abscissas of Developed Parallel. dm 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
25°00′	68.821	5.227	10.455	15.682	20.910	26.137	31.365
10	11.472	5.220	10.441	15.661	20.881	26.101	31.322
20	22.943	5.213	10.426	15.639	20.852	26.065	31.279
30	34.415	5.206	10.412	15.618	20.824	26.029	31.235
40	45.886	5.199	10.397	15.596	20.795	25.993	31.192
50	57.358	5.191	10.383	15.575	20.766	25.958	31.149
26 00	68.830	5.184	10.369	15.553	20.737	25.922	31.106
10	11.473	5.177	10.354	15.531	20.708	25.884	31.061
20	22.946	5.169	10.339	15.508	20.678	25.847	31.017
30	34.419	5.162	10 324	15.486	20.648	25.810	30.972
40	45.892	5.154	10.309	15.463	20.618	25.772	30.927
50	57.365	5.147	10.294	15.441	20.588	25.735	30.882
27 00	68.838	5.140	10.279	15.419	20.558	25.698	30.838
10	11.475	5.132	10.264	15.396	20.528	25.659	30.791
20	22.950	5.124	10.248	15.373	20.497	25.621	30.745
30	34.424	5.116	10.233	15.349	20.466	25.582	30.699
40	45.899	5.109	10.218	15.326	20.435	25.544	30.653
50	57.374	5.101	10.202	15.303	20.404	25.505	30.607
28 00	68.849	5.093	10.187	15.280	20.374	25.467	30.560
10	11.476	5.085	10.171	15.256	20.342	25.427	30.513
20	22.953	5.077	10.155	15.232	20.310	25.387	30.465
30	34.430	5.069	10.139	15.208	20.278	25.347	30.417
40	45.906	5.061	10.123	15.185	20.246	25.308	30.369
50	57.383	5.054	10.107	15.161	20.214	25.268	30.321
29 00	68.859	5.046	10.091	15.137	20.182	25.228	30.274
10	11.478	5.037	10.075	15.112	20.150	25.187	30.224
20	22.957	5.029	10.058	15.087	20.117	25.146	30.175
30	34.435	5.021	10.042	15.063	20.084	25.105	30.126
40	45.913	5.013	10.025	15.038	20.051	25.064	30.076
50	57.391	5.004	10.009	15.013	20.018	25.022	30.027
30 00	68.870	4.996	9.993	14.989	19.985	24.981	29.978

ORDINATES OF DEVELOPED PARALLEL. dp

Longitude Interval.	25°	26°
	inches.	inches.
5′	0.002	0.002
10	.006	.007
15	.014	.015
20	.026	.026
25	.040	.041
30	.058	.059
	27°	28°
5	0.002	0.002
10	.007	.007
15	.015	.016
20	.027	.028
25	.042	.043
30	.061	.063
	29°	30°
5	0.002	0.002
10	.007	.007
15	.016	.016
20	.028	.029
25	.044	.045
30	.064	.065

TABLE XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridinal Distances from Even-degree Parallel. *dl*	ABSCISSAS OF DEVELOPED PARALLEL. *dm* 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
30°00′	68.870	4.996	9.993	14.989	19.985	24.981	29.978
10	11.480	4.988	9.976	14.963	19.951	24.939	29.927
20	22.960	4.979	9.959	14.938	19.917	24.896	29.876
30	34.440	4.971	9.942	14.912	19.883	24.854	29.825
40	45.920	4.962	9.925	14.887	19.849	24.812	29.774
50	57.400	4.954	9.908	14.862	19.815	24.769	29.723
31 00	68.880	4.945	9.891	14.836	19.782	24.727	29.672
10	11.482	4.937	9.873	14.810	19.747	24.683	29.620
20	22.964	4.928	9.856	14.784	19.712	24.640	29.568
30	34.446	4.919	9.838	14.758	19.677	24.596	29.515
40	45.927	4.910	9.821	14.731	19.642	24.552	29.463
50	57.409	4.902	9.804	14.705	19.607	24.509	29.411
32 00	68.891	4.893	9.786	14.679	19.572	24.465	29.358
10	11.484	4.884	9.768	14.652	19.536	24.420	29.305
20	22.967	4.875	9.750	14.625	19.500	24.376	29.251
30	34.451	4.866	9.732	14.598	19.465	24.331	29.197
40	45.934	4.857	9.714	14.572	19.429	24.286	29.143
50	57.418	4.848	9.696	14.545	19.393	24.241	29.089
33 00	68.902	4.839	9.679	14.518	19.357	24.196	29.036
10	11.485	4.830	9.660	14.490	19.320	24.150	28.980
20	22.971	4.821	9.642	14.462	19.283	24.104	28.925
30	34.456	4.812	9.623	14.435	19.246	24.058	28.870
40	45.942	4.802	9.605	14.407	19.210	24.012	28.814
50	57.427	4.793	9.586	14.379	19.173	23.966	28.759
34 00	68.913	4.784	9.568	14.352	19.136	23.920	28.704
10	11.487	4.774	9.549	14.323	19.098	23.872	28.647
20	22.975	4.765	9.530	14.295	19.080	23.825	28.590
30	34.462	4.755	9.511	14.267	19.022	23.778	28.533
40	45.949	4.746	9.492	14.238	18.984	23.730	28.476
50	57.437	4.737	9.473	14.210	18.946	23.683	28.420
35 00	68.924	4.727	9.454	14.181	18.908	23.636	28.363

ORDINATES OF DEVELOPED PARALLEL. *dp*

Longitude Interval.	30°	31°
	inches.	inches.
5′	0.002	0.002
10	.007	.007
15	.016	.017
20	.029	.030
25	.045	.046
30	.065	.067
	32°	**33°**
5	0.002	0.002
10	.007	.008
15	.017	.017
20	.030	.031
25	.047	.048
30	.068	.069
	34°	**35°**
5	0.002	0.002
10	.008	.008
15	.017	.018
20	.031	.031
25	.049	.049
30	.070	.071

Table XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even-degree Parallels. *dl*	ABSCISSAS OF DEVELOPED PARALLEL. *dm* 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
35°00′	68.924	4.727	9.454	14.181	18.908	23.636	28.363
10	11.489	4.717	9.435	14.152	18.870	23.587	28.305
20	22.978	4.708	9.415	14.123	18.831	23.539	28.246
30	34.468	4.698	9.396	14.094	18.792	23.490	28.188
40	45.957	4.688	9.377	14.065	18.753	23.442	28.130
50	57.446	4.679	9.357	14.036	18.714	23.393	28.072
36 00	68.935	4.669	9.338	14.007	18.676	23.345	28.014
10	11.491	4.659	9.318	13.977	18.636	23.295	27.954
20	22.983	4.649	9.298	13.947	18.596	23.245	27.894
30	34.474	4.639	9.278	13.917	18.556	23.195	27.835
40	45.965	4.629	9.258	13.887	18.517	23.146	27.775
50	57.457	4.619	9.238	13.858	18.477	23.096	27.615
37 00	68.948	4.609	9.219	13.828	18.437	23.046	27.656
10	11.493	4.599	9.198	13.797	18.396	22.995	27.594
20	22.986	4.589	9.178	13.767	18.356	22 944	27 533
30	34.480	4.579	9.157	13.736	18.315	22.894	27.472
40	45.973	4.568	9.137	13.706	18.274	22.843	27.411
50	57.466	4.558	9.117	13.675	18.234	22.792	27.350
38 00	68.959	4 548	9.096	13.645	18.193	22.741	27.289
10	11.495	4.538	9.076	13.613	18.151	22.689	27.227
20	22.990	4.527	9.055	13.582	18.109	22.637	27.164
30	34.485	4.517	9.034	13.551	18.068	22.585	27.102
40	45.980	4.506	9.013	13.520	18.026	22.533	27.039
50	57.475	4.496	8.992	13.488	17.984	22.481	26.977
39 00	68.970	4.486	8.971	13.457	17.943	22.429	26.914
10	11.497	4.475	8.950	13.425	17.900	22.375	26.851
20	22.994	4.464	8.929	13.393	17.858	22.322	26.787
30	34.491	4.454	8.908	13.361	17.815	22.269	26.723
40	45.988	4 443	8.886	13.330	17.773	22.216	26.659
50	57.485	4.433	8.865	13.298	17.730	22.163	26.595
40 00	68.982	4.422	8.844	13.266	17.688	22.110	26.532

ORDINATES OF DEVELOPED PARALLEL. *dp*

Longitude Interval.	35°	36°
	inches.	inches.
5′	0.002	0.002
10	.008	.008
15	.018	.018
20	.031	.032
25	.049	.050
30	.071	.072
	37°	**38°**
5	0.002	0.002
10	.008	.008
15	.018	.018
20	.032	.033
25	.050	.051
30	.073	.073
	39°	**40°**
5	0.002	0.002
10	.008	.008
15	.018	.019
20	.033	.033
25	.051	.052
30	.074	.074

TABLE XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even-degree Parallel. *dl*	ABSCISSAS OF DEVELOPED PARALLEL. *dm* 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
40°00′	68.982	4.422	8.844	13.266	17.688	22.110	26.532
10	11.499	4.411	8.822	13.233	17.644	22.055	26.466
20	22.998	4.400	8.800	13.201	17.601	22.001	26.401
30	34.497	4.389	8.779	13.168	17.557	21.947	26.336
40	45.996	4 378	8.757	13.135	17.514	21.892	26.271
50	57.495	4.368	8.735	13.103	17.470	21.838	26.206
41 00	68.994	4.357	8.713	13.070	17.427	21.784	26.140
10	11.501	4.346	8.691	13.037	17.383	21.728	26.074
20	23.002	4.335	8.669	13.004	17.338	21.673	26.007
30	34.503	4 324	8.647	12.971	17.294	21.618	25.941
40	46.004	4.312	8.625	12.937	17.250	21.562	25.875
50	57.506	4.301	8.603	12.904	17.205	21.507	25.808
42 00	69.007	4.290	8.581	12.871	17.161	21.451	25.742
10	11.503	4.279	8.558	12.837	17.116	21.395	25.674
20	23.006	4.268	8.535	12.803	17.071	21.338	25.606
30	34.510	4.256	8.513	12.769	17.025	21.282	25.538
40	46.013	4.245	8.490	12.735	16.980	21.225	25.470
50	57.516	4.234	8.467	12.701	16.935	21.169	25.402
43 00	69.019	4.222	8.445	12.667	16.890	21.112	25.334
10	11.505	4.211	8.422	12.633	16.844	21.054	25.265
20	23.010	4.199	8.399	12.598	16.798	20.997	25.196
30	34.515	4.188	8.376	12.564	16.751	20.939	25.127
40	46.020	4.176	8.353	12.529	16.705	20.882	25.058
50	57.525	4.165	8.330	12.494	16.659	20.824	24.989
44 00	69.030	4.153	8.307	12.460	16.613	20.767	24.920
10	11.507	4.142	8.283	12.425	16.566	20.708	24.849
20	23.014	4.130	8.260	12.390	16.519	20.649	24.779
30	34.522	4.118	8.236	12.354	16.473	20.591	24.709
40	46.029	4.106	8.213	12.319	16.426	20.532	24.638
50	57.536	4.095	8.189	12.284	16.379	20.473	24.568
45 00	69.043	4.083	8.166	12.249	16.332	20.415	24.498

ORDINATES OF DEVELOPED PARALLEL. *dp* Longitude Interval.	40°	41°
	inches.	inches.
5′	0.002	0.002
10	.008	.008
15	.019	.019
20	.033	.033
25	.052	.052
30	.074	.075
	42°	43°
5	0.002	0.002
10	.008	.008
15	.019	.019
20	.033	.033
25	.052	.052
30	.075	.075
	44°	45°
5	0.002	0.002
10	.008	.008
15	.019	.019
20	.034	.034
25	.052	.053
30	.075	.076

TABLE XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even-degree Parallels. *dl*	ABSCISSAS OF DEVELOPED PARALLEL. *dm* 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
45°00′	69.043	4.083	8.166	12.249	16.332	20.415	24.498
10	11.509	4.071	8.142	12.213	16.284	20.355	24.426
20	23.018	4.059	8.118	12.177	16.236	20.295	24.354
30	34.528	4.047	8.094	12.141	16.188	20.236	24.283
40	46.037	4.035	8.070	12.105	16.141	20.176	24.211
50	57.546	4.023	8.046	12.070	16.093	20.116	24.139
46 00	69.055	4.011	8.023	12.034	16.045	20.056	24.068
10	11.511	3.999	7.998	11.997	15.997	19.996	23.995
20	23.023	3.987	7.974	11.961	15.948	19.935	23.922
30	34.534	3.975	7.950	11.925	15.899	19.974	23.849
40	46.045	3.963	7.925	11.888	15.851	19.813	23.776
50	57.557	3.951	7.901	11.852	15.802	19.753	23.703
47 00	69.068	3.938	7.877	11.815	15.754	19.692	23.630
10	11.513	3.926	7.852	11.778	15.704	19.630	23.556
20	23.027	3.914	7.827	11.741	15.655	19.569	23.482
30	34.540	3.901	7.803	11.704	15.606	19.507	23.408
40	46.053	3.889	7.778	11.667	15.556	19.445	23.334
50	57.567	3.877	7.753	11.630	15.507	19.383	23.260
48 00	69.080	3.864	7.729	11.593	15.457	19.322	23.186
10	11.516	3.852	7.704	11.555	15.407	19.259	23.111
20	23.031	3.839	7.679	11.518	15.357	19.196	23.035
30	34.546	3.827	7.653	11.480	15.307	19.134	22.960
40	46.062	3.814	7.628	11.442	15.257	19.071	22.885
50	57.577	3.802	7.603	11.405	15.206	19.008	22.810
49 00	69.093	3.789	7.578	11.367	15.156	18.945	22.734
10	11.517	3.776	7.553	11.329	15.105	18.882	22.658
20	23.035	3.764	7.527	11.291	15.054	18.818	22.581
30	34.552	3.751	7.502	11.253	15.003	18.754	22.505
40	46.070	3.738	7.476	11.214	14.952	18.690	22.429
50	57.587	3.725	7.451	11.176	14.901	18.627	22.352
50 00	69.105	3.713	7.425	11.138	14.850	18.563	22.276

ORDINATES OF DEVELOPED PARALLEL. *dp* Longitude Interval.	45°	46°
	inches.	inches.
5′	0.002	0.002
10	.008	.008
15	.019	.019
20	.034	.034
25	.053	.053
30	.076	.076
	47°	**48°**
5	0.002	0.002
10	.008	.008
15	.019	.019
20	.034	.033
25	.052	.052
30	.075	.075
	49°	**50°**
5	0.002	0.002
10	.008	.008
15	.019	.019
20	.033	.033
25	.052	.052
30	.075	.075

TABLE XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even-degree Parallels. dl	ABSCISSAS OF DEVELOPED PARALLEL. dm 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
50°00′	69.105	3.713	7.425	11.138	14.850	18.563	22.276
10	11.520	3.700	7.399	11.099	14.799	18.499	22.198
20	23.039	3.687	7.374	11.060	14.747	18.734	22.121
30	34.558	3.674	7.348	11.021	14 695	18.369	22.043
40	46.078	3.661	7.322	10.983	14.644	18.305	21.965
50	57.598	3.648	7.296	10.944	14.592	18.240	21.888
51 00	69.117	3.635	7.270	10.905	14.540	18.176	21.811
10	11.521	3.622	7.244	10.866	14.488	18.110	21.732
20	23.043	3.609	7.218	10.827	14.436	18.045	21.653
30	34.564	3.596	7.191	10 787	14.383	17.979	21.574
40	46.086	3.583	7.165	10.748	14.330	17.913	21.496
50	57.607	3.570	7.139	10.709	14.278	17.848	21.417
52 00	69.128	3.556	7.113	10.669	14.226	17.782	21.338
10	11.523	3.543	7.086	10.629	14.172	17.716	21.259
20	23.047	3.530	7.060	10.589	14.119	17.649	21.179
30	34.570	3.516	7.033	10.550	14.066	17.583	21.099
40	46.094	3.503	7.006	10.510	14.013	17.516	21.019
50	57.617	3 490	6.980	10.470	13.960	17.450	20.939
53 00	69.140	3 477	6.953	10.430	13.906	17.383	20.860
10	11.525	3.463	6.926	10.389	13.852	17.316	20.779
20	23.051	3.450	6.899	10.349	13.798	17.248	20.698
30	34.576	3.436	6.872	10.309	13.745	17.181	20.617
40	46.102	3.423	6.845	10.268	13.691	17.114	20.536
50	67.627	3.409	6.818	10.228	13.637	17.046	20.455
54 00	69.152	3.396	6.791	10.187	13.583	16.979	20.374
10	11.527	3.382	6.764	10.146	13.528	16.910	20.292
20	23.055	3.368	6.737	10.105	13.474	16.842	20.210
30	34.582	3.355	6 709	10.064	13.419	16.774	20 128
40	46.109	3.341	6.682	10.023	13.364	16.706	20.047
50	57.636	3 327	6.655	9.982	13.310	16.637	19.964
55 00	69.164	3.314	6.628	9.941	13.255	16.569	19.883

ORDINATES OF DEVELOPED PARALLEL. dp

Longitude Interval.	50°	51°
	inches.	inches.
5′	0.002	0.002
10	.008	.008
15	.019	.019
20	.033	.033
25	.052	.051
30	.075	.074

Longitude Interval.	52°	53°
5	0.002	0.002
10	.008	.008
15	.018	.018
20	.033	.032
25	.051	.050
30	.073	.073

Longitude Interval.	54°	55°
5	0.002	0.002
10	.008	.008
15	.018	.018
20	.032	.032
25	.050	.049
30	.072	.071

Table XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even-degree Parallels. dl	Abscissas of Developed Parallel. dm 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
55°00′	69.164	3.314	6.628	9.941	13.255	16.569	19.883
10	11.529	3.300	6.600	9.900	13.200	16.500	19.800
20	23.059	3.286	6.572	9.859	13.145	16.431	19.717
30	34.588	3.272	6.545	9.817	13.089	16.362	19.634
40	46.117	3.258	6.517	9 776	13.034	16.293	19.551
50	57.646	3.245	6.489	9.734	12.979	16.224	19.468
56 00	69.176	3.231	6.462	9.693	12.924	16.155	19.385
10	11.531	3.217	6.434	9.651	12 868	16.085	19.301
20	23.063	3.203	6.406	9.609	12.812	16.015	19.217
30	34.594	3.189	6.378	9.567	12.756	15.945	19.134
40	46.125	3.175	6.350	9.525	12.700	15.875	19.050
50	57.656	3.161	6.322	9.483	12.644	15.805	18.966
57 00	69.188	3.147	6.294	9.441	12.588	15.735	18.882
10	11.533	3.133	6.266	9.398	12.531	15.664	18.797
20	23.066	3.119	6.237	9.356	12.475	15.594	18.712
30	34.599	3.104	6.209	9.314	12.418	15.523	18.627
40	46.132	3.090	6.181	9.271	12 362	15.452	18.542
50	57.666	3 076	6.152	9.229	12.305	15.381	18.457
58 00	69.199	3.062	6.124	9.186	12.248	15.311	18.373
10	11.535	3.048	6.096	9 143	12.191	15.239	18.287
20	23.074	3.034	6.067	9.101	12.134	15.168	18.201
30	34.605	3.019	6.038	9.058	12.077	15.096	18.115
40	46.140	3.005	6.010	9.015	12 020	15.025	18.029
50	57.675	2.991	5.981	8.972	11.962	14.953	17.944
59 00	69.210	2.976	5.953	8.929	11.905	14.882	17.858
10	11.537	2.962	5.924	8.885	11.847	14.809	17.771
20	23.074	2.947	5.895	8.842	11.790	14.737	17.684
30	34.610	2.933	5.866	8.799	11.732	14.665	17.597
40	46.147	2 918	5.837	8.755	11.674	14.562	17.510
50	57.684	2.904	5.808	8.712	11.616	14.520	17.424
60 00	69.221	2.890	5.779	8.669	11.558	14 448	17.337

Ordinates of Developed Parallel. dp

Longitude Interval.	55°	56°
	inches.	inches.
5′	0.002	0.002
10	.008	.008
15	.018	.018
20	.032	.031
25	.049	.049
30	.071	.070

Longitude Interval.	57°	58°
5	0.002	0.002
10	.008	.008
15	.017	.017
20	.031	.030
25	.048	.047
30	.069	.068

Longitude Interval.	59°	60°
5	0.002	0.002
10	.007	.007
15	.017	.016
20	.030	.029
25	.046	.045
30	.067	.065

TABLE XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even-degree Parallels. dl	ABSCISSAS OF DEVELOPED PARALLEL. dm 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
60°00′	69.221	2.890	5.779	8.669	11.558	14.448	17.337
10	11.539	2.875	5.750	8.625	11.500	14.375	17.249
20	23.077	2.860	5.721	8.581	11.441	14.302	17.162
30	34.616	2.846	5.691	8.537	11.383	14.229	17.074
40	46.154	2.831	5.662	8.493	11.324	14.156	16.987
50	57.693	2.816	5.633	8.450	11.266	14.083	16.899
61 00	69.232	2.802	5.604	8.406	11.208	14.010	16.811
10	11.540	2.787	5.574	8.361	11.148	13.936	16.723
20	23.081	2.772	5.545	8.317	11.090	13.863	16.634
30	34.621	2.758	5.115	8.273	11.030	13.788	16.546
40	46.162	2.743	5.486	8.229	10.972	13.715	16.457
50	57.702	2.728	5.456	8.184	10.912	13.641	16.369
62 00	69.242	2.713	5.427	8.140	10.854	13.567	16.280
10	11.542	2.699	5.397	8.096	10.794	13.493	16.191
20	23.084	2.684	5.367	8.051	10.734	13.418	16.102
30	34.626	2.669	5.337	8.006	10.675	13.344	16.012
40	46.168	2.654	5.308	7.961	10.615	13.269	15.923
50	57.710	2.639	5.278	7.917	10.556	13.195	15.833
63 00	69.253	2.624	5.248	7.872	10.496	13.120	15.744
10	11.544	2.609	5.218	7.827	10.436	13.045	15.654
20	23.087	2.594	5.188	7.782	10.376	12.970	15.564
30	34.631	2.579	5.158	7.737	10.316	12.895	15.473
40	46.175	2.564	5.128	7.692	10.256	12.820	15.383
50	57.718	2.549	5.098	7.647	10.196	12.745	15 293
64 00	69.262	2.534	5.068	7.602	10.136	12.670	15.203
10	11.545	2.519	5.037	7.556	10.075	12.594	15.112
20	23.091	2.504	5.007	7.511	10.014	12.518	15.022
30	34.636	2.488	4.977	7.465	9.954	12.452	14.930
40	46.182	2.473	4.947	7.420	9.893	12.367	14.840
50	57.727	2.458	4.916	7.374	9.832	12.291	14.749
65 00	69.272	2.443	4.886	7.329	9.772	12.215	14.658

ORDINATES OF DEVELOPED PARALLEL. dp

Longitude Interval.	60°	61°
	inches.	inches.
5′	0.002	0.002
10	.007	.007
15	.016	.016
20	.029	.029
25	.045	.045
30	.065	.064

Longitude Interval.	62°	63°
5	0.002	0.002
10	.007	.007
15	.016	.015
20	.028	.027
25	.044	.043
30	.063	.061

Longitude Interval.	64°	65°
5	0.002	0.002
10	.007	.006
15	.015	.014
20	.026	.026
25	.041	.040
30	.060	.058

Table XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even-degree Parallels. dl	Abscissas of Developed Parallel. dm 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
65°00	69.272	2.443	4.886	7.329	9.772	12.215	14.658
10	11.547	2.428	4.855	7.283	9.711	12.139	14.566
20	23.094	2.412	4.825	7.237	9.650	12.062	14.474
30	34.641	2.397	4.794	7.191	9.588	11.986	14.383
40	46.188	2.382	4.764	7.145	9.527	11.909	14.291
50	57.735	2.366	4.733	7.100	9.466	11.833	14.199
66 00	69.282	2.351	4.702	7.054	9.405	11.756	14.107
10	11.548	2.336	4.672	7.007	9.343	11.679	14.015
20	23.097	2.320	4.641	6.961	9.282	11.602	13.922
30	34.646	2.305	4.610	6.915	9.220	11.525	13.830
40	46.194	2.290	4.579	6.869	9.158	11.448	13.738
50	57.742	2.274	4.548	6.823	9.097	11.371	13.645
67 00	69.291	2.259	4.518	6.776	9.035	11.294	13.553
10	11.550	2.243	4.487	6.730	8.973	11.217	13.460
20	23.100	2.228	4.455	6.683	8.911	11.139	13.366
30	34.650	2.212	4.424	6.637	8.849	11.061	13.273
40	46.200	2.197	4.393	6.590	8.787	10.984	13.180
50	57.750	2.181	4.362	6.543	8.724	10.906	13.087
68 00	69.300	2.166	4.331	6.497	8.662	10.828	12.994
10	11.552	2.150	4.300	6.450	8.600	10.750	12.900
20	23.103	2.134	4.269	6.403	8.538	10.672	12.806
30	34.654	2.119	4.237	6.356	8.475	10.594	12.712
40	46.206	2.103	4.206	6.309	8.412	10.516	12.619
50	57.758	2.088	4.175	6.263	8.350	10.438	12.525
69 00	69.309	2.072	4.144	6.216	8.288	10.360	12.431
10	11.553	2.056	4.112	6.169	8.225	10.281	12.337
20	23.106	2.040	4.081	6.121	8.162	10.202	12.242
30	34.659	2.025	4.049	6.074	8.099	10.124	12.148
40	46.212	2.009	4.018	6.027	8.036	10.045	12.054
50	57.764	1.993	3.986	5.980	7.973	9.966	11.959
70 00	69.317	1.977	3.955	5.932	7.910	9.888	11.865

Ordinates of Developed Parallel. dp

Longitude Interval.	65°	66°
	inches.	inches
5′	0.002	0.002
10	.006	.006
15	.014	.014
20	.026	.025
25	.040	.039
30	.058	.056
	67°	**68°**
5	0.001	0.001
10	.006	.006
15	.014	.013
20	.024	.023
25	.038	.036
30	.054	.053
	69°	**70°**
5	0.001	0.001
10	.006	.005
15	.013	.012
20	.022	.022
25	.035	.034
30	.051	.049

TABLE XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even-degree Parallels. dl	ABSCISSAS OF DEVELOPED PARALLEL. dm 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches	inches.	inches	inches.	inches.	inches.
70°00′	69.317	1.977	3.955	5.932	7.910	9.888	11.865
10	11.554	1.962	3.923	5.885	7.846	9.808	11.770
20	23.109	1.946	3.892	5.837	7.783	9.729	11.675
30	34.663	1.930	3.860	5.790	7.720	9.650	11.579
40	46.217	1.914	3.828	5.742	7.656	9.571	11.485
50	57.772	1.898	3.796	5.695	7.593	9.491	11.389
71 00	69.326	1.882	3.765	5.647	7.530	9.412	11.294
10	11.556	1.866	3.733	5.600	7.466	9.333	11.199
20	23.111	1.850	3.701	5.552	7.402	9.253	11.103
30	34.667	1.835	3.669	5.504	7.338	9.173	11.008
40	46.222	1.819	3.637	5.456	7.275	9.094	10.912
50	57.778	1.803	3.605	5.408	7.211	9.014	10.816
72 00	69.334	1.787	3.574	5.360	7.147	8.934	10.721
10	11.557	1.771	3.542	5.312	7.083	8.854	10.625
20	23.114	1.755	3.509	5.264	7.019	8.774	10.528
30	34.670	1.739	3.477	5.216	6.955	8.694	10.432
40	46.227	1.723	3.445	5.168	6.891	8.614	10.336
50	57.784	1.707	3.413	5.120	6.826	8.533	10.240
73 00	69.341	1.691	3.381	5.072	6.762	8.453	10.144
10	11.558	1.674	3.349	5.024	6.698	8.373	10.047
20	23.116	1.658	3.317	4.975	6.634	8.292	9.950
30	34.674	1.642	3.284	4.927	6.569	8.211	9.853
40	46.232	1.626	3.252	4.878	6.504	8.131	9.757
50	57.790	1.610	3.220	4.830	6.440	8.050	9.660
74 00	69.348	1.594	3.188	4.782	6.376	7.970	9.563
10	11.559	1.578	3.155	4.733	6.311	7.889	9.466
20	23.118	1.562	3.123	4.685	6.246	7.808	9.369
30	34.677	1.545	3.091	4.636	6.181	7.727	9.272
40	46.236	1.529	3.058	4.587	6.116	7.645	9.175
50	57.796	1.513	3.026	4.539	6.052	7.565	9.077
75 00	69.355	1.497	2.993	4.490	5.987	7.484	8.980

ORDINATES OF DEVELOPED PARALLEL. dp

Longitude Interval.	70°	71°
	inches	inches.
5′	0.001	0.001
10	.005	.005
15	.012	.012
20	.022	.021
25	.034	.032
30	.049	.047
	72°	**73°**
5	0.001	0.001
10	.005	.005
15	.011	.011
20	.020	.019
25	.031	.029
30	.044	.042
	74°	**75°**
5	0.001	0.001
10	.004	.004
15	.010	.009
20	.018	.017
25	.028	.026
30	.040	.038

TABLE XXIII.

COORDINATES FOR PROJECTION OF MAPS.

Scale $\frac{1}{63360}$, or one inch to one mile.

Latitude of Parallel.	Meridional Distances from Even-degree Parallels. *dl*	Abscissas of Developed Parallel. *dm* 5′ long.	10′ long.	15′ long.	20′ long.	25′ long.	30′ long.
	inches.	inches.	inches.	inches.	inches.	inches.	inches.
75°00′	69.355	1.497	2.993	4.490	5.987	7.484	8.980
10	11.560	1.480	2.961	4.441	5.922	7.402	8.882
20	23.120	1.464	2.928	4.392	5.856	7.321	8.785
30	34.681	1.448	2.896	4.344	5.792	7.240	8.687
40	46.241	1.432	2.863	4.295	5.726	7.158	8.590
50	57.801	1.415	2.831	4.246	5.661	7.077	8.492
76 00	69.361	1.399	2.798	4.197	5.596	6.995	8.394
10	11.561	1.383	2.765	4.148	5.530	6.913	8.296
20	23.122	1.366	2.733	4.099	5.465	6.832	8.198
30	34.683	1.350	2.700	4.050	5.400	6.750	8.099
40	46.244	1.334	2.667	4.001	5.334	6.668	8.002
50	57.806	1.317	2.634	3.952	5.269	6.586	7.903
77 00	69.367	1.301	2.602	3.903	5.204	6.505	7.805
10	11.562	1.284	2.569	3.854	5.138	6.423	7.707
20	23.124	1.268	2.536	3.804	5.072	6.341	7.609
30	34.686	1.252	2.503	3.755	5.006	6.258	7.510
40	46.248	1.235	2.470	3.706	4.941	6.176	7.411
50	57.810	1.219	2.438	3.656	4.875	6.094	7.313
78 00	69.373	1.202	2.405	3.607	4.810	6.012	7.214
10	11.563	1.186	2.372	3.558	4.744	5.930	7.115
20	23.126	1.169	2.339	3.508	4.678	5.847	7.016
30	34.689	1.153	2.306	3.459	4.612	5.765	6.918
40	46.252	1.136	2.273	3.410	4.546	5.683	6.819
50	57.814	1.120	2.240	3.360	4.480	5.600	6.720
79 00	69.377	1.104	2.207	3.311	4.414	5.518	6.621
10	11.564	1.087	2.174	3.261	4.348	5.435	6.522
20	23.127	1.070	2.141	3.211	4.282	5.352	6.422
30	34.691	1.054	2.108	3.162	4.216	5.270	6.323
40	46.255	1.037	2.075	3.112	4.150	5.187	6.224
50	57.818	1.021	2.042	3.062	4.083	5.104	6.125
80 00	69.382	1.004	2.009	3.013	4.017	5.022	6.026

ORDINATES OF DEVELOPED PARALLEL. *dp*

Longitude Interval.	75°	76°
	inches.	inches.
5′	0.001	0.001
10	.004	.004
15	.009	.009
20	.017	.016
25	.026	.025
30	.038	.036

Longitude Interval.	77°	78°
5	0.001	0.001
10	.004	.003
15	.008	.008
20	.015	.014
25	.023	.021
30	.033	.031

Longitude Interval.	79°	80°
5	0.001	0.001
10	.003	.003
15	.007	.006
20	.013	.011
25	.020	.018
30	.028	.026

186. Use of Projection Tables.—Where it is proposed to project a map on a scale which bears a decimal ratio in inches to linear miles, the quantities to be laid off can be derived directly from Table XXIII. This table is arranged on the scale of one mile to one inch, and the quantities to be laid off for meridians or parallels are given in inches. For any other scale, as that of two miles to one inch, and, for example, for a 30′ projection between latitudes 31° and 31° 30′, and say in longitude 98° to 98° 30′ (Fig. 134), the quantities to be laid off on the projection are to be obtained in inches from the table for every 5′ by halving the amounts in the table. Quantities required for projections ruled at shorter intervals than 5′ may be obtained by moving the decimal point. Thus for parallels 3′ apart the quantity corresponding to differences in latitude of 30′ is sought and the decimal point moved one place to the left, etc.

Where it is desired to make a projection on any other scale than that bearing an even decimal relation of inches to miles, projection tables, XXIV, XXV, and XXVI, should be used. The first of these, Table XXIV, gives the exact lengths of degrees of parallels and meridians in meters and in statute miles, and these may be reduced to inches or other scale. Tables XXV and XXVI may be used in projecting large-scale maps, approximately within the limits of the United States, between latitudes 24° and 51° north. The first of each pair of columns in Table XXV gives the latitude, and opposite to it the corresponding length of one minute of parallel in meters. These may be reduced to any map scale by consultation of reduction tables (Chap. XXX.) Corresponding values less than one minute may be obtained by moving the decimal point one place, which will give the value for six seconds. Thus, in Table XXV, for latitude 28° the length of one minute of parallel is 1639.4 meters. The length of six seconds of the same parallel is obtained by moving the decimal point one place to the left, 163.94 meters.

For the lengths of meridional arcs the quantities *dm* are obtained for a given latitude from Table XXVI in the following manner: For the latitude and for the number of degrees of longitude included in the projection, the length of *dm* as given in meters, which is to be found in the first column, is to be laid off both to right and left of the vertical central meridian. At each of the points thus found perpendiculars are to be erected which will be parallel to the central meridian, and the lengths of the corresponding ordinates *dp* are to be laid off upon them. Through the extremities of each of these perpendiculars draw lines which will give the confining outlines of the curves of the parallels and meridians. Spaces between the extremities *dp* may now be divided into convenient equal parts of the same value, 5′ or 15′, etc., as was given the spaces between the meridians. Curved lines drawn between these will represent the parallels of the completed projection according to the number of equal parts used.

187. Areas of Quadrilaterals of Earth's Surface.—It is sometimes desirable to determine the areas of quadrilaterals of the earth's surface, and these may be found directly from Table XXVII. Areas of quadrilaterals of less or greater extent than one degree may be found by simple division or multiplication.

188. Platting Triangulation Stations on Projection.—The projection of the map being now constructed, it is necessary to plat upon it the exact positions of the triangulation stations. These must, of course, have been previously computed, so that their *geodetic coordinates* (Chap. XXIX) are exactly known. These coordinates are given in degrees, minutes, and seconds of arc. Assume that the projection has been so platted that meridian and parallel lines are shown for every ten minutes; then the nearest degrees and ten minutes of latitude and longitude of each position are taken out and the corresponding rectangle found in which the point will fall. The odd minutes and seconds, those greater than ten min-

TABLE XXIV.—FOR PROJECTION OF MAPS OF LARGE AREAS.

(Extracted from Appendix No. 6, U. S. Coast and Geodetic Survey Report for 1884, by Henry Gannett.)

LENGTHS OF DEGREES OF THE MERIDIAN.

Lat.	Meters.*	Statute Miles.*	Lat.	Meters.	Statute Miles.	Lat.	Meters.	Statute Miles.
0°	110,567.2	68.704	30°	110,848.5	68.879	60°	111,414.5	69.230
1	110,567.0	68.704	31	110,865.7	68.890	61	111,431.5	69.241
2	110,568.6	68.705	32	110,883.2	68.901	62	111,448.2	69.251
3	110,570.3	68.706	33	110,901.1	68.912	63	111,464.4	69.261
4	110,572.7	68.70	34	110,919.2	68.923	64	111,480.3	69.271
5	110,572.8	68.710	35	110,937.6	68.935	65	111,495.7	69.281
6	110,579.5	68.712	36	110,956.2	68.946	66	111,510.7	69.290
7	110,583.9	68.715	37	110,975.1	68.958	67	111,525.3	69.299
8	110,589.0	68.718	38	110,994.1	68.969	68	111,539.3	69.309
9	110,594.7	68.721	39	111,013.3	68.981	69	111,552.9	69.316
10	110,601.1	68.725	40	111,032.7	68.993	70	111,565.9	69.324
11	110,608.1	68.730	41	111,052.2	69.006	71	111,578.4	69.332
12	110,615.8	68.734	42	111,071.7	69.018	72	111,590.4	69.340
13	110,624.1	68.739	43	111,091.4	69.030	73	111,601.8	69.347
14	110,633.0	68.744	44	111,111.1	69.042	74	111,612.7	69.354
15	110,642.5	68.751	45	111,130.9	69.054	75	111,622.9	69.360
16	110,652.6	68.757	46	111,150.6	69.066	76	111,632.6	69.366
17	110,663.3	68.764	47	111,170.4	69.079	77	111,641.6	69.372
18	110,674.5	68.771	48	111,190.1	69.091	78	111,650.0	69.377
19	110,686.3	68.778	49	111,209.7	69.103	79	111,657.8	69.382
20	110,698.7	68.786	50	111,229.3	69.115	80	111,664.9	69.386
21	110,711.6	68.794	51	111,248.7	69.127	81	111,671.4	69.390
22	110,725.0	68.802	52	111,268.0	69.139	82	111,677.2	69.394
23	110,738.8	68.811	53	111,287.1	69.151	83	111,682.4	69.397
24	110,753.2	68.820	54	111,306.0	69.163	84	111,686.9	69.400
25	110,768.0	68.829	55	111,324.8	69.175	85	111,690.7	69.402
26	110,783.3	68.839	56	111,343.3	69.186	86	111,693.8	69.404
27	110,799.0	68.848	57	111,361.5	69.197	87	111,696.2	69.405
28	110,815.1	68.858	58	111,379.5	69.209	88	111,697.9	69.407
29	110,831.6	68.869	59	111,397.2	69.220	89	111,699.0	69.407
30	110,848.5	68.879	60	111,414.5	69.230	90	111,699.3	69.407

LENGTHS OF DEGREES OF THE PARALLEL.

Lat.	Meters.	Statute Miles.	Lat.	Meters.	Statute Miles.	Lat.	Meters.	Statute Miles.
0°	111,321	69.172	30°	96,488	59.956	60°	55,802	34.674
1	111,304	69.162	31	95,506	59.345	61	54,110	33.623
2	111,253	69.130	32	94,495	58.716	62	52,400	32.560
3	111,169	69.078	33	93,455	58.071	63	50,675	31.488
4	111,051	69.005	34	92,387	57.407	64	48,934	30.406
5	110,900	68.911	35	91,290	56.725	65	47,177	29.315
6	110,715	68.795	36	90,166	56.027	66	45,407	28.215
7	110,497	68.660	37	89,014	55.311	67	43,622	27.106
8	110,245	68.504	38	87,835	54.579	68	41,823	25.988
9	109,959	68.326	39	86,629	53.829	69	40,012	24.862
10	109,641	68.129	40	85,396	53.063	70	38,188	23.729
11	109,289	67.910	41	84,137	52.281	71	36,353	22.589
12	108,904	67.670	42	82,853	51.483	72	34,506	21.441
13	108,486	67.410	43	81,543	50.669	73	32,648	20.287
14	108,036	67.131	44	80,208	49.840	74	30,781	19.127
15	107,553	66.830	45	78,849	48.995	75	28,903	17.960
16	107,036	66.510	46	77,466	48.136	76	27,017	16.788
17	106,487	66.169	47	76,058	47.261	77	25,123	15.611
18	105,906	65.808	48	74,628	46.372	78	23,220	14.428
19	105,294	65.427	49	73,174	45.469	79	21,311	13.248
20	104,649	65.026	50	71,698	44.552	80	19,394	12.051
21	103,972	64.606	51	70,200	43.621	81	17,472	10.857
22	103,264	64.166	52	68,680	42.676	82	15,545	9.659
23	102,524	63.706	53	67,140	41.719	83	13,612	8.458
24	101,754	63.228	54	65,578	40.749	84	11,675	7.255
25	100,952	62.729	55	63,996	39.766	85	9,735	6.049
26	100,119	62.212	56	62,395	38.771	86	7,792	4.842
27	99,257	61.676	57	60,774	37.764	87	5,846	3.632
28	98,364	61.122	58	59,135	36.745	88	3,898	2.422
29	97,441	60.548	59	57,478	35.716	89	1,949	1.211
30	96,488	59.956	60	55,802	34.674	90	0	0.000

* These quantities express the number of meters and statute miles contained within an arc of which the degree of latitude named is the middle; thus, the quantity, 111,032.7, opposite latitude 40°, is the number of meters between latitude 39° 30′ and latitude 40° 30′.

TABLE XXV.

FOR PROJECTION OF MAPS OF LARGE AREAS.

(Extracted from Appendix No. 6, U. S. Coast and Geodetic Survey Report for 1884.)

ARCS OF THE PARALLEL IN METERS.

Latitude.	Value of 1'.	Latitude.	Value of 1'.	Latitude.	Value of 1'.	Latitude.	Value of 1'.
24° 00'	1695.9	31° 00'	1591.8	38° 00'	1463.9	45 00	1314.2
10	3.7	10	89.0	10	60.6	10	10.3
20	1.5	20	6.2	20	57.3	20	06.5
30	1689.3	30	3.4	30	53.9	30	02.7
40	7.0	40	0.6	40	50.6	40	1298.8
50	4.8	50	77.8	50	47.2	50	95.0
25 00	1682.5	32 00	1574.9	39 00	1443.8	46 00	1291.0
10	80.3	10	72.1	10	40.4	10	87.2
20	1678.0	20	69.2	20	37.0	20	83.3
30	5.7	30	6.3	30	33.6	30	79.4
40	3.3	40	3.4	40	30.2	40	75.5
50	1.0	50	0.5	50	26.7	50	71.6
26 00	1668.7	33 00	1557.6	40 00	1423.3	47 00	1267.6
10	6.3	10	4.7	10	19.8	10	63.7
20	3.9	20	1.7	20	16.3	20	59.7
30	1.5	30	48.7	30	12.8	30	55.8
40	1659.1	40	5.8	40	09.3	40	51.8
50	6.7	50	2.8	50	05.8	50	47.8
27 00	1654.3	34 00	1539.8	41 00	1402.3	48 00	1243.8
10	51.8	10	6.8	10	1398.8	10	39.8
20	1649.4	20	3.7	20	95.2	20	35.8
30	6.9	30	0.7	30	91.6	30	31.7
40	4.4	40	27.6	40	88.1	40	27.7
50	1.9	50	4.6	50	84.5	50	23.6
28 00	1639.4	35 00	1521.5	42 00	1380.9	49 00	1219.6
10	6.9	10	18.4	10	77.3	10	15.5
20	4.3	20	15.3	20	73.7	20	11.4
30	1.8	30	12.2	30	70.0	30	07.3
40	29.2	40	09.1	40	66.4	40	03.2
50	6.6	50	05.9	50	92.7	50	1199.1
29 00	1624.0	36 00	1502.8	43 00	1359.1	50 00	1195.0
10	21.4	10	1499.6	10	55.4	10	90.8
20	18.8	20	6.4	20	51.7	20	86.7
30	6.1	30	3.2	30	48.0	30	82.5
40	3.5	40	0.0	40	44.3	40	78.4
50	0.8	50	86.8	50	40.5	50	74.2
30 00	1608.1	37 00	1483.6	44 00	1336.8		
10	5.4	10	80.3	10	33.1		
20	2.7	20	77.1	20	29.3		
30	0.0	30	73.8	30	25.5		
40	1597.3	40	70.5	40	21.7		
50	4.5	50	67.2	50	18.0		

TABLE XXVI.—FOR PROJECTIONS OF MAPS OF LARGE AREAS.

(Extracted from Appendix No. 6, U. S. Coast and Geodetic Survey Report for 1884, by Henry Gannett.)

MERIDIONAL ARCS. COORDINATES OF CURVATURE.

NATURAL SCALE.—VALUES OF *dm* AND *dp* IN METERS.

Latitude 24°.			Latitude 25°.			Latitude 26°.			Latitude 27°.			Latitude 28°.		
Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*
1° 00′	101,753	361	1° 00′	100,951	372	1° 00′	100,118	383	1° 00′	99,256	393	1° 00′	98,363	403
2 00	203,500	1,445	2 00	201,896	1,489	2 00	200,231	1,532	2 00	198,505	1,573	2 00	196,719	1,612
3 00	305,237	3,250	3 00	302,831	3,351	3 00	300,332	3,447	3 00	297,742	3,539	3 00	295,062	3,627
4 00	406,959	5,778	4 00	403,749	5,957	4 00	400,416	6,128	4 00	396,960	6,291	4 00	393,385	6,447
5 00	508,660	9,028	5 00	504,645	9,307	5 00	500,476	9,574	5 00	496,154	9,829	5 00	491,682	10,073
6 00	610,336	13,001	6 00	605,514	13,401	6 00	600,507	13,787	6 00	595,316	14,154	6 00	589,945	14,505
7 00	711,981	17,695	7 00	706,349	18,239	7 00	700,501	18,763	7 00	694,440	19,264	7 00	688,168	19,741
8 00	813,590	23,109	8 00	807,146	23,821	8 00	800,456	24,505	8 00	793,522	25,159	8 00	786,347	25,782
9 00	915,159	29,245	9 00	907,899	30,146	9 00	900,364	31,011	9 00	892,554	31,839	9 00	884,472	32,627
10 00	1,016,681	36,102	10 00	1,008,603	37,215	10 00	1,000,218	38,282	10 00	991,528	39,303	10 00	982,537	40,276
11 00	1,118,152	43,679	11 00	1,109,252	45,926	11 00	1,100,015	46,316	11 00	1,090,442	47,551	11 00	1,080,537	48,728
12 00	1,219,566	51,977	12 00	1,209,841	53,578	12 00	1,199,747	55,114	12 00	1,189,287	56,583	12 00	1,178,464	57,983
13 00	1,320,919	60,994	13 00	1,310,364	62,837	13 00	1,299,409	64,675	13 00	1,288,057	66,398	13 00	1,276,312	68,040
14 00	1,422,205	70,731	14 00	1,410,815	72,909	14 00	1,398,994	74,998	14 00	1,386,746	76,995	14 00	1,374,075	78,899
15 00	1,523,420	81,186	15 00	1,511,190	83,685	15 00	1,498,498	86,082	15 00	1,485,348	88,374	15 00	1,471,745	90,558
16 00	1,624,558	92,360	16 00	1,611,483	95,202	16 00	1,597,914	97,928	16 00	1,583,857	100,534	16 00	1,569,315	103,017
17 00	1,725,614	104,251	17 00	1,711,688	107,458	17 00	1,697,237	110,534	17 00	1,682,267	113,474	17 00	1,666,781	116,275
18 00	1,826,583	116,859	18 00	1,811,800	120,453	18 00	1,796,460	123,899	18 00	1,780,570	127,193	18 00	1,764,135	130,331
19 00	1,927,460	130,184	19 00	1,911,813	134,186	19 00	1,895,578	138,023	19 00	1,878,762	141,690	19 00	1,861,371	145,185
20 00	2,028,240	144,225	20 00	2,011,722	148,656	20 00	1,994,585	152,905	20 00	1,976,836	156,966	20 00	1,958,481	160,835
21 00	2,128,918	158,981	21 00	2,111,522	162,862	21 00	2,093,475	168,544	21 00	2,074,786	173,018	21 00	2,055,460	177,280
22 00	2,229,488	174,451	22 00	2,211,207	179,805	22 00	2,192,243	184,939	22 00	2,172,606	189,845	22 00	2,152,302	194,518
23 00	2,329,946	190,634	23 00	2,310,771	196,482	22 00	2,290,882	202,089	23 00	2,270,289	207,447	23 00	2,248,998	212,550
24 00	2,430,287	207,530	24 00	2,410,210	213,894	24 00	2,389,387	219,993	24 00	2,367,830	225,823	24 00	2,345,544	231,374
25 00	2,530,505	225,138	25 00	2,509,518	232,038	25 00	2,487,453	238,650	25 00	2,465,222	244,970	25 00	2,441,932	250,988
26 00	2,630,596	243,458	26 00	2,608,689	250,914	26 00	2,585,973	258,061	26 00	2,562,459	264,889	26 00	2,538,156	271,391
27 00	2,730,554	262,487	27 00	2,707,718	270,521	27 00	2,684,042	278,222	27 00	2,659,535	285,577	27 00	2,634,210	292,582
28 00	2,830,374	282,225	28 00	2,806,600	290,859	28 00	2,781,953	299,132	28 00	2,756,445	307,035	28 00	2,730,087	314,559
29 00	2,930,052	302,671	29 00	2,905,329	311,025	29 00	2,879,702	320,788	29 00	2,853,181	329,259	29 00	2,825,779	337,321
30 00	3,029,582	323,825	30 00	3,003,900	333,718	30 00	2,977,281	343,197	30 00	2,949,739	352,249	30 00	2,921,284	360,866

TABLE XXVI.—FOR PROJECTIONS OF MAPS OF LARGE AREAS.

(Extracted from Appendix No 6, U. S. Coast and Geodetic Survey Report for 1884, by Henry Gannett.)

MERIDIONAL ARCS. COORDINATES OF CURVATURE.

NATURAL SCALE.—VALUES OF *dm* AND *dp* IN METERS.

Latitude 29°.			Latitude 30°.			Latitude 31°.			Latitude 32°.			Latitude 33°.		
Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*
1° 00′	97,439	412	1° 00′	96,487	421	1° 00′	95,505	429	1° 00′	94,494	437	1° 00′	93,454	444
2 00	194,872	1,649	2 00	192,967	1,684	2 00	191,002	1,717	2 00	188,980	1,748	2 00	186,899	1,777
3 00	292,291	3,710	3 00	289,432	3,789	3 00	286,484	3,863	3 00	283,449	3,933	3 00	280,328	3,997
4 00	389,689	6,595	4 00	385,875	6,735	4 00	381,943	6,867	4 00	377,894	6,991	4 00	373,731	7,106
5 00	487,059	10,305	5 00	482,288	10,523	5 00	477,371	10,729	5 00	472,307	10,922	5 00	467,100	11,102
6 00	584,394	14,838	6 00	578,665	15,153	6 00	572,760	15,450	6 00	566,680	15,727	6 00	560,428	15,986
7 00	681,687	20,194	7 00	674,098	20,623	7 00	668,103	21,027	7 00	661,004	21,404	7 00	653,704	21,757
8 00	778,931	26,374	8 00	771,279	26,934	8 00	763,392	27,461	8 00	755,272	27,954	8 00	746,922	28,414
9 00	876,120	33,376	9 00	867,502	34,084	9 00	858,619	34,751	9 00	849,475	35,375	9 00	840,072	35,957
10 00	973,246	41,199	10 00	963,658	42,074	10 00	953,777	42,897	10 00	943,605	43,667	10 00	1,933,146	44,385
11 00	1,070,302	49,845	11 00	1,059,741	50,503	11 00	1,048,858	51,898	11 00	1,037,655	52,829	11 00	1,026,036	53,697
12 00	1,167,282	59,313	12 00	1,155,744	60,570	12 00	1,143,854	61,753	12 00	1,131,616	62,861	12 00	1,119,033	63,893
13 00	1,264,178	69,601	13 00	1,251,658	71,074	13 00	1,238,758	72,462	13 00	1,225,480	73,761	13 00	1,211,829	74,971
14 00	1,360,983	80,706	14 00	1,347,477	82,415	14 00	1,333,561	84,024	14 00	1,319,239	85,529	14 00	1,304,515	86,931
15 00	1,457,691	92,631	15 00	1,443,193	94,591	15 00	1,428,257	96,437	15 00	1,412,885	98,164	15 00	1,397,083	99,771
16 00	1,554,295	105,375	16 00	1,538,800	107,603	16 00	1,522,837	109,701	16 00	1,506,411	111,664	16 00	1,489,526	113,491
17 00	1,650,787	118,935	17 00	1,634,290	121,449	17 00	1,617,294	123,815	17 00	1,599,808	126,029	17 00	1,581,834	128,089
18 00	1,747,161	133,311	18 00	1,729,654	136,127	18 00	1,711,621	138,777	18 00	1,693,067	141,256	18 00	1,673,998	143,564
19 00	1,843,410	148,502	19 00	1,824,887	151,637	19 00	1,805,810	154,586	19 00	1,786,182	157,346	19 00	1,766,011	159,914
20 00	1,939,527	164,506	20 00	1,919,982	167,977	20 00	1,899,852	171,241	20 00	1,879,144	174,296	20 00	1,857,866	177,138
21 00	2,035,505	181,324	21 00	2,014,930	185,147	21 00	1,993,740	188,741	21 00	1,971,946	192,105	21 00	1,949,553	195,234
22 00	2,131,338	198,953	22 00	2,109,725	203,143	22 00	2,087,468	207,085	22 00	2,064,579	210,772	22 00	2,041,062	214,201
23 00	2,227,020	217,392	23 00	2,204,359	221,966	23 00	2,181,027	226,270	23 00	2,157,035	230,295	23 00	2,132,387	234,037
24 00	2,322,539	236,640	24 00	2,298,825	241,616	24 00	2,274,411	246,295	24 00	2,249,305	250,672	24 00	2,223,521	254,740
25 00	2,417,893	256,695	25 00	2,393,116	262,089	25 00	2,367,610	267,159	25 00	2,341,385	271,901	25 00	2,314,453	276,309
26 00	2,513,074	277,558	26 00	2,487,224	283,383	26 00	2,460,618	288,860	26 00	2,433,264	293,981	26 00	2,405,175	298,741
27 00	2,608,075	299,224	27 00	2,581,144	305,498	27 00	2,553,427	311,396	27 00	2,524,935	316,910	27 00	2,495,680	322,034
28 00	2,702,890	321,694	28 00	2,674,867	328,432	28 00	2,646,029	334,765	28 00	2,616,390	340,686	28 00	2,585,961	346,187
29 00	2,796,511	344,964	29 00	2,768,385	352,183	29 00	2,738,418	358,966	29 00	2,707,621	365,307	29 00	2,676,007	371,197
30 00	2,891,931	369,036	30 00	2,861,694	376,749	30 00	2,830,585	383,997	30 00	2,798,621	390,770	30 00	2,765,812	397,061

TABLE XXVI.—FOR PROJECTIONS OF MAPS OF LARGE AREAS.

(Extracted from Appendix No. 6, U. S. Coast and Geodetic Survey Report for 1884, by Henry Gannett.)

MERIDIONAL ARCS. COORDINATES OF CURVATURE.

NATURAL SCALE.—VALUES OF *dm* AND *dp* IN METERS.

Latitude 39°.			Latitude 40°.			Latitude 41°.			Latitude 42°.			Latitude 43°.		
Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*
1° 00′	86,627	476	1° 00′	85,394	479	1° 00′	84,136	482	1° 00′	82,851	484	1° 00′	81,541	485
2 00	173,243	1,903	2 00	170,778	1,916	2 00	168,260	1,927	2 00	165,691	1,935	2 00	163,071	1,941
3 00	259,839	4,281	3 00	256,140	4,311	3 00	252,363	4,335	3 00	248,508	4,354	3 00	244,578	4,367
4 00	346,403	7,611	4 00	341,470	7,663	4 00	336,432	7,706	4 00	331,292	7,739	4 00	326,050	7,763
5 00	432,925	11,891	5 00	426,757	11,972	5 00	420,457	12,039	5 00	414,030	12,092	5 00	407,476	12,129
6 00	519,396	17,121	6 00	511,990	17,238	6 00	504,428	17,335	6 00	496,712	17,410	6 00	488,844	17,464
7 00	605,803	23,300	7 00	597,158	23,460	7 00	588,332	23,591	7 00	579,325	23,693	7 00	570,143	23,766
8 00	692,138	30,428	8 00	682,252	30,637	8 00	672,159	30,807	8 00	661,861	30,941	8 00	651,361	31,036
9 00	778,388	38,504	9 00	767,260	38,768	9 00	755,897	38,983	9 00	744,305	39,152	9 00	732,486	39,272
10 00	864,545	47,527	10 00	852,171	47,852	10 00	839,537	48,118	10 00	826,648	48,325	10 00	813,508	48,474
11 00	950,598	57,496	11 00	936,975	57,888	11 00	923,067	58,209	11 00	908,879	58,459	11 00	894,415	58,639
12 00	1,036,536	68,409	12 00	1,021,661	68,875	12 00	1,006,475	69,256	12 00	990,985	69,553	12 00	975,195	69,766
13 00	1,122,349	80,266	13 00	1,106,218	80,811	13 00	1,089,752	81,258	13 00	1,072,956	81,605	13 00	1,055,837	81,854
14 00	1,208,027	93,064	14 00	1,190,636	93,695	14 00	1,172,886	94,212	14 00	1,154,781	94,614	14 00	1,136,329	94,901
15 00	1,293,559	106,802	15 00	1,274,904	107,525	15 00	1,252,866	108,117	15 00	1,236,449	108,577	15 00	1,216,661	108,905
16 00	1,378,934	121,479	16 00	1,359,012	122,300	16 00	1,338,681	122,971	16 00	1,317,948	123,493	16 00	1,296,820	123,864
17 00	1,464,144	137,093	17 00	1,442,949	138,017	17 00	1,421,321	138,773	17 00	1,399,267	139,360	17 00	1,376,795	139,777
18 00	1,549,177	153,642	18 00	1,526,704	154,675	18 00	1,503,775	155,520	18 00	1,480,395	156,175	18 00	1,456,575	156,640
19 00	1,634,023	171,124	19 00	1,610,267	172,272	19 00	1,586,031	173,210	19 00	1,561,321	173,937	19 00	1,536,148	174,451
20 00	1,718,671	189,537	20 00	1,693,628	190,805	20 00	1,668,079	191,841	20 00	1,642,035	192,642	20 00	1,615,505	193,209
21 00	1,803,113	208,878	21 00	1,776,775	210,272	21 00	1,749,909	211,409	21 00	1,722,524	212,289	21 00	1,694,632	212,909
22 00	1,887,337	229,146	22 00	1,859,698	230,671	22 00	1,831,509	231,914	22 00	1,802,779	232,874	22 00	1,773,519	233,551
23 00	1,971,333	250,337	23 00	1,942,387	251,998	23 00	1,912,869	253,352	23 00	1,882,788	254,396	23 00	1,852,155	255,129
24 00	2,055,091	272,450	24 00	2,024,833	274,252	24 00	1,993,978	275,719	24 00	1,962,540	276,850	24 00	1,930,528	277,642
25 00	2,138,602	295,481	25 00	2,107,023	297,430	25 00	2,074,826	299,014	25 00	2,042,024	300,234	25 00	2,008,628	301,087
26 00	2,221,854	319,429	26 00	2,188,948	321,528	26 00	2,155,402	323,233	26 00	2,121,230	324,544	26 00	2,086,443	325,459
27 00	2,304,838	344,289	27 00	2,270,597	346,543	27 00	2,235,695	348,374	27 00	2,200,146	349,778	27 00	2,163,963	350,750
28 00	2,387,545	570,059	28 00	2,351,961	372,473	28 00	2,315,695	374,432	28 00	2,278,762	375,932	28 00	2,241,176	376,974
29 00	2,469,963	396,736	29 00	2,433,029	399,314	29 00	2,395,392	401,404	29 00	2,357,067	403,002	29 00	2,318,071	404,109
30 00	2,552,084	424,317	30 00	2,513,790	427,063	30 00	2,474,774	429,287	30 00	2,435,052	430,985	30 00	2,394,639	432,157

TABLE XXVI.—FOR PROJECTIONS OF MAPS OF LARGE AREAS.

(Extracted from Appendix No. 6, U. S. Coast and Geodetic Survey Report for 1884, by Henry Gannett.)

MERIDIONAL ARCS. COORDINATES OF CURVATURE.

NATURAL SCALE.—VALUES OF *dm* AND *dp* IN METERS.

Latitude 34°.			Latitude 35°.			Latitude 36°.			Latitude 37°.			Latitude 38°.		
Long.	*dm*.	*dp*.	Long.	*dm*.	*dp*.	Long.	*dm*.	*dp*.	Long.	*dm*.	*dp*.	Long.	*dm*.	*dp*.
1° 00′	92,385	451	1° 00′	91,289	457	1° 00′	90,164	462	1° 00′	89,012	467	1° 00′	87,833	472
2 00	184,762	1,803	2 00	182,568	1,828	2 00	180,319	1,850	2 00	178,015	1,870	2 00	175,656	1,888
3 00	277,121	4,057	3 00	273,830	4,112	3 00	270,455	4,162	3 00	266,997	4,207	3 00	263,458	4,247
4 00	369,454	7,212	4 00	365,064	7,310	4 00	360,562	7,399	4 00	355,951	7,479	4 00	351,230	7,549
5 00	461,751	11,268	5 00	456,261	11,421	5 00	450,631	11,560	5 00	444,865	11,685	5 00	438,962	11,795
6 00	554,004	16,225	6 00	547,412	16,445	6 00	540,653	16,645	6 00	533,730	16,824	6 00	526,643	16,983
7 00	646,205	22,082	7 00	638,509	22,381	7 00	630,618	22,652	7 00	622,536	22,896	7 00	614,263	23,112
8 00	738,344	28,839	8 00	729,542	29,229	8 00	720,517	29,583	8 00	711,273	29,901	8 00	701,812	30,183
9 00	830,413	36,494	9 00	820,501	36,987	9 00	810,340	37,435	9 00	799,932	37,838	9 00	789,280	38,195
10 00	922,403	45,048	10 00	911,379	45,656	10 00	900,078	46,209	10 00	888,503	46,706	10 00	876,657	47,145
11 00	1,014,305	54,499	11 00	1,002,165	55,234	11 00	989,720	55,903	11 00	976,975	56,503	11 00	963,933	57,034
12 00	1,106,110	64,846	12 00	1,092,850	65,721	12 00	1,079,259	66,515	12 00	1,065,340	67,229	12 00	1,051,098	67,860
13 00	1,197,809	76,089	13 00	1,183,426	77,115	13 00	1,168,684	78,046	13 00	1,153,587	78,882	13 00	1,138,141	79,622
14 00	1,289,395	88,227	14 00	1,273,884	89,415	14 00	1,257,987	90,494	14 00	1,241,707	91,462	14 00	1,225,052	92,319
15 00	1,380,858	101,258	15 00	1,364,214	102,619	15 00	1,347,156	103,856	15 00	1,329,690	104,967	15 00	1,311,823	105,949
16 00	1,472,190	115,180	16 00	1,454,407	116,728	16 00	1,436,184	118,133	16 00	1,417,526	119,395	16 00	1,398,441	120,511
17 00	1,563,381	129,993	17 00	1,544,454	131,738	17 00	1,525,061	133,323	17 00	1,505,206	134,745	17 00	1,484,899	136,002
18 00	1,654,423	145,696	18 00	1,634,347	147,650	18 00	1,613,777	149,423	18 00	1,592,721	151,015	18 00	1,571,185	152,421
19 00	1,745,308	162,287	19 00	1,724,076	164,460	19 00	1,702,324	166,433	19 00	1,680,059	168,203	19 00	1,657,289	169,767
20 00	1,836,026	179,763	20 00	1,813,632	182,168	20 00	1,790,691	184,350	20 00	1,767,211	186,307	20 00	1,743,202	188,037
21 00	1,926,569	198,124	21 00	1,903,006	200,772	21 00	1,878,870	203,173	21 00	1,854,169	205,326	21 00	1,828,914	207,229
22 00	2,016,929	217,368	22 00	1,992,190	220,268	22 00	1,966,851	222,899	22 00	1,940,922	225,258	22 00	1,914,415	227,341
23 00	2,107,097	237,493	23 00	2,081,174	240,657	23 00	2,054,625	243,527	23 00	2,027,462	246,099	23 00	1,999,694	248,370
24 00	2,197,065	258,497	24 00	2,169,949	261,936	24 00	2,142,183	265,055	24 00	2,113,777	267,849	24 00	2,084,743	270,315
25 00	2,286,823	280,378	25 00	2,258,507	284,102	25 00	2,229,516	287,479	25 00	2,199,860	290,503	25 00	2,169,551	293,172
26 00	2,376,363	303,134	26 00	2,346,838	307,154	26 00	2,316,613	310,798	26 00	2,285,699	314,061	26 00	2,254,109	316,939
27 00	2,465,677	326,763	27 00	2,434,934	331,089	27 00	2,403,467	335,009	27 00	2,371,287	338,519	27 00	2,338,406	341,613
28 00	2,554,756	351,262	28 00	2,522,787	355,905	28 00	2,490,068	360,111	28 00	2,456,612	363,874	28 00	2,422,433	467,192
29 00	2,643,591	376,629	29 00	2,610,386	381,598	29 00	2,576,407	386,099	29 00	2,541,667	390,125	29 00	2,506,181	393,662
30 00	2,732,175	402,863	30 00	2,697,724	408,158	30 00	2,662,475	412,971	30 00	2,626,441	417,267	30 00	2,589,639	421,050

TABLE XXVI.—FOR PROJECTIONS OF MAPS OF LARGE AREAS.

(Extracted from Appendix No. 6, U. S. Coast and Geodetic Survey Report for 1884, by Henry Gannett.)

MERIDIONAL ARCS. COORDINATES OF CURVATURE.

NATURAL SCALE.—VALUES OF *dm* AND *dp* IN METERS.

Latitude 44°.			Latitude 45°.			Latitude 46°.			Latitude 47°.			Latitude 48°.		
Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*
1° 00′	80,206	486	1° 00′	78,847	486	1° 00′	77,464	486	1° 00′	76,057	485	1° 00′	74,626	484
2 00	160,401	1,945	2 00	157,682	1,946	2 00	154,915	1,945	2 00	152,100	1,942	2 00	149,239	1,936
3 00	240,572	4.375	3 00	236,493	4,378	3 00	232,342	4,342	3 00	228,119	4,368	3 00	223,827	4.355
4 00	320,708	7.778	4 00	315,269	7,783	4 00	309,732	7,779	4 00	304,101	7,765	4 00	298,377	7,742
5 00	400.797	12,152	5 00	393.996	12,160	5 00	387,074	12,153	5 00	380,034	12,131	5 00	372,877	12,095
6 00	480,820	17.496	6 00	472,663	17,508	6 00	464,354	17,498	6 00	455,904	17,467	6 00	447,314	17,414
7 00	560,786	23,811	7 00	551,258	23,826	7 00	541,562	23,813	7 00	531,700	23,770	7 00	521,677	23,698
8 00	640,662	31,094	8 00	629,769	31,114	8 00	618,684	31,096	8 00	607,410	31,040	8 00	595,951	30,946
9 00	720,445	39,345	9 00	708,184	39,370	9 00	695,708	39,347	9 00	683,020	39,276	9 00	670,125	39,157
10 00	800,122	48,563	10 00	786,492	48,594	10 00	772,623	48,565	10 00	758,520	48,477	10 00	744,186	48,329
11 00	879,681	58,746	11 00	864,679	58,782	11 00	849,416	58,747	11 00	833,895	58,640	11 00	818,123	58,461
12 00	959.110	69,893	12 00	942,735	69,936	12 00	926,075	69,893	12 00	909,135	69,765	12 00	891,921	69,552
13 00	1,038,309	82,002	13 00	1,020,647	82,051	13 00	1,002,588	82,000	13 00	984,227	81,849	13 00	964.570	81,598
14 00	1,117,535	95,072	14 00	1,098,404	95.127	14 00	1,078,943	95,067	14 00	1,059,158	94,890	14 00	1,039,056	94,598
15 00	1.196.507	109,100	15 00	1,175,994	109,162	15 00	1,155,128	109,091	15 00	1,133,917	108,887	15 00	1,112,367	108,551
16 00	1,275,303	124,084	16 00	1,253,404	124,153	16 00	1,231,131	124,071	16 00	1,208,491	123,837	16 00	1,185,491	123,453
17 00	1,353.911	140,023	17 00	1,330,624	140,699	17 00	1,306,940	140,003	17 00	1,282,868	139,738	17 00	1,258,416	139,302
18 00	1,432,320	156,913	18 00	1,407,640	156,996	18 00	1,382,543	156,887	18 00	1,357,036	156,587	18 00	1,331,129	156,096
19 00	1,510,519	174,753	19 00	1,484,443	174,842	19 00	1,457,928	174,718	19 00	1,430,984	174,381	19 00	1,403,618	173,832
20 00	1,588,496	193.540	20 00	1,561,019	193,635	20 00	1,533,083	193,494	20 00	1,504,697	193,118	20 00	1,475,871	192,596
21 00	1,666,240	213.270	21 00	1,637,358	213,371	21 00	1,607,997	213,212	21 00	1,578,166	212,793	21 00	1,547,876	212,116
22 00	1,743,738	233,942	22 00	1,713,447	234,048	22 00	1,682,657	233,869	22 00	1,651,377	233,405	22 00	1,619,620	232,658
23 00	1,820,980	255,552	23 00	1,789,276	255,663	23 00	1,757,052	255,462	23 00	1,724,320	254,950	23 00	1,691,091	254,128
24 00	1,897,955	278,096	24 00	1,864,831	278,211	24 00	1,831,170	277,987	24 00	1,796,982	277,425	24 00	1,762,279	276,524
25 00	1,974,650	301,572	25 00	1,940,103	301,690	25 00	1,904,999	301,441	25 00	1,869,351	300,824	25 00	1,833,170	299,842
26 00	2,051,055	325,977	26 00	2,015,079	326,097	26 00	1,978,528	325,820	26 00	1,941,415	325,146	26 00	1,903,752	324,077
27 00	2,127,159	351,306	27 00	2,089,749	351,427	27 00	2,051,745	351,120	27 00	2,013,163	350,386	27 00	1,974,015	349,225
28 00	2,202,950	377,555	28 00	2,164,100	377,677	28 00	2,124,639	377,337	28 00	2,084,583	376,539	28 00	2,043,945	375,283
29 00	2,278,417	404,722	29 00	2,238,121	404,841	29 00	2,197,197	404,468	29 00	2,155,663	403,602	29 00	2,113,531	402,245
30 00	2,353,550	432,801	30 00	2,311,802	432,918	30 00	2,269,410	432,507	30 00	2,220,392	431,569	30 00	2,182,762	430,107

TABLE XXVI.

FOR PROJECTIONS OF MAPS OF LARGE AREAS.

(Extracted from Appendix No. 6, U. S. Coast and Geodetic Survey Report for 1884, by Henry Gannett.)

MERIDIONAL ARCS. COORDINATES OF CURVATURE.

NATURAL SCALE.—VALUES OF *dm* AND *dp* IN METERS.

Latitude 49°.			Latitude 50°.		
Long.	*dm.*	*dp.*	Long.	*dm.*	*dp.*
1° 00′	73,172	482	1° 00′	71,696	479
2 00	146,331	1,928	2 00	143,379	1,917
3 00	219,465	4,337	3 00	215,037	4,313
4 00	292,561	7,709	4 00	286,656	7,667
5 00	365,606	12,044	5 00	358,224	11,978
6 00	438,588	17,340	6 00	429,727	17,246
7 00	511,493	23,598	7 00	501,154	23,469
8 00	584,310	30,815	8 00	572,492	30,646
9 00	657,626	38,991	9 00	643,727	38,777
10 00	729,627	48,123	10 00	714,847	47,859
11 00	802,102	58,213	11 00	785,839	57,891
12 00	874,438	69,254	12 00	856,691	68,872
13 00	946,622	81,248	13 00	927,389	80,798
14 00	1,018,642	94,191	14 00	997,922	93,669
15 00	1,090,485	108,082	15 00	1,068,277	107,482
16 00	1,162,138	122,918	16 00	1,138,440	122,234
17 00	1,233,591	138,097	17 00	1,208,400	137,923
18 00	1,304,829	155,416	18 00	1,278,144	154,546
19 00	1,375,840	173,071	19 00	1,347,660	172,099
20 00	1,446,613	191,660	20 00	1,416,934	190,581
21 00	1,517,135	211,180	21 00	1,485,956	209,987
22 00	1,587,394	231,623	22 00	1,554,711	230,314
23 00	1,657,378	252,998	23 00	1,623,189	251,559
24 00	1,727,073	275,288	24 00	1,691,377	273,717
25 00	1,796,470	298,495	25 00	1,759,262	296,785
26 00	1,865,554	322,614	26 00	1,826,833	320,758
27 00	1,934,315	347,640	27 00	1,894,077	345,633
28 00	2,002,740	373,570	28 00	1,960,983	371,404
29 00	2,070,817	400,399	29 00	2,027,538	398,068
30 00	2,138,536	428,123	30 00	2,093,731	425,619

Table XXVII.

AREA OF QUADRILATERALS OF EARTH'S SURFACE OF 1° EXTENT IN LATITUDE AND LONGITUDE.

(Prepared by R. S. Woodward.)

Middle Latitude of Quadrilateral.	Area in Square Miles.	Middle Latitude of Quadrilateral.	Area in Square Miles.	Middle Latitude of Quadrilateral.	Area in Square Miles.	Middle Latitude of Quadrilateral.	Area in Square Miles.
0° 00′	4752.33	25° 30′	4300.17	50° 30′	3047.37	75° 30′	1205.13
0 30	52.16	26 00	4282.50	51 00	15.34	76 00	1164.49
1 00	51.63	26 30	64.51	51 30	2983.08	76 30	23.75
1 30	50.75	27 00	46.20	52 00	50.58	77 00	1082.91
2 00	49.52	27 30	27.56	52 30	17.85	77 30	41.99
2 30	47.93						
3 00	46.00	28 00	08.61	53 00	2884.88	78 00	1000.99
3 30	43.71	28 30	4189.33	53 30	51.68	78 30	959.90
4 00	41.07	29 00	69.74	54 00	18.27	79 00	18.73
4 30	38.08	29 30	49.83	54 30	2784.62	79 30	877.49
5 00	34.74	30 00	29.60	55 00	50.76	80 00	36.18
5 30	31.04	30 30	4109.06	55 30	16.67	80 30	794.79
6 00	27.00	31 00	4088.21	56 00	2682.37	81 00	53.34
6 30	22.61	31 30	67.05	56 30	47.85	81 30	11.83
7 00	17.86	32 00	45.57	57 00	13.13	82 00	670.27
7 30	12.66	32 30	23.79	57 30	2578.19	82 30	28.64
8 00	07.32	33 00	01.69	58 00	43.05	83 00	586.97
8 30	01.52	33 30	3979.30	58 30	07.70	83 30	45.24
9 00	4695.38	34 00	56.59	59 00	2572.16	84 00	03.47
9 30	88.89	34 30	33.49	59 30	36.42	84 30	461.66
10 00	82.05	35 00	10.28	60 00	00.48	85 00	19.81
10 30	74.86	34 30	3889.67	60 30	2364.34	85 30	377.93
11 00	67.32	36 00	62.76	61 00	28.02	86 00	36.02
11 30	59.43	36 30	38.56	61 30	2291.51	86 30	294.08
12 00	51.20	37 00	14.06	62 00	54.82	87 00	52.11
12 30	42.63	37 30	3789.26	62 30	17.94	87 30	10.12
13 00	33.71	38 00	64.18	63 00	2180.89	88 00	168.12
13 30	24.44	38 30	38.80	63 30	43.66	88 30	126.10
14 00	14.82	39 00	13.14	64 00	06.26	89 00	84.07
14 30	04.87	39 30	3687.18	64 30	2068.68	89 30	42.04
15 00	4594.57	40 00	60.95	65 00	30.94	90 00	00.00
15 30	4583.92	40 30	34.42	65 30	1993.04		
16 00	72.94	41 00	07.62	66 00	54.97		
16 30	61.61	41 30	3580.54	66 30	16.75		
17 00	49.94	42 00	53.17	67 00	1878.37		
17 30	37.93	42 30	25.54	67 30	39.84		
18 00	25.59	43 00	3497.62	68 00	1801.16		
18 30	12.90	43 30	69.44	68 30	1762.33		
19 00	4499.87	44 00	40.98	69 00	23.36		
19 30	86.51	44 30	12.26	69 30	1684.24		
20 00	72.81	45 00	3383.27	70 00	45.00		
20 30	58.78	45 30	3354.01	70 30	05.62		
21 00	44.41	46 00	24.49	71 00	1566.10		
21 30	29.71	46 30	3204.71	71 30	26.46		
22 00	14.67	47 00	64.68	72 00	1486.70		
22 30	4399.30	47 30	34.39	72 30	46.81		
23 00	83.60	48 00	03.84	73 00	06.81		
23 30	67.57	48 30	3173.04	73 30	1366.69		
24 00	51.21	49 00	41.90	74 00	26.46		
24 30	34.52	49 30	10.69	74 30	1286.12		
25 00	17.51	50 00	3079.15	75 00	45.68		

utes, are then reduced to minutes and their decimals. These are multiplied by the corresponding distances, taken directly from Table XXIII or Tables XXV and XXVI, and corresponding to one minute for the map scale selected.

Having found these quantities, they are platted as differentials of latitude northward from the last latitude line ruled on the projection, and as differentials of longitude westward from the last longitude line platted on the projection, and perpendiculars are erected, the intersections giving the exact position of the point. When all the points falling within the area of the map have been platted in this manner, the accuracy of the plat may be tested by computing the differences of latitude and longitude backward by subtracting the minutes and fractions from the next greatest ten-minute projection line. They are also to be checked by measuring the distance between the various points as given in the computed geodetic coordinates and reduced to the map scale.

189. **Scale Equivalents.**—The proper scale to employ for a topographic map is dependent wholly upon the purposes to which that map is to be put. Where it is desirable to show the topography of a large area of country on a single map, the *smallest scale should be employed* which will permit of representing the features it is desired to show, for the reason that the smaller the scale the larger the area brought in view of the eye on one piece of paper. Again, the scale is affected materially by the method of representing relief. If contours are employed, such a scale must be used as will permit of all the contours being shown in the proper places without crowding them too closely upon the map, on the one hand, and yet without their being so far apart, on the other hand, as to detract from the expression which they give to the surface relief. In general it may be stated that for a given contour interval the smallest scale should be chosen which will permit of properly platting the contours, for thus, by bringing them closer together, the best effect is obtained

in depicting the forms mapped, and the largest area is shown on a given map sheet.

For *exploratory maps* scales as small as one to one-millionth may be employed. For *geographic maps* scales of 1 : 63,360 to 1 : 500,000 may be most satisfactorily employed. For general *topographic maps* scales of 1 : 10,000 to 1 : 63,360 will be sufficiently large to permit of properly representing the terrane. For *cadastral maps* scales of 1 : 2,500 to 1 : 10,000 may be used, and for these as well as for *detailed topographic maps* for the working out of engineering problems scales as large even as 100 or 200 feet to the inch may be employed.

Table XXVIII gives in fractional form the ratio of inches corresponding to a given distance in feet, miles, meters, or kilometers, as represented by the reduction from nature

TABLE XXVIII.

SCALE EQUIVALENTS FOR VARIOUS RATIOS.

Feet to One Inch.	Miles to One Inch.	Meters to One Inch.	Kilometers to One Inch.	Ratio (Number Inches).
100	0.019	30.578	0.031	1 : 1,200
400	0.076	121.882	0.122	1 : 4,800
500	0.095	152.854	0.184	1 : 6,000
800	0.151	243.533	0.244	1 : 9,6c0
883⅓	0.158	254.177	0.254	1 : 10,000
1,000	0.189	305.708	0.306	1 : 12,000
2,500	0.473	761.9	0.762	1 : 30,000
2,640	0.5	804.6	0.805	1 : 31,680
3,333⅓	0.631	1,015.9	1.016	1 : 40,000
3,750	0.710	1,143	1.143	1 : 45,000
5,000	0.947	1,523.9	1.524	1 : 60,000
5,208	0.988	1,587	1.587	1 : 62,500
5,280	1	1,609.3	1.609	1 : 63,360
6,666⅔	1.263	2,031.9	2.032	1 : 80,000
7,500	1.42	2,285.8	2.286	1 : 90,000
8,333⅓	1.578	2,539.9	2.54	1 : 100,000
10,416	1.976	3,174.9	3.175	1 : 125,000
10,560	2	3,218.6	3.219	1 : 126,720
16,666⅔	3.156	5,079.8	5.08	1 : 200,000
20,832	3.952	6,349.8	6.35	1 : 250,000
21,120	4	6,437.3	6.437	1 : 253,440
31,680	6	9,655.9	9.656	1 : 380,160
41,666⅔	7.891	12,699.6	12.7	1 : 500,000
83,333⅓	15.783	25,399.2	25.4	1 : 1,000,000

to maps of different scales. This table gives a number of those equivalents corresponding to the more usual map scales employed both in engineering topography and in the topographic and geographic atlases published by State and Government organizations.

Table XXIX gives equivalent ratios showing the number of inches corresponding to one mile.

Table XXIX.

RATIOS EQUIVALENT TO INCHES TO ONE MILE.

1	inch	to 1	mile.	Equivalent	1 : 63,360
$1\frac{1}{4}$	inches	" 1	"	"	1 : 50,688
$1\frac{1}{3}$	"	" 1	"	"	1 : 47,520
$1\frac{1}{2}$	"	" 1	"	"	1 : 42,240
$1\frac{3}{5}$	"	" 1	"	"	1 : 39,600
$1\frac{2}{3}$	"	" 1	"	"	1 : 38,016
2	"	" 1	"	"	1 : 31,680
$2\frac{1}{2}$	"	" 1	"	"	1 : 25,344
3	"	" 1	"	"	1 : 21,120
4	"	" 1	"	"	1 : 15,840
5	"	" 1	"	"	1 : 12,672
6	"	" 1	"	"	1 : 10,560

CHAPTER XX.

TOPOGRAPHIC DRAWING.

190. Methods of Map Construction.—There are two general modes of representing artificially in map form the various topographic features surveyed. These are:

1. Representation on paper by means of various conventional signs used in topographic drawing; and

2. Representation of the relief in a miniature model, special features of culture or drainage being denoted by conventional signs painted thereon.

A third method, and one which is most graphic in the depiction of surface forms, is by making a photograph of a model map, the result being a *relief map*.

The various processes employed in indicating the results of a topographic survey on paper are described as *topographic drawing* (Art. 191). Those employed in representing the same on a *model map* are known under the general expression modeling (Art. 198).

Relief maps are photomechanical copies of model maps (Art. 198).

191. Topographic Drawing.—In drawing topographic or geographic maps, a few conventional signs have been generally accepted for the representation of the various features of the terrane. Wavy lines, corresponding in plan to their positions upon the surface of the earth, are employed to represent outlines of seacoast or lakes, margins of streams, etc. In representing a *stream* it is customary to begin at the headwaters, where the stream is smallest, with the finest possible line, increasing its width as the stream increases in size, until toward the mouth, if the map scale will permit it—a single line failing to be sufficient for its representation—it becomes necessary to double-line it, the two lines representing as nearly as possible to scale the outlines of the shores of the stream. *Water surfaces*, such as those of oceans or lakes or of broad rivers, are indicated by parallel lines called water lines, somewhat like contour lines, and the distance between them at the shore is about equivalent to the width of the line, this distance being increased rapidly away from the shore until the lines disappear entirely. (Figs. 43 and 146.)

Surface forms of relief are represented by two general systems:

1. The vertical system, by *hachures* (Figs. 19, 141, and 143), which are short lines parallel to the direction of flow of water on the slope and based upon a scale of shades so graduated as to represent the relative amount of light which may be reflected from various degrees of slope; and

2. The horizontal system, by *contours* (Figs. 4, 135, and 139), which are continuous lines representing equal vertical intervals and which are in fact projected plans of the line at which a water surface (of the ocean, for instance) would intersect the surface of the earth were it raised successively by equal amounts. These contours are, then, curved lines which represent in plan the country as it would appear if it were cut by a series of equidistant horizontal planes. The *contour interval*, as it is called, or the distance between two contour lines, may be assumed at pleasure, but must be constant for the same map.

Still another method of representing surface slopes is by *crayon or brush shading* (Fig. 138), so as to give the effect produced by hachures, but in a uniform tint; and still another and perhaps the best method of all is that of *shading a contour map* in such manner as to produce the graphic relief effect resulting from hachures, while at the same time it retains the quantitative property given by contours. (Fig. 136.)

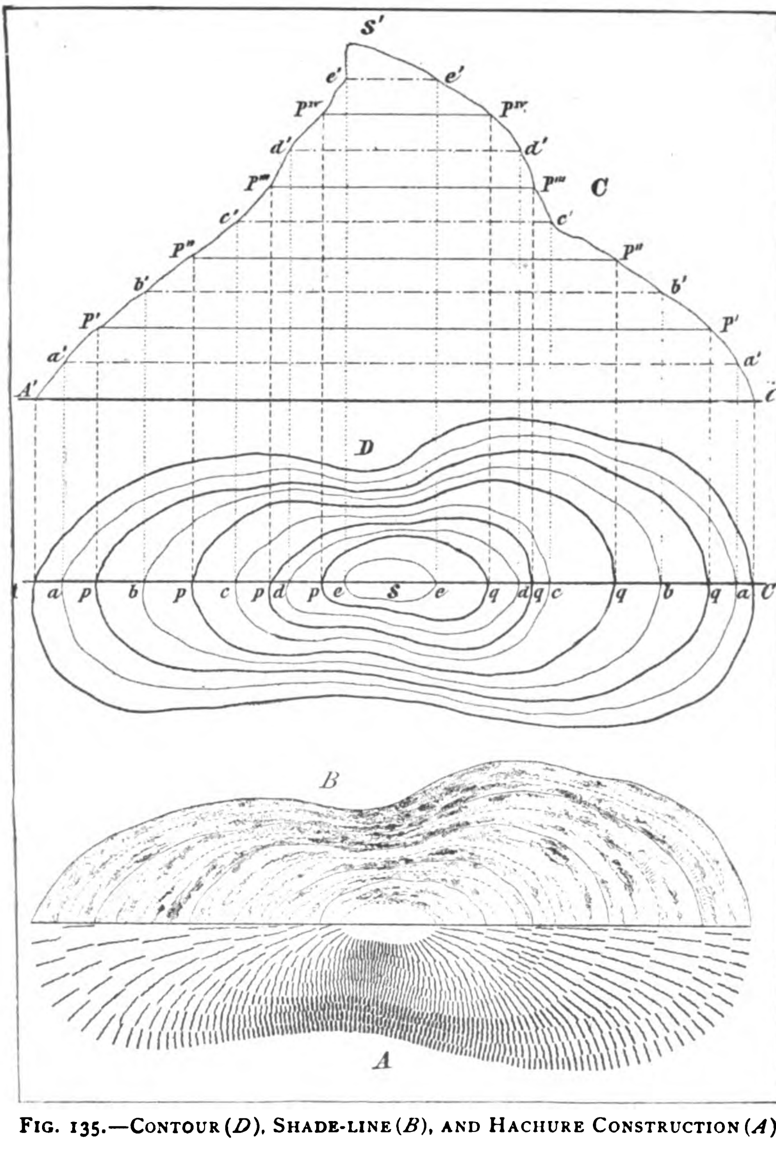


FIG. 135.—CONTOUR (*D*), SHADE-LINE (*B*), AND HACHURE CONSTRUCTION (*A*).

The representation of relief by *hachures* is *graphic only*. By this method quality of relief is the first consideration, and quantity or relative amount of relief is secondary. (Figs. 135 and 143.) Where quantitative relief is necessary, as in the work of the engineer or geologist, a contour map is essential. While such a map is possessed of mathematical qualities and clearness that are lacking in the hachured map, it fails to a large degree in representing the details of the surface, and, moreover, its representation of surface forms is difficult of interpretation by the inexpert. The representation of relief by hachures has been characterized as a graphic system with a conventional element, and the contour method of representing relief as a conventional system with a graphic element, the latter being obtained when the contour interval is so small as to produce a shading in the map, as when the scale selected is relatively small. (Figs. 4, 34, and 35.)

In any form of map-shading the lighting may be taken from one of two directions. If vertical, that is, from above, no high lights are introduced, but the highest summits have the lightest tint. The better and more commonly accepted method of shading is to assume that the light comes from an angle of 45° to the left, or, in other words, from the upper left-hand corner of the map; the northwest corner (Figs. 136, 138, and 143) and the high lights are, therefore, those which are tangent to this direction.

Two effective methods of representing relief which are most useful in sketching in the field on a plane-table board are:

1. By means of sketch or broken contours; and
2. By means of crayon shading.

Sketch contours have the general form of continuous contour lines and represent the slopes in plan pictorially. They also give differences of altitude relatively, but the quantity of relief is not accurate over any great space on the map, (Figs. 15, 23, and 137). When the final drawing is made in

FIG. 136.—SHADED CONTOUR MAP.

office from such a sketch map, the altitudes which have been determined everywhere give points upon which connected contour lines can be constructed by following the forms indicated by the sketch contours.

Crayon or *brush shading* bears about the same relation to hachure drawing as do sketch contours to continuous contour

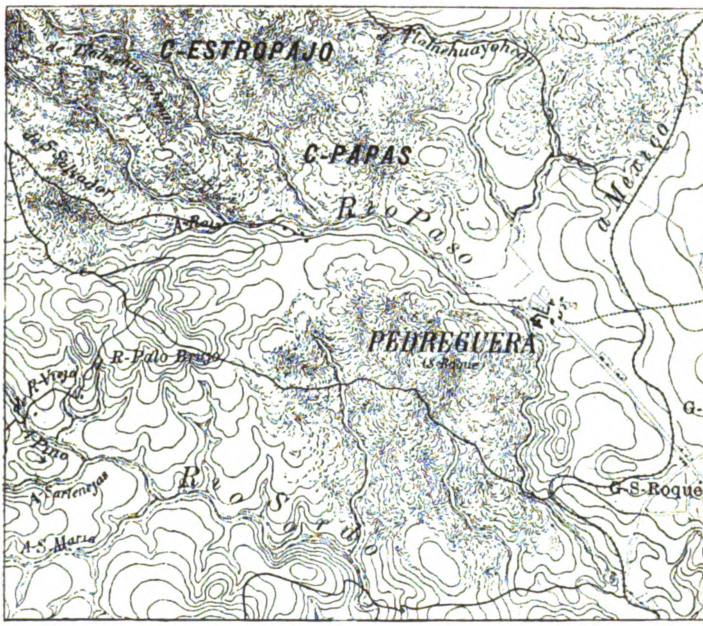


FIG. 137.—SKETCH CONTOURS. XALAPA, MEXICO.

lines. By the means of a soft crayon or pencil the shapes and steepness of slope of the terrane can rapidly be represented in the field, and, if it is desired, the same can afterwards be worked up into a finished hachure map, or, providing elevations are sufficiently abundant, into a contour map.

192. Contour Lines.—In order to represent the terrane quantitatively, that is, to show not only the slopes and shapes of the country and the relative steepness of the hills, but also amounts and differences in elevation at any point, a system of map construction is employed called contouring. *Contour lines* are lines of equal elevation above some datum as the mean sea-level. They are drawn at regular vertical intervals, their distances apart being dependent upon the horizontal scale of the map, and they thus indicate not only actual differences of elevation, but degrees of steepness or grades.

Contour lines express three degrees of relief:

1. Elevation;
2. Horizontal form;
3. Degree of slope.

The manner in which they express these is illustrated in the following perspective view and contour sketch (Fig. 139), taken from the explanatory text accompanying the atlases of the U. S. Geological Survey.

The sketch represents a valley between two hills. In the foreground is the sea with a bay which is partly closed by a hooked sand-bar. On either side of the valley is a terrace; from that on the right a hill rises gradually with rounded forms, whereas from that on the left the ground ascends steeply to a precipice which presents sharp corners. The western slope of the higher hill contrasts with the eastern by its gentle descent. In the map each of these features is indicated, directly beneath its position in the sketch, by contours. The following explanation may make clearer the manner in which contours delineate height, form, and slope:

1. A contour indicates approximately height above sea-level: in this illustration the contour interval is 50 feet; therefore the contours occur at 50, 100, 150, 200 feet, and so on, above sea-level. Along the 250-foot contour lie all points of the surface 250 feet above sea; and so with any other contour. In the space between any two contours occur all elevations above the lower and below the higher contour. Thus the contour at 150 feet falls just below the edge of the

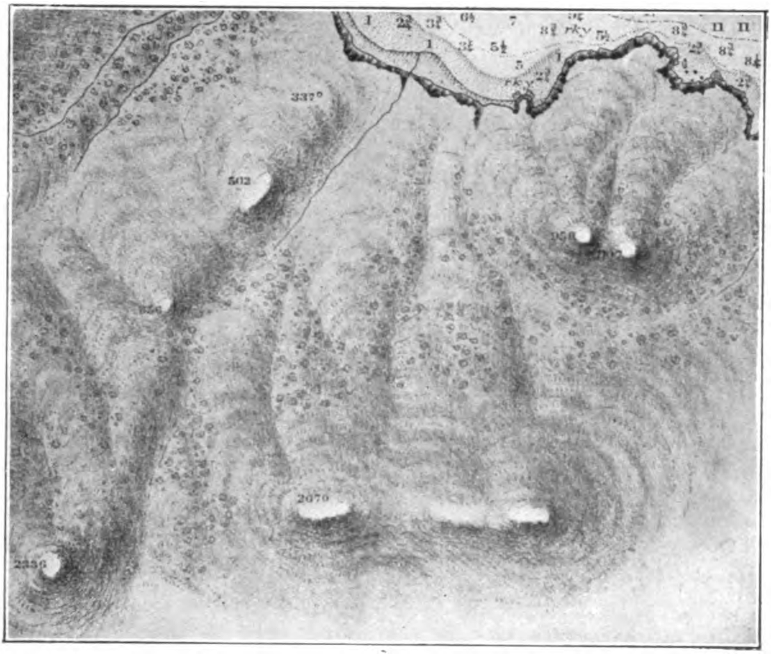


Fig. 138.—Relief by Crayon Shading.

terrace, while that at 200 feet lies above the terrace; therefore all points on the terrace are shown to be more than 150 but less than 200 feet above sea. The summit of the higher hill is stated to be 670 feet above sea; accordingly the contour at 650 feet surrounds it. In this illustration nearly all

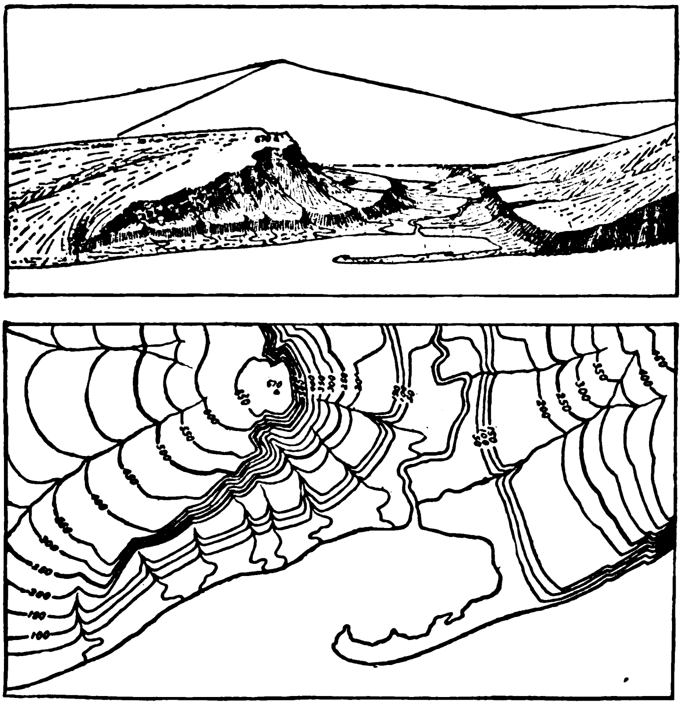


FIG. 139.—CONTOUR SKETCH.

the contours are numbered. Where this is not possible, certain contours, as every fifth, are made heavy and are numbered; the heights of others may then be ascertained by counting up or down from a numbered contour.

2. Contours define the horizontal forms of slopes: since contours are continuous horizontal lines conforming to the surface of the ground, they wind about the surfaces, recede into all re-entrant angles of ravines, and define all promi-

nences. The relations of contour characters to forms of the landscape can be traced in the sketch and map.

3. Contours show the approximate grade of any slope: the vertical space between two contours is the same whether they lie along a cliff or on a gentle slope; but to rise a given height on a gentle slope one must go farther than on a steep slope. Therefore contours are far apart on the gentle slopes, and near together on steep ones.

193. Contour Construction.—In representing on a map the relief or configuration of the surface of the land by means of contour lines two objects are to be kept constantly in mind.

1. The contour lines must be so constructed as to always maintain with accuracy relative and absolute elevations.

2. They must be so drawn as to give a distinct picture of the shapes of the country as though viewed from a considerable altitude above the surface.

Contour lines are actual mathematical horizontal projections, to a given scale, of the intersection of the surface of the terrane by imaginary horizontal planes. Moreover, these imaginary planes are, for any given map, accepted as being at equal and uniform vertical distances apart.

Contour lines are drawn during the processes of eye "sketching" (Fig. 139) by making an imaginary projection in plan of numerous sections through the hill viewed. This is illustrated in Fig. 140, which represents a section through a hill and indicates graphically the manner in which the contours are projected. Each individual contour line is the projection of the intersection of the horizontal plane through the hill. In *learning to sketch contours* the topographer will do well at first to keep in mind clearly this form of construction, and wherever in doubt as to the mode of representing any feature he should construct a profile sketch of it, draw horizontal section lines through it and project them in plan. In this manner he will soon learn to mechanically perform this operation in imagination, and later, as his skill develops, will draw contour lines without performing any intermediate mental operations.

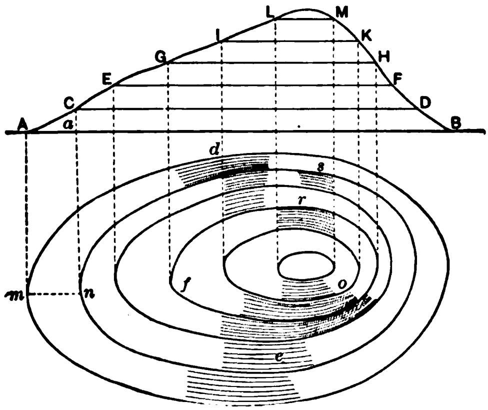


FIG. 140.—CONTOUR PROJECTION.

194. Hachures.—Hachuring is a conventional method of representing the relief of a country by shading, in short disconnected lines, in such manner as to indicate its slopes and relative steepness. The distance apart of the lines, their weight or thickness, and accordingly the degree of density which they produce on the map give qualitative and not quantitative results. These lines are of varying lengths, and are drawn in the direction of the slopes and in such manner as to horizontally encircle them in bands, and the width of these is intended to represent equal vertical heights.

In order, therefore, to *construct a hachure map* it is necessary to sketch approximate contour lines (Fig. 141), the distance between any two of these representing approximately a fixed vertical distance. Between these contour lines and at right angles to them are drawn the hachure lines, the contours being only penciled in and the hachures inked so that ultimately the contour lines disappear. The hachure lines, as already stated, are not made continuous, but rest on the horizontal or contour curves, which are thus indicated by the termini of the hachure lines. In *hachuring a map* the following general directions are suggested:

Commence with the lighter slopes in the lightest line, in order that the intensity of the tint may be increased with more regularity. (Fig. 141.) When the projections of the

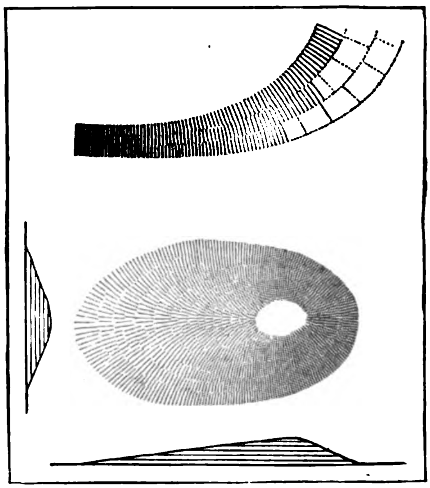


FIG. 141.—HACHURE CONSTRUCTION.

horizontal sections or contours are parallel the hachures are at right lines normal to both curves, but when they are not parallel the hachures radiate, their extremities being respectively *normal to the curves* at which they terminate. Hachures are in sections or bands which should not be continuous with the adjoining ones, but should terminate in the spaces between them, thus accentuating the contour lines without inking them. When the slope suddenly becomes abrupt the tint must be deepened by increasing the width of the hachure near the extremities or by interpolating short lines between the original hachures. Hachures are made shorter and wider for steep slopes, and are lenghtened and narrowed as the inclination decreases.

The first principle upon which hachures are constructed is that the steeper the slope the less light is received in the inverse ratio of its length. Various methods of expressing the degree of shade, or the *ratio of black to white*, have been adopted by various draftsmen. The Enthoffer or American method is to indicate the degree of slope by varying the distance between the hachure lines, the distance between the centers of lines to be .02 of an inch plus one-fourth of the denominator of the fraction denoting the declivity, expressed in hundredths of an inch. The lines are accordingly made heavier as the slope is steeper, and finer for gentle slopes, increasing in width until the blank spaces between them equal one-half the breadth of the lines. (Fig. 142.) The German

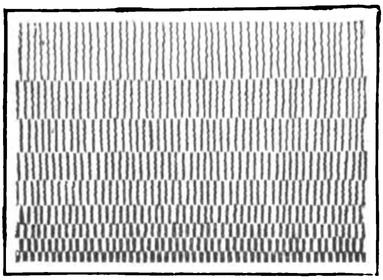


FIG. 142.—SHADED HACHURES.

or Lehmann's method consists in using nine widths of lines for slopes from zero to 45°, the first being white and the last black. For the intermediate slopes the proportion of white to black is as 45° minus the angle of slope is to the angle of slope. Steeper slopes than 45° are represented by short, heavy lines, parallel to the contour lines.

195. Conventional Signs.—Various conventional signs are employed in topographic drafting to represent roads, houses, woods, marshes, the shapes of hills, etc. These signs may be divided into three general classes:

1. Signs to represent culture or the works of man.

2. Signs to represent hydrography or water.

3. Signs to represent hypsography or relief.

In the making of a geographic map or of a topographic map for the use of a government or State, only such *culture* should be represented as is of a permanent or public nature. This includes all highways, bridges, railways, political boundary lines, and houses. (Figs. 144 and 145.) Though the latter are not of a public nature, yet they are comparatively permanent and are prominent topographic features.

For purposes of legibility in deciphering a map it is desired to use various colors in representing different features, and the color scheme selected by the U. S. Geological Survey, which is one of the best, employs (1) *black* for all culture and lettering; (2) *blue* for hydrography; and (3) *brown* for surface relief.

In the representation of *hydrography*, or water forms, conventional signs must be adopted (Fig. 146) for streams, lakes, marshes, canals, glaciers, etc.

For the representation of *hypsography*, or surface relief, conventional signs must be adopted (Fig. 147) for the representation of slopes, by means of contours or hachures, with separate symbols to indicate depressions of the surface, also sand-dunes, cliffs, etc.

In addition to the above conventional signs used in depicting public culture and the more usual topographic forms, a great variety of signs are used to represent minor forms, as lighthouses, mines, quarries, churches, different kinds of houses, as barns, private residences, mills, also to represent different kinds of woods and cultivated fields. These are described and illustrated in various works on topographic drawing. The only one of these of importance which may be further characterized here are *woods*, and for these conventional signs may be adopted to indicate the wooded character of the country, or, better, a light *green tint* may be washed over the wooded portion of the map.

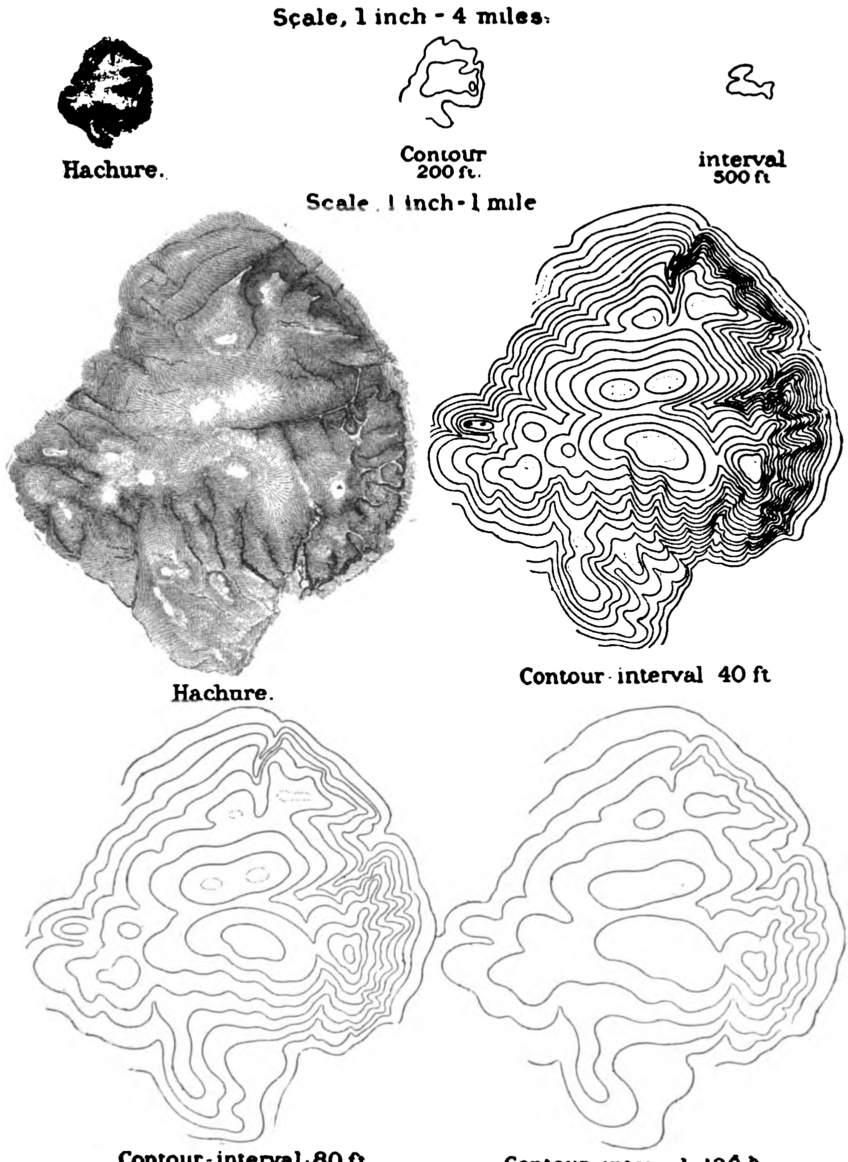


FIG. 143.—HACHURED AND CONTOURED HILL ON DIFFERENT SCALES.
(After S. Enthoffer.)

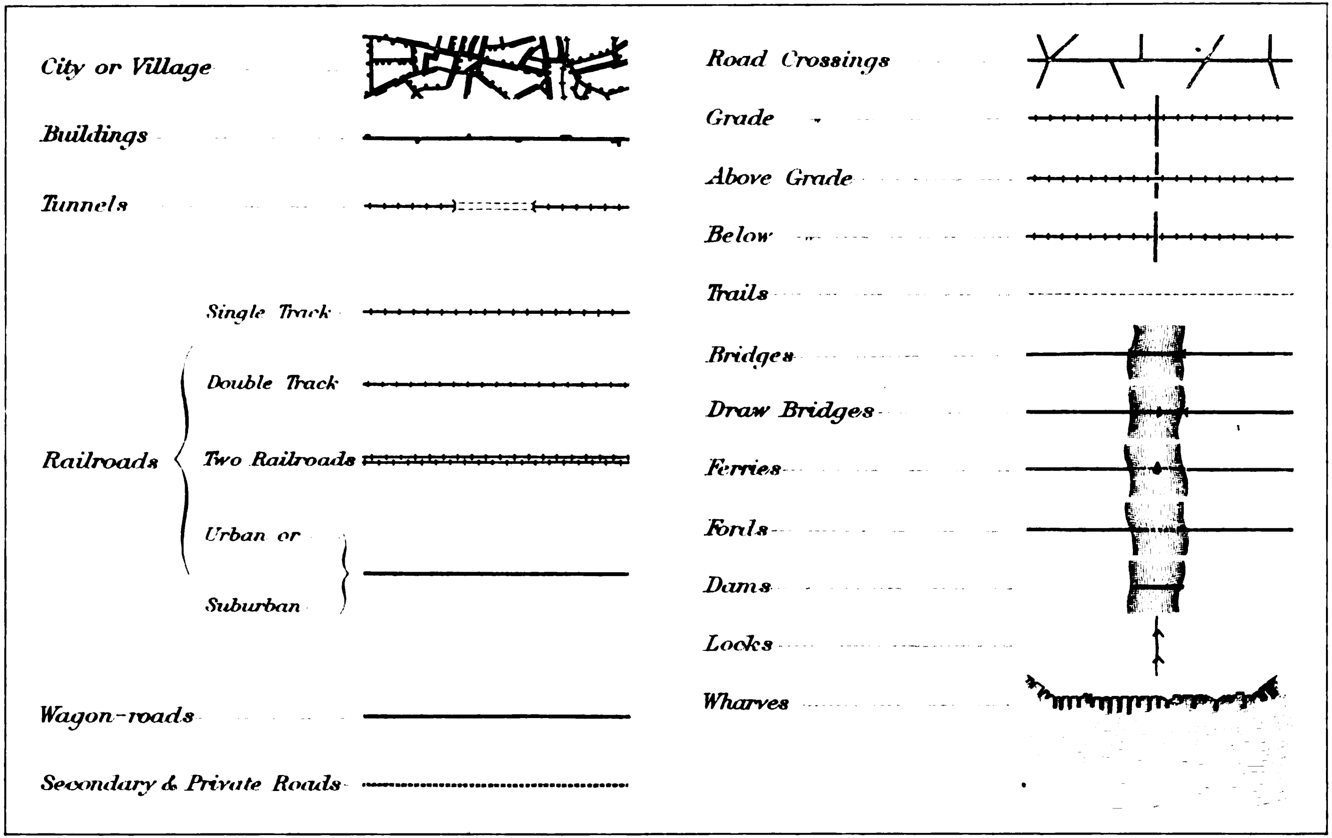


FIG. 144.—CONVENTIONAL SIGNS; PUBLIC AND PRIVATE CULTURE.

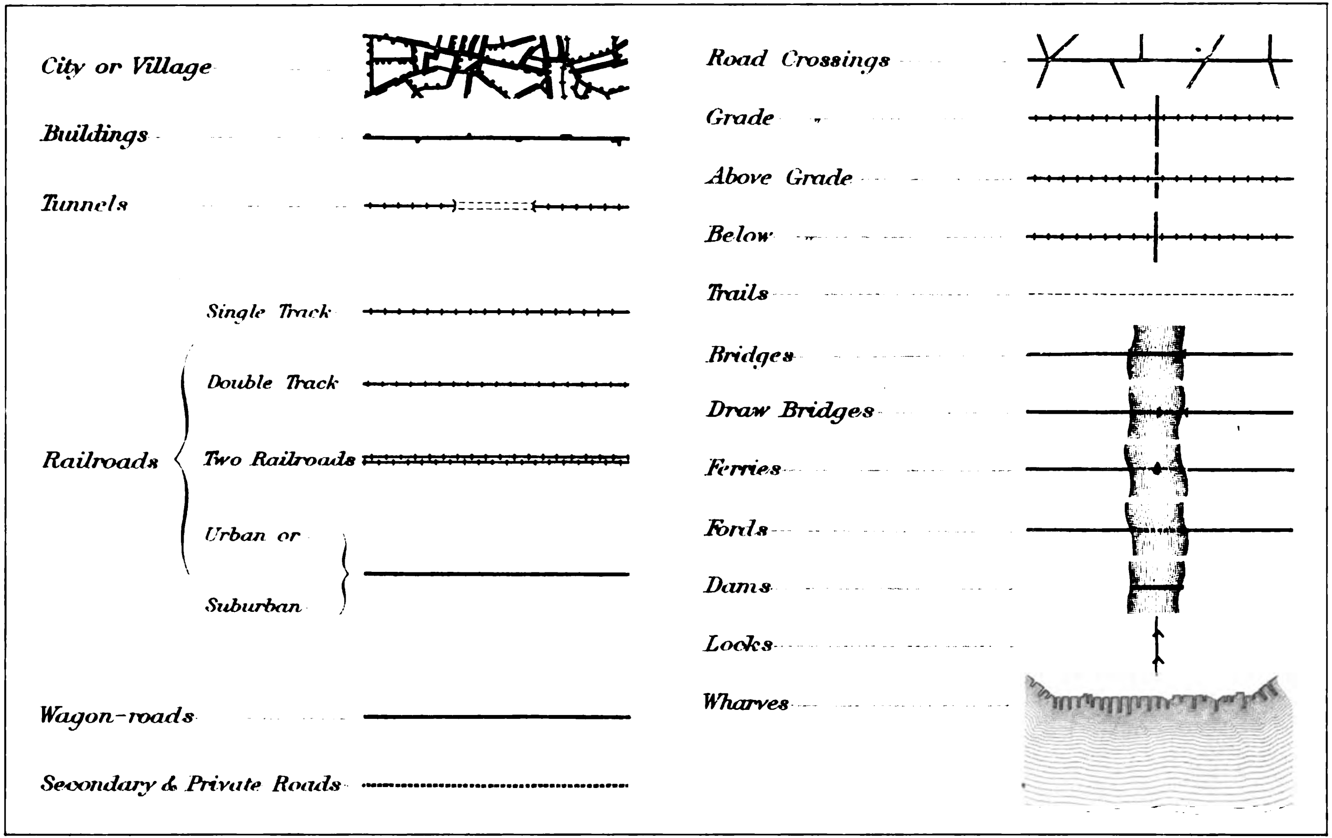


FIG. 144.—CONVENTIONAL SIGNS; PUBLIC AND PRIVATE CULTURE.

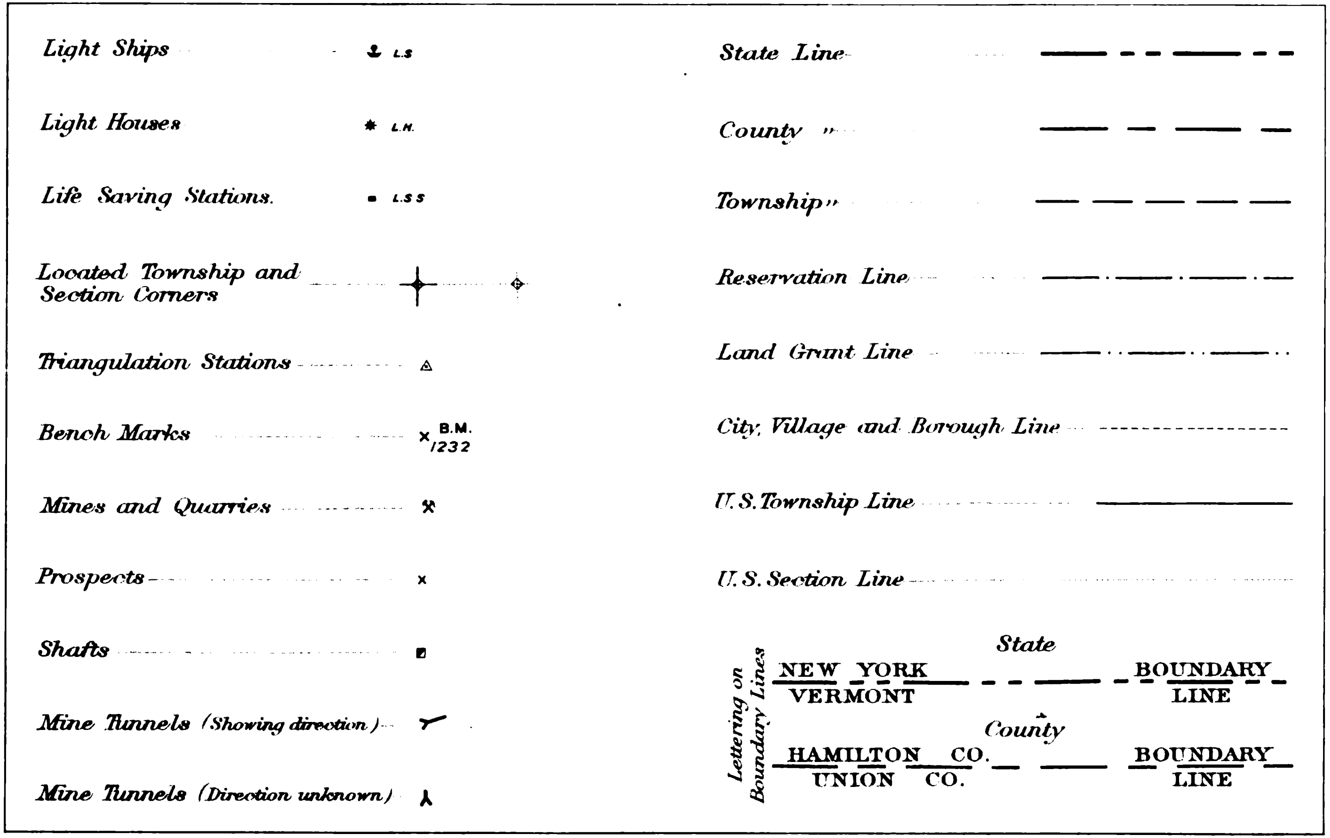


FIG. 145.—CONVENTIONAL SIGNS; MISCELLANEOUS SYMBOLS AND BOUNDARY LINES.

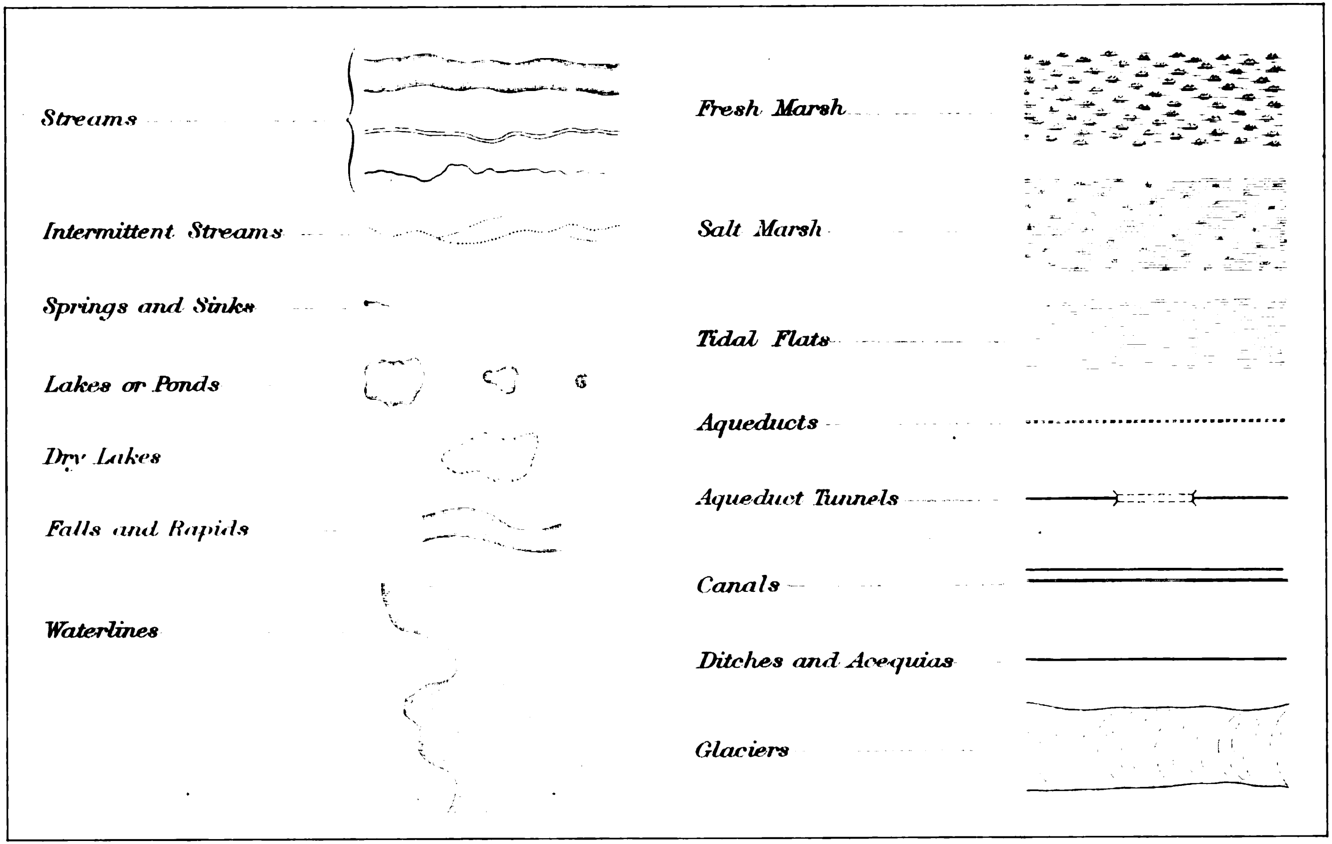


FIG. 146.—CONVENTIONAL SIGNS; HYDROGRAPHY.

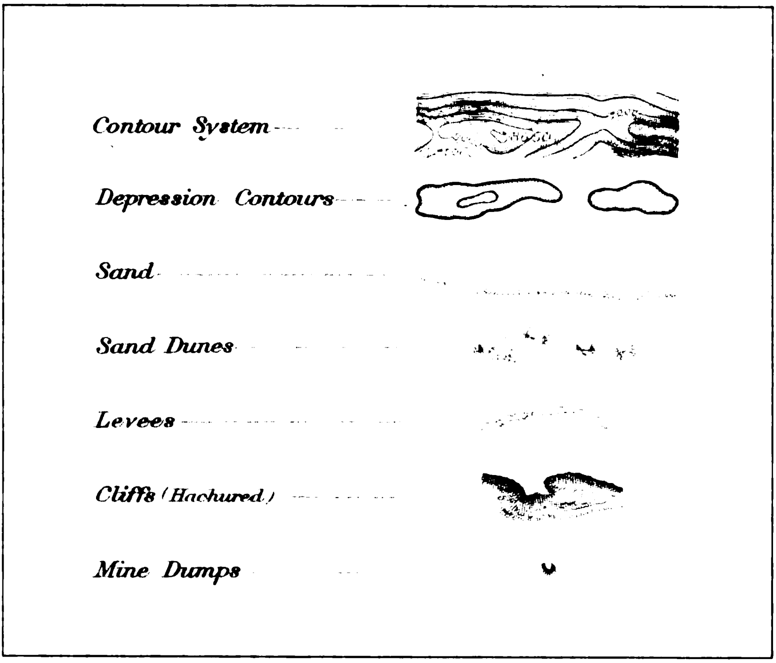


FIG. 147.—CONVENTIONAL SIGNS: RELIEF OR HYPSOGRAPHY.

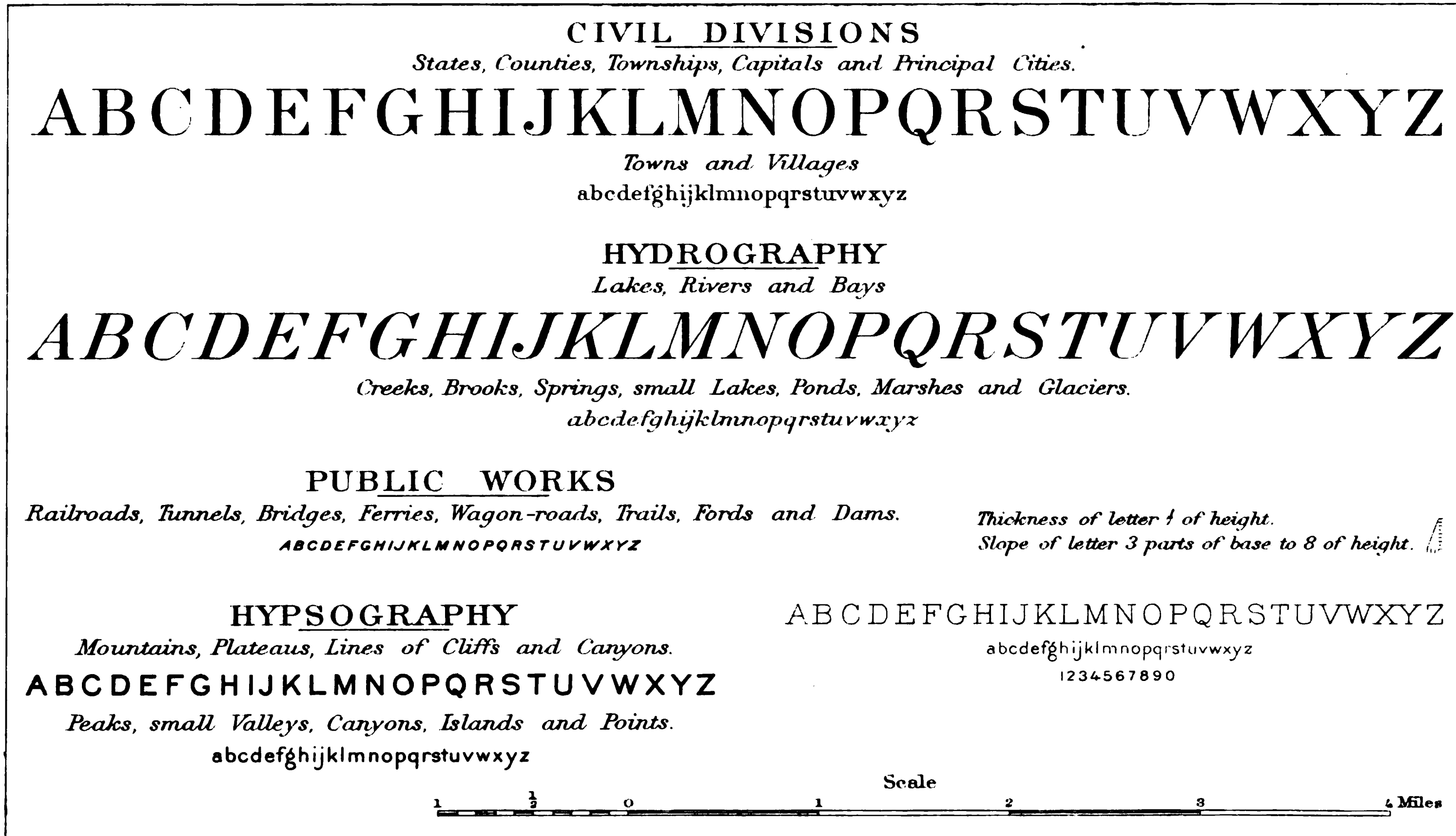


FIG. 148.—CONVENTIONAL SIGNS; LETTERING.

196. Lettering.—As with conventional signs, so with lettering. Various books are published describing the mode of constructing different kinds of letters. It is desirable here, therefore, only to give a general outline of the principles on which topographic lettering should be executed. Letters used in describing different topographic forms may be divided into four corresponding classes, and there should be, therefore, as many different styles of letters. Those preferred by the author and shown in Fig. 148 consist of—

1. Roman letters of various sizes for the names of civil and political divisions, as cities, States, etc.
2. Italic and script letters of various sizes for the names of hydrographic features, as lakes, rivers, etc.
3. Vertical block of various sizes to represent hypsographic features, as mountains, plateaus, valleys, etc.
4. Slanting block to represent public work, as railroads, trails, etc.

197. Drafting Instruments.—It is not deemed desirable to describe in detail the various instruments commonly used in topographic drawing. These can be found fully described in catalogues of instrument-makers and in works on mechanical and topographical drawing, lettering, etc. A few instruments, however, which are less common and which are peculiarly serviceable to the topographic draftsman will be briefly characterized.

The *pantograph* is a parallel linked-motion apparatus for enlargement or reduction of maps. It is of occasional assistance in the reduction or enlargement of compiled map material, and is constructed on the theory of parallelograms. (Fig. 149.) The pantograph is, however, a comparatively inaccurate instrument because of the great play between the various parts. If accuracy is attempted, none but the most expensive and heavily constructed instruments should be used. There is an inconvenient variety of combinations in the adjustment and use of this instrument. The essentials are that the fixed, the tracing, and the copying points shall lie in a straight line on at least three sides of the jointed parallelogram.

Two very useful instruments to the topographer are *proportional dividers* and *three-legged dividers*, the first of which is very serviceable in the reduction or enlargement of small portions of maps, and the second in transferring work

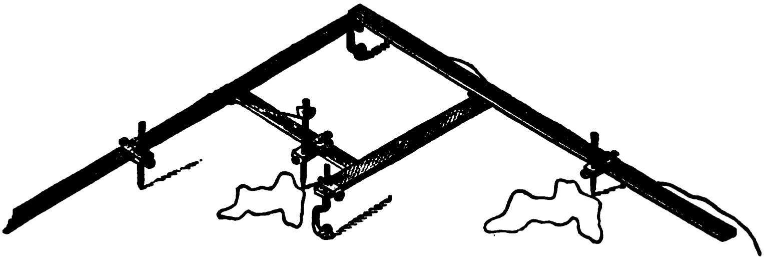


FIG. 149.—PANTOGRAPH.

from one map to another. In this operation two of the legs are set on fixed points common to both maps, as the intersections of projection lines, and the third is used as a pointer to transfer the position desired. This instrument is especially useful in the transferring and adjustment of lines from the traverse sheets (Fig. 2) to the sketch sheets on which they are to be adjusted to the triangulation positions (Fig. 3).

For the construction of projections the topographer needs a first-class *beam compass* and an accurately graduated *steel scale*. The ordinary triangular boxwood scale is well graduated and is useful in the projection of very small scale-maps; but for larger ones a long steel scale, preferably divided to the scale of the map work, will give much more satisfactory results.

The use of *vernier protractors* is fully described in Article 89, while *plane-table paper* and like accessories are discussed in Chapter VIII.

198. Model and Relief Maps.—Relief maps are of two general kinds:

1. The model, which is not a map in that it has three

dimensions, is bulky, and cannot be inserted in atlases or books; and

2. The reproduction of the model by some photo-mechanical process which results in a print in map form of the model.

Models have certain striking advantages over maps of all kinds, because they represent graphically the surface relief in a manner superior to that of hachure or shaded contour maps, besides which they represent quantitatively the relative relief in a more simple and legible manner than do contour maps.

There are two general varieties of models:

1. Those in which surface slopes are smoothed out in such manner as to practically represent the surface of the country as it appears in nature, and which, while possessing inertly relative relief, lack the element of absolutely quantitative relief possessed by contour maps; and

2. Those in which the slopes are represented by steps, each of which is a contour interval apart on some scale; and while this does not imitate nature as exactly as the first form, it possesses an absolute quantitative element which makes it superior for many purposes.

Models are of especial value for *educational purposes*, in teaching those who are not familiar with maps something of geography which they would not appreciate by looking at the flat surface of a map. They are *nature in miniature*. In addition they have great *economic value* to the mining engineer and the geologist: to the former in obtaining a true appreciation of the differences in level and direction of the numerous shafts and tunnels which permeate the ground in the mining districts; to the latter because many important structural features and relations are presented to the eye at a glance, and because both the surface topography and its relation to the underground topography are brought together in their proper relationship. For *exhibition purposes* they are unsurpassed in that they possess the quality, next best to that of moving objects, which catches the attention of the beholder and attracts him to a further study of the subject represented in a way which no map can.

Relief maps possess numerous advantages over hachure or contour maps chiefly because they give a more graphic idea of the surface relief than can be had from any artificial method of map construction. They are made by photographing a model which is set up in such manner as to get the proper lighting, that which will bring out lights and shadows most effectively. For the successful reproduction of a relief map it is essential that the model should not be colored and its surface should be dull, not glossy; there should be a slight yellow tint in the material composing it, the effect of which is to produce a smoother, more subdued lighting and shading, and to do away with the glaring high lights coming from a white model. (Fig. 150.) Relief maps can be given certain quantitative values if they are reproduced from contour models (Fig. 151), and they may be given certain further values by simple lettering in black.

The groundwork for the construction of a model is a good contour map, in addition to which the modeler should possess a fair knowledge of the topography of the country, obtained by personal inspection, and he should have at hand good hachured maps and photographs which will aid him in interpreting the topographic forms. It is the personal expression that is brought into a model, by the appreciation of the country obtained from a knowledge of it, which results in the difference between a good and a bad model of the same region as produced by two modelers from the same data. The treatment of the vertical or relief element required to represent the individuality of a given district is especially important.

199. Modeling the Map.—The amount of relief to be given a model, that is, the amount of exaggeration in vertical scale as compared with the horizontal, is a question of great importance. The tendency is always to exaggerate the ver-

FIG. 150.—RELIEF MAP FROM CATSKILL MODEL.
Vertical and horizontal scales, both 1 inch to 1 mile. Modeled by E. E. Howell.

tical element too much, the result of which is to produce a false effect by diminishing the proportionate width of valleys, thus making the country seem more rugged and mountainous than it is. Another effect is to make the area of the region represented appear small, all idea of the extent of the country being lost. Messrs. E. E. Howell and Cosmos Mindeleff, of Washington, D. C., two of the most expert model-map-makers, agree that it is almost impossible under most circumstances to use too low relative relief. Mr. Mindeleff says that on a scale of six inches to a mile no exaggeration at all is necessary, the ratio of vertical to horizontal scale being as 1 to 1. For smaller scales than this the vertical exaggeration may be 2 to 1 or 3 to 1. He says further that "the absolute and not the relative amount of relief is the desideratum. For small-scale models I have found half an inch of absolute relief ample." In a very handsome model of the United States, made by Mr. Mindeleff, a proportion of 10 to 1 was used, but it is believed from the appearance of the resulting model that 6 to 1 would have been even more satisfactory. Some of the most effective of recent models are made to natural scale, i.e., without any exaggeration of vertical scale. (Fig. 150.)

For the *making* of *model maps* a number of methods have been employed, the majority of which are so crude or so inferior to the better methods as to call for scant recital here. One of the first employed consists in drawing cross-section lines at regular intervals over a contoured map, and, if the topography is intricate, corresponding lines at right angles. These sections are transferred to thin strips of cardboard or similar material and cut down to the surface line, thus forming the cross-section. These are mounted on a suitable base-board and the cavities between them filled in with plaster or wax or other easily worked material. The topography is then carved down to the form of the country as indicated by the upper edges of the strips. This method is crude and laborious.

Where *no contour map* is *obtainable* as a base and the known elevations are few and scattered, one of the simplest methods of producing a model map is by driving pins into a base-board, each to a height corresponding to the elevation of the point it represents. The map is then built up in wax or moist clay by laying this on the base-board and bringing it up to the level of the summits of the pins, and then working in the details of the map by practically sketching it in as a sculptor would, following a hachure or other map of the country as a guide.

Another method, practically the converse of that last described, may be satisfactorily employed where the *map material* is *scanty*. A tracing of the map enlarged to the required size is mounted on a frame. Another but deeper frame, large enough to contain the mounted tracing, is made and laid upon a suitable base-board, upon which is mounted a copy of the map. Upon this base-board the model is then built up in clear wax, the low areas first. Horizontal control is obtained by pricking through the mounted tracing with a needle-point, and vertical control by measuring down with a straight-edge, sliding on the top of the deep frame.

Model maps are sometimes made by carving or *cutting down* instead of modeling *or building up*, a solid block of plaster being used, and this being carved down so as to produce a series of steps similar to those made by building up contours.

The best and most *modern method* of making map models is that now more generally employed by the professional model-makers. This consists of building up the model and modeling instead of carving the detail. The ratio of relief or vertical to horizontal scale having been determined, thin cardboard or wooden boards are procured of the exact thickness of the contour interval which the modeler proposes using. He then takes a contour map, enlarged or reduced, as the case may be, to the scale of his model and traces on the boards the outlines of each separate contour. Then with a knife, scissors or scroll-saw, following the contour line on the board, he cuts out each contour and lays each of these outline contour boards one upon the other, thus building them up in steps, the height of each of which bears the proper relation, because of the thickness of the material, to the vertical scale. The result is a completed model in steps. The re-entrant angles of the steps are then filled in with modeling clay or wax or some similar substance, so as to produce a smooth outline.

The best *material for modeling* is wax; but if much modeling material is to be used, clay may be kept sufficiently moist to be worked. Some modelers find clay mixed with glycerine instead of water works most satisfactorily, because it does not dry. The filling-in process is the most important in the making of a model map, for in this the modeler must show his knowledge of and feeling for topographic forms, in the interpretation of not only such hachured and other maps as he has to guide him, but of the country, if he has examined it as he should.

200. Duplicating the Model by Casting.—The model resulting from the above operations is practically the base only of the completed model map. It is the common practice to make a replica by taking from the first a *mould with plaster of Paris*, and from this a *plaster cast*. The common mistake is made of making a solid cast by filling in a frame which has been built around a model, the result being so heavy and cumbersome as to be of little use. The best modelers say that it is wholly unnecessary to make a cast which is more than ½ or 1½ inches in thickness of plaster. This is procured by incorporating in the plaster tow or bagging or netting of various kinds, the result being to make the cast light and strong, though the expense is slightly increased. Such casts can be readily and even roughly handled without breakage.

In making the final *casting from the mould* the process is repeated. The model for the making of the mould or the latter for the after-process of casting should not be varnished, as the finer details are thus lost. The mould should be prepared with a solution of soap, so that nothing is left on the surface but a thin coat of oil, which is taken up by the plaster of the cast. With care and skill a cast may be thus produced which is but little inferior in point of sharpness to the original model.

The plaster model being completed, only such little painting of names and places as may be necessary to make it intelligible should be done before photographing for the production of the relief map, after which it may be colored as desired to represent any other subject and varnished. (Fig. 150.)

Other materials than plaster of Paris have been used for making models. Some modelers, after cutting the wooden contours and fitting these together with wooden pegs, carve away the steps left by the contours with graver's tools. This is an exceedingly laborious and difficult process, and the resulting model lacks expression and looks as wooden as the material from which it is made. Many efforts have been made to use papier maché, but owing to the distortion and warping in this, because of the varying degrees of moisture in the atmosphere and the material itself, no success has as yet attended its use.

The form of model used in depicting underground workings in a mine is by making a *skeleton model* of cardboard and glass. A rectangular box of glass is made of such size to scale as to include the cubic contents to be modeled. In this are glued or suspended by wires, etc., painted sheets of cardboard at such inclinations as to graphically represent the various tunnels, shafts, etc., or the ore-bearing strata, as desired.

A very effective form of model is made by pasting

Fig. 151.—Relief Map from Contour Model.
Scale 1 mile to 1 inch; contour interval 20 ft. Modeled by Wm. Stranahan.

together *paper contours.* Fig. 151 shows such a model, made by taking the printed sheets of the U. S. Geological Survey 20-foot contour map of the area depicted. One sheet had to be taken for each contour interval, and in all 30 to 50 sheets were used. The modeler followed carefully with scissors each contour line, and then superimposed each sheet on the next lower. By having printed paper bearing a fixed relation in thickness to the contour interval an exact quantitative reproduction of each 20-foot contour in nature is obtained.

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PART V.

TERRESTRIAL GEODESY.

CHAPTER XXI.

FIELD-WORK OF BASE MEASUREMENT.

201. Geodesy.—Geodesy has been defined as a system of the most exact land measurements, extended in the form of a triangulation over a great area, controlled in its relation to the meridian by astronomic azimuths computed by formulæ expressed in the dimensions of the spheroid, and placed in its true position on the surface of the earth by astronomic latitudes and differences of longitude from an established meridian.

Geodesy in its most general sense may be more *briefly defined* as the solution of problems which are conditioned by considerations of the figure and dimensions of the earth. Those particular problems which occur in plane and topographic surveying are solved without regard to the form of the earth. (Art. 52.)

Geodetic operations include, in the order given:

1. The determination of the exact length, by measurements reduced to mean sea-level, of a line several miles in length, which is the base line of the triangulation;

2. The determination of the latitude and longitude of one

end of the base line and of the azimuth of the line by astronomic observations;

3. The expansion of the base by triangulation executed with theodolite; and

4. The computation of the triangulation, whereby the geodetic coordinates of each of the trigonometric points are determined.

To these may be added:

5. The measurement and computation of geodetic coordinates of controlling points upon a route traverse, adjusted or reduced to one or more astronomic positions.

One of the primary *objects of geodetic operations* is to furnish data for the exact reference of a topographic map to its corresponding position upon the surface of the earth.

This consists of the *measurement of a base line* (Art. 202), which is an arbitrary distance upon the surface of the earth, to which the remaining surveyed positions may be referred in standard units, as meters or miles. Also the determination of the *astronomic position* upon the earth's surface (Part VI) of some initial point on this line, and its *azimuth*, that it may be platted in correct relation upon the map.

Geodetic operations are also executed for the purpose of checking astronomic positions determined by systems of primary triangulation or traverse extended from some point the geodetic coordinates of which have been already determined.

Astronomic checks on the quality of geodetic triangulation by a single determination of astronomic position are far less accurate than the positions obtained and computed by trigonometric operations. It is unnecessary, however, to introduce astronomic checks upon primary triangulation at frequent intervals, though these should be sufficiently numerous to eliminate station error. The positions determined by *primary triangulation* (Chap. XXV) are not likely to be in error by amounts larger than those introduced by astronomic observation after the triangulation has been extended a distance of 150 to 250 miles. Therefore a system of primary triangulation should be checked by astronomic observation at intervals not greater than this.

Primary traverse is far more liable to errors than is primary triangulation, because of the greater number of courses sighted and the consequent opportunity for the accumulation of error both in angular and in distance measurement. Primary traverse (Chap. XXIII) must therefore be more frequently checked by astronomic determination, and such checks should not exceed 100 miles apart.

Whereas primary triangulation and primary traverse may be executed with various degrees of accuracy, according to the distance to which a system of such control is to be propagated and according to its objects, astronomic determinations should be of the highest order of accuracy. Only the most refined instruments and methods known to science for use in the field and in permanent observatory work give results of sufficiently high quality to fulfill their purposes.

202. Base Measurement.—The *selection of a site* for a base line is the first step towards the making of a trigonometric survey, and on its proper selection depends much of the quality of the subsequent work of triangulation.

1. The site should be reasonably level;

2. It should afford room for a base from 4 to 8 miles in length;

3. Its ends must be intervisible and so situated as to permit of the expansion of a system of primary triangulation which will form the best-conditioned figures.

The *degree of accuracy* with which the base measurement is to be made depends upon the uses to which the resulting triangulation is to be put.

1. If intended for geodetic purposes, the measurement must be made with the greatest attainable precision.

2. If intended only as a base for the expansion of triangu-

lation over a comparatively limited area and for the making of a topographic map, this measurement should be made only with such care as will attain an accuracy such that its errors will not affect the map, although multiplied in the resulting triangulation as many times as there are stations.

3. If intended only as a base for a large-scale topographic map of but a few square miles, it will be unnecessary to determine its geodetic coordinates, as the resulting map may depend upon a plane survey.

The early method of measuring base lines consisted in the employment of *wooden rods*, varnished and tipped with metal, which were supported upon trestles and between the ends of which contacts were made with great care. The advantage of wooden rods consisted in the fact that their length is but slightly affected by temperature, and as they were thoroughly varnished they were only slightly affected by moisture. Later a more accurate method of base measuring was adopted, consisting in the employment of various forms of *compensated rods*, as the Contact-Slide Apparatus (Art. 210) of the U. S. Coast Survey and the Repsold primary base bars of the U. S. Lake Survey (Art. 212). More recently the use of *steel tapes* (Art. 204) has become popular, as the accuracy attainable with these has become better appreciated. The latest approved base bar apparatus is the *Eimbeck duplex-bars* of the U. S. Coast Survey. Finally the *iced bar* (Art. 211) is the highest development of base-measuring apparatus adopted by the same Survey.

203. Accuracy of Base Measurement.—The chief *sources of error* in base measurement, by whatsoever means made, are due to—

1. Changes of temperature;
2. Difficulties of making contact; and
3. Variations of the bars or tape from the standards.

The refinements of measurement consist especially in—

a. Standardizing the measuring apparatus or its comparison with a standard of length.

b. Determination of temperature or its neutralization by the use of compensating bars; and

c. Means adopted for reducing the number of contacts to the fewest possible, and of making these with the greatest degree of precision.

The *inherent difficulties* of measurement *with bars* of any kind are:

1. Necessity of measuring short bases because of the number of times which the bars must be moved.

2. Their use is expensive, requiring a considerable number of men; and

3. The measurement proceeds slowly, often occupying from a month to six weeks.

The *advantages of* measurements made by a *steel tape* are:

1. Great reduction in the number of contacts, as the tapes are about three hundred feet long as compared with bars of about twelve feet;

2. Comparatively small cost because of the few persons required;

3. Shortness of the time employed, an hour to a mile being an ordinary record in actual measurement; and

4. Errors in trigonometric expansion may be reduced by increasing the length of the base from 5 miles, the average length of a bar-measured base, to 8 miles, not an uncommon length for tape-measured bases.

Prof. T. C. Mendenhall, in reviewing the qualities of the various base apparatus, stated that "The use of an *iced bar* applied to the measurement of considerable distances is unquestionably the *method of highest precision*, and its cost is not believed to be greater than that of other primary methods in use in Europe, but it will not be found necessary to resort to it in ordinary practice except for purposes of standardization." He then goes on to state that "the *metallic tape* is capable of giving a result of great accuracy in the hands of experts, and that this is evidently the best device for rapid base measurement when no great precision is aimed at."

It seems that the steel tape is capable of giving a precision indicated by a *probable error* of $\frac{1}{2000000}$ part of a measured line, while $\frac{1}{1000000}$ appears to be easily and cheaply attainable with long tapes after they are standardized. This is amply sufficient for the present purposes of geodesy, and the sole obstacle in the way of much higher precision, should it be deemed essential, appears to be only the difficulty of measuring the temperature of the tape.

Bases are not measured solely for the accuracy attainable within themselves, but to attain the greatest accuracy which, when expanded through a scheme of triangulation, will not introduce into it errors of appreciable amount. Therefore it is scarcely economic to strive at an accuracy which will be greatly in excess of that attainable in the succeeding triangulation (Art. 240). Precision of measurement represented by probable errors of $\frac{1}{300000}$ to $\frac{1}{500000}$ is sufficient for all practical requirements of good primary triangulation not required in the solution of geodetic problems.

204. Base Measurement with Steel Tapes. — Steel tapes offer a means of measuring base lines which is superior to that obtained by measuring bars because (Art. 203) it combines the advantages of great length and simplicity of manipulation, with the precision of the shorter laboratory standards, providing only that means be perfected for eliminating the errors of temperature and of sag in the tape. Base lines can be so conveniently and rapidly measured with long steel tapes as to permit of their being made of greater length than has been the practice with lines measured by bars, and as a result still greater errors may be introduced in tape-measured bases and yet not affect the ultimate expansion any more than will the errors in the latter, because of the greater length of the base. Primary base lines have been measured by means of long steel tapes within recent years by the U. S.

Geological Survey and the U. S. Coast and Geodetic Survey, as well as by the Missouri and the Mississippi River Commissions, and in each case with satisfactory results.

As showing the *quality* of this work, some measures made by Prof. R. S. Woodward of the St. Albans base, previously measured with base bars by the U. S. Coast and Geodetic Survey, showed a range of several measurements of the whole base of 24.1 millimeters or $\frac{1}{160000}$ of the whole, the greatest divergence from the adopted mean being 13 millimeters or $\frac{1}{2980000}$. The *probable error* of one measure of a kilometer made at this time was $\pm$ 1.88 mm. It is believed that the probable error of a single tape length did not exceed $\pm$ 0.05 mm., while the probable error of the measurement of the whole base was $\frac{1}{2000000}$.

205. Steel Tapes.—The tapes used for this work are of steel, either 300 feet or 100 meters in length. The meter tapes used by the Coast Survey are 101.01 meters in length, 6.34 millimeters by 0.47 millimeters in cross-section, and weigh 22.3 grams per meter of length. They are subdivided into 20-meter spaces by graduations ruled on the surface of the tape, and their ends terminate in loops obtained either by turning back and annealing the tape on itself or by fastening them into brass handles. When not in use the tapes are rolled on reels for easy transportation.

The steel tapes used by the Geological Survey are similar to those used by the Coast Survey, excepting in their length, which is a little over 300 feet. They are graduated for 300 feet and are subdivided every 10 feet, the last 5 of which at either end is subdivided to feet and tenths. The various instrument-makers now carry such tapes in stock, wound on hand-reels. All tapes must be standardized before and after use by comparison with laboratory standards, and, if possible, thereafter frequently in the field by means of an iced bar apparatus. (Art. 211.)

206. Tape-stretchers.—In measuring with steel tapes a uniform tension must be given. In order to get a uniform tension of 20 to 25 pounds some form of stretcher should be used. That used by the U. S. Coast Survey consists of a

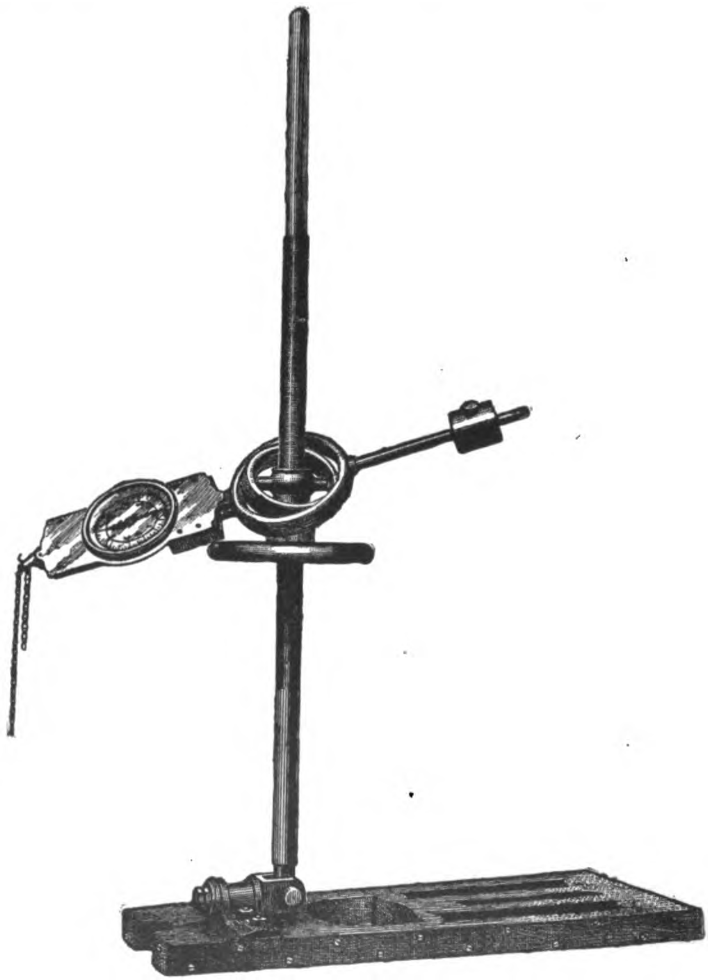


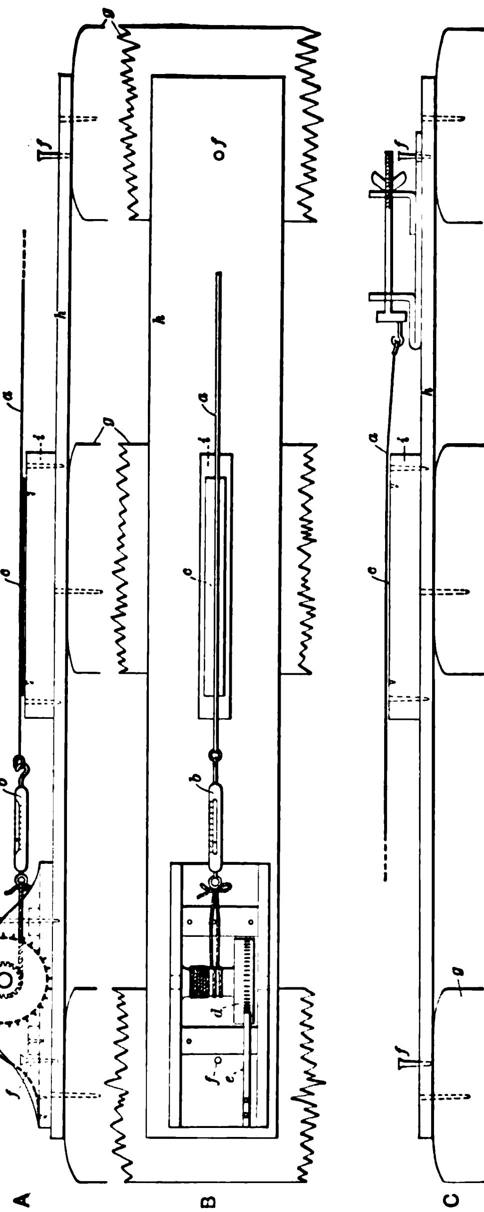
FIG. 152.—COAST SURVEY TAPE-STRETCHER.

base of brass or wood, 2 or 3 feet in length by a foot in width, upon which is an upright metallic standard, and to this is attached by a universal joint an ordinary spring-balance, to which the handle of the tape is fastened. (Fig. 152.) The upright standard is hinged at its junction with the base, so that when the tape is being stretched the tapeman can put the proper tension on it by taking hold of the upper end of the upright standard and using it as a lever, and by pulling it back toward himself he is enabled to use a delicate leverage on the balance and attain the proper pull.

The *thermometers* used are ordinary glass thermometers, around the bubbles of which should be coiled thin annealed steel wire, so that by passing that in the air adjacent to the tape a temperature corresponding to that of the tape can be obtained. Experience with such thermometers shows that they closely follow the temperature of the steel tape. For the best results two thermometers should be used, each at about one-fourth of the distance from the extremities of the tape.

The stretching device used by the U. S. Geological Survey is much simpler and more quickly manipulated than that of the Coast Survey. It is also more simple than, and, it is believed, equally as satisfactory as, that employed by the Mississippi River Commission, in which a series of weights are employed to give a proper uniform tension. The chief object to be attained in tension is steadiness and uniformity of tension; the simplest device which will attain this end is naturally the best. Two general forms of such devices are employed by the U. S. Geological Survey, one for measurement of base lines along railways, where the surface of the ties or the roadbed furnishes support for the tape, and the device must therefore be of such kind as to permit of the ends being brought close to the surface; the other is employed in measure made over rough ground, where the tape may frequently be raised to considerable heights above the surface and be supported on pegs.

The stretcher used by the Geological Survey for measuring on railways is illustrated in Fig. 153, and was devised by Mr. H. L. Baldwin. It consists of an ordinary spring-



A = FORWARD END, PROFILE
B = " PLAN
C = REAR " PROFILE

a = TAPE; b = SPRING BALANCE; c = ZINC STRIPS; d = COGWHEEL FOR STRAINING TAPE; e = RATCHET; f = NAIL FOR HOLDING GEAR IN PLACE; g = RAILROAD TIE; h = PINE STRIP; i = BLOCK SUPPORT FOR ZINC STRIP.

FIG. 153.—TAPE-STRETCHER FOR USE ON RAILROADS.

balance attached to the forward end of the tape, where a tension of twenty pounds is applied, the rear end of the tape being caught over a hook which is held steadily by a long screw with a wing-nut, by which the zero of the tape may be exactly adjusted over the mark scratched on the zinc plate. The spring-balance is held by a wire running over a wheel, which latter is worked by a lever and held by ratchets in any desired position, so that by turning the wheel a uniform strain is placed on the spring-balance, which is held at the desired tension by the ratchets.

The tape-stretcher used by the U. S. Geological Survey off railways consists of a board about 5 feet long, to the forward end of which is attached by a strong hinge a wooden lever about 5 feet in length, through the larger portion of the length of which is a slot (Fig. 154). Through the slot is a bolt with wing-nut, which can be raised or lowered to an elevation corresponding with the top of the hub over which measurement is being made, and hung from the bolt is the spring-balance, to which the forward tapeman gives the proper tension by a direct pull on the lever, the weight of the lever and the friction in the hinge being such as to make it possible to bring about a uniform tension and to hold that tension without difficulty. The zero on the rear end of the tape is adjusted over the contact mark on the zinc by means of a similar lever with hook-bolt and wing-nut, but without the use of spring-balance.

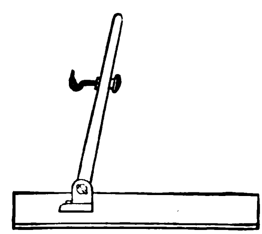


FIG. 154.—SIMPLE TAPE-STRETCHER.

207. Laying out the Base.—The most laborious operation in base measurement is its preliminary preparation or the "laying out" of the base, as it is called, which consists of—

1. Aligning it with a theodolite;

2. Careful preliminary measurement for the placing of stakes on rough ground; and

3. Placing of zinc marking-strips on the stakes and, in the case of railway measurement, on the ties.

Where a base is measured on a railway tangent, no alignment is needed beyond a provision for keeping the tape at a uniform distance from one rail. In *measuring along railways*, a number of boards 5 feet in length, and equal to the number of tape-lengths to be laid down, are provided, and nailed across the ties at the proper distances. Numbered *strips of zinc*, 6 to 18 inches in length and an inch in width, are tacked to blocks of wood nailed on the boards. The latter form the support for the tension device, and the contacts are scratched upon the strips of zinc. The *thermometers* by which the temperature is observed are wound with fine wire, and at least two are used by which careful readings are made for each tape-length. The base is invariably measured at least twice, and the two results are compared by sections of at least four tape-lengths. The measurements are preferably made at night or on very dull and cloudy days, and after the line has been once prepared a base of about 5 miles in length can be measured in as many hours.

Base lines measured with steel tapes *across country* are aligned by theodolite, and are *laid out* by driving large hubs of 3 × 6 scantling into the ground, the tops of the same projecting to such a height as will permit a tape-length to swing free of obstructions. These large hubs are placed by careful preliminary measurement at exact tape-lengths apart, and between them, as supports, long stakes are driven at least every 50 feet. Into the sides of these near their tops are driven, horizontally, long nails, which are placed at the same level by eye, by sighting from one terminal hub to the next. On these nails the tape rests, and on the surface of the terminal hubs are tacked strips of zinc on which to make the contact-marks. A careful *line of spirit-levels* must be run over the base line, and whether measured on a railroad or on rough ground the elevation of the hub or contact-mark of each tape-length must be determined in order to furnish the data for reduction, both for slope and to sea-level.

208. Measuring the Base.—A party for the measurement of a base line *along a railway* consists of *four men:* the chief of party, who marks the front extremity of the tape and has general supervision of the work; a rear chairman, who adjusts the rear end of the tape to the contact-marks, and reads one thermometer; the head chainman, who adjusts the forward end of the tape, applies the requisite tension, and reads a second thermometer; and a recorder. In measuring *over rough ground* off railways *six men* are necessary, namely, two tape-stretchers, two markers, two observers of thermometers, one of whom will record. The cooperation of these men is obtained by a *code of signals*, the first of which calls for the application of the tension, then the two tape-stretchers by signal announce when the proper tension has been applied; then the rear observer adjusts the rear graduation over the determining mark on the zinc plate and gives a signal, upon hearing which the thermometer-recorder near the middle of the tape lifts it a little and lets it fall on its supports, thus straightening the tape. Immediately thereafter the front observer marks the position of the tape graduation on the zinc plate, and at the same time the thermometers are read and recorded. By this method a *speed* can be obtained as great as six to eight miles per day.

209. Compensated Base Bars.—Compensated base apparatus consists of two bars of different metals which have different rates of expansion, laid close together, parallel and firmly fastened together at the center, from or to which point they are free to expand or contract. At a fixed temperature they are taken of the same length, so that if they experience an equal change in temperature the lines drawn parallel to their extremities will remain always at the same constant distance apart. The *two bars*, one of iron and one of brass, are each 10 feet long, $\frac{1}{2}$ inch in thickness, and $1\frac{1}{2}$ inches in width, and are placed 1.1 inches apart, connected at the centers by two transverse steel cylinders not quite in contact. At each extremity of the bars is a metal tongue so connected by pivots to the bars as to admit of free expansion. These tongues are each 6 inches long, and on a silver pivot at one end is marked the compensation point. This compound bar is *placed in* a *wooden box* and is kept from moving lengthwise by means of a brass stay firmly fixed to the bottom of the box at the center. A long level is fixed to the upper surface of the brass bar and is read by means of a glass-covered opening in the top of the box. The tongues carrying the compensation points project beyond the box, but are carefully protected, and these points lie in the line of measurement.

In measuring a base six sets of bars are used, and each when in use is *supported* at one-fourth and three-fourths of its length by means of strong brass tripods having rollers on their upper surface and provided with a tangent screw for communicating a longitudinal motion to the bar, and other screws for communicating a transverse motion, and an elevating-screw for final adjustment of the level. These tripods rest on trestles which are at various heights according to the nature of the ground. The interval between two adjacent compensating points lying in a line is brought to exactly 6 inches by means of a compensation microscope.

210. Contact-slide Base Apparatus.—The contact-slide base-measuring apparatus, made by Saegmuller & Co., consists of two measuring-bars, each 4 meters in length and supported on trestles. (Fig. 155.) The *measurement is made* by bringing these bars successively in contact, which is effected by means of a screw motion and defined by the coincidence of lines on the rod and contact-slide. Each bar consists of two pieces of wood about 8 × 14 cm. square and a little less than 4 meters long, firmly screwed together. Between the pieces of wood is a brass frame carrying three rollers, on the central one of which rests a steel rod about 8 mm. in diam-

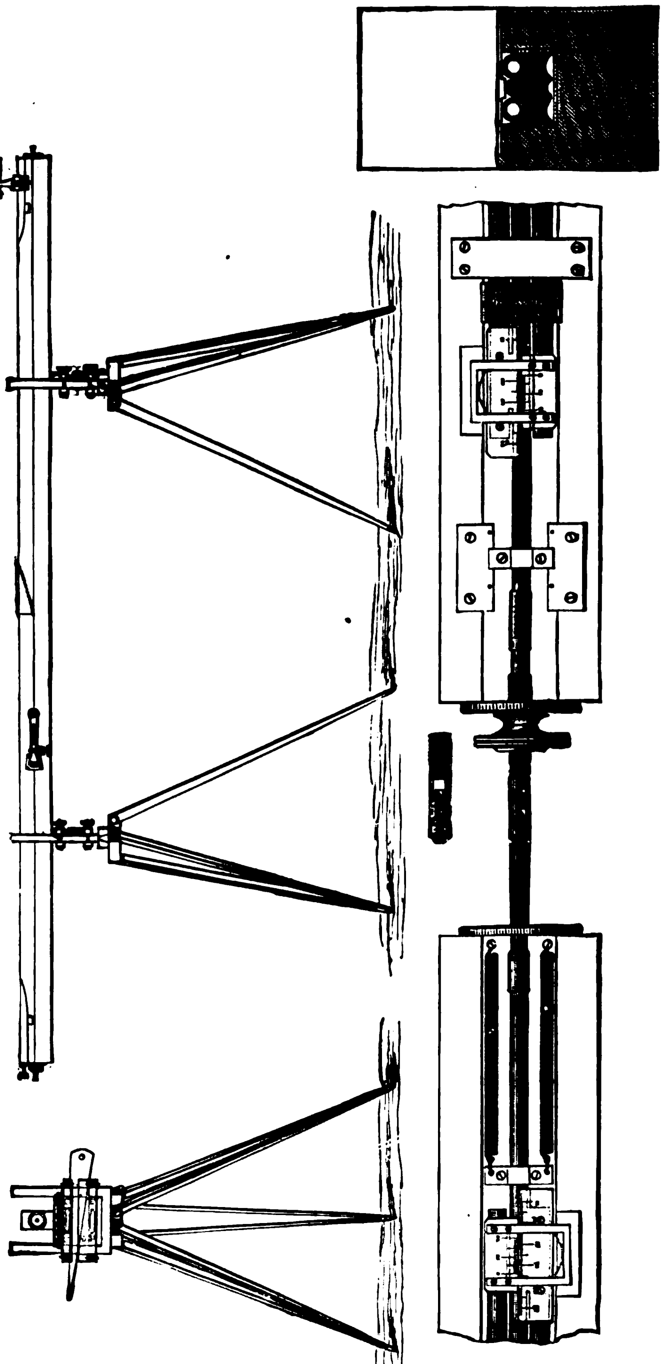


FIG. 155.—CONTACT-SLIDE BASE APPARATUS.

eter. On each side there is a zinc tube 9 mm. diameter. The rod and tubes are supported throughout their length on similar systems of rollers. The zinc tubes form with the steel rod a metallic differential thermometer, and are so arranged that one tube is secured to one end of the rod, being free to expand in the other direction, the other tube being in a like manner fastened to the other end of the rod. The *zinc tubes*, therefore, with any change of temperature, expand or contract in opposing directions, and the amount by which the expansion of the zinc exceeds that of the steel is measured by a fine scale attached to the rod, while the zinc tube carries a corresponding vernier. The cut shows this arrangement, which is identical on both ends of the bars; a perforation in the wood of the bar allows this scale to be read. In addition to these metallic thermometers a mercurial thermometer is attached to the bar about midway of its length.

The *rods and tubes* thus forming a united whole are *movable lengthwise* on the rollers by means of a milled nut working in threads cut on the steel rod, which passes through a circular opening in the brass plate screwed to the wooden bar, and against which the nut presses. Two strong spiral springs pull the rods back, and the nut is always pressed against the plate. One *end of the rod* is defined by a plain agate securely fastened to it; the other end carries the contact-slide, having an agate with a horizontal knife-edge. This slide is a short tube, fitting over the end of the rod and pushed outward by a spiral spring. A slot in the tube shows an index-plate, with a ruled line fastened to the rod.

To *align* the *bars* properly a small telescope is placed on each bar, and can be adjusted to bring the line of collimation over the axis of the rod. The trestle, shown in the upper left-hand corner of the illustration, consists of a strong tripod stand, carrying a frame with two upright guides for two cross-slides, which are separated by a movable wedge. These cross-slides can be clamped in any position. By moving the wedge, the bar resting between the uprights is either elevated or depressed. To obtain smooth movements, friction rollers are provided. To move the bars sideways, a coarse screw takes hold of a projection on the lower side of the bar, by turning which the bar can be moved laterally.

There are *three pairs of trestles*, alike in construction, with the exception that the upper slide of the trestle intended for the forward end of the bar carries a roller on which the bar rests, while the other has a fixed semi-cylindrical surface for the support of the bar. In making the measurement, the bars being 4 meters in length, the stands are set up at a distance of 2 meters, each bar being supported at one-fourth its length from the ends, as indicated by painted black bands. Each bar has a sector with level alidade attached to one side, by which its inclination can be read off to single minutes.

The U. S. Coast and Geodetic Survey has recently used with much success a new form of bimetallic contact-slide or *duplex apparatus* designed by Mr. Wm. Eimbeck. This consists of two disconnected bars of brass and steel, of precisely similar construction, each 5 meters in length. These are reversible and are contained in double metallic truss tubes the inner of which is reversible on its axis. They are so arranged as to indicate the accumulated difference of length of the measures of the brass and steel components.

211. Iced-bar Apparatus.—This apparatus, which is of recent invention and is capable of work of the highest precision, was designed by Prof. R. S. Woodward. It belongs to that type in which a *single rigid bar* is used as the element of length along with micrometer microscopes to mark its successive positions. Fig. 157 shows the iced bar in cross-section. The measuring-bar is carried in a *Y-shaped trough*, where it is kept surrounded with melting ice. The trough is mounted on two cars which move on tracks, and the *microscopes* are mounted on wooden posts which are

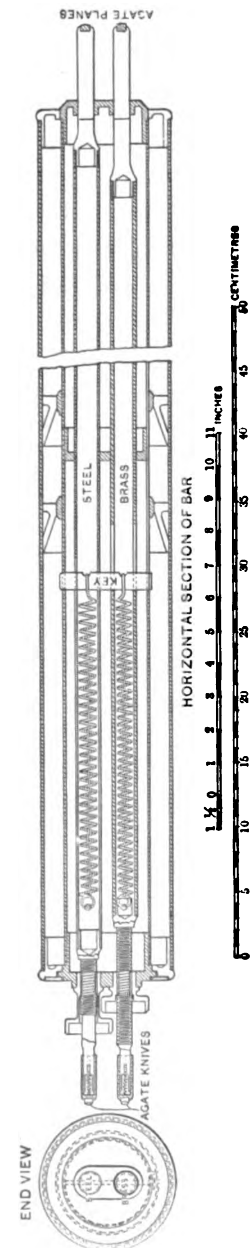


FIG. 156.—EIMBECK DUPLEX BASE APPARATUS.

ranged out and set firmly in the ground beforehand, the microscopes being clamped or detached from the posts easily in moving forward as the measuring of the line progresses, and the Y trough being likewise rolled forward on cars over a temporary track. The apparatus is 5 meters long, the microscope posts being set 5 meters apart, and the supports for the car-track a like distance. In field-work the microscopes are shielded by umbrellas instead of by temporary sheds, as in the illustration.

The *measuring-bar* is a rectangular bar of tire-steel 5.02 meters long, 8 mm. thick, and 32 mm. deep. The upper half of the bar is cut away for about 2 cm. at either end to receive the graduated plates of platinum-iridium, which are inserted so that their upper surface lies in the neutral surfaces of the bar. Three lines are ruled on each of these plugs, two in the direction of and one transverse to the length of the bar. The Y trough supports the bar, keeps it aligned, and carries the ice essential to the control of the temperature of the bar. It is made of two steel plates 5.14 m. long, 25.5 cm. wide, and 3 mm. thick. They are bent to a Y shape, angle of 60 degrees, and riveted together at the stem. The bar is supported at every half-meter of its length by saddles, as shown in the illustrations, and these are rigidly attached to the sides of the trough by screws, each saddle carrying two lateral and one vertical adjusting-screw.

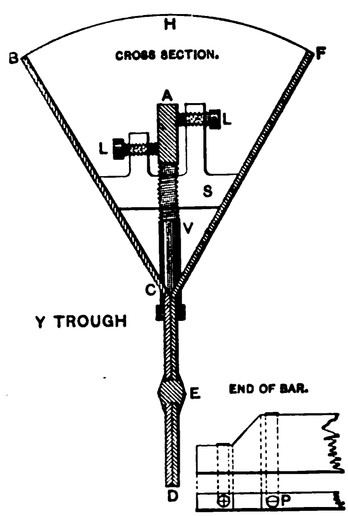


FIG. 157.—CROSS-SECTION OF ICED-BAR APPARATUS.

When the apparatus is in use the Y trough is completely filled with pulverized ice, the upper surface of which is rounded to about the height shown by the curve in the diagram, that is, to the top of the trough. The amount of ice required for this purpose is about 8 kg. per meter of the bar's length. By reason of the sloping sides of the trough, the ice is kept in close contact with the bar, especially as the trundling of the car produces sufficient jarring to overcome any tendency of the ice to pack. An essential auxiliary to the apparatus is an *ice-crusher* and a plane for shaving fine ice to pack the ends of the bar. The micrometer microscopes which define the successive positions of the bar in measuring a line are similar to those used in the Repsold base-measuring apparatus. (Art. 212.)

212. Repsold Base Apparatus.—This is an unusual apparatus and has been used in this country in measuring primary base lines of the U. S. Lake Survey. The following description of it is copied from the final report of that organization:

This consists of a *measuring-bar of steel* approximately 4 meters long. (Fig. 158.) Its exact length at any temperature is known. By the side of the steel bar is a similar zinc bar. The two are fastened firmly together in the middle. Their unequal expansion is observed upon scales at both ends, making a metallic thermometer by which the temperature of the steel bar becomes known. These two bars are placed within a hollow iron cylinder, called the *tube-cylinder*, which supports them rigidly and protects them from sudden changes of temperature. The bars are supported in the cylinder by a system of rollers which keeps them straight, parallel, and at constant distance from each other. The combination of the two bars and the tube-cylinder is called a *tube*. The tube is provided with a sector which indicates the deviation of the tube from the horizontal, so that a base can be measured upon slightly inclined as well as upon level surfaces. A telescope is also attached, which points in the same direction as the tube and enables consecutive tube measurements to be kept in the same vertical plane.

In *measuring a base* the rear end of the tube is placed at the beginning of the line and the position of the front end is marked. Then the tube is carried forward and the rear end is placed at the mark and the front end is marked again, and so on, in the same way that a line is measured with a chain and pins. In order that the tube may stand firmly it is supported upon iron stands, one at each end. These stands have three legs, which rest upon iron pins driven in the

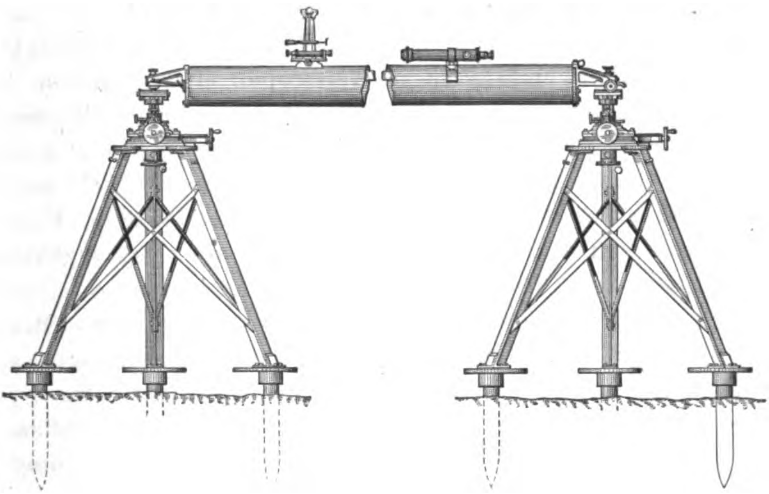


FIG. 158.—REPSOLD BASE APPARATUS.

ground. To place the tube exactly in the line and at a proper height, the tops of the tube-stands are provided with movements in three directions, by means of which the tube can be moved sidewise, lengthwise, and up and down. For convenience there are four tube-stands, so that two can be placed in position while the tube is resting on the other two.

The positions of the ends of the tube are marked with *microscopes.* Thus while the tube does the work of a chain, the microscopes do that of the pins. The microscopes are mounted upon iron stands, which, like the tube-stands, are supported upon iron pins driven into the ground. The microscope-stands are so constructed that the microscope can be placed directly over the end of the tube. The microscopes are provided with two motions, so that they can be moved a short distance along the line or at right angles to it. They also have levels attached, so that they can be made vertical. For convenience there are four microscopes, so that two can be placed in position while two are standing over the ends of the tube.

To *measure a tube-length*, the rear end of the tube is placed under the microscope which marks the position of the front end of the preceding tube-length. The tube is then brought into the line by means of its telescope. Its inclination is found by reading its sector, and the temperature of the steel bar is found by microscope readings on the scales at front and rear ends. From the temperature the length of the steel bar is found at the instant the measurement is made. From its inclination the horizontal projection of this length is found, and thus the actual advance becomes known.

213. Base Lines: Cost, Speed, and Accuracy.—Base lines of the highest attainable accuracy of measurement, as those measured by the U. S. Coast and Geodetic Survey, cost from twenty-five hundred to thirteen thousand dollars, according to the methods employed and the precision aimed at. The speed of measurement by the U. S. Coast Survey, using base bars, is from two to six months, including preparation and actual measurement. The probable error of the result attained is from $\frac{1}{1000000}$ to $\frac{1}{1500000}$.

Base lines as measured by the U. S. Geological Survey with sufficient accuracy for the expansion of primary triangulation which is to be developed to distances of 200 to 400 miles, cost from one hundred to two hundred dollars per base. This work is executed with steel tapes and requires from seven to ten days for preparation and measurement of the base. The accuracy attained has an average probable error of $\frac{1}{300000}$.

CHAPTER XXII.

COMPUTATION OF BASE MEASUREMENT.

214. Reduction of Base Measurement.—After the measurement of the base line has been completed in the field, the results of the measurement have to be reduced for various corrections, among which are:

1. Comparison with standard of measure;
2. Corrections for inclination and sag of tape if such is used;
3. Correction for temperature;
4. Reduction to sea-level.

As an example of the method of making such corrections and of keeping the records of base measurement, the following has been selected from the measurement of the Spearville base in Kansas by the U. S. Geological Survey.

215. Reduction to Standard.—The first correction to be applied is that of reducing the tape-line or the base-bars to a standard. The data for this reduction is best obtained by a comparison with the international standards of the U. S. Coast and Geodetic Survey, that at their office in Washington, or the standard kilometer measured at the Holton base in Indiana. Or, if these are not accessible, comparisons may be made with standards in the possession of the U. S. Mississippi and Missouri River Commissions and of one or two of the more reliable instrument-makers of the country.

The reduction to standard may be positive if the tape is longer than the standard, or negative if shorter, and this reduction is proportioned to the entire length of the line, and is generally made by multiplying the length of the tape or bar as obtained from comparison with the standard into the number of times which the same is applied on various sections or the whole of the base.

An example of this reduction is given in Art. 217, which contains a record of the measurement of the Spearville base.

216. Correction for Temperature.—As the length of a steel tape or a metal bar varies with the temperature, one of the most uncertain elements in the measurement of a base by means of a steel tape is its length as compared with the standard because of variations due to expansion and contraction from changes in temperature. As already shown, the most accurate mode of measurement obtainable is that in which the temperature is fixed, as in the case of the iced-bar apparatus (Art. 211). In any other form of base-measuring apparatus every effort must be made to obtain the greatest uniformity of temperature, and in using a tape corrections must be made for every tape-length, as derived from readings of one or more thermometers applied to the tape in the course of the measurement.

Steel expands .0000063596 of its length for each degree Fahrenheit of temperature. This fraction, multiplied by the average number of degrees of temperature above or below 62 degrees at the time of the measurement, gives the proportion by which the base is to be diminished or extended on account of temperature changes. This correction is applied usually by obtaining with great care the mean of all thermometric readings taken at uniform intervals of distance during the measurement. An example of the record of temperature and of reduction for temperature is given in the last two columns of the table in the following article.

217. Record of Base Measurement.—The following is a sample page from the note-book containing record of measurement of the Spearville Base as made by Mr. H. L. Baldwin of the U. S. Geological Survey in 1889. This base was measured along a railroad and therefore has no correction for sag as the tape rested on the ties. It was measured in a number of sections, and the following is that of two measurements of the first section. In the last column is shown the method of making corrections to the tape to reduce to standard (Art. 215), and in the next to the last column the method of making temperature correction (Art. 216).

RECORD OF BASE MEASUREMENT AND REDUCTION.

(First measurement, section 1, October 16, 1889.)

H. L. Baldwin, Topographer.

No. of Tape.	Time, A.M.	Tension.	Thermometers. A.	Thermometers. B.	Temperature Correction.	Total Length of Section.
	h. m.	Pounds	°	°		
1	10 13	19.75	50.5	50.0		
2	20	20.00	50.5	50.0		
3	26	20.00	50.5	50.0		
4	31	20.25	50.5	50.0	Mean temp. = 50°.51	
5	37	20.00	50.7	50.5		1 tape-length....... = 300.0617
6	42	20.125	51.5	50.6	62° − 50°.51 = 11°.49	
7	47	20.25	51.0	50.8		10 × 300'.0617....... = 3,000'.617
8	51	20.00	50.8	50.2	− 11°.49 × 3000'.	Temperature corr.. — .207
9	55	20.125	50.8	50.0	× .000006	
10	58	20.00	50.7	50.5	= − '.207	Result first meas... = 3,000.410

(Second measurement, October 17, 1889.)

No. of Tape.	Time, P.M.	Tension.	Thermometers. A.	Thermometers. B.	Temperature Correction.	Total Length of Section.
	h. m.	Pounds	°	°		
1	12 13	20.00	52.3	52.4		
2	21	20.25	53.3	52.9		
3	25	20.00	53.8	54.0		Tape set back from sta. 0
4	29	19.75	55.0	54.8	Mean = 53°.96	.85 inch. = .071 foot.
5	33	20.00	55.0	53.2		
6	36	20.00	53.8	54.0	62° − 53°.96 = 8°.04	10 × 300'.0617...... = 3,000'.617
7	38	20.00	54.0	54.0		Set back........ — .071
8	41	20.12	54.5	54.0	− 8°.04 × 3000'.	Temperature corr.. — .145
9	45	19.75	55.1	54.4	× .000006	
10	50	20.13	54.5	54.1	= − '.145	Result sec. meas.. = 3,000.401

218. Correction for Inclination of Base.—The data for this correction are obtained by running a careful line of spirit-levels over the base line (Chap. XV). In the course of this leveling, elevations are obtained for every plug upon which the tape rests. The result of this leveling is to give a profile showing rise or fall in feet and fractions thereof between the points of change in inclination of the tape-line. From this and measured distances between these points the angle of inclination is computed by the formula

$$\sin\theta = \frac{h}{D}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (40)$$

in which D = length of tape or measured base;
h = difference in height of the two ends of the tape or measured base, both in feet; and
θ = angle of slope expressed in minutes.

The correction in feet to the distance is that computed by the equation

$$\text{Cor.} = D\frac{\sin^2 1'}{2}\theta^2. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (41)$$

Since, however, $\dfrac{\sin^2 1'}{2} = 0.0000000423 1$, we have

$$\text{Cor.} = 0.00000004231\,\theta^2 D. \quad . \quad . \quad . \quad . \qquad (42)$$

As the logarithm of the constant is equal to 2.6264222, the above may be expressed in logarithms, thus:

$$\text{Cor.} = \log 2.6264222 + 2\log\theta + \log D. \quad . \qquad (43)$$

An example of the record and mode of making corrections for inclination is given in the following, taken from Spearville Base measurement:

S. S. Gannett, Computer.

Approximate Distance.	Difference of Elevation	Angle θ	log θ	2 log θ	log $\frac{\sin^2 1'}{2}$	log dist.	log correction.	Correction.
feet.	feet.	′ ″						
0,200	0.8	13 34	1.1326	2.2652	2.6264	2.3010	7.1926	.0015
4,200	4.2	2 22	0.3674	0.7348		4.6232	6.9844	.0010
4,000	12.0	10 08	1.0052	2.0104		3.6021	8.2389	.0173
1,000	1.0	3 23	0.5250	1.0501		3.0000	6.6765	.0005
2,000	3.0	5 04	0.7024	1.4049		3.3010	7.3323	.0021
4,020	22.0	12 23	1.0917	2.1834		3.6232	8.4330	.0271
2,800	7.0	8 27	0.9263	1.8527		3.4472	7.9263	.0084
1,000	0.0	0 00	0.0000	0.0000	Constant.	3.0000	0.0000	.0000
1,000	1.0	3 23	0.5250	1.0500		3.0000	6.6764	.0005
4,200	20.0	11 16	1.0504	2.1008		3.6232	8.3504	.0224
3,800	6.0	5 20	0.7267	1.4535		3.5798	7.6597	.0046
2,000	4.0	6 45	0.8293	1.6586		3.3010	7.5860	.0038
5,400	31.4	19 39	1.2934	2.5867		3.7324	8.9455	.0882
2,000	2.6	4 24	0.6437	1.2874		3.3010	7.2148	.0016
0,135	0.05	1 18	0.1072	0.2144		2.1303	4.9712	.0000
								.1790

An *approximate formula* for reducing distances measured on sloping ground to horizontal is expressed by the rule: Divide the square of the difference of level by twice the measured distance, subtract the quotient thus found from the measured distance, the remainder equals the distance required; or,

$$d = D - \frac{h^2}{2D}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

in which $d =$ horizontal or reduced length. This formula may be used in reducing the various inclined measures made over rough ground in primary traverse (Art. 227).

Example: Let 50 = length in feet of distance measured on slope, 3 = difference in height in feet between two ends of measured line, then

$3^2 = 9; \quad 9 \div (50 \times 2) = .09$ (exact formula (41) gives .0908).

$50 - .09 = 49.91 = d =$ horizontal distance required.

219. Correction for Sag.—When the base measurement is made with steel tape across country, and is, accordingly, not supported in every part of its length as on a railway, there will occur some change in its length due to sag. As explained in Art. 207, where measurement is made off a line of railroad, the tape should be rested on supports placed not less than 50 feet apart. With supports placed even this short distance apart, however, a change of length will occur between them, while even greater changes will occur should one or more supports be omitted, as in crossing a road, ravine, etc. As tapes are standardized by laying them on a flat standard, it is essential to determine the amount of shortening from the above causes. The following formulas apply:

Let $w =$ weight per unit length of tape;

$t =$ tension applied;

$a = \frac{w}{t}$;

n = number of sections in which tape is divided by supports;

l = length of any section;

L = normal length of tape or right-line distance between n marks when under tension = nl, approximately;

μ = reciprocal of product of modulus of elasticity of tape by area of its cross-section.

If a tape be divided by *equidistant supports*, the difference in distance between the end graduations, due to sag, or the correction for sag = dL, becomes

$$dL = \tfrac{1}{24}a^2(n_1l_1^3 - n_2l_2^3).$$

If one or more *supports* are *omitted*, then the omission of m consecutive supports shortens the tape by

$$\tfrac{1}{24}m(m+1)(m+2)a^2l^3;$$

where l is the length of the section when no supports are omitted.

Example: Let $n = 6$; $l = 50$ feet; $w = .0145$ = weight in pounds per foot found by dividing whole weight of tape by whole length; and $t = 20$ pounds;—then

$$dL = \frac{nl}{24}\left(\frac{wl}{t}\right)^2 = \frac{6 \times 50}{24}\left(\frac{0.0145 \times 50}{20}\right)^2 = 0.0162 \text{ feet},$$

which is the length of sag or shortening of each tape-length. This correction is always negative.

If in a certain measure of a base there were 86 full tape-lengths, the total correction for sag would be

$$\text{Cor.} = 86 \times .0162 = 1.393 \text{ feet},$$

which is quite an appreciable quantity.

220. Reduction of Base to Sea-level.—The base is always measured on a circle parallel to the mean sea-surface and raised above it at an elevation the amount of which must be known, at least approximately. This circle with radii drawn therefrom to the center of the earth forms a triangle approximately similar to that formed by the radii of the earth with the sea-surface. The length of the base at sea-level is therefore derived with a sufficient approximation to correctness by the proportion

$$r : H :: D : d, \text{ or Cor.} = \frac{DH}{r}, \quad . \quad . \quad . \quad (45)$$

in which $r =$ the radius of the earth;
$H =$ the height of base line above mean sea-level;
$D =$ the measured length of the base line;
$d =$ the correction to reduce this measured length to length at mean sea-level.

An example of the form of such reduction is the following, taken from the Spearville Base:

REDUCTION TO SEA-LEVEL.

Correction..................	$= \dfrac{DH}{r}$
log D (meters)...............	= 4.05956
log H (meters)..............	= 2.87599
Co log r.....................	= 3.19660
log 1.356 meters.............	= 0.13215
log. meters to feet...........	= 0.51599
4.448 feet (always subtractive)....	.0.64814

221. Summary of Measures of Sections.—Corrections for temperature and standard having been made to each of the sections of the measured base (Arts. 215 and 216), the mean of the several measures of each section must be obtained and the total length of the base will then be obtained by summation of the reduced lengths of sections. The table on page 524 is an example of the record of such summary of sections.

222. Corrected Length of Base.—The foregoing corrections and summations having been made, the correct length of the base may now be obtained by applying the corrections for inclination and reduction to sea-level both of which are

SPEARVILLE BASE: SUMMARY BY SECTIONS.

(Corrected for Temperature.)

S. S. GANNETT, Computer.

Stations.		First Measure.	Second Measure.	Difference. *First – Second.*
		feet.	feet.	feet.
1 to	10	3,000.410	3,000.401	+.009
10	20	.418	.393	+.025
20	30	.431	.431	+.000
30	40	.426	.446	−.020
40	50	.437	.478	−.041
50	60	.417	.455	−.038
60	70	.369	.392	−.023
70	80	.366	.356	+.010
80	90	.955	.938	+.017
90	100	.676	.667	+.009
100	110	3,000.899	3,000.898	+.001
110	119	2,700.581	2,700.571	+.010
119	126	2,100.244	2,100.234	+.010
		37,806.629	37,806.660	−.031

always negative (Arts. 218 and 220). This is done in the following manner:

MEAN OF TWO MEASUREMENTS.

Correction for temperature and standard.....	37,806.645 ft.
" " inclination..................	0.179 "
" " reduction to sea-level.........	4.448 "
Final corrected length of measured base......	37,802.018 ft.

223. Transfer of Ends of Base to Triangulation Signals.—In the foregoing article the corrected length of the base, 37,802.018 feet, would be the final length of the base as determined under ordinary circumstances—that is, when the ends of the base line are also the astronomic pier and triangulation station from which the expansion of the base is made. Occasionally, and especially where a base is measured on a railroad, it is impossible to erect the astronomic pier or the triangulation stations (Art. 243) over the extremities of the base, and it then becomes necessary to transfer the measured length of the base to the triangulation signals and pier, which are erected as near as possible to each end.

In the case of the Spearville Base, the astronomic pier was first erected at the northeast extremity of the base and a triangulation signal was erected near the railroad track at the point selected for the southwestern extremity of the base, and triangulation was started by reconnaissance and erection of signals prior to the measurement of the base line. Accordingly, after the base had been measured its length had to be transferred to the triangulation signals. The following is an example of the elements of this reduction.

The end O (Fig. 159) was not selected at exactly right

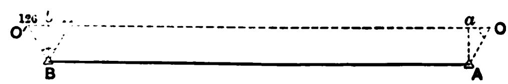


FIG. 159.—TRANSFER OF MEASURED BASE (OO') TO MARKED BASE (AB).

angles to the pier and station, but at a distance a little beyond the extremity of the pier so that the angle between the base and the pier was less than 90°. These angles were carefully measured at the extremities of the base O and O', also the distance from O to the pier at A, and the distance a to O. Solution of the right-angled triangle AaO gave $aO = D = 2.864$ feet. At the southwest extremity of the base marks were left at the 125th and 126th tape-lengths, and the angles read at these points between the measured base line and signal B, also the angles at the signal B to those marks, the distance between them also being noted as an exact tape-length. These data gave the elements necessary for the solution of the right-angled triangles into which the main triangle was divided by the projection of B at right angles to the base line at the point b, and the amount determined by which the measured base was to be reduced was $b + 126 = 168.235$ feet.

The following is the mode of applying this correction to the corrected measured base length to get the secondary base or the distance between the triangulation signals:

Corrected length of measured base..........................	37,802.018 ft.
Reduction from southwestern base to triangulation signal...	168.235 "
Reduction from northeastern base to triangulation station..	2.864 "
Corrected length of triangulation base.....................	37,630.919 ft.

224. Other Corrections to Base Measurements.—In addition to the simple corrections above given, which are always made to the measurement of base lines, it is sometimes desirable to determine the *modulus of elasticity* of the metal in order to make corrections for the pull in stretching the tape. This correction is, however, often of doubtful application, because the exact amount of pull at any time may be carelessly noted. It is far better and quite as simple to *eliminate such corrections* by giving a uniform pull at all times, thus doing away with the correction for modulus of elasticity. Another correction is for *metallic thermometers*; but as glass thermometers can be purchased without difficulty and almost anywhere, it seems unnecessary to make provision for such correction.

225. To Reduce Broken Base to Straight Line.—Occasionally, because of some obstacle to the straight alignment of the base or in order that either extremity may terminate in the most desirable position for the expansion of triangulation, it becomes necessary to introduce one or more angles in a base measurement. This, however, should never be done unless absolutely unavoidable, and then such angles should never deviate greatly from 180°. The correction for such a broken base may be expressed as follows:

If the measured base be in two lengths, A and B and it being necessary to find the third side of the triangle which they form, the latter being the straight line L; then, θ, being the difference between the angle and 180°, we have

$$L = A + B - 0.0000000423 1 \frac{AB\theta^2}{A + B}. \quad . \quad . \quad (46)$$

This formula cannot be employed, however, where θ is greater than 5°, in which case the unknown side will have to be computed by the ordinary sine formula for the solution of triangles. (Chap. XXVII.)

CHAPTER XXIII.

FIELD-WORK OF PRIMARY TRAVERSE.

226. Traverse for Primary Control.—It is frequently inexpedient, because of the relative expense, to procure primary control for topographic mapping by means of triangulation. Sometimes, especially in heavily forested and level country, it is practically impossible to execute primary triangulation within any reasonable limits of time or cost. The means adopted for securing sufficient primary control under such conditions is by the running of traverse lines of a high degree of accuracy.

Primary traverse does not differ from secondary traverse, such as is executed for the determination of topographic details (Art. 87) in the general methods of its execution. It does differ therefrom materially in the quality of the instruments employed and the elaborateness of detail with which the field and office computations are conducted. It may, therefore, be likened rather to the measurement of a series of long and broken but connected base lines measured in a manner rather similar to that explained in Art. 208, but with less care.

As primary traverse is executed for control of topographic mapping, it furnishes the initial coordinates by which the topographic map is fixed in astronomic position. One or more points of the primary traverse must therefore have their geodetic coordinates determined by astronomic observation (Part VI) or by connection with a system of primary triangulation (Chap. XXV). Primary traverse is materially inferior in quality as control to primary triangulation (Art.

201). In order that errors occuring in its execution may be reduced, it is desirable that the two extremities, and, if very long, a middle point on the primary traverse line, be checked either by closing the traverse back on itself or on some other adjusted primary traverse.

The best and in fact the only satisfactory means of distributing by adjustment the errors inherent in the primary traverse is to connect at least two of its more remote points with primary triangulation stations or astronomic positions. (Chap. XXV and Part VI.)

227. Errors in Primary Traverse.—The errors inherent in primary traverse are of three general classes:

1. Those due to measurement of deflection angles or azimuth errors;
2. Those due to linear measurement or errors of distance;
3. Instrumental errors.

Probably the most serious errors introduced in primary traverse are those due to the measurement of the deflection angles or the *azimuth errors*. These are of several kinds and are affected—

1. By the quality and graduation of the instrument;
2. By the shortness of the sights;
3. By the relative dimensions and plumbing of the flag;
4. By the care exercised in centering the instrument over stations.

The first is to be provided against only by use of such an instrument as is best suited to the work to be done and by keeping it in perfect adjustment. The second is the most important source of error and is not to be confused with the irregularity and *sinuosity of the traverse* run. This may be ever so winding, yet, if the sights are sufficiently long and the angles not great, the errors by such an irregularity will not be of serious moment. These errors are affected by the third and fourth factors, and in sinuous traverses of *short sights* the errors in bisecting a large signal or any centering over a station become matters of considerable moment.

Where such ordinary care is exercised as is indicated in Articles 228 and 229, and in the instructions Art. 231, the errors of measurement will be relatively small. Likewise, the *errors of instrument* should be small providing the ordinary precautions designated for care and adjustment of instruments are exercised.

In the *measurement of distance* the most important source of error is:

1. Failure to keep the tape horizontal;

2. Carelessness in plumbing down to center points where there is much inclination and short tape-lengths are used;

3. Failure to apply a uniform tension; and

4. Errors in count or record of number of tape-lengths.

Where the traverse is over a good line of railroad having easy grades and long tangents the best results may be expected. In such case it is unnecessary to keep the tape horizontal by lifting it above the ground, it being sufficient to rest it upon the ties. The error of slope on a good railway grade is less than that of sag when the tape is held horizontally. Under such circumstances the chief source of error is likely to be in the measurement rather than in the azimuth. On the other hand, where the traverse is run over rough ground the inaccuracies introduced are greatest in amount. Then, as in running railroads having short tangents and consequently short sights, a considerable source of error is in the azimuths, and even a greater source of error arises from the necessity of taking short tape-lengths on sloping ground and plumbing down to center marks. It is evident, therefore, that not only is greater precision obtained in measuring over good lines of railway, but also the speed is materially increased and the cost reduced proportionately.

228. Instruments used in Primary Traverse.—The instrument used for measuring *azimuths* should be a *transit* of high grade, having a six- to eight-inch circle and reading to 20 or 30 seconds. Such an instrument should have a hollow telescope axis and be provided with a lamp and other attachments for night-work. As an important source of error in such work is in the azimuth, this should be checked nightly, if weather permits, by observations on a circumpolar star (Art. 312) at or near elongation. When the line of traverse is crooked such observations should never be at intervals greater than ten to fifteen miles. When the route traversed has long tangents, distances between check azimuths may be increased.

There are two methods of measuring the horizontal angles:

1. By transiting the telescope and reading forward deflection angles as with an engineer's transit (Art. 87);

2. By reading full circle or deflection angles from backsight to foresight.

The former is preferable when the instrument is kept in good adjustment, as it is more rapid and more accurate. The process consists in sighting on rear flag, transiting telescope, and revolving on upper circle until fore flag is bisected by the cross-hairs. The angle read is the deflection from the last sight prolonged to the new sight (Fig. 67). Then the upper circle is again revolved through nearly 180° until the rear flag is again bisected. Once more the telescope is transited and the fore flag bisected. The result is two separate records and two measures of the angle, one a single measure, and the other double. Moreover, one pointing is with telescope direct, and the other with it reversed.

The second method of measuring the horizontal reflections is to point the telescope at the rear flag and read both verniers as before. Then with lower motion clamped, the instrument is revolved horizontally on the upper plates without transiting, and pointed at the fore flag, and both verniers are again read. The difference between the two readings is the deflection or arc through which the telescope has been revolved. By repeating this operation at least two measures are made, one with telescope direct, and the other with it reversed and should be on different parts of the circle. (Art. 252.)

Distances in primary traversing should be measured with a three-hundred-foot *steel tape* of kind similar to those employed in measuring base lines (Art. 205). The tape should be tested by a standard and be corrected for average temperature somewhat as in measuring base lines (Art. 216). Every effort should be made to use only even tape-lengths. As the tapemen, however, approach the instrument point a tape-length must necessarily be broken, and care must be exercised in the precautions employed to measure the fractional tape. A good way of doing this is by having a three-hundred-foot tape divided by clear markings into hundred-foot lengths and then to use a standardized one-hundred-foot steel tape for measuring the fraction less than one hundred feet.

229. Method of Running Primary Traverse.—The party organization for running a primary traverse should consist of five or six persons; namely, the chief as transitman, one recorder, two tapemen, and one or two flagmen. The *transitman* directs the movements of the other members of the party and determines directions by reading angles on the transit instrument. He also reads the compass-needle as a check on the azimuth computation. The *recorder* keeps a record of the angles called off by the transitman, works up the mean pointing as the work advances, notes by observation of the angles recorded whether any gross error has been made in reading of the transit vernier, and calls the attention of the transitman to such errors if any exist. He or the chief of party also checks the measurement of distance by the tapemen by counting rail-lengths or by pacing, recording the same in the first or station column as shown in the example (Art. 230), thus checking the liability of making gross errors.

About once an hour he also reads a thermometer held beside the tape at an instrument station.

The *tapemen* measure the distance with the steel tape, which is stretched by a twenty-pound tension on the front end by the fore tapeman with a spring-balance. Temperature is also read and recorded by one of the tapemen, and both tapemen keep a record of the number of tape-lengths between stations. These distances are worked up daily into the notes kept by the recorder. The rear *flagman* gives backsight for the transitman, who aligns one of the tapemen as a fore flagman. Or a fore flagman may be employed, when the speed will be increased somewhat.

The initial and terminal points of the primary traverse must be well indicated by *permanent marks*, as should numerous intermediate points on the line of the traverses, especially at such places as may be used as tie points for other primary or secondary control (Art. 248). All road crossings, stream crossings, railway stations, and other permanent objects should be indicated in the note-books, that they may furnish check points for the control of the topographic or secondary traverse (Arts. 14 and 16), and connection with the leveling (Chap. XV).

230. Record and Reduction of Primary Traverse.—The following is an example of record and reduction of a portion of a primary traverse run near Traskwood, Arkansas, by Mr. George T. Hawkins of the U. S. Geological Survey. In the first column are given the distances between stations in feet, checked by counting rail-lengths; in the following three columns are given the readings of the angles recorded by the separate verniers and their mean; in next to the last column is recorded the deflection angle; and in the last column are given the computed and corrected azimuths.

The azimuth recorded in this column in plain type is that carried forward by computation from the last station to the station occupied, being the algebraic addition to the former of the deflection angle at the latter. Underneath, in italicized figures, is given the reduced or corrected azimuth which is to be used in the further computations. This is obtained by distributing the error found between the last and the next observed astronomic azimuth. (Chap. XXXIII.)

Distance.	Ver. A.	Ver. B.	Mean.	Angle.	Azimuth.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
					44 30 17
Sta. 107	96 09 00	276 09 00	96 09 00	0 00 00	
	96 09 00	276 09 00	96 09 00	0 00 00	
(60 rails)			96 09 00	0	44 30 17
1800 feet	96 09 00	276 09 00	96 09 00		*44 30 10*
	96 09 00	276 09 00	96 09 00	0 00 00	
Sta. 108	96 09 00	276 09 00	96 09 00	0 00 00	
			96 09 00	0	44 30 17
					44 30 04
(100 rails)					
3000 feet					
	Brought forward from Sta. 108 to Sta. 132.				34 43 15
Sta. 132	76 27 00	256 26 30	76 26 45		
	84 05 00	264 04 30	84 04 45	7 38 00	
			84 04 45	7 38 30	42 21 30
(40 rails)	91 43 30	271 43 00	91 43 15	+ 7 38 15	*42 18 45*
1200 feet					
Sta. 133	91 43 30	271 43 00	91 43 15		
	92 20 00	272 19 30	92 19 45	0 36 30	
			92 19 45	0 36 45	42 58 07
	92 56 30	272 56 30	92 56 30	+ 0 36 37	*42 55 15*
	Station 133 + 130 feet is in front of middle window in Traskwood Depot.		*Observed azimuth at Traskwood between stations 133 and 134.*		

231. Instructions for Primary Traverse.—The details in running primary traverse are best explained in the following instructions, which govern the execution of such work by the U. S. Geological Survey:

1. The instruments to be used are a 20″ or 30″ transit; one 300-foot steel tape graduated to feet for five feet at either end; one spring-balance; one 100-foot steel tape; two ther-

mometers; four hand-recorders; two flagpoles; and one good watch.

2. The party should consist of: One chief, as transitman; one recorder; two tapemen, either of whom may act as front or rear flagman; and one flagman.

3. At each station the transitman should proceed as follows: Set telescope on rear flag, read both verniers, transit telescope, set on front flag, and read both verniers. Shift the circle and remeasure the same angle with telescope reversed. If the two angles thus measured differ more than 60″, repeat the operation.

4. Along a railroad the operation of measuring is to be conducted as follows: The front tapeman puts a 20-lb. tension on the front end of the 300-foot tape with a spring-balance. He makes a chalk-mark on the rail, or places a tack or nail on a tie, stake, or measuring-board, under the 300-foot mark for full tape-lengths, and under the fractional graduation at stations. The distance which he records is checked by the transitman and at least one other member of the party. The tack or nail is left, surrounded by conspicuous chalk-marks, and the same process is continued.

5. The rails should be counted by two others of the party, who also check the number of tape-lengths at the first opportunity. Each station should be marked by a small-headed tack or pricking-needle through a piece of white paper or cloth, its number being chalked on the rail near where it falls. The distance between stations should be limited to the visibility of the flagpoles. Rails or center of track must not be used as alignment sights.

6. Along highways or open country the tape should be kept level. On steep slopes a plumb-bob must be used, either to bring the tape vertically over an established point or to establish a new one, as the case may be. Tape-lengths are marked on the measuring-board, tie, or stake with the pricking-needle. Where slopes are so steep as to render the level-

ing of the 300-foot tape impracticable a shorter tape must be used.

7. The chief and two other members of the party must keep an independent count of tape-lengths. The temperature of the tape must be taken every hour in the day. Stations should be made at even tape-lengths whenever practicable.

8. Observations for azimuth must be made at close of each day's work when possible, and azimuth stations should not be more than ten miles apart, except on long tangents.

9. An azimuth observation must consist of not less than three direct and three reverse measures on three parts of the circle between Polaris and an azimuth mark, to be made at any hour, but preferably near elongation, and the place, date, time, and watch error should be recorded.

10. The watch should be compared with standard time often enough to determine its error within ten seconds.

11. Where the line traversed is very crooked the instrument should be fitted for observation of solar azimuths, and these should be made at least twice in each day, weather permitting, in addition to Polaris observations.

12. The record must contain a description of the starting-point of the line and the beginning and ending of each day's work; also, location of each railroad station, mile-post, and switch passed, and wagon-road, stream, land or county line crossed, and connection with corners of the public-land surveys.

13. Two permanent marks, either the copper bolts or the standard bronze tablets of the Survey, should be placed not less than 500 feet apart at the beginning and end of each line, also at prominent junction points from which other primary control lines may be started. A complete description and detailed sketch of these should be entered in the note-book.

14. Permanent marks of some kind should be left at such points passed during cloudy or unfavorable weather as it may be necessary to return to for the observation of azimuths.

15. Meridian marks, consisting of two of the standard bronze tablets let into dressed stone or masonry posts and placed 500 feet or more apart, must be established at each county seat passed in the progress of the work.

16. Observations for magnetic declination must be made at several points in the course of a season's work, especially at county seats.

17. A complete record must be kept by the transitman in book No. 9–905, and a separate record of tape-lengths by the front tapeman.

18. The transit notes should be entered and worked up in the following manner:

Station.	Pointings: Back and Transited.		Mean Pointing.	Deflection Angle.	Comp. az.	Remarks.
	Ver. A.	Ver. B.				
392	65° 48′ 00″	245° 48′ 00″	65° 48′ 00″	0° 46′ 00″		11 A.M. 72°.
	65 02 00	245 02 00	65 02 00			
100 rails.			65 02 00	0 46 00		76 road crossing.
2,975 ft.	64 16 00	244 16 00	64 16 00			
				0 46 00		
393	64 16 00	244 16 00	64 16 00	4 41 30		12 M. 73°.
	68 57 30	248 57 30	68 57 30			
39 rails.			68 57 30	4 41 30		On road crossing.
1,197 ft.	73 39 00	253 39 00	73 39 00			
				4 41 30		

232. Cost, Speed, and Accuracy of Primary Traverse.—Primary traverse executed by the U. S. Geological Survey *costs* from three to five dollars per linear mile, according to the topography of the country, and has averaged generally about $3.50 per linear mile. The *speed* made varies between two and ten miles per day, also depending upon the topography. With parties of from five to seven men the daily cost is from $15 to $25.

The primary traverse lines of the above organization are from 50 to 300 miles in length, averaging 150 to 200 miles. The *closure errors* of such lines vary within a wide range, and there seems to be no accounting for their erratic character.

They have been found to average from 10 to 200 feet per 100 miles of traverse, and their *probable error*, therefore, varies between 1 : 3000 and 1 : 50,000. Where the topographic control is to be platted on a scale of 1 mile to 1 inch, it will thus be seen that the error in a primary traverse of 100 miles length may be a perceptible quantity if the error be in excess of one foot in a mile. Ordinarily in a distance of 100 miles the error is so small that it can be practically eliminated by the adjustment to the points which control the extremities. (Art. 226.)

CHAPTER XXIV.

COMPUTATION OF PRIMARY TRAVERSE.

233. Computation of Primary Traverse.—The computation of the primary traverse involves:

1. Correction of tape-lengths for temperature;
2. Correction of tape-lengths for inclination;
3. Reduction of measured distance to sea-level;
4. Determination of mean angle;
5. Computation of deflection angle;
6. Correction to reduce to observed astronomic azimuths;
7. Computation of latitudes and longitudes of controlling points on the traverse; and
8. Correction from adjustment to check astronomic positions.

The correction of tape-lengths for temperature and inclination are explained in Articles 216 and 219, as is the mode of reducing the distances to sea-level in Article 220. The derivation of the mean deflection angle is clearly indicated in the example given in Article 230, as is the method of computing and correcting the azimuth.

Under ordinary circumstances the *temperature correction* is a negligible quantity, as it is far less in amount than the other avoidable errors. So also is the *correction for inclination*, providing the tape is held horizontal as it should be.

The *computed azimuths* are corrected by the nightly azimuth observations, the new astronomic azimuth being adopted in place of that brought forward. In case of a small discrepancy between the two such correction should be uniformly distributed between two consecutive azimuth stations (Art. 309). The above corrections having been made, latitudes and departures may now be computed (Arts. 90 and 235) for each station commencing at the initial point or that for which geodetic coordinates have been previously obtained. Such latitudes and departures are computed one from the other, dimensions being in feet, the sum of latitudes being converted into seconds to give differences in latitude, and the sum of departures into seconds to give seconds in longitude.

234. Correction for Observed Check Azimuths.—The method of correcting the computed azimuths by the observed check azimuths is illustrated in the example in Art. 230. Azimuth was observed at Traskwood between stations 133 and 134, and was found to be 42° 55′ 15″. The azimuth brought forward from the last azimuth station, 26 instrument stations distant, was 42° 58′ 07″. The difference between the observed and the computed azimuth at Traskwood was 2′ 52″. There were accordingly 172″ to be distributed between the 26 stations, or, giving the proper algebraic signs, the correction amounted to 06″.5 per station.

The *convergence* of *meridians* subtracted from the apparent error in azimuth 2′ 52″, gives the actual error of the computed azimuth. Table XXX gives the convergence of meridians for every six miles. As the distance in longitude in the above example was three miles, the convergence for the mean latitude, 34° 30′, amounts to 1′ 45″. This from the apparent error 2′ 52″ shows the actual azimuth error to be 1′ 07″.

Convergence of meridians, which is the amount by which they approach from their greatest distance apart at the equator until they intersect at the pole, may be determined approximately by the rule: "A change of longitude of one degree changes the azimuths of the straight line by as many minutes as there are degrees from the latitude of the place."

TABLE XXX.

CONVERGENCE OF MERIDIANS SIX MILES LONG AND SIX MILES APART.

(From U. S. Land Survey Manual.)

Latitude.	Convergence.		Difference of Longitude per Range.		Longitude.	Difference of Latitude for 1 Mile in Arc.
	On the Parallel.	Angle.	In Arc.	In Time.	Arc of 1″.	
°	Feet.	′ ″	′ ″	Seconds.	Feet.	
30	24.7	3 0	6 0.36	24.02	87.9	0′.871
31	28.8	3 7	6 4.02	24.27	87.1	
32	30.0	3 15	6 7.93	24.53	86.1	
33	31.2	3 23	6 12.00	24.80	85.1	
34	32.4	3 30	6 16.31	25.09	84.2	
35	33.6	3 38	6 20.95	25.40	83.2	0.′870
36	34.8	3 46	6 25.60	25.71	82.2	
37	36.1	3 55	6 30.59	26.04	81.1	
38	37.5	4 4	6 35.81	26.39	80.1	
39	38.8	4 13	6 41.34	26.76	78.9	
40	40.2	4 22	6 47.13	27.14	77.8	0.′869
41	41.6	4 31	6 53.22	27.55	76.7	
42	43.2	4 41	6 59.62	27.97	75.5	
43	44.7	4 51	7 6.27	28.42	74.3	
44	46.3	5 1	7 13 44	28.90	73.1	
45	47.9	5 12	7 20.93	29.39	71.9	0.′869
46	49.6	5 23	7 28.81	29.92	70.6	
47	51.3	5 34	7 37.10	30.47	69.3	
48	53.2	5 46	7 45.79	31.05	68.0	
49	55.1	5 59	7 55.12	31.67	66.7	
50	57.1	6 12	8 4.90	32.33	65.3	0.′868

235. Computation of Latitudes and Longitudes.—The following is an example of the form employed in the United States Geological Survey in computing the differential latitudes and longitudes of the several traverse stations. This is almost identical with the method already described for computing latitudes and departures (Art. 90).

While the form here used is apparently complicated, it is in reality very simple and condensed. Instead of the logarithms of the distances being arranged as separate columns for addition to the sines and cosines of the azimuth, to obtain respectively the logarithms of distances in longitude and latitude, they are arranged here in one column. When the logarithm of cosine is to be added to the logarithm of distance the logarithm of the sine is covered by a lead-pencil or slip of

Station.	Distance and Azimuth.	Logarithms	
107 to 108	1800 44° 30′ 10″	3.25527 9.84568 9.85322	Log of dist. Sine of az. Cosine of az.
	South 1284 ft. *West 1262 "*	*3.10849* *3.10095*	Log of dist. + log cos az. Log of dist. + log sine az.
108 to 109	3000 44° 30′ 04″	3.47712 9.84567 9.85323	Log of dist. Sine of az. Cosine of az.
	South 2140 ft. *West 2103 "*	*3.33035* *3.32279*	Log of dist. + log cos az. Log of dist. + log sine az.

* * * * * * * * *
* * * * * * * * *

132 to 133	1200 42° 18′ 45″	3.07918 9.82813 9.86893	Log of dist. Sine of az. Cosine of az.
	South 887 ft. *West 808 "*	*2.94811* *2.90731*	Log of dist. + log cos az. Log of dist. + log sine az.
133 + 130 ft.	130 42° 55′ 15″	2.11394 9.83314 9.86468	Log of dist. Sine of az. Cosine of az.
	South 95 ft. *West 89 "*	*1.97862* *1.94708*	Log of dist. + log cos az. Log of dist. + log sine az.

paper, and the sum is placed in the fourth line, marked "Logarithm Distance + Logarithm Cosine." Likewise when the logarithms of sines and distances are added the logarithm of cosine is blocked out and the result put in the last line. The number corresponding to the logarithms in the last two lines, as obtained from a table of logarithms (Tables V and VI), is given on the left end of this line opposite the word "South" or "West" as the case may be, and is in feet. In computing primary traverse a five- or seven-place table of logarithms should be used as in all other primary computations.

236. Corrected Latitudes and Longitudes.—The latitudes and longitudes as computed in the last article are their respective amounts in feet, and may be used in this form in platting. That these may be reduced to their geodetic coordinates and finally corrected by adjustment to observed astronomic positions (Art. 309) the computation is continued as follows:

All northings and southings and all eastings and westings are added together, and the differences of north and south and of east and west are obtained by algebraic summation. In the following example the line was run in one direction; and, accordingly, there is no sum of norths to be subtracted from a sum of souths, nor easts to be subtracted from wests. Thus:

South.	West.
1284	1262
2140	2103
x	x
x	x
887	808
95	89
37,860 = Total South	20,644 = Total West

The *total south and west* is changed to arc by adding the arithmetic complement of the logarithm of the value of one second of arc in meters to the constant for reducing feet to meters. To this sum is finally added the logarithm of the total distance in feet of southing to get latitude. The number corresponding to the sum gives the correction or change of latitude in seconds. The same operation is performed to obtain the corrections in longitudes or departures, by adding to the total westing in feet the logarithmic constant of feet to meters, and the a. c. log. value of one second in meters. Thus:

a. c. log. value 1″ in meters..	8.51125	log. total west (Long.) 20644.......	4.31479
log. reduc. ft. to meters. .	9.48401	log. ft. to meters..................	9.48401
log. total south (Lat.) 37860..	4.57818	log A, Table XXXVII............	8.50926
Lat. cor. = 374″.49 (No.).....	2.57344 (log.)	Log. Secant L'.............	0.08374
		Long. correction = 246″.49 (No.)..	2.39180 (log.)

The *logarithmic value of one second* is obtained for the example by finding in Table XXX, opposite the approximate latitude 34° 30′, the length in meters of one minute of arc, 1848.81. This divided by 60 gives the value of one second, 30.813. The logarithm of this is 1.48875, and its arithmetic complement is 8.51125. The logarithm of the constant for reducing feet to meters is derived from Table XLIII.

The latitude and longitude of the last station, say Benton Depot, being known, those of the next station, as Traskwood Depot, are obtained by adding to or subtracting from the former according as it is north or south, i.e., plus or minus, the amount of change in minutes and seconds between the two. Thus:

Latitude.		Longitude.
34° 33′ 11″.92	Benton Depot (middle window).............	92° 35′ 18″.63
− 6 14. 49	Correction for new position	+ 4 06. 49
34° 26′ 57″.43	Traskwood Depot (middle window)..........	92° 39′ 25″.12

If now this primary traverse has been run between an observed astronomic position at Little Rock and another astronomic position, say at Fort Smith, Arkansas, the latitudes and longitudes as brought through by the primary traverse computation from Little Rock to Fort Smith will be in error by a certain number of seconds. This error may be distributed by dividing its amount in seconds by the distance in miles, and the correction per mile applied to the various computed positions. Or, providing known portions of the traverse are for any reason more liable to be in error than others, arbitrary weights may be applied in distributing the error. A rigid adjustment by the method of least squares is scarcely warranted by the quality of the work and the number of conditions. (Art. 264.)

CHAPTER XXV.

FIELD-WORK OF PRIMARY TRIANGULATION.

237. Primary Triangulation.—The *purpose of primary triangulation* is the determination of the relative position upon the face of the earth of various commanding points. This operation involves a knowledge of the astronomic position of some initial point, as an extremity of a base line or some other known point; and of the distance in standard measure between the initial and some other intervisible point, as the two extremities of a base line or the imaginary line joining two known triangulation positions.

The *operations of triangulation* involve the measurement of the angle at the initial point, between two intervisible points, the position of one of which is known; also the angles at the other two points, thus giving the three angles of the triangle. Finally, with the known length of one side and the three measured angles of the triangle, the other sides of the triangle may be computed. (Art. 259.)

Triangulation, as executed in connection with geodetic and topographic operations, may be divided into three kinds according to its precision:

1. Primary triangulation, with sides varying from 15 to 100 miles or more in length, and executed with the best instruments in the most accurate manner;

2. Secondary triangulation, with sides from 5 to 40 miles in length, executed with surveyors' transit or with plane-table; and

3. Tertiary triangulation, with sides less than 10 miles in length, executed in the progress of the second, and consisting of unoccupied locations or of resections from primary and secondary locations.

Primary triangulation should, if possible, be expanded in such manner as to form quadrilateral, pentagonal, hexagonal, and other standard figures (Art. 238) by which combinations of angles may be obtained in order to strengthen the component angles in the course of the computations. These figures are verified by the angles, and at intervals the precision of the whole scheme of triangulation is verified by the measurement of additional primary base lines with the accompanying astronomic determinations of coordinates.

238. Reconnaissance for Primary Triangulation.—The term *reconnaissance* is generally meant to embrace all those investigations of a region about to be triangulated which precede the actual field-work of base measurement (Art. 202) and the measurement of angles (Art. 252). Where the reconnaissance is preliminary to the execution of triangulation of the highest degree of precision, as for geodetic investigations, it should be thorough and exhaustive and should develop every possible scheme of triangulation.

Ordinarily a reconnaissance, while hastily made, should develop the most practical scheme of figures and afford such information as will add to the economy and rapidity of the work of angle measurement. The reconnaissance should be made with a view to avoiding as far as possible the necessity of occupying elevated structures, as observing scaffolds (Art. 245); also the longest lines or the highest peaks, as the clouds which surround the latter will impede progress, while lines having greater length than one hundred miles invariably delay progress of the work. Lines of sight should be avoided which pass closely to the ground or to the vertical surface of any object, as a building, because of the liability to lateral refraction. In the course of the reconnaissance sites for check base lines should be sought every 150 to 200 miles, or, say, every eight or ten figures, depending on the length of the bases.

In the *conduct of a reconnaissance* little difficulty will be encountered where the elevations are great and the summits comparatively clear of timber. If, on the other hand, the summits are heavily wooded and comparatively uniform in height, the greatest skill will be required and the slowest progress made in selecting intervisible points which are most favorably situated for the extension of the triangulation and the formation of the most satisfactory figures. In the flat, comparatively level country of the plains of Kansas, Nebraska, and thereabouts, triangulation may be practically laid off as on paper, the points selected being made intervisible by means of high signals (Art. 244), and the length of the sides being limited only by the curvature of the earth (Art. 239) and the height of the signals. If the same class of country is heavily covered with forests, the lengths of the lines will be governed by the labor and expense of clearing them.

The *outfit required* in making a reconnaissance includes (1) a small theodolite, with circle reading by vernier to minutes; (2) a compass-needle attached to the theodolite for determining the magnetic direction; (3) an aneroid barometer; (4) a 100-foot steel tape; (5) a prismatic compass; and (6) a protractor, scale, and paper for platting the reconnaissance triangulation as it progresses, in order that the position of the points sighted may be approximately ascertained. Also the best available existing maps of the country, and only such camp outfit (Chap. XXXVIII) and assistants as will permit of the execution of the work and least impede the rate of progress.

In making the reconnaissance it is necessary to keep steadily in view not only the limitations imposed by the necessity of selecting well-conditioned figures, but also that the most rigid requirements may be modified in accordance with the special features of the country as they are developed by the reconnaissance. Hence the *best plan of triangulation* will be that which not only satisfies the conditions prescribed, but will be most effective in its results and economic in its execution.

Among the more important of these requirements are:

1. Assured intervisibility of stations;

2. Selection of the higher summits;

3. Maximum length of line within a limit of 100 miles;

4. Angles not in excess of 120 or less than 30 degrees;

5. The formation of the simplest and strongest figures practicable;

6. The greatest area in view in order that the largest number of intermediate stations may be sighted; and

7. A consideration of the altitude to which the instrument must be raised in order that the visual ray may pass above intermediate obstacles.

The first thing to be borne in mind in planning a triangulation is that the stations chosen shall form well-conditioned or standard *figures* (Art. 273), and that each triangle in the figures shall be as nearly equilateral as possible. To this end no angle should be smaller than 30 or greater than 120 degrees, except in quadrilaterals, where a few degrees more latitude may be permitted in the size of the angles. Hexagonal figures cover the largest areas, while quadrilaterals secure the greatest degree of accuracy. On the other hand hexagonal figures, because of the disposition of the stations, tend to retard linear progress and should be avoided, as should quadrilaterals with open diagonals when direct and not areal progress is sought. Quadrilaterals with observable diagonals, while giving the strongest figures, adapt themselves least to the topography and are found to be relatively difficult figures. *Pentagons* and *quadrilaterals with central stations* readily conform to the configuration of the country however complex or difficult, and give figures of considerable strength.

In the course of the *reconnaissance* extensive *notes* should be made of the character of the country, the difficulties of travel, facilities for transportation, methods of climbing the various summits, the routes to them, stopping-places, etc. Horizontal and vertical angles should be taken on all prominent peaks and objects, even though it is not expected to include them in the triangulation scheme. The reconnaissance scheme should be kept platted up each day to scale in order to facilitate the finding of such stations as have already been selected, by determining the angles to them from known points and to thus aid in their recognition from new stations.

239. Intervisibility of Triangulation Stations.—The following table, prepared by Mr. R. D. Cutts of the U. S. Coast Survey, is of use in reconnaissance in deciding upon the height of signals and observing scaffolds to be erected. The line of sight from the telescope to the signal should never

TABLE XXXI.

DIFFERENCE IN HEIGHT BETWEEN THE APPARENT AND TRUE LEVEL.

Distance, miles.	Difference in Feet for—			Distance, miles.	Difference in Feet for—			Distance, miles.	Difference in Feet for—		
	Curvature.	Refraction.	Curvature and Refraction		Curvature.	Refraction.	Curvature and Refraction		Curvature.	Refraction.	Curvature and Refraction
1	0.7	0.1	0.6	23	353.0	49.4	303.6	45	1351.2	189.2	1162.0
2	2.7	0.4	2.3	24	384.3	53.8	330.5	46	1411.9	197.7	1214.2
3	6.0	0.8	5.2	25	417.0	58.4	358.6	47	1474.0	207.3	1267.7
4	10.7	1.5	9.2	26	451.1	63.1	388.0	48	1537.3	215.2	1322.1
5	16.7	2.3	14.4	27	486.4	68.1	418.3	49	1602.0	224.3	1377.7
6	24.0	3.4	20.6	28	523.1	73.2	449.9	50	1668.1	233.5	1434.6
7	32.7	4.6	28.1	29	561.2	78.6	482.6	51	1735.5	243.0	1392.5
8	42.7	6.0	36.7	30	600.5	84.1	516.4	52	1804.2	252.6	1551.6
9	54.0	7.6	46.4	31	641.2	89.8	551.4	53	1874.3	262.4	1611.9
10	66.7	9.3	57.4	32	683.3	95.7	587.6	54	1945.7	272.4	1673.3
11	80.7	11.3	69.4	33	726.6	101.7	624.9	55	2018.4	282.6	1735.8
12	96.1	13.4	82.7	34	771.3	108.0	663.3	56	2092.5	292.9	1799.6
13	112.8	15.8	97.0	35	817.4	114.4	703.0	57	2167.9	303.5	1864.4
14	130.8	18.3	112.5	36	864.8	121.1	743.7	58	2244.6	314.2	1930.4
15	150.1	21.0	129.1	37	913.5	127.9	785.6	59	2322.7	325.2	1997.5
16	170.8	23.9	146.9	38	963.5	134.9	828.6	60	2402.1	336.3	2065.8
17	192.8	27.0	165.8	39	1014.9	142.1	872.8	61	2482.8	347.6	2135.2
18	216.2	30.3	185.9	40	1467.6	149.5	918.1	62	2564.9	359.1	2205.8
19	240.9	33.7	207.2	41	1121.7	157.0	964.7	63	2648.3	370.8	2277.5
20	266.9	37.4	229.5	42	1177.0	164.8	1012.2	64	2733.0	382.6	2350.4
21	294.3	41.2	253.1	43	1233.7	172.7	1061.0	65	2819.1	394.7	2424.4
22	322.9	45.1	277.7	44	1291.8	180.8	1111.0	66	2906.5	406.9	2499.6

pass less than 6 feet above the earth's surface at the tangent point, and should be higher, if possible, to reduce errors from unequal refraction.

$$\text{Curvature} = \frac{\text{square of distance}}{\text{mean diameter of earth}} = \frac{K^2}{2R}. \quad . \quad . \quad (47)$$

Log curvature = log square of distance in feet − 7.6209807.

$$\text{Refraction} = \frac{K^2}{R}m, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

where K = the distance in feet;

R = mean radius of the earth ($\log R = 7.3199507$); and

m = the coefficient of refraction, assumed at .070, its mean value, seacoast and interior.

$$\text{Curvature and refraction} = (1 - 2m)\frac{K^2}{2R}. \quad . \quad (49)$$

Or, calling h the height in feet, and K the distance in statute miles, at which a line from the height h touches the horizon, taking into account refraction, assumed to be of the same value as in Table XXXI (0.70 for one mile), we have

$$K = \frac{\sqrt{h}}{.7575}, \quad \text{and} \quad h = \frac{K^2}{1.7426}.$$

An *approximate*, yet comparatively accurate, empirical formula for determining the combined effect of curvature and refraction is

Curvature and refraction, in feet = 0.574 (distance in miles)^2.

The following examples will serve to illustrate the use of the preceding table:

I. *Elevation of Instrument required to Overcome Curvature and Refraction.*—Let us suppose that a line, A to B, was 18 miles in length over a plain, and that the instrument could be elevated at either station, by means of a tripod, to a height of 20 or 30 or 50 feet. If we determine upon 36.7 feet at A, the tangent would strike the curve at the distance represented by that height in the table, viz., 8 miles, leaving the curvature (decreased by the ordinary refraction) of 10 miles to be overcome. Opposite to 10 miles we find 57.4 feet, and a signal at that height erected at B would, under favorable refraction, be just visible from the top of the tripod at A, or be on the same apparent level. If we now add 8 feet to tripod and 8 feet to signal-pole, the visual ray would certainly pass 6 feet above the tangent point, and 20 feet of the pole would be visible from A.

II. *Elevations required at given distances.*—If it is desired to ascertain whether two points in the reconnaissance, estimated to be 44 miles apart, would be visible one from the other, the natural elevations must be at least 278 feet above mean tide, or one 230 feet, and the other 331 feet, etc. This supposes that the intervening country is low, and that the ground at the tangent point is not above the mean surface of the sphere. If the height of the ground at this point should be 200 feet above mean tide, then the natural elevations should be 478, or 430, and 531 feet, etc., in height, and the line barely possible. To insure success, the theodolite must be elevated, and at both stations, to avoid high signals.

III. *To determine whether the line of sight between two stations would pass above or below the summit of an intervening hill, and how much in either case.* (Fig. 160.)

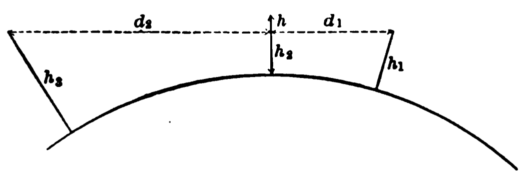


FIG. 160.—INTERVISIBILITY OF OBJECTS.

h_1 = height of lower station.
h_3 = height of higher station.
h_2 = height of intervening hill.

d_1 = distance h_1 to h_2.
d_2 = distance h_2 to h_3.

Example I.

$h_1 =$ 600 feet.	600 feet strikes horizon at	32.3 miles,
$h_2 =$ 2000 feet.	$64 - 32.3 = 31.7$ miles	$=$ 577 feet,
$h_3 =$ 1340 feet.	$31.7 - 10 = 21.7$ miles	$=$ 270 feet,
$d_1 =$ 54 miles.	$2000 = 577$ feet	$=$ 1423 feet,
$d_2 =$ 10 miles.	$\frac{64}{10} = 6.4$ and $\frac{1423}{6.4}$	$=$ 222.3 feet,

and h_1, or height of line at $h_3 = 1423 + 270 - 222.3 = 1470$ feet.

Hence the line passes 130.7 feet above the intervening hill and the stations are intervisible.

Example II.

$h_1 =$ 900 feet.	900 feet strikes horizon at	39.4 miles.
$h_2 =$ 3600 feet.	80 miles $-$ 39.4 miles $=$ 40.6 miles	$=$ 946.0 feet,
$h_3 =$ 1980 feet.	$40.6 - 25.0 = 15.6$ miles	$=$ 139.8 feet,
$d_1 =$ 55 miles.	3600 feet $-$ 946 feet	$=$ 2654.0 feet,
$d_2 =$ 25 miles.	$\frac{80}{25} = 3.2$ and $\frac{2654}{3.2}$	$=$ 829.4 feet,

and $h = 2654 + 139.8 - 829.4 = 1964.4$ feet.

Hence the summit at h_3 is 15.6 feet higher than the line of sight, and the two stations are not intervisible.

If we elevate the instrument 60 feet at h_2, the line would pass clear of h_3, or its height at that point would be 2006 feet.

The question of intervisibility may be also determined by the following formula, in which the coefficient of refraction is reduced to .065:

$$h = h_1 + (h_2 - h_1)\frac{d_1}{d_1 + d_2} - 0.5803 d_1 d_2. \quad . \quad . \quad (50)$$

Example III. Same data employed as in Example I.

				logs.
$(h_2 - h_1) =$ 1400 feet.	$\log (h_2 - h_1)$	$= 3.14613$	$d_1 = 54$ miles	1.73239
$d_1 =$ 54 miles.	$\log d_1$	$= 1.73239$	$d_2 = 10$ miles	1.00000
$d_1 + d_2 =$ 64 miles.	Co. log $(d_1 + d_2)$	$= 8.19382$	Constant	9.76365
1181.2 feet		3.07234	313.4 feet	2.49604

and hence $h =$ 600 feet $+ 1181.2 - 313.4 = 1467.8$ feet.

240. Accuracy of Triangulation.—The precision of a scheme of primary triangulation is dependent upon several related quantities; notably—

1. The precision of the astronomic determination of the geodetic coordinates of the initial point.

2. The precision with which the base line on which the triangulation is dependent has been measured.

3. The care taken in gradually transferring the short length of a base through expansion to the longer sight lines of the triangles.

4. The errors inherent in the measurement of the angles of the triangulation, including those due to instrument as well as signal.

The *probable error* of an astronomic determination (Art. 327) is considerably greater than that of the measure of a base line and nearly as great as that introduced within the expansion of the triangulation. A base line can readily be measured with a probable error far less than that which can be maintained in the execution of the triangulation (Art. 213). It is impossible in triangulation to maintain an accuracy at all approaching that of the base measured with a probable error of $\frac{1}{2000000}$, while an accuracy of even $\frac{1}{500000}$ is difficult to maintain in an extended triangulation. The accuracy of the base is lost partly in the base figure and is rapidly dissipated in the adjacent expansion. The figures used in expanding a base line through the net of triangles to the outer triangles are, therefore, of great importance. Ideally they should be a series of quadrilaterals with diagonals intersecting at right angles. The lengths of their sides should be increased in a ratio of 1 to 2 or 3, thus requiring two or three steps or series of figures in the expansion to reach the outer triangulation scheme. (Fig. 161).

241. Instruments.—The various tools employed in the measurement of angles of a scheme of primary triangulation may be classed under two general heads:

1. Instruments for measuring horizontal angles; and
2. Signals or objects upon which to observe.

There is practically only one form of instrument employed in the measurement of horizontal angles, and this is known as a *theodolite*. In its general characteristics it resembles an engineer's transit (Art. 85), from which it differs chiefly—

1. In the fact that the telescope does not transit or revolve vertically through 180 degrees in the wyes;
2. In the special care exercised in making the instrument, particularly in the accuracy of centering and fitting the vertical axis and strict uniformity of graduation; and
3. In the size of the telescope and horizontal circle, and the mode of reading the graduations upon the latter.

The earlier theodolites were so constructed that the circle was read with *verniers*, and in order that a proper degree of precision might be attained they were made excessively large, having circles from 16 to 30 inches in diameter. Experience has proven, however, that with the aid of *micrometer microscopes* (Art. 242) greater accuracy of measurement can be had by use of an instrument having a horizontal limb not exceeding 12 inches in diameter; while work which is sufficiently accurate for all the purposes of an ordinary triangulation can be executed with theodolites having circles of 8 inches diameter.

In Fig. 161 are indicated two typical *expansions of the base*, *AB*, by means of nearly ideal figures. The first is by enlargement to the quadrilateral *ACBD*; the second would be employed when signals could not be observed in the direction *C*, and would be by expansion to the quadrilateral *ABDF*. An especially strong pentagonal figure is formed when all these directions can be sighted. From these expansion may be made to the still longer sides of the quadrilateral *GHIF*, or of the pentagonal figure *GHIFE*.

Where superior theodolites are employed it has been found that the *probable error* of the measure of a direction may vary between 1 and 5 seconds of arc. Some of the best work of the U. S. Coast Survey has given results varying between .6″ and .75″. Any triangulation in which this does not exceed 1″ may be classed as of the highest order.

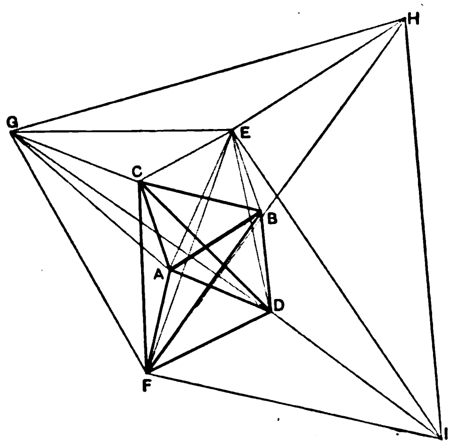


FIG. 161.—BASE EXPANSION.

Measures attaining accuracy of from 2 to 5 seconds are sufficiently refined for all the ordinary purposes of a primary triangulation outside of those required in geodetic investigations.

The latest *direction theodolites* used by the *U. S. Coast Survey* have circles of 12 inches diameter with double centers, the outer center of cast iron, and the inner of hardened steel. The inner center and socket are made with great precision; the outer center and socket are well made, though constructed with less precision, as this center serves only for shifting the position of the circle and not for the reading of angles. The *alidade*, by which is meant everything above and including the graduated circle, the wyes, and the telescope, is supported on the inner center and is made of aluminum as far as practicable, and so constructed that the friction upon the center is exceedingly small. The center is 8 inches long, its bearing surfaces being cones. The circle is covered to protect it from dust, and is made especially heavy and the centers long, to give stability. In spite of this the weight is but 41 pounds, due to the extensive use of aluminum. The circle is graduated on coin silver and is divided to five minutes, and is read to two seconds of arc by means of three equidistant micrometer microscopes. Each degree of the graduation is numbered. The telescope objective is 2.4 inches aperture and 29 inches focal length. A striding-level which rests on the axis supporting the telescope is graduated to 4 seconds of arc.

The *direction theodolites* used in the *U. S. Geological Survey* are supported on heavy split-leg wooden tripods and rest upon aluminum tripod heads. (Fig. 162.) The circles of these instruments have a diameter of 8 inches and are divided to 10 minutes, though they can be read to 2 seconds of arc by means of two micrometer microscopes placed on opposite sides of the alidade. The object-glass is 2 inches in diameter and has a focal length of 16½ inches, with an eyepiece having a magnifying power of about 30 diameters.

242. Micrometer Microscope.—This is a device for the measurement of smaller parts of an arc than are indicated by the graduations upon it. Micrometer microscopes are used in high-power angle-reading instruments in place of the verniers used on engineering instruments, since they give more accurate results and finer subdivisions of the arc. They consist of a microscope generally supported upon the standards of a theodolite with the objective end in close contact with the horizontal circle, and lighted by a cylindrical glass extension. It is sometimes called the *filar micrometer*, because the small measurements are made by means of fine threads

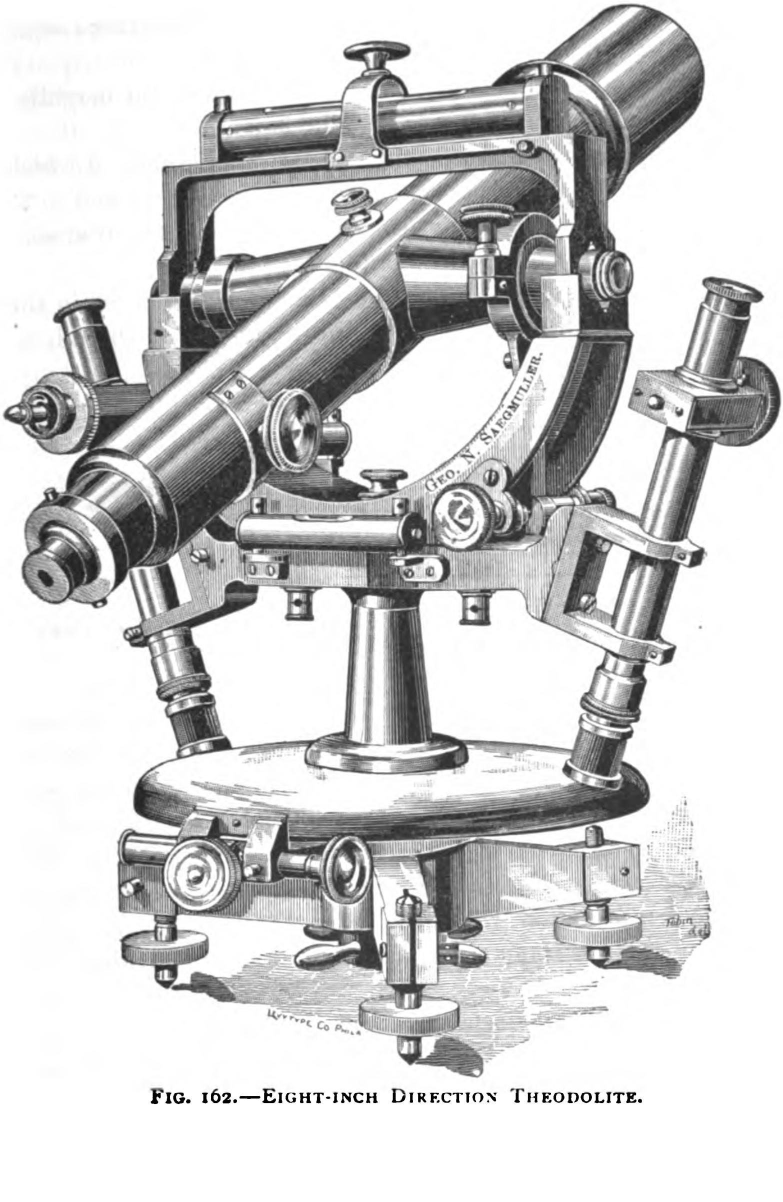


FIG. 162.—EIGHT-INCH DIRECTION THEODOLITE.

or films. These are similar to the cross-hairs of the eye-piece of a telescope. The instrument consists of three separate parts:

1. The microscope tube, carrying the lenses for magnifying the divisions on the circle and the hairs;

2. A large-headed screw the outer circumference of which is divided, and is read by means of a fixed pointer; and

3. A comb-scale and cross-hairs by which the divisions of the circle are read and subdivided.

The micrometer *cross-hairs and comb-scale* are fixed in the plane of the image produced by the objective of the micro-

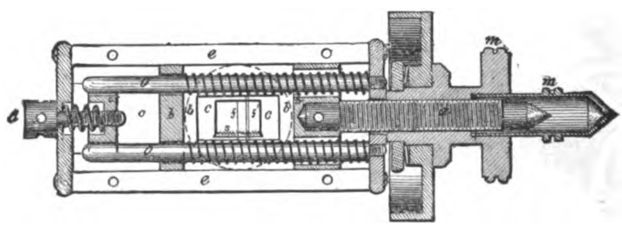


FIG. 163.—SECTION OF MICROMETER THROUGH SCREW SHOWING COMB AND CROSS-HAIRS IN CENTRAL PLAN.

scope. This image is larger than the object seen in the microscope, therefore a given amount of the micrometer cross-hairs corresponds to a much less distance on the object sighted. The cross-hairs are held in a frame which is moved by a screw having a very fine thread, called the *micrometer screw.* (Fig. 163.) This is caused to revolve by a large head, called the *micrometer head,* which is cylindrical or hollow, its outer circumference being divided into sixty.

The relation between the comb-scale of the microscope, and the graduations on the micrometer head which denote the fractions of a revolution of the screw, is such that one full revolution of the screw corresponds to one tooth of the comb-scale. The number of whole revolutions of the screw are recorded by noting how many teeth of the comb-scale are passed over; the fractional parts of a revolution being read on the graduated micrometer head.

If the circle of the instrument is divided to 10 minutes and the micrometer read to 2 seconds, as in the case of the *8-inch theodolite* of the U. S. Geological Survey, the heads of the micrometers are divided into sixty parts numbered 0, 10, 20, 0, 10, 20. One revolution of the micrometer screw is equivalent to 2 minutes, and one division of the head to 2 seconds. The comb-scale of the micrometer consists of ten parts, each of which corresponds to the space of 10 minutes on the circle and to five revolutions of the micrometer screw. The slide by which it is read carries two cross-hairs close together. The micrometer records zero near the middle when one of the two cross-hairs is on the middle line of the comb-scale and the head of the screw is at zero. Degrees and minutes are read directly in the microscope. The readings of the micrometer head are recorded in the notes as divisions. The sum of the readings of the heads of the two separate micrometer microscopes gives the mean reading of the two in seconds.

Five revolutions of the screw should move the cross-hairs from one graduation to the next. If this is not exactly true, then the value of the ten-minute space should be measured a number of times by running the cross-hairs backward and forward. The mean of these five revolutions should give the mean value of one revolution of the micrometer screw, and this is called the *run of the screw*. When reading the instrument a correction is to be applied called the *correction for run*, and this is determined as described above for various parts of the micrometer screw. (Art. 251.)

243. Triangulation Signals.—There are three general forms of signals upon which to observe or point the cross-hairs of the telescope in the measurement of angles of a primary triangulation. These are:

1. Opaque signals, usually tripods or poles of wood with flag or other opaque device attached thereto;

2. Reflecting signals; and

3. Lights or night-signals.

Opaque signals should generally be employed where the conditions of the atmosphere and the lengths of the sights will permit. A smaller probable error results from observing upon them than upon any other form of signal.

Reflecting signals are of two general types:

1. Tin reflecting cones or other stationary objects of conical or cylindrical shape; and

2. Heliotropes, or instruments by which sunlight is reflected by a mirror towards the observer.

Neither heliotropes nor *tin* reflecting *cones* permit of as accurate results in observing as do opaque signals, because of the *phase* or displacement of the reflected beam of light, which is often considerable. The most satisfactory reflecting signal upon which to observe, because of the certainty of its being seen in hazy and foggy weather or on timber-covered summits, is the *heliotrope*. The flash of the reflected sunlight from this instrument can be seen from the most distant points which can be observed, as well as on those partially obscured by atmospheric conditions.

In smoky and hazy weather the atmosphere is clearest at *night*, and it may be necessary to use reflectors illuminated by ordinary kerosene lamps or, on very long lines, by magnesium tape burned in and reflected by a special apparatus.

The *correction for phase in tin cones*, or reduction to the center of the signal, is

$$\text{Cor.} = \pm \frac{r \cos^2 \frac{1}{2}Z}{D \sin 1''},$$

in which r = radius of signal,

Z = angle at point of observation between the sun and the signal, and

D = distance from observer to signal.

244. Tripod and Quadripod Signals.—Various forms of opaque signals are employed, according to the length of sight and the availability of materials for construction. Where the circumstances will permit, the simplest form is a tripod from the center of and above which projects a pole carrying cross-pieces to which are fastened strips of cloth at right angles in target form, while the whole may be surmounted by a flag. (Fig. 164.) The object of the flag is that, in waving in the sunlight, its white flash is more readily distinguished and more quickly attracts the observer than do the stationary tripod and targets. This form of signal must be accurately centered over the station mark, the bottom of the pole being cut off so that the theodolite can be set beneath the tripod and over the station mark.

The various details of the tripod signal, such as—

1. Height of pole;
2. Dimensions of the tripod;
3. Boarding-in of the lower part or covering it with canvas to shade the instrument and protect it from high winds;
4. Whether the pole or the cloth upon it shall be white or black;
5. Character of the signal to be employed;

and other matters of detail vary with—

a. Length of the line observed;

b. Altitude of the station;

c. Background of the station; and

d. Atmospheric conditions encountered.

The *diameter of* the *signal-pole* must not be greater than will just permit of its being distinctly seen, so that it may be accurately bisected by the vertical cross-hair. The latter must not cover the pole, but must permit a portion of it to show on either side of the hair. The diameter which, with averaged-sized cross-hair and magnifying power, subtends an angle of 1 second at 1 mile is .307 of an inch; hence at 20 miles it is 6.1 inches, at 40 miles it is 12.3 inches, at 60 miles it is 18.4 inches, and at 80 miles it is 24.6 inches. The above proportions show that for lines exceeding 16 miles the diameter of the signal should not exceed one second in value. The solid part of the pole should never be greater than about 4 to 6 inches in diameter, in order that it may not be too heavy to raise. Its visible dimensions may be increased by nailing slats of wood upon it and covering these with cloth or with a reflecting cone of metal.

Such signals may be constructed of any material which is convenient to hand, as poles cut in the woods, old rails, etc. It is preferable, however, to build them of 2×4 or 3×4 sawed scantling, and this should be procured in 12- to 16-foot lengths, according to the height to which it is necessary to raise the flag above the ground surface in order that it may be seen over intervening obstructions. Moreover, the upper portions of such a signal should be painted white. The additional expense incurred in using such materials and in painting will be more than counterbalanced by the added immunity from destruction by vandalism, a well-built and attractive-looking signal being far less liable to such injury than one crudely put together. Moreover, squared scantling can be more easily cut to abut against the central pole, or to be pieced together where it is necessary to have a signal of greater height than the average length of the scantling. The spread of the legs should be about two-thirds of the height of the pyramid to give stability.

To *anchor the signal*, holes should be dug about 2 feet in depth into which the legs of the scantling should be sunk. Stakes 4 feet in length at least should then be driven into the

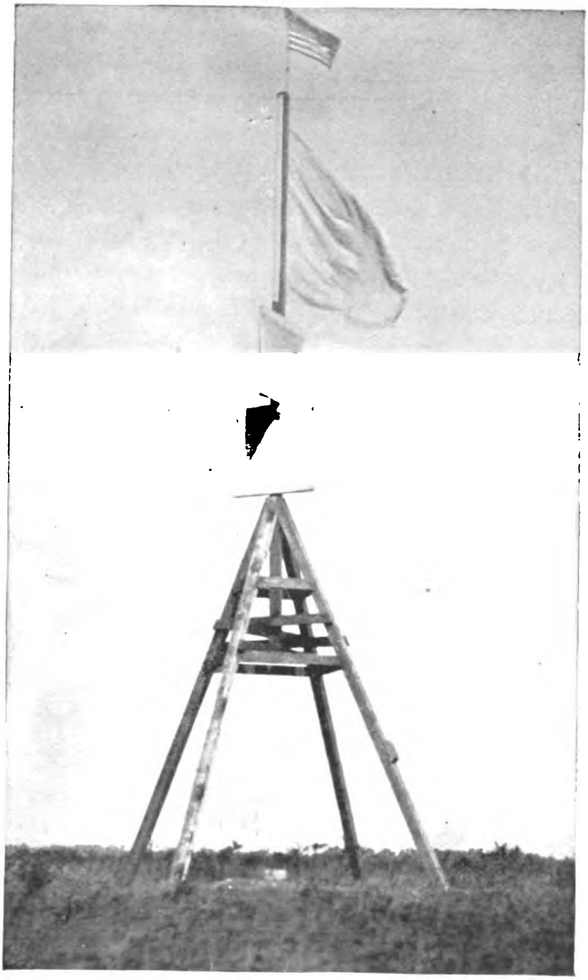


FIG. 164.—QUADRIPOD SIGNAL.

ground in contact with the foot of each pole and approximately at right angles to them, and these should be nailed to the scantling. This will practically insure the signal against being blown over.

245. Observing Scaffolds.—Where it becomes necessary to elevate the instrument, a wooden scaffold must be erected, and this must be so constructed that the instrument can rest on a central scaffold entirely independent of the platform upon which the observer stands, in order to avoid jarring the instrument. The inner scaffold for the support of the instrument should be a tripod triangular in cross-section, and the inclination of its sides should be such as to give it rigidity and to bring the main frames together in one cap-piece at the summit on which the instrument will rest. The outer scaffold for the support of the platform on which the observer will stand should be square in plan, and the sides should be inclined so as to give stability. The width of the platform at the top should be sufficiently great to permit of the free movement of the observer about the instrument, and he should be protected by a railing around the outer edge of the platform. (Fig. 165.)

Such scaffolds may be constructed of the crudest material at hand, and it may be necessary often to use such only. On inaccessible mountains the writer has erected scaffolds in the tops of trees, and a central tree has been used as an observing stand for the instrument. In such cases all limbs should be cut off the tree as well as the top, in order to offer the least obstruction to the wind, which will otherwise jar the instrument. In other instances satisfactory observing scaffolds have been built in the limbs of a single large tree, and another growing close to it has been used as the observing stand. Wherever sawed scantling is available, however, it should be employed, for the reasons given in the last article. The scaffold should be erected much as is the framework of a building. Each length should be framed on the ground until as many bents have been fastened together as can be readily raised with the force available, perhaps two bents of 8 or 12 feet each. Those two which are opposite should be erected at the same time, and then the cross-bracing be nailed to place them at the proper distance. Thereafter boards are laid across the tops of each set of bents, and the workmen, standing on these, frame the next higher bent.

The whole should be strengthened by diagonal bracing with one-inch planks. It should be anchored as described in the last article, and braced, moreover, by long planks, leaning against it as struts and suitably grounded, or by guying it with long wires to neighboring stakes or trees.

246. Heliotrope.—This is an instrument designed to reflect sunlight by a mirror from the station sighted upon to that occupied by the observer. The beam of reflected light is pointed upon as on a signal. There are three specific objects to be aimed at in the design and use of the heliotrope:

1. The reflecting surface should be as near the center of the station as possible;

2. The method of aligning or directing the reflected beam toward the observer's station should be the most precise and simple attainable;

3. The method of maintaining the direction of the reflected beam, while following the apparent movement of the sun, should be the simplest possible.

There are three general types of heliotropes for the accomplishment of the above objects. These are:

1. Simple hand-mirrors provided with screw for attachment to a wooden support;

2. Telescopes carrying revolving mirror and aligning sights;

3. Steinheil heliotrope having mirror aligned by the reflected image of the sun.

Heliotropes should rarely be used as signals for distances

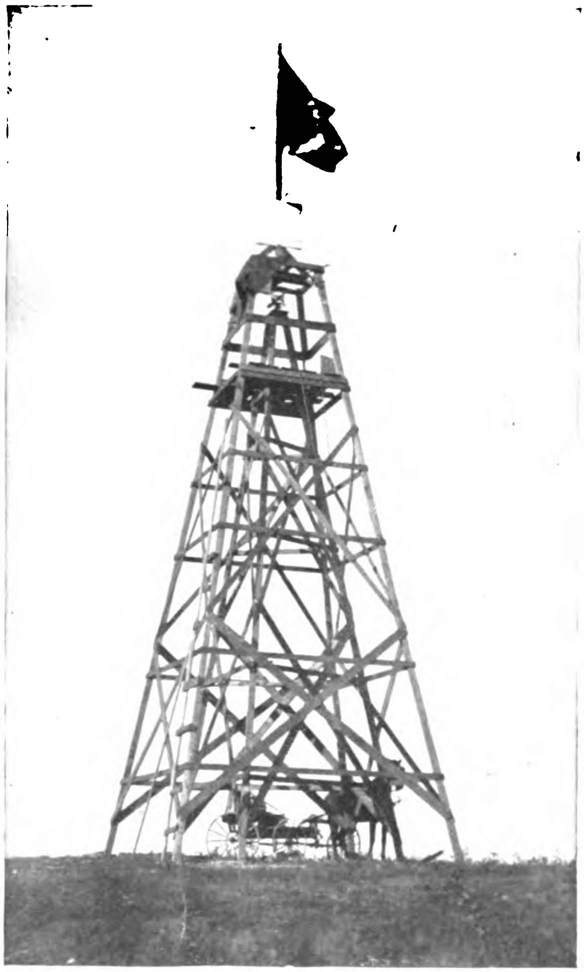


FIG. 165.—OBSERVING SCAFFOLD AND SIGNAL.

less than twenty miles, excepting for very smoky or hazy weather, or because of the difficulty of making visible an opaque signal in dense wood; or at any greater distance providing an opaque signal can be seen. This chiefly because (1) opaque signal gives better definition; (2) the beam reflected from the heliotrope is too large when observed at short range; (3) it is difficult to arrange a satisfactory understanding between the observer and the distant heliotroper.

The *dimensions of a heliotrope* should be the smallest which will produce a clearly defined and visible star of light at the distance observed. In order, therefore, to secure images of uniform size at all distances, the size of the mirror must be varied according to the distance. For ordinary atmospheric conditions and distances of ten miles and over, the following formula may be used to determine the size of the mirror:

$$x = .046d,$$

in which x = the length of the sides of the mirror in inches;
d = the distance observed in miles.

In accordance with this formula the following table gives the length of the side of the mirror for various distances.

TABLE XXXII.

SIZES OF HELIOTROPE MIRRORS.

Distance, Miles.	Side, Inches.	Distance, Miles.	Side, Inches.	Distance, Miles.	Side, Inches.
10	0.46	60	2.8	120	5.5
20	0.92	70	3.2	140	6.4
30	1.37	80	3.7	160	7.3
40	1.83	90	4.1	180	8.3
50	2.3	100	4.6	200	9.2

While the *alignment of the mirror* must be relatively precise, such accuracy is only required as may be obtained by relatively crude methods. The cone of incident and reflected rays subtends equal angles the amount of which is about 32 minutes. The base of this cone is about 50 feet in diameter per mile of distance. Thus for a distance of 20 miles the reflected ray is visible over a vertical area of about 1000 feet diameter. It is thus evident that the alignment may vary as much as 15 minutes of arc on either side of the true direction, or nearly .01 of a foot in a distance of 2 feet.

The simplest, most useful, and most practical heliotrope for all ordinary usage is a small *hand-mirror* (Fig. 166, *b*) similar to the reflecting mirror used with the telescopic helio-

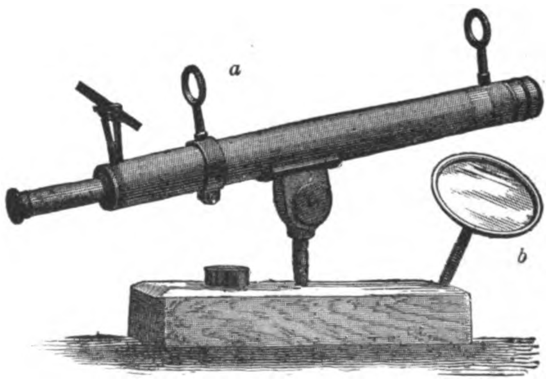


FIG. 166.—TELESCOPIC HELIOTROPE.

trope. The hand-mirror or the *telescopic heliotrope* (Fig. 166, *a*) is used by inserting into the side of a tree or post or other wooden support the screw to which it is attached by means of a hinged joint working with friction. Thus, by screwing or unscrewing it into the wood and moving the joint, it can be made to follow the path of the sun. A similar mirror at a distance of 10 or 15 feet is used to reflect the sunlight into the heliotrope mirror when the position of the sun is such that a direct reflection cannot be cast towards the observing station. The *reflecting* or second *mirror* (Fig. 166, *b*) is of similar construction to the hand mirror and likewise may be screwed into a stake, board, or tree.

The *alignment of* the *reflected beam* from a hand mirror is procured by ranging the pointed tops of two small stakes or sticks by eye from it to the observer's station. In order that this alignment may be sufficiently accurate these stakes must be set up at some considerable distance apart, the first about 10 feet and the second 20 to 50 feet distant from the heliotrope. Such alignment having been once made by the triangulator or an assistant, a heliotroper or man who shall move the mirror so as to keep the sunbeam on the tops of the two stakes may be employed, and any near-by resident of ordinary intelligence will be capable of performing such simple labor. To make sure that the reflected image is cast upon the tops of the stakes some dark object, as the trousers or hat of the heliotroper, should be placed behind them at intervals in order that the shadow cast by the reflected beam over the top of the stake may be clearly noted. Or a square of tin with a hole cut in it of size proportioned to the distance may be held in front of the mirror as a stop.

The *telescopic* or Coast Survey *heliotrope* (Fig. 166) consists of a telescope of moderate magnifying power attached to a screw moving with friction, by which it is fastened into a wooden support. Near the eye end is a mirror supported by a horizontal axis, and the latter may be rotated vertically so as to give two motions. A few inches in front of this and at the objective end of the telescope are two rings, so placed that the axes of the center of the mirror and of the rings are parallel to the line of sight of the telescope. The telescope being directed upon the observing station, the mirror is so turned as to reflect the sunlight through the rings and thus to the observing station. This instrument is less simple than the hand mirror because of its liability to get out of *adjustment*. One of the rings may be adjusted by raising or lowering it, and the adjustment should be frequently tested to assure that the alignment of the mirror and both rings is perfect. This operation consists in pointing the telescope at some object 100 or 200 feet away and noting if the reflected light is cast exactly upon it. If not, the adjustable ring must be moved to correct the error.

The *Steinheil heliotrope* (Fig. 167) is a far more compact and serviceable instrument than the telescopic heliotrope. It requires, however, for its manipulation an assistant of some intelligence. On the other hand telescopic and hand heliotropes can scarcely be safely used by an ordinary laborer; therefore where hand-mirrors aligned by stakes are not used the Steinheil will give the greatest satisfaction. It is but three or four inches in length and can be carried easily in the pocket or in a light leather case. This instrument, in addition to its portability, has the advantage that there are no movable parts to get out of adjustment by jarring in carrying.

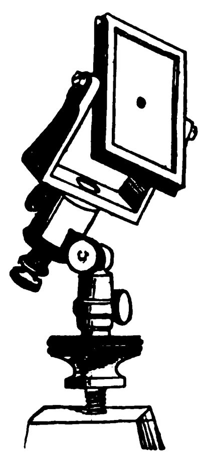


FIG. 167.—STEINHEIL HELIOTROPE.

The Steinheil heliotrope consists of a small sextant mirror, the two surfaces of which are as nearly absolutely parallel as possible. This mirror has a small hole in the center of the reflecting surface, below which is a small lens in the shaft carrying the mirror, and below the lens is some white reflecting material, as plaster of Paris. The mirror is so mounted that it has four different motions, two about its horizontal axis and two about its vertical axis, each of which can be separately controlled by clamps or friction joints. *To use the Steinheil*, it is screwed into some wooden support, as the side of a tree or post, in such a position that the main axis carrying the lens and plaster-of-Paris reflector can be kept parallel to the sun's rays. The heliotroper, standing behind the mirror and looking through the central hole towards the instrument station, sees an imaginary sun produced by the reflection of the true sun from the plaster of Paris and focused by the lens on the surface of the glass. The mirror should then be slowly moved until this imaginary sun, moving with it, appears to rest on the object towards which the flash is to be cast. As both surfaces of the mirror are parallel, the true reflection of the sun from the surface of the mirror will also be cast on the object sighted.

Various attempts have been made to design *heliotropes which shall automatically follow* the path of *the sun* in a manner similar to the clockwork mechanism employed in observatories for following the movement of stars with large telescopes. None of these have proven satisfactory, however, because of their complexity and weight. Other attempts at effecting a similar result have been made by using rectangular polished steel bars made to revolve about a horizontal axis, and the latter to revolve about a vertical axis through hand mechanism. An automatic motion for the same has been attempted by use of cup-shaped wind vanes, similar to those used in anemometers, whereby a many-sided heliotrope is made to revolve in all directions continuously by the wind. Such an apparatus flashes light in every direction, but as yet such flashes have not been procured of sufficient duration and certainty to serve the purposes desired.

The greatest *objections to* the *use of heliotropes* as signals are due to the uncertainty of the atmosphere and the difficulties of communication between observer and heliotroper. Where any attempt is made to observe on several heliotroped stations at one time, if the sun be occasionally or partly obscured by cloud, uncertainty arises on the part of the heliotroper as to whether the observer has measured all the angles required, and he may prematurely leave his station, thus causing considerable delay in conveying directions to him to return. To obviate this a brief code of *heliograph signals* should be arranged where much heliotroping is to be done. The fewest possible sentences should be devised and practiced by heliotropers and observer. The method of conveying such signals is by intermittent flashes and blank spaces, corresponding in general to the dots and dashes of the Morse telegraphic code. The interval between the period during which the heliotrope is permitted to shine and that during which its light is cut off by interposing the hand or some other object before the mirror to produce a blank, should be of not less than ten seconds' duration nor as great as one minute. Thus a sentence may be conveyed by a flash of ten seconds followed by a blank of ten seconds, or another by a flash of ten seconds followed by a blank of thirty seconds and a flash of ten seconds. Any number of similar combinations may be prearranged.

247. **Night-signals.**—Where the observation of angles is impeded during the daytime by dense smoke, the best results are procured by signalling with lights used at night. The moisture in the air at night carries the smoke down into the valleys, thus clearing the atmosphere between the higher summits from which observations are made. Several forms of night-signals have been employed with some success both in India and France, and experimentally by the U. S. Coast Survey. These lights are practically of three kinds only: (1) electric arc light, (2) magnesium tape, and (3) kerosene-oil lamp. All should be used with a parabolic reflector 12 or more inches in diameter, depending on the distance.

The chief conditions in connection with a suitable night-light are that it should be (1) cheap, (2) capable of manipulation by persons of ordinary intelligence, (3) light enough to be easily transported to mountain-tops, and (4) simple of construction and adjustment.

The form of light which fulfills these conditions best, excepting that of cost, is *magnesium tape*. Experiments with this by the Coast Survey indicate that its cost is about $2.25 per oz. of 40 yards length, and its consumption 12 to 18 inches per minute if sufficient brilliancy and steadiness is maintained. This is at the rate of about $2\frac{1}{8}$ cents per minute, or \$1.40 per hour if burned steadily. Accordingly, on the assumption of the average period for observing, which is about two hours, such night-signals will cost about \$3 per night for each signal burned. Another form of night-signal, which is difficult to transport, but is perhaps even more satisfactory, is the ordinary *kerosene-oil headlight* of a locomotive.

248. Station- and Witness-marks.—Primary triangulation stations should be so permanently marked as to render them possible of identification at any future time. This class of marking should include both surface and underground marks. The surface mark, being visible, is readily found, and in searching for a station its position can be verified by the discovery of the underground mark. Should the surface mark be disturbed, which is not unlikely, the witness-marks will indicate the positions of the underground mark, the discovery of which will again locate the station.

Underground marks should be buried below frost and plow line, say at least three feet beneath the surface. Their chief characteristics should be: (1) indestructibility; (2) peculiarity of shape and appearance; (3) cheapness and lack of value as a protection against cupidity. Some of the following are excellent underground marks: a stoneware tile or tablet, dish or cone; a short chiseled block of granite, sandstone, or other stone not indigenous to the locality; a brick or block of hydraulic cement stamped with suitable lettering.

Surface marks depend largely upon the nature of the material composing the ground surface. Where this is soil, they may consist (1) of iron posts sunk into the ground (Fig. 100) with a few inches projecting and bearing a suitable inscription on a metal cap; (2) of stone posts dressed to a square cross-section and appropriately marked; or (3) where the surface is of rock a small eminence or a cross-mark should be chiseled on it and a copper bolt sunk therein.

Witness-marks are established by measurement and magnetic bearings, which are recorded from the station-mark to projecting rocks, houses, trees, or to witness monuments which may be planted for the purpose. The *record of a station* should include descriptions of its summit, underground station-marks, and witness-marks, with a sketch of the whole.

CHAPTER XXVI.

MEASUREMENT OF ANGLES.

249. Precautions in Measuring Horizontal Angles.—The following are some of the more important precautions to be observed in occupying a station and setting up the instrument preparatory to the measurement of horizontal angles, namely:

1. Stability of support of instrument;
2. Stability of foot-screws;
3. Freedom of motion of alidade;
4. Knowledge of signals; and
5. Avoidance of gross errors in record.

The instrument should have a *stable support*, which may be a stone pier, a wooden post, or a good tripod. If a portable tripod is used, its legs should be set firmly in the ground and clamped tightly to the tripod head. On this the instrument should rest freely without being held by center clamp.

The *foot-screws of* the *instrument* should be tightly clamped after it is leveled for work. Looseness of the foot-screws and tripod is a common source of error.

The *alidade*, or part of the instrument carrying the telescope, circle, and verniers or microscopes, should move freely on the vertical axis. Clamps should likewise move freely when loosened. Whenever either of these moves tightly, the instrument needs cleaning, oiling, or adjusting.

The observer should always have a definite preliminary *knowledge of* the *signals* or objects observed. The lack of it may lead to serious error and entail cost much in excess of that involved in procuring such knowledge.

Great care should be taken to insure *correctness in the record* of degrees and minutes of an observed angle. The removal of an ambiguity in them is sometimes a troublesome and expensive task.

250. **Observer's Errors and their Correction.**—The general directions and precautions given in Articles 250 to 252 regarding instrument errors and the measurement of angles in a system of primary triangulation were prepared by Prof. R. S. Woodward for the guidance of the observers of the U. S. Geological Survey. They are supplemented by memoranda from the Proceedings of the Geodetic Conference held at the U. S. Coast Survey office in Washington in 1893, and from the experience of the author.

The errors to which measured angles are subject may be divided into two classes, viz.:

1. Those dependent on the instrument used, or instrumental errors; and
2. Those arising from all other sources, which may be called observer's or extra-instrumental errors.

Extra-instrumental errors may be divided into four classes, namely:

1. Errors of observation;
2. Errors from twist of tripod or other support;
3. Errors from centering; and
4. Errors from unsteadiness of the atmosphere.

Barring blunders or mistakes, the *errors of observation* are in general relatively small or unimportant. With observers practiced in measuring angles such errors are the least formidable of all the unavoidable errors, and the methods devised for their elimination result in practical perfection The recognition of this fact is very important, for observers are prone to attribute unexpected discrepancies to bad observation rather than to their much more probable causes.

After learning how to make good observations the observer should place the utmost confidence in them, and never yield to the temptation of changing them because they disagree with some preceding observations. Such discrepancies are in general an indication of good rather than of poor observations.

Stations or tripods which have been unequally *heated by* the *sun* or other source of heat usually *twist* more or less *in azimuth.* The rate of this twist is often as great as a second of arc per minute of time, and it is generally quite uniform for intervals of ten to twenty minutes. The effect of twist is to make measured angles too great or too small, according as they are observed by turning the microscopes in the direction of increasing graduation or in the opposite direction. This effect is well eliminated, in general, in the mean of two measures, one made by turning the alidade in the direction of increasing graduation, followed immediately by turning the alidade in the opposite direction. Such means are called *combined measures* or combined results, and all results used should be of this kind. As the uniformity in rate of twist cannot be depended on for any considerable interval, the more rapidly the observations of an angle can be made the more complete will be the elimination of the twist. The observer should not wait more than two or three minutes after pointing on one signal before pointing on the next. If for any reason it should be necessary to wait longer than such short interval, it will be best to make a new reading on the first signal.

The precision of *centering an instrument or signal* over the station or geodetic point increases in importance inversely as the length of the triangulation sides. Thus if it is desired to exclude errors from this source as small as a second, one must know the position of the instrument within one-third of an inch for lines a mile long, or within six inches for lines twenty miles long. The following easily remembered relations will serve as a guide to the required precision in any case:

1 second	is	equivalent	to	0.3 inch	at	the	distance of	1 mile;
1 "	"	"	"	3.0 inches	"	"	"	10 miles;
1 "	"	"	"	6.0 "	"	"	"	20 " ;
1 minute	"	"	"	1.5 feet	"	"	"	1 mile.

The notes should always state explicitly the relative positions of instrument and signal, and give their coordinates (preferably polar coordinates) if they are not centered.

Objects seen through the atmosphere appear unsteady, and sometimes this *boiling of* the *atmosphere* is so great as to render the identity of objects doubtful. This is usually greatest during the middle of the day, and generally subsides or ceases for a considerable period between 2 P.M. and sundown. There is frequently also a short interval of quietude about sunrise, and on cloudy days many consecutive hours of steady atmosphere may occur. For the best work, observations should be made only when the air causes small or imperceptible displacements of signals. In applying this rule, however, the observer must use his discretion. Errors of pointing increase rapidly with increase of unsteadiness, but it will frequently happen that time may be saved by counterbalancing errors from this source by making a greater number of observations. Thus, if signals are fairly steady, it may be economical to make double the number of observations rather than wait for better conditions.

251. Instrumental Errors and their Correction.—The best instruments are more or less defective, and all adjustments on which precision depends are liable to derangement. Hence results the general practice of arranging observations in such a manner that the errors due to instrumental defects will be eliminated. The principal errors of this kind and the methods of avoiding their effects are:

1. Periodic errors;
2. Accidental errors;

3. Collimation errors;
4. Errors due to inequality of pivots;
5. Errors due to inequality in height of wyes;
6. Errors due to inclination of telescope axis;
7. Errors due to parallax of cross-hairs;
8. Errors of run of micrometer-screw; and
9. Errors from loose tangent or micrometer screws.

Measurements made with a graduated circle are subject to certain *systematic errors* commonly called *periodic*. Certain of these errors are always eliminated in the mean or sum of the readings of the equidistant verniers or microscopes, and both or all of these should be read with equal care in precise work. Certain other errors of this class are not eliminated in the mean of the microscope readings, and only these need consideration. Their effect on the mean of all the measures of an angle may be rendered insignificant by making the same number of individual measures with the circle in each of n equidistant positions separated by an interval equal to $\frac{360°}{nm}$, where m is the number of equidistant verniers or microscopes. Thus, if $m = 2$, the circle should be shifted after each measure by an amount equal to $\frac{180°}{n}$, which, for example, is 45° for $n = 4$ and 30° for $n = 6$. The degree of approximation of this elimination increases rapidly with n. (Art. 252.) Other things being equal, therefore, the measures of such special angles should show less range than the measures of other angles.

Besides the instrumental errors of the periodic class, there are also *accidental errors of graduation*. These are in general small, however, in the best modern circles, and their effect is sufficiently eliminated by shifting the circle in the manner explained above for periodic errors.

The effect of an *error of collimation* on the circle reading for any direction varies as the secant of the altitude of the object observed. The effect on an angle between two objects varies as the difference between the secants of their altitudes. This effect is eliminated either by reversing the telescope in its wyes, or by transiting it without changing the pivots in the wyes, the same number of measures being obtained in each of the two positions of the telescope. The latter method is the better, especially in determining azimuth, since it eliminates at the same time *errors due to inequality of pivots and inequality in height of wyes.*

The effect of *errors due to inclination of telescope axis* on the circle reading for any direction varies as the tangent of the altitude of the object observed. If the inclination is small, as it may always be by proper adjustment, its effect will be negligible in most cases. But if the objects differ much in altitude, as in azimuth work, the inclination of the axis must be carefully measured with the striding-level, so that the proper correction can be applied. The following formula includes the corrections to the circle reading on any object for collimation and inclination of telescope axis:

$$\text{Cor.} = c \sec h + b \tan h; \quad . \quad . \quad . \quad . \quad (51)$$

in which c = collimation in seconds of arc;
b = inclination of axis in seconds of arc;
h = altitude of object observed.

Parallax of cross-hairs occurs when they are not in the common focal plane with the eyepiece and objective. It is detected by moving the eye to and fro sidewise while looking at the wires and image of the object observed. If the wires appear to move in the least, an adjustment is necessary. The eyepiece should always be first adjusted to give distinct vision of the cross-hairs. This adjustment is entirely independent of all others, and requires only that light enough to illuminate the wires enter the telescope or microscope tube. It is dependent on the eye, and is in general different for different persons. Hence bad adjustment of the eyepiece cannot be corrected by moving the cross-hairs with reference to the objective. Having adjusted the eyepiece, the image of the object observed may be brought into the plane of the cross-hairs by means of the rack-and-pinion movement of the telescope. A few trials will make the parallax disappear.

When circles are read by micrometer microscopes it is customary to have them so adjusted that an even number of revolutions of the screw will carry the wires over the image of a graduation space. If the adjustment is not perfect, an *error of run* will be introduced. This may in all cases be made small or negligible, since by means of the independent movements of the whole microscope and the objective with respect to the circle the image may be given any required size. In making this adjustment some standard space, or space whose error is known, should be used. At least once at each station where angles are read observations should be made for run of micrometers.

READINGS FOR RUN OF MICROSCOPES ON SPACE 359° 50′ TO 360°.

(Ideal case showing microscopes in need of adjustment.)

	A		B	
	359° 50′	360°	359° 50′	360°
	4.0	3.1	1.7	0.2
	4.0	2.2	2.1	1.1
	3.9	2.4	2.0	0.7
	3.3	2.6	1.7	0.0
	4.1	2.7	2.1	0.1
Means	3.86	2.60	1.92	0.42
Difference		−1.26		−1.50
Error of space		−0.37 known		−0.37 known
Error of run		−1.63 for 5 revs.		−1.87 for 5 revs.

Hence readings of microscope A should be diminished by 0.33 div. per revolution, and those of B by 0.37 div. per revolution, which is one-fifth of the error of run in each case.

Errors from loose tangent or micrometer screws are due to their moving too freely or loosely. In making a pointing with the telescope the tangent screw should always move against or push the opposing spring. Likewise, bisections with the micrometer wires must always be made by making the screw pull the micrometer frame against the opposing spring or springs.

252. Methods of Measuring Horizontal Angles.—Two general methods are employed for reading parts of the angle less than the smallest space graduated on the horizontal limb, namely:

1. By means of verniers; and
2. By micrometer microscopes.

Where it is unnecessary to read angles to lesser amounts than 10 seconds of arc verniers may be successfully employed. If greater accuracy is to be attempted by reading to smaller fractions of the arc, micrometer microscopes must be employed. Primary angles are read with verniers by the *method of repetition*, and with micrometer microscopes by the *method of directions*.

Vernier or *repeating theodolites* are not used now to any extent on primary work of high order. In order that the best results may be had from such an instrument it must have a very large circle, as from 16 to 20 inches diameter, and be proportionately heavy and cumbersome. Such instruments are now generally employed only on secondary triangulation, where a circle not greater than six or seven inches in diameter will give satisfactory results.

The *method of reading angles* with the repeating instrument consists in pointing at the first station, *n*, and with the lower circle clamped revolving the graduated limb and pointing at the second station, *o* (Fig. 175). Then with the upper circle clamped the instrument is revolved on its lower circle in the reverse direction so as to point back again at *n*, and the operation is repeated. By this means the direct reading of the angle between the two stations is increased or added on the vernier in proportion to the number of repetitions of the angle made. Thus if six such angles are read, the single angle will be the total recorded on the circle divided by 6, and the result will be a reading possibly $\frac{1}{6}$ smaller than the amount by which a single angle could be read on the vernier. In the use of such an instrument each *set of repetitions* consists of a fixed number of measures of the angle, say three, followed by an equal number of measures with the telescope reversed. Two sets of six repetitions, as 3 direct + 3 reversed, are preferable to one set of twelve repetitions, as 6 direct + 6 reversed, because something may occur to interrupt the observations during the longer time. In like manner the various angles between the adjacent stations observed are each separately read.

The *best results* procurable in the measurement of horizontal angles are obtained *with direction instruments*. Such instruments, with circles as small as 8 inches, will give more accurate readings of the angle than a corresponding repeating instrument of 16 to 18 inches circle. In observing with a direction instrument the more usual method is to divide the circle into a number of equal parts known as positions. This number should be such that no microscope may fall upon the same graduation in pointing upon the same object in different positions or after reversal of the telescope. Having established the initial direction, one or more series are observed in each position, each consisting of the pointing and reading upon each of the signals in order and reading of the graduation of the circle. Then the *telescope* is *reversed*, the alidade turned 180° in azimuth, and another pointing and reading made upon the various signals in order. The *number of* the various *positions* depends upon (1) the accuracy of the graduation, and (2) upon the degree of refinement desired. For geodetic work of a high order from twenty-four to thirty positions or series should be observed. For primary triangulation of a sufficiently high order for map-making purposes, however, six to eight positions are sufficient.

Angles may be read with a direction instrument by two general methods, namely:

1. Method of independent measures; and
2. Method of measurement by series.

The *best results* are obtained by measuring the angles separately and independently. Thus if the signals in sight around the horizon are in order n, o, p, etc. (Fig. 175), the angles n to o, o to p, etc., are by this method observed separately, and whenever there is sufficient time at the disposal of the observer this method should be followed.

In order to secure the elimination of the errors of observation (Arts. 250 and 251) the following programmes should be strictly adhered to.

When *direction instruments* are used the following is the *programme for independent measurement of angles:*

Pointing on n and readings of both micrometers.
" " o " " " " "
Transit telescope and turn alidade 180°.
Pointing on o and readings of both micrometers.
" " n " " " " "

Shift circle by $\frac{180°}{N}$ and proceed as before until N such sets of measures have been obtained.

Then measure the angles o to p, p to q, etc., including the angle necessary to close the horizon, in the same manner. A form for record and computation of the results is given in Article 253.

When *repeating instruments* are used the same programme will be followed, except that there should be five pointings instead of one each on n and o, the circle being read for the first pointing on n and the fifth on o, and again for the sixth pointing on o and the tenth on n.

The importance of having the measures of a set follow in quick succession must be constantly borne in mind. Under ordinarily favorable conditions an observer can make a pointing and read the microscopes once a minute, and a set of five repetitions should be made in five minutes or less.

When several stations or signals are visible and a *direction instrument* is used, time may be saved without material loss of precision in the angles by observing on all the signals successively according to the following *programme for measurement by series*, the signals being supposed in the order n, o, p, etc., as above:

Pointing on n with micrometer readings.
" " o " " "
" " p " " "
. . . .
Pointing on n with micrometer readings.
Transit telescope and turn alidade 180°.
Pointing on n with micrometer readings.
" " r " " "
" " q " " "
. . . .
Pointing on n with micrometer readings.

Shift circle by $\dfrac{180^\circ}{N}$ and proceed as before until N such sets have been obtained.

The angles n to o, o to p, etc., read in this way may be computed as in the first method, always combining the measure n to o with the immediately succeeding measure o to n to eliminate twist. There is a theoretical objection to this process of deriving the angles founded on the fact that they are not independent, but in secondary work this objection may be ignored as of little weight.

In observing horizontal angles the *number of sets of measures* of any angle is dependent upon the character of instrument and the precision desired. For the primary triangulation of the U. S. Geological Survey with 8-inch direction theodolite read by micrometer microscopes, four sets of measures on as many different parts of the circle will be required. For repeating theodolites six sets of measures will be required, all made according to the programmes given above. Only under specially unfavorable conditions will it be necessary to increase the number of sets of measures. Care should always be taken to shift the circle so as to eliminate periodic errors.

When there is ample time at the disposal of the observer, or need for additional measures, the work may be strengthened by *measuring sum-angles*. This is done in such manner as to introduce additional conditions which will thus strengthen the least-square adjustment. Thus, after reading the separate angles n to o, o to p, p to q, etc., the intermediate pointings may be skipped by reading from n to p, p to r, etc., and the conditions are introduced that n to $o + o$ to $p = n$ to p, and o to $p + p$ to $q = o$ to q.

The practice of starting the measurement of an angle or series of angles with the microscopes reading 0° and 180°, 90° and 270°, etc., must be avoided; otherwise the errors of these particular divisions will affect many angles. In shifting the circle it is neither necessary nor desirable to have the new position differ from the preceding one by exactly $\frac{180^\circ}{N}$. A difference of half a degree either way is unimportant as respects periodic errors, and it is advantageous to have the minutes and seconds differ for the different settings.

253. Record of Triangulation Observations.—In recording the angles read at any primary station with the theodolite, the first page of the notes should give a concise description of the station, how it is reached, character of station-mark, description of witness-points, and a topographic sketch of station and surroundings. There should also be, in case of the necessity of reduction to center (Art. 267), (1) a diagram showing the relation of the signal to the position of the instrument, (2) the distance between the two, and (3) the angle read at the instrument position between one or more of the observed stations and the signal, that full data for reduction may be available. (Fig. 174.) Another diagram should show the directions to the various stations observed, and the arrangement or groupings of the angles. (Fig. 175.) The date and time of observation should be noted at intervals, to show that the instrument has not stood too long between pointings.

The following is an *example of* the *record* made of pointings from the triangulation station "Township" occupied in Kansas by the U. S. Geological Survey in 1889. This is the record of one pair of pointings only, that determining the angle observed between the stations Newt and Walton.

RECORD OF MEASUREMENT OF HORIZONTAL ANGLE.

H. L. Baldwin, Observer.

(Station: Township corner, Kansas, July 1, 1889. Fauth 8-inch theodolite No. 362; one division of micrometer head = 2 seconds.)

Station.	Micr. A.	Micr. B.	Mean Reading.	Angle.	Mean.
	Telescope direct.				
	° ' *Div.*	° ' *Div.*	° ' "	° ' "	"
Walton	93 12 11.3	273 12 09.9	93 12 21.2	36 29 03.9	
Newt.............	129 41 11.9	309 41 13.2	129 41 25.1		05.9
Newt.............	129 41 15.6	309 41 12.1	129 41 27.7	08.0	
Walton............	93 12 10.6	273 12 09.1	93 12 19.7		
	Telescope reversed.				
Walton............	138 27 03.2	318 26 28.0	138 27 01.2		
Newt..............	174 56 02.8	354 55 28.9	174 56 01.7	00.5	
Newt..............	174 56 06.2	354 55 29.5	174 56 05.7		01.8
Walton............	138 27 05.2	318 26 27.4	138 27 02.6	03.1	
	Telescope reversed.				
Walton............	183 07 03.0	3 06 27.2	183 07 00.2		
Newt..............	219 36 05.0	39 35 29.8	219 36 04.8	04.6	
Newt..............	219 36 08.1	39 35 29.5	219 36 07.6		03.9
Walton............	183 07 06.4	3 06 28.1	183 07 04.5	03.1	
	Telescope direct.				
Walton............	228 24 28.1	48 24 22.6	228 24 50.7		
Newt..............	264 53 27.4	84 53 26.1	264 53 53.5	02.8	
Newt..............	264 54 01.1	84 53 26.1	264 53 57.2		04.3
Walton............	228 24 29.3	48 24 22.1	228 24 51.4	05.8	

4|15".9

Mean of four combined measures.. 36° 29' 03".98

254. Instructions for Field-work of Primary Triangulation.—The following instructions are those governing the field-work of primary triangulation in the U. S. Geological Survey.

1. Signals should be of sawed lumber whenever it can be obtained, and great care must be taken to secure perfect centering of instrument and target over station-mark.

2. All stations should be selected with a view to their adaptability to topographic expansion, and when the exact location of a station is decided upon one of the standard iron posts, copper plugs, or bronze tablets must be set as a permanent mark. In light soil a bottle or similar object must be left as a subsurface mark. These marks should be at exact center of station, and in addition there should be left one or more reference marks. At base-line stations there should be left at least two reference marks.

3. Whenever practicable, set the theodolite over center of station while reading angles, to obviate reduction to center.

4. The theodolite when in use must be sheltered from the sun and wind. When setting the theodolite tripod, leave the tripod-head thumb-screws loose until the legs are firmly placed.

5. Never, under any circumstances, attempt to place the circle so that when pointing at any particular station the micrometers will be set to even degrees.

6. Use book No. 9–912 for all field records, and do not crowd notes. Have notes plainly written with No. 4 pencil or with ink, and never erase, but draw a single line through erroneous records.

7. On page immediately preceding record of angles, write a minute and complete description of the station occupied, giving nearest trails or roads, camping-places, station-marks, etc., as well as ownership of land when possible. Write this description before leaving the station. In addition plat a

rough diagram of pointings, showing also plan of eccentric location of instrument, if there be such.

8. Before observations are commenced at a station, test all adjustments of theodolite, and correct such as are found in error, paying special attention to micrometers to avoid the errors of run.

9. For micrometer theodolites, angles must be measured either by the method of circle readings (directions) or by single angles, and in either case each set of angles must be kept on a single page of note-book. If the method of directions be adopted, each complete set must consist of pointings with telescope direct, and reverse pointings with telescope inverted, always closing horizon.

10. No angle should be considered finally determined that has not been measured on at least four different parts of the circle.

11. The error of closure of any triangle in primary schemes should not exceed 5″.

12. Opposite each angle recorded give any necessary information in regard to visibility of signals or atmospheric conditions.

13. Do not trust to memory for notes. Make all notes as complete as though it were expected another person would compute them.

14. Magnetic declination must be determined at each azimuth station and at each county seat.

15. Observations for azimuth on Polaris before and after elongation must be made on two nights from at least one station in each square degree, to consist of not less than 6 angles between mark and star with telescope direct and reversed. See Monograph above referred to for form of record. Great care must be taken in adjusting and leveling the horizontal axis of theodolite. Watch error must be determined by telegraphic comparison of time or by astronomic observations.

16. Two marks of dressed stone or masonry, about 500 feet apart on a true north-and-south line, must be established at each county seat, the center of each to be the cross-mark on one of the standard bronze tablets.

17. Angles at each station must be reduced to center of permanent mark in order to test triangle closures. Arbitrary adjustments and preliminary computations should be made in the field. All computations except distances and coordinates must be in book No. 9–889.

18. Keep a careful plot of the work on a scale of 10 miles to an inch, and each month send a copy with monthly report, indicating angles measured by the usual signs.

19. On fly-leaf of each note-book write an index of contents of book, and state make and number of theodolite used.

20. The observer should always endeavor to locate prominent points that may be of use to the topographer, or that may be used for future stations.

21. Especial attention must be paid to the location of county court-houses, section and county corners, and State-line marks.

22. Useful locations can often be made by the "three-point method," the theodolite being set up for the purpose while going to or from stations.

23. Keep in view the fact that station names are to be published, and select such as have local significance.

255. Primary Triangulation—Cost, Speed, and Accuracy.—Triangulation of the highest geodetic precision, as executed by the U. S. Coast and Geodetic Survey, costs at the average rate of $1500 per station occupied and from $10 to $30 per square mile, according to the character of the topography; the daily cost of a party of from five to fifteen individuals averaging $65. The speed of the work, or, in other words, the length of time which is required to occupy a station, is indicated by the rate of $\frac{3}{4}$ station per month. In this work the average closure error of a triangle is $0''.7$, the probable error of an extended triangulation being $\frac{1}{150000}$. Or, stated otherwise, a line 10 miles in length would have a probable error of 0.35 ft.

In the primary triangulation executed by the U. S. Geological Survey, not for geodetic purposes, but with sufficient accuracy to safely control topographic maps, the average cost per station is $170, and the cost per square mile controlled about 90 cents. The cost per day for working-parties of from two to five members has averaged $18. The speed has been at the rate of six stations per month. The accuracy is shown by closure errors averaging 3″.0. The probable error of this triangulation has averaged $\frac{1}{40000}$, which may be otherwise expressed as 1.32 feet in a line 10 miles in length.

CHAPTER XXVII.

SOLUTION OF TRIANGLES.

256. Trigonometric Functions.—Let $\alpha =$ angle GAB $=$ arc GB, and let radius $AB = AG = 1$; then

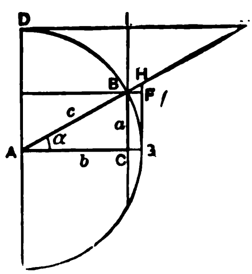


FIG. 168.—TRIGONOMETRIC FUNCTIONS.

$$\sin \alpha = CB;$$
$$\cos \alpha = AC;$$
$$\tan \alpha = HG;$$
$$\text{cotan } \alpha = DE;$$
$$\sec \alpha = AH;$$
$$\text{cosec } \alpha = AE;$$
$$\text{versin } \alpha = BF;$$
$$\text{coversin } \alpha = BI;$$
$$\text{chord } \alpha = GB.$$

257. Fundamental Formulas for Trigonometric Functions.—The fundamental formulas are:

$$\sin^2 \alpha + \cos^2 \alpha = 1; \qquad \tan \alpha \cot \alpha = 1;$$

$$\cos \alpha \sec \alpha = 1; \qquad \sin \alpha \text{ cosec } \alpha = 1;$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}; \qquad \cot \alpha = \frac{\cos \alpha}{\sin \alpha};$$

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha} = \sec^2 \alpha; \quad 1 + \cot^2 \alpha = \frac{1}{\sin^2 \alpha} = \text{cosec}^2 \alpha;$$

$$\text{versed sin } \alpha = 1 - \cos \alpha.$$

258. Formulas for Solution of Right-angled Triangles.—In the right-angled triangle, Fig. 168,

Let a = altitude,
b = base, and
c = hypothenuse; and let
α, β, and γ = the angles opposite a, b, and c, respectively; also let
A = area of triangle, and
R = radius of circumscribed circle.

For a right-angled triangle $\gamma = 90°$; the fundamental values of a, b, and A are then

$$a = c \sin \alpha = c \cos \beta = b \tan \alpha = b \operatorname{cotan} \beta;$$
$$b = c \sin \beta = c \cos \alpha = a \tan \beta = a \operatorname{cotan} \alpha; \text{ and}$$
$$A = \tfrac{1}{2}ab = \tfrac{1}{2}a^2 \operatorname{cotan} \alpha = \tfrac{1}{2}b^2 \tan \alpha = \tfrac{1}{4}c^2 \sin 2\alpha.$$

Fig. 169 furnishes a method of graphically stating the formulas relating to the solution of right-angled triangles.

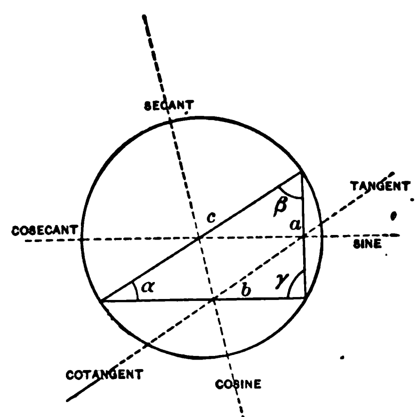


FIG. 169.—GRAPHIC STATEMENT OF FORMULAS FOR SOLUTION OF RIGHT-ANGLED TRIANGLES.

Let P = perpendicular in a right-angled triangle, the angle between the base of which, B, and the hypothenuse, H, is denoted by α.

Then the diagram is applied by the use of the following rules, the order of sequence being to follow either the names written around the circumference of the circle or by following the names along the intersecting lines in the order written; thus:

1. Any trigonometric function or part equals the adjacent part divided by the following part. Example:

$$\sin \alpha = \frac{\cos \alpha}{\cot \alpha};$$

also,
$$\sin \alpha = \frac{a}{c},$$

and
$$a = \frac{c}{\cos \alpha}.$$

2. Any part equals the product of the adjacent parts. Example:

$$a = c \sin \alpha = b \tan \alpha; \quad \text{cosin } \alpha = \sin \alpha \text{ cotan } \alpha.$$

3. Each part equals the reciprocal of the opposite part Example:

$$\tan = \frac{1}{\text{cotan } \alpha}; \quad \sec \alpha = \frac{1}{\text{cosin } \alpha}.$$

4. The product of opposite parts equals 1. Example:

$$\tan \alpha \text{ cotan } \alpha = 1.$$

259. Solution of Plane Triangles.—In the solution of geodetic triangulation there arise a few simple problems which involve the solution of triangles in accordance with the principles of trigonometry. These occur when one or more angles or sides have been measured in the field and the dimensions of the remaining parts are desired. In the following articles are illustrated by practical examples those problems most likely to arise in actual practice.

Table XXXIII, from Smithsonian Tables, gives all the more important formulas for finding unknown parts of a triangle with three parts given.

TABLE XXXIII.

SOLUTION OF OBLIQUE PLANE TRIANGLES.

Given.	Sought.	Formula.
a, b, c	α	$\sin \frac{1}{2}\alpha = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $s = \frac{1}{2}(a+b+c)$,
		$\cos \frac{1}{2}\alpha = \sqrt{\frac{s(s-a)}{bc}}$,
		$\tan \frac{1}{2}\alpha = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$.
	A	$A = \sqrt{s(s-a)(s-b)(s-c)}$.
a, b, α	β	$\sin \beta = b \sin \alpha / a$.
		When $a > b$, $\beta < 90°$ and but one value results. When $b > a$, β has two values.
	γ	$\gamma = 180° - (\alpha + \beta)$.
	c	$c = a \sin \gamma / \sin \alpha$.
	A	$A = \frac{1}{2}ab \sin \gamma$.
a, α, β	b	$b = a \sin \beta / \sin \alpha$.
	γ	$\gamma = 180° - (\alpha + \beta)$.
	c	$c = a \sin \gamma / \sin \alpha = a \sin (\alpha + \beta) / \sin \alpha$.
	A	$A = \frac{1}{2}ab \sin \gamma = \frac{1}{2}a^2 \sin \beta \sin \gamma / \sin \alpha$.
a, b, γ	α	$\tan \alpha = \frac{a \sin \gamma}{b - a \cos \gamma}$.
	α, β	$\frac{1}{2}(\alpha + \beta) = 90° - \frac{1}{2}\gamma$,
		$\tan \frac{1}{2}(\alpha - \beta) = \frac{a-b}{a+b} \cot \frac{1}{2}\gamma$.
	c	$c = (a^2 + b^2 - 2ab \cos \gamma)^{\frac{1}{2}}$,
		$= \{(a+b)^2 - 4ab \cos^2 \frac{1}{2}\gamma\}^{\frac{1}{2}}$,
		$= \{(a-b)^2 + 4ab \sin^2 \frac{1}{2}\gamma\}^{\frac{1}{2}}$,
		$= (a-b)/\cos \phi$, where $\tan \phi = 2\sqrt{ab} \sin \frac{1}{2}\gamma / (a-b)$,
		$= a \sin \gamma / \sin \alpha$.
	A	$A = \frac{1}{2}ab \sin \gamma$.

260. Given Two Sides and Included Angle, to Solve the Triangle.

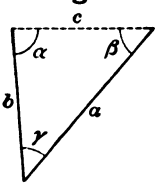


FIG. 170.

$$\tan \alpha' = \log \left(\frac{a}{b}\right) \text{ (less from greater)}. \quad . \quad . \quad (52)$$

$$\tan(\alpha' - 45^\circ) = \tan \delta \text{ (less from greater)}. \quad (53)$$

$$\tan \alpha \times \tan \tfrac{1}{2}(\alpha + \beta) = \tan \tfrac{1}{2}(\alpha - \beta). \quad . \quad (54)$$

$$\tfrac{1}{2}(\alpha + \beta) + \tfrac{1}{2}(\alpha - \beta) = \alpha. \quad . \quad . \quad (55)$$

$$\tfrac{1}{2}(\alpha + \beta) - \tfrac{1}{2}(\alpha - \beta) = \beta. \quad . \quad . \quad (56)$$

Knowing α and β, compute remaining parts.

Check is that the sum of the angles or $\alpha + \beta + \gamma = 180^\circ$. For convenience always call greater side a; then as greater side is always opposite greater angle, α is opposite a.

EXAMPLE.

$$\begin{aligned}
\text{Let } \log a &= 4.1361976 \\
\log b &= 4.1726495 \\
\hline
\tan \alpha' &= 0.0364519 \\
\alpha' &= 47^\circ\ 24'\ 06''.1 \\
& \quad 45^\circ \\
\hline
\delta &= \ \ 2^\circ\ 24'\ 06''.1
\end{aligned}$$

$$\begin{aligned}
& \quad 180^\circ \\
\text{Let } \gamma &= \ \ 51^\circ\ 43'\ 27''.3 \\
& 2)128^\circ\ 16'\ 32''.7 \\
\tfrac{1}{2}(\alpha + \beta) &= \ \ 64^\circ\ 08'\ 16''.35 \\
+\tfrac{1}{2}(\alpha - \beta) &= \ \ \ \ 4^\circ\ 56'\ 42''.06 \\
\hline
\alpha &= \ \ 69^\circ\ 04'\ 58''.41
\end{aligned}$$

$$\begin{aligned}
\log \tan \alpha &= 8.6226496 \\
\tan \tfrac{1}{2}(\alpha + \beta) &= 0.3144757 \\
\hline
\tan \tfrac{1}{2}(\alpha - \beta) &= 8.9371253 \\
\tfrac{1}{2}(\alpha - \beta) &= \ \ 4^\circ\ 56'\ 42''.06 \\
\hline
\tfrac{1}{2}(\alpha + \beta) &= 64^\circ\ 08'\ 16''.35 \\
-\tfrac{1}{2}(\alpha - \beta) &= \ \ 4^\circ\ 56'\ 42''.06 \\
\hline
\beta &= 59^\circ\ 11'\ 34''.29
\end{aligned}$$

Check.

$$\begin{aligned}
\alpha &= 69^\circ\ 04'\ 58''.41 \\
\beta &= 59^\circ\ 11'\ 34''.29 \\
\gamma &= 51^\circ\ 43'\ 27''.30 \\
\hline
& 180^\circ\ 00'\ 00''.00
\end{aligned}$$

261. Given Certain Functions of a Triangle, to Find Remainder.

The sides of a triangle are proportional to the sines of their opposite angles, hence

$$\sin \beta = \frac{b \sin \alpha}{a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

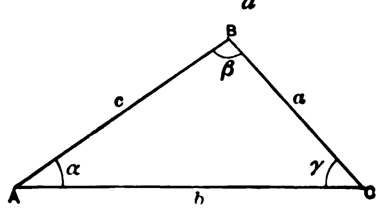


FIG. 171.

EXAMPLE.—Let $a = 12.92$ miles; $b = 153$ ft. 7 in.; $\alpha = 130° \ 58' \ 18''.3$.

Almost all field measures made in the United States are in miles, feet, etc., while all geodetic tables are prepared on the metric system; hence the former must be reduced to the latter for computation. Reducing miles and feet to the same unit, meters, and finding the corresponding logarithms, we have

$$\begin{aligned}
\log a &= 4.31790 \\
\text{a. c. } \log a &= 5.68210 \\
\log b &= 1.67035 \\
\log \sin \alpha &= \underline{9.87796} \\
\log \sin \beta &= 7.23041 \\
\beta &= 00° \ 05' \ 50''.62
\end{aligned}$$

262. Given Three Sides of a Triangle, to Find the Angles.

s = one-half the sum of the three sides. For convenience designate $\sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}}$ by H, then

$$\tan \tfrac{1}{2}\alpha = \frac{H}{s-a}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

$$\tan \tfrac{1}{2}\beta = \frac{H}{s-b}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

and

$$\tan \tfrac{1}{2}\gamma = \frac{H}{s-c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

EXAMPLE.

$$\begin{aligned}
c &= 4.1908 \\
b &= 40.8954 \\
a &= \underline{43.7566} \\
2s &= 88.8428 \\
s &= 44.4214 & \log s &= 1.6475922 \\
s-a &= 0.6648 & \log &= 9.8226910 \\
s-b &= 3.5620 & \text{''} &= 0.5472823 \\
s-c &= 40.2306 & \text{''} &= \underline{1.6045564} \\
& & \text{Sum of logs} &+ 1.9745297 \\
& & \log s &= -\underline{1.6475922} \\
& & H^2 &= 0.3269375 \\
& & H &= 0.1634688
\end{aligned}$$

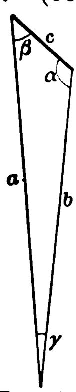


FIG. 172.

$0.1634688 = H$	
$9.8226910 = \log s - a$	
$0.3407778 = \log \tan \frac{1}{2}\alpha$	$\frac{1}{2}\alpha = 65° \; 28' \; 27''$; $\therefore \alpha = 130° \; 56' \; 54''$
$0.1634688 \; (H) \log$	
$.5472823 \; (s - b) \log$	
$9.6161865 = \log \tan \frac{1}{2}\beta$	$\frac{1}{2}\beta = 22° \; 27' \; 06''$; $\therefore \beta = 44° \; 54' \; 12''$
$0.1634688 = \log H$	
$1.6045564 = \log s - c$	
$8.5589124 = \log \tan \frac{1}{2}\gamma$	$\frac{1}{2}\gamma = 2° \; 4' \; 27''$; $\therefore \gamma = 4° \; 8' \; 54''$
	Check $180° \; 00' \; 00''$

263. Three-point Problem.—The object sought in the solution of this problem is the determination of the unknown position of an occupied station P, when the positions of three other stations, A, B, and C, are known. (See Graphic Solution, Art. 75.)

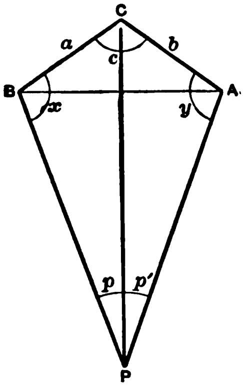


FIG. 173.—THREE-POINT PROBLEM.

The problem is indeterminate when P is on the circumference of a circle passing through A, B, and C. This is known by the sum of the angles $p + p' + c$, being equal to 180°, and also by the radius of the circumference passing PAC, being equal to that for PBC.

$$\cot x = \cot R = \left(\frac{a \sin p'}{b \sin p \cos R} + 1\right), \quad . \quad . \quad (61)$$

in which

1. $R = 360° - p - p' - c$ or $R = x - y$ or $R - x = y$.

   If $p + p' = c$ or nearly, the solution is impossible.

2. $\log \dfrac{a \sin p'}{b \sin p \cos R} =$ log of a number taking the sign of $\cos R$.

3. Add algebraically $+ 1$ to the above number.

4. Take out the log of this number, annexing the proper sign.

5. Then add this log to log cot R, remembering that this is in effect multiplying one by the other, and the rule of the signs must be attended to; this gives the log of the cot of x. Then $R - x = y$.
6. If $R < 90°$, cos R is $+$ and cot R is $+$.
   " $R < 270°$ and $R > 180°$, cos R is $-$, cot R is $+$.
   " $R < 180°$ and $R > 90°$, cos R is $-$, cot R is $-$.
   " $R < 360°$ and $R > 270°$, cos R is $+$, cot R is $-$.
7. p' is at opposite side of quadrilateral to a, and p to b.
8. The angle c is always the interior angle of the quadrilateral $PBCA$, and C is the middle point as seen from P.

EXAMPLE.—The following quantities are known from observation or computation, since the positions of B, C, and A are known, namely:

$a = 6672.47$ ft.; $\qquad p = 20° \, 05' \, 53''$;
$p' = 35° \, 06' \, 08''$; $\qquad c = 152° \, 23' \, 22''$.
$b = 12481.66$ ft.;

Then

$$R = 152° \, 24' \, 37'' = 360° - p - p' - c.$$

log a = 3.8242868
sin p' = 9.7596958
a.c. log b = 5.9037277
a.c. sin p = 0.4639117
a.c. cos R = 0.0524258 −
(cos R gives sign) 0.0040478 − = the number − 1.009364

$$\frac{a \sin p'}{b \sin p \cos R} + 1 \qquad \begin{array}{r} +\ 1.000000 \\ \hline -\ 0.009364 \end{array}$$

$$\begin{array}{rl} R = & 152° \, 24' \, 37'' \\ x = & 88° \, 58' \, 24'' \\ \hline y = & 63° \, 26' \, 13'' \end{array}$$

number − 0.009364 = log − 7.9714614
$R = 152° \, 24' \, 37''$ = cot − 0.2818637
$x = 88° \, 58' \, 24''$ = cot x = + 8.2533261

CHAPTER XXVIII.

ADJUSTMENT OF PRIMARY TRIANGULATION.

264. Method of Least Squares.—By the *method of least squares* is understood a process by which observations are adjusted and compared. When several precise measurements have been made of a given quantity, no matter how similar the conditions may appear, the results do not agree and it becomes necessary to adjust the various measures or observations in order to get a mean or apparent agreement. The result is not necessarily the true value, but is used and accepted as such since it is a mean derived from the combination and adjustment of all the measures taken which are most probably and apparently correct.

Errors of observation are of two kinds, (1) systematic and (2) accidental; the former, resulting from unknown causes, affect all observations alike, while accidental errors are of a kind which produce discrepancies between observations: and it is this kind of errors alone, and not the systematic errors, which are considered in the so-called "theory of errors" and which it is the object of adjustment to minimize. The error of observation is truly the difference between the observed and true value, and may be plus or minus according as it exceeds or is less than the true value. The object of the *theory of errors* is to obtain from a number of discordant observations the best obtainable result. The *fundamental principle* of the method of least squares is the rule of Legendre, that, *in observations of equal precision the most probable values* *of observed quantities are those that render the sum of the squares of the residual errors a mininum.*

The *probable value of an observed quantity* is that which we are justified in considering as the more likely to be the true value than any other. As stated by Prof. Mansfield Merriman, the probability is expressed by an abstract fraction, which measures numerically the degree or likelihood in the happening or failing of an event; as confidence may range from improbability to certainty, so this measure may range from zero to one. If the figure 6, for example, occurs once on a die of six faces, the probability of its turning up when thrown is $\frac{1}{6}$; likewise, if the same figure occurs on each face, the probability of its turning up when the die is thrown is $\frac{6}{6}$, or unity, which is certainty.

When a *number of unknown quantities* are to be determined by means of equations involving unknown quantities, the quantities sought are said to be *indirectly observed.* It is necessary to have as many such indirectly observed equations as there are unknown quantities, and the discovery of these unknown quantities by solution of equations is the *method of least squares.* The differences between the several observed values and that which is taken as the true value of an observation are called the *residuals*, and these are the *apparent errors of observation.* When observations are not made under the same conditions and the computer is aware of reasons which prevent them being equally good, a greater relative importance may be given to better observations by treating them as equivalent to more than one occurrence of the same value in a set of equal observations; in other words, they may be weighted. *Weights* (Art. 284) may therefore be regarded as numerical measures of the influence of the observation upon the arithmetical mean.

In observing a series of angles, the angles read at station Walton, between points n and $o = a$, o and $p = b$, p and $q = c$, and q and $r = d$ (Fig. 175, p. 613) are rendered functions of an adjustment equation. If combinations of these angles are observed, as the angles between n and $p = g$, and between o and $q = i$, and between n and $q = g + c$, then the means of the various angles separately measured between n and $o = a$, and o and $p = b$, should be equal to the mean of the angle read between n and $p = g$. Similarly with the others, and by thus observing all of the separate and many of the combined angles it becomes possible to arrange a number of *equations of condition*, as they are called. In these, however, the mean observed angles never exactly sum up as they should theoretically, and the differences are called the *residuals*.

265. Rejection of Doubtful Observations.—When theodolites or other angle-measuring instruments are used, there occur among a number of observations for the value of a particular angle, one or more which differ greatly from the mean of all. It is not advisable to depend entirely on judgment as to which of these observations shall be retained and which rejected. The least objectionable criterion by which to judge as to the rejection of doubtful observations, and one based on mathematical principles, has been stated by Mr. T. A. Wright thus:

Where an observation differs from the general run of the series by more than five times the probable error or three times the mean square error, attention should be called to it.

An excellent fixed rule for the rejection of doubtful observations is *Peirce's Criterion*, which is applied in the following manner:

Let m = number of measures;

n = number of doubtful observations to be rejected (to be found by trial);

e = mean error of one observation in the set of m;

v, v', etc. = residuals of the observations or the difference of each value from the mean; and

x = ratio of required limit of error for the rejection of n observations, to the mean error e; so that xe is the limiting error.

The value of x^2 for $n = 1$, $n = 2$, etc., and for various values of m is found from Table XXXIV. All observations in which $xe > v$ are to be rejected, stopping when xe for any value of n does not reject any observation for a value of n numerically one less.

EXAMPLE.

No.	Observed Angles.	v	v^2
1	34° 09′ 17″.0	0″.0	0
2	34° 09′ 22″.0	5″.0	25.0
3	34° 09′ 20″.0	3″.0	9.0
4	34° 09′ 16″.5	0″.5	0.3
5	34° 09′ 17″.0	0″.0	0.0
6	34° 09′ 09″.5	7″.5	56.2
Mean =	34° 09′ 17″.0	$\Sigma v^2 =$	90.5

In the above column v contains the differences between consecutive observations, and the sum of the squares of the differences $= \Sigma v^2 = 90.5$. This number divided by the number of observations less 1, or

$$\frac{\Sigma v^2}{m - 1} = \frac{90.5}{6 - 1} = 18.1,$$

gives a quotient which, multiplied by the number from the table of constants for six observations (Table XXXIV), gives $2.592 \times 18.1 = 46.9$. Should any number in column v^2 exceed this product, the observation from which it is found must be rejected. This rule requires the rejection of observed angle No. 6 and no other, and a new mean must now be found for the remaining angles, giving 34° 09′ 18″.5. Peirce's Criterion is also employed in determining the probable error and in rejecting doubtful observations in astronomic work.

TABLE XXXIV.

PEIRCE'S CRITERION.

Values of x^2 for $n = 1$.

m	n								
	1	2	3	4	5	6	7	8	9
3	1.480								
4	1.912	1.163							
5	2.278	1.439							
6	2.592	1.687	1.208						
7	2.866	1.910	1.409	1.045					
8	3.109	2.112	1.589	1.229					
9	3.327	2.295	1.753	1.388	1.091				
10	2.526	2.464	1.904	1.531	1.242				
11	3.707	2.621	2.045	1.662	1.373	1.122			
12	3.875	2.766	2.176	1.785	1.492	1.249	1.018		
13	4.029	2.902	2.299	1.901	1.604	1.362	1.145		
14	4.173	3.030	2.416	2.009	1.709	1.465	1.255	1.053	
15	4.309	3.151	2.526	2.111	1.807	1.561	1.354	1.163	
16	4.436	3.264	2.630	2.207	1.898	1.651	1.445	1.259	1.080
17	4.555	3.371	2.729	2.300	1.985	1.736	1.529	1.347	1.176
18	4.668	3.475	2.824	2.389	2.069	1.817	1.609	1.428	1.261
19	4.776	3.571	2.914	2.474	2.150	1.895	1.685	1.504	1.341
20	4.878	3.664	3.001	2.556	2.227	1.970	1.757	1.576	1.415
21	4.975	3.755	3.084	2.634	2.301	2.041	1.827	1.644	1.483
22	5.068	3.840	3.164	2.709	2.373	2.109	1.893	1.710	1.549
23	5.157	3.923	3.240	2.782	2.442	2.176	1.957	1.773	1.612
24	5.242	4.002	3.315	2.852	2.509	2.240	2.019	1.833	1.671
25	5.324	4.078	3.387	2.920	2.573	2.302	2.079	1.892	1.729
26	5.403	4.151	3.456	2.986	2.636	2.362	2.137	1.948	1.784
27	5.479	4.222	3.523	3.049	2.697	2.420	2.194	2.003	1.838
28	5.552	4.291	3.588	3.111	2.756	2.477	2.249	2.056	1.891
29	5.622	4.358	3.651	3.171	2.813	2.532	2.302	2.108	1.941
30	5.690	4.422	3.712	3.229	2.869	2.586	2.354	2.158	1.990
31	5.756	4.484	3.772	3.285	2.923	2.638	2.404	2.207	2.038
32	5.820	4.545	3.829	3.340	2.976	2.689	2.454	2.255	2.085
33	5.882	4.604	3.884	3.394	3.028	2.738	2.502	2.302	2.130
34	5.942	4.661	3.939	3.446	3.078	2.787	2.549	2.347	2.175
35	6.001	4.717	3.992	3.497	3.127	2.834	2.594	2.392	2.218
36	6.058	4.771	4.044	3.547	3.174	2.880	2.639	2.436	2.261
37	6.113	4.823	4.095	3.595	3.221	2.926	2.683	2.478	2.302
38	6.167	4.874	4.144	3.643	3.267	2.970	2.726	2.520	2.343
39	6.219	4.925	4.192	3.689	3.312	3.013	2.768	2.561	2.383
40	6.270	4.974	4.239	3.734	3.356	3.055	2.809	2.601	2.422
41	6.320	5.022	4.285	3.779	3.398	3.097	2.849	2.640	2.460
42	6.369	5.069	4.331	3.822	3.440	3.138	2.888	2.678	2.497
43	6.416	5.114	4.375	3.865	3.481	3.178	2.927	2.716	2.534
44	6.463	5.159	4.418	3.906	3.521	3.217	2.965	2.753	2.570
45	6.508	5.202	4.460	3.947	3.561	3.255	3.002	2.789	2.606
46	6.552	5.245	4.501	3.987	3.600	3.293	3.039	2.825	2.641
47	6.596	5.287	4.542	4.026	3.638	3.330	3.075	2.860	2.675
48	6.639	5.328	4.581	4.065	3.675	3.366	3.110	2.894	2.708
49	6.681	5.368	4.620	4.103	3.712	3.401	3.145	2.928	2.741
50	6.720	5.408	4.657	4.140	3.748	3.436	3.179	2.962	2.774
51	6.761	5.447	4.695	4.176	3.784	3.471	3.213	2.994	2.806
52	6.800	5.484	4.732	4.212	3.819	3.505	3.246	3.027	2.838
53	6.838	5.522	4.768	4.247	3.853	3.538	3.279	3.059	2.869
54	6.876	5.559	4.804	4.282	3.887	3.571	3.311	3.090	2.896
55	6.913	5.595	4.839	4.316	3.920	3.603	3.342	3.121	2.829
56	6.950	5.630	4.873	4.349	3.952	3.635	3.373	3.151	2.959
57	6.986	5.665	4.907	4.382	3.984	3.666	3.404	3.181	2.988
58	7.021	5.699	4.941	4.415	4.016	3.697	3.434	3.210	3.017
59	7.050	5.733	4.974	4.447	4.047	3.728	3.463	3.239	3.046
60	7.990	5.766	5.006	4.478	4.078	3.758	3.492	3.268	3.074

266. Probable Error of Arithmetic Mean.—It is sometimes desirable to determine the relative precision of different series of observations or their *probable error*. The probable error of the arithmetic mean of a number of measures is given by the formula

$$R = \frac{0.6745}{\sqrt{m(m-1)}}\sqrt{\Sigma v^2}, \quad . \quad . \quad . \quad . \quad (62)$$

in which R = probable error of arithmetic mean;
0.6745 = a constant given by theory;
Σ = a symbol denoting "the sum of."

The probable error of a single observation in the series is given by the formula

$$r = 0.6745\sqrt{\frac{\Sigma v^2}{m-1}}. \quad . \quad . \quad . \quad . \quad (63)$$

EXAMPLE.—The application of the foregoing formula is illustrated in the following tabular form:

Between Observations.	Reduced Intervals.	v	v^2
2	40° 35′ 32″.5	+ 6″.423	41″.216
3	31 .6	+ 5 .523	30 .470
4	26 .2	+ 0 .123	.015
5	25 .0	− 1 .077	1 .166
6	25 .0	− 1 .077	1 .166
7	27 .5	+ 1 .423	2 .016
8	28 .1	+ 2 .023	4 .080
9	18 .8	− 7 .277	52 .853
10	20 .0	− 6 .077	36 .966
		$\Sigma v^2 = 169''.948$	
Sum = 234″.7		$\frac{\Sigma v^2}{m-1} = \frac{169''.948}{8} = 21''.243$	
Mean = 26″.077		$R = \frac{0.6745}{\sqrt{m(m-1)}}\sqrt{\Sigma v^2} = \pm 1''.041$	

The factors $\dfrac{0.6745}{\sqrt{m(m-1)}}$ and $\dfrac{0.6745}{\sqrt{m-1}}$ are tabulated below.

TABLE XXXV.

FACTORS FOR COMPUTING PROBABLE ERROR BY BESSEL'S FORMULAS.

m	Single Observations. $\dfrac{0.6745}{\sqrt{m-1}}$	Set of Observations. $\dfrac{0.6745}{\sqrt{m(m-1)}}$
2	.6749	.4769
3	.4769	.2754
4	.3894	.1947
5	.3372	.1508
6	.3016	.1231
7	.2754	.1041
8	.2549	.0901
9	.2385	.0795
10	.2248	.0711
11	.2133	.0643
12	.2034	.0587
13	.1947	.0540
14	.1871	.0500
15	.1803	.0465
16	.1742	.0435
17	.1686	.0409
18	.1636	.0386
19	.1590	.0365
20	.1547	.0346

267. Reduction to Center.—The first operation in the computation of a system of triangulation is that of reducing to the center of the station such observations as were taken with the instrument not centered over it. The mode of making such reduction is best illustrated by the following example taken from the triangulation of the U. S. Geological Survey in Kansas, in which the position of the instrument on the station Walton was eccentric to the station.

In Fig. 174 let

P = place of instrument;
C = center of station;
O = angle at P between two objects, A and B;
y = angle at P between C and the *left*-hand object, B;
r = distance CP;
C = unknown and required angle at C;
D = distance AC;
G = distance BC; and
A = angle at A between P and C.

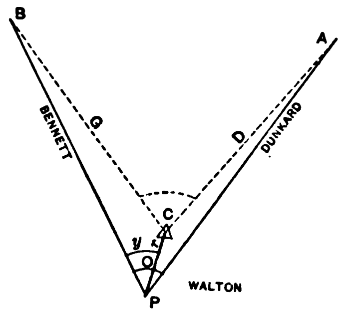


FIG. 174.—REDUCTION TO CENTER.

Then, from the relation between the parts of the triangle,

$$D : r :: \sin y : \sin A; \quad . \quad . \quad . \quad . \quad (64)$$

hence

$$\sin A = \frac{r \sin y}{D}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

As the angles at A and B are very small, they may be regarded as equal to $A \sin 1''$ and $B \sin 1''$; hence

$$A = (\text{in seconds}) \frac{r \sin y}{D \sin 1''}, \quad . \quad . \quad . \quad (66)$$

and

$$C = O + \frac{r \sin (O + y)}{D \sin 1''} - \frac{r \sin y}{G \sin 1''} \quad . \quad . \quad . \quad (67)$$

In the use of this formula proper attention should be paid to the signs of $\sin (O + y)$ and $\sin y$; for the first term will be *positive* when $(O + y)$ is less than 180° (the reverse with $\sin y$); D being the distance of the *right*-hand object, the graduation of the instrument running from left to right.

r being relatively very small, the lengths of D and G are approximately computed with the angle O.

The *following quantities must be known* in addition to the measured angles in order to find the correction for reducing to center:

1. The angle measured at the instrument, P, between the center of the signal or station, C, and the first observed station to the left of it, B.

2. The distance from the center of the instrument to the center of the station $= r$.

3. The approximate distances, D, G, etc., from the station occupied to the stations observed. The latter may be computed from the uncorrected angles.

The practical mode of determining the correction to each angle read at the instrument on Walton to a corresponding angle at the center of the signal or station (Fig. 175) is illustrated as follows:

EXAMPLE.—REDUCTION TO CENTER OF STATION AT WALTON △.

(See explanation: Appendix No. 9, page 167, U. S. Coast and Geodetic Survey report for 1882.)

Distance, inst. to center $= 0'.48$ log $= 9.6812$;
log feet to meters $= 0.5160$;
Distance, inst. to center log meters $= 9.1652 = \log r$.

Direction.	x to n 7°.	x to o 73°.	x to p 105°.	x to q 185°.	x to r 273°.	x to s 306°.
log sin angle............	9.0859	9.9806	9.9849	8.9403	9.9994	9.9080
a.c. log, distance.	5.9321	5.9182	6.4228	6.2434	6.0079	6.2514
log r	9.1652	9.1652	9.1652	9.1652	9.1652	9.1652
a.c. log sin 1″..........	5.3144	5.3144	5.3144	5.3144	5.3144	5.3144
	9.4976	0.3784	0.8873	9.6633	0.4869	0.6390
Correction to direction..	0″.31	2″.39	7″.71	0″.46	3″.06	4″.36

			+	−
Correction to angle	$a = n$ to $o - 0.31'' + 2.39'' = +2.08''$	Check	2″.08	8″.17
	$b = o$ to $p - 2.39 + 7.71 = +5.32$		5 .32	2 .60
	$g = n$ to $p - 0.31 + 7.71 = +7.40$		4 .67	1 .30
	$c = p$ to $q - 7.71 - 0.46 = -8.17$		——	——
	$d = q$ to $r + 0.46 - 3.06 = -2.60$		12″.07 =	12″.07
	$e = r$ to $s + 3.06 - 4.36 = -1.30$			
	$h = q$ to $s + 0.46 - 4.36 = -3.90$			
	$f = s$ to $n + 4.36 + 0.31 = +4.67$			

The corrections $+ 2'.08$, $+ 5'.32$, etc., found in the last column above, are those which are applied to the observed angles (Example, Art. 268) to reduce them to center of station.

268. Station Adjustment.—Doubtful observations having been eliminated and the observed angles having been reduced to center of station, the next step is the station adjustment. The sum of all the angles closing on the horizon and observed at the center of any stations should equal 360°, and the sum of any two angles, as *a* and *b* (Fig. 175), should equal their combined observed angle *g*. In fact, it will be found that this is not the case owing to errors of observation due to various causes (Chap. XXVI). The *object of* the *station adjustment* is to so distribute these errors among the angles *a*, *b*, and *g* as to give the most probable values which will satisfy these conditions. The following example is taken from the same station Walton, as is the example of reduction to center (Art. 267).

EXAMPLE.

	Obs. Angles.	Station Adjustment.	Reduction to Center.	Angles Locally Adjusted and Reduced to Center.
	° ′ ″	″	″	° ′ ″
a Dunkard—Peabody..................	65 45 28.37	+ .51	+ 2.08	65 45 30 96
b Peabody—Newt....................	31 47 58.50	+ .52	+ 5.32	31 48 04.34
Sum..................... =	97 33 26 87	....	....	97 33 35.30
g Dunkard—Newt (meas.)........	97 33 28.39	− .49	+ 7.40	97 33 35.30
Difference.................. =	− 1.52	....	....	00.00
d Township cor.—Royer..........	87 44 57.41	− .56	− 2.60	87 44 54 25
e Royer—Bennett	34 00 03.35	− .56	− 1.30	34 00 01.49
Sum .... =	121 44 60.76	....	....	121 44 55.74
h Township cor.—Bennett.......	121 44 59.05	+ .59	− 3 90	121 44 55.74
	+ 1.71	....	....	00.00
f Bennett—Dunkard................	61 09 26.17	+ .02	+ 4.67	61 09 30.86
g Dunkard—Newt....................	97 33 28.39	− .49	+ 7.40	97 33 35.30
i Newt—Township cor.............	79 32 06.25	+ .02	− 8.17	79 31 58.10
h Tp. corner.—Bennett............	121 44 59 05	+ .59	− 3.90	121 44 55 74
Sum...................... =	359 59 59.86	....	....	360 00 00.00
	− 0.14	.. ..	....	00.00

269. Routine of Station Adjustment.—In the solution of a station adjustment a certain fixed routine is followed which furnishes the simplest arrangement for determining unknown corrections to the angles read around the horizon. The various operations performed in the course of this solution are elaborated in the following articles; some are identical with those performed in the solution of a figure adjustment (Art. 273), to which latter operation reference is made in the proper places. The routine consists of the following:

1. The determination of the differences between separate observed angles and their combined observed angle as between $a + b$ and g, as shown in the second column of the preceding example. These furnish the equations of condition. (Art. 270.)

2. The formation of a table of correlates from the equations of condition. (Art. 271.)

3. The transfer of the table of correlates into normal equations. (Art. 272.)

4. The solution of the normal equations for the determination of the unknown quantities. (Arts. 272 and 281.)

5. The substitution of the corrections found back into the table of correlates. (Art. 272.)

6. The placing of the corrections to the angles found by the last operation in the proper place. (Example, Art. 268, column three, also Art. 272.)

7. The addition or subtraction of the corrections to or from the observed angles. (Example, Art. 268, column three.)

8. The addition or subtraction of the correction resulting from reduction to center (Art. 267) to or from the corrected observed angles. (Example, Art. 268, column four.)

270. Equations of Condition.—In the column of "Observed Angles" (Example, Art. 268) occur the following three *equations of condition:*

$$
\begin{aligned}
&(A) \quad a + b - g - 1''.52 &&= 0; \\
&(B) \quad d + c - h + 1''.71 &&= 0; \\
&(C) \quad f + g + c + h - 0''.14 &&= 0;
\end{aligned}
$$

in which the letters represent not the angles, as in the diagram, but unknown corrections to the angles. The method of solving these equations is briefly described in the next Article. The description is elaborated in the example of a figure adjustment (Arts. 273 to 284), which may be consulted in this connection.

The number of equations of condition which may be arranged is limited only by the number of single and combined angles observed (Fig. 175). In fact, however, provid-

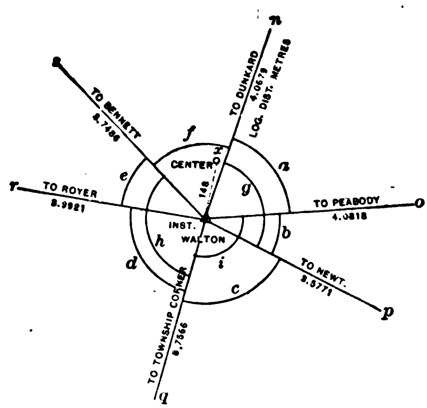


FIG. 175.—STATION ADJUSTMENT.

ing all of the possible combinations have been observed, only a portion of them are employed in the adjustment and used to make additional conditions. This number is limited by the angles used which enter into the figure adjustment (Arts. 276 and 278). Thus it is unnecessary to introduce conditions by the adjustment of angles which will not form a part of some one of the figures which is to be adjusted later. In the case considered the figures resting on the station Walton are so disposed about it that it is necessary to solve but three equations of condition.

The above equations result from the fact that the sum of the angles a and b, as separately observed, fails to equal their corresponding sum angle g, as it was measured. The difference $1''.50$ is the amount which is to be divided among the three observed angles as a correction to each. The object of the least-square station adjustment is to so apply these corrections that the resulting angles will be such that $a + b - g = 0$; and so for the others.

271. Formation of Table of Correlates.—By this mode of solution corrections are found which fulfill the conditions expressed in the equations of condition (Art. 270). The method of least squares is described in Article 264, and no attempt will be made to explain it theoretically here. The lists of works of reference (p. 809) show where the theory may be studied, the more important books on the subject being those of Chauvenet, Wright, and Merriman. Its application in the simpler geodetic operations is best explained by examples, of which this is typical.

In the solution of the above equations by the method of least squares, they are first written in the form of a *table of correlates*, the letters at the top designating the equations, as follows:

	A	B	C
a	1		
b	1		
c			1
d		1	
e		1	
f			1
g	−1		1
h		−1	1

Thus a occurred in equation (A), now called column A, + 1 time, and the figure 1 is written opposite a in column A. So g occurs − 1 time in equation (A), and is written in column

A; also + 1 time in equation (C), and is written so in column C opposite g, etc.

The above table of correlates is now solved according to the algebraic formula:

$$\left.\begin{array}{l} a^2 + ab + ac + ad, \text{ etc.}; \\ ba + b^2 + bc + bd, \text{ etc.}; \\ ca + cb + c^2 + cd, \text{ etc.}; \end{array}\right\} \quad . \quad . \quad . \quad . \qquad (68)$$

272. Formation of Normal Equations and Substitution in Table of Correlates.—This is accomplished by multiplying each coefficient in the above table by itself and by every other in the same horizontal line and summing them. Then, by substituting the result back in the equations of condition, there are formed the following three *normal equations*. Thus

A.	+ 3.00A	....	− 1.00C	− 1″.52 = 0
B.	....	+ 3.00B	− 1.00C	+ 1″.71 = 0
C.	− 1.00A	− 1.00B	+ 4.00C	− 0″.14 = 0

column A is multiplied vertically into itself 3 times; into B, horizontally, no time, as neither column has coefficients on the same horizontal line; and into C, horizontally, once.

These three equations, involving three unknown quantities, are then *solved by elimination* (Art. 281) with results as follows:

$$A = +\overset{''}{.515};$$
$$B = -.562;$$
$$C = +.023.$$

These values of A, B, and C can now be substituted in the table of correlates (p. 164), columns A, B, and C; the algebraic sum of lines a, b, c, d, etc., giving *corrections to the angles* a, b, c, d, etc.

	A	B	C	Corrections to Angles.
				"
a	+ .515			+ .515
b	+ .515			+ .515
c			+ .023	+ .023
d		− .562		− .562
e		− .562		− .562
f			+ .023	+ .023
g	− .515		+ .023	− .492
h		+ .562	+ .023	+ .585

The above are the corrections which have been entered in the example (p. 611) under the column heading "Station Adjustment." The algebraic summation of the corrections in that column and those in the column headed "Reducction to Center" give the column of final *locally adjusted angles.* In these it will be noted that the sums of the various observed angles exactly equal their corresponding combined observed angle.

273. Figure Adjustment.—In primary triangulation computation for *figure adjustment* means the fulfilling of the conditions imposed by the various triangles which form geometric figures. (Art. 238.) The length of any side in any triangle in a *triangulation net* being known and all the angles measured, the length of any other side may be computed by following at least two independent routes through the intervening triangles.

The *object of a figure adjustment* is to find from a given set of measured angles the values which will remove the contradictions among them and will satisfy the following two classes of conditions:

1. The *local conditions*, or those arising at each station from the relations of the angles to one another at that station. These are satisfied by the station adjustment. (Art. 268.)

2. The *general conditions*, or those arising from the geometrical relations necessary to form a closed figure. These are satisfied by the figure adjustment, and are of the following three kinds:

(*a*) The sum of the angles of each triangle must be equal to 180° plus spherical excess.

(*b*) The length of a side must be the same by whatever route it is computed from the given base.

(*c*) The adjusted values of the angles must be the most probable that can be found from the observations.

Ordinarily the angles have been measured by instruments and methods better than the requirements of mapping, and in such cases it is not necessary to make a figure adjustment other than an arbitrarily equal or perhaps weighted distribution of the error of each triangle among the three triangles which compose it. The necessity of a more elaborate adjustment may arise where the computations are to be carried through a long scheme of triangulation connecting distant points, when it becomes desirable to make so rigid an adjustment that any connection with this scheme of triangulation from any direction will not alter the computed quantities.

Rigid figure adjustment is made by the method of least squares, and the simplest mode of explaining such adjustment is by an example taken from actual practice rather than by algebraic formulas. The latter may be found fully elaborated by Chauvenet, Merriman, etc. Such an example is the following, taken from a scheme of triangulation executed with vernier theodolite for the U. S. Geological Survey and computed and in part elaborated by Mr. E. M. Douglas:

274. Routine of Figure Adjustment.—In making a figure adjustment a certain tabular routine is followed, because it furnishes the simplest arrangement of solving a complicated series of algebraic problems and in the most mechanical manner. This consists practically of seven separate operations, which are elaborated in the following articles. The order presented is, after a description of the notation, as follows:

1. The formation of the angle equations. (Art. 276.)

2. The determination and application of the spherical excess to each triangle. (Art. 277.)

3. The formation of side equations. (Art. 278.)

4. The solution of the angle and side equations, which is performed as one operation. (Art. 279.)

5. The summation of angle and side equations (Art. 279), which consists of the following separate operations:

(*a*) The formation of a table of correlatives. (Art. 280.)

(*b*) The formation of normal equations from the table of correlatives. (Art. 280.)

(*c*) The algebraic solution of the normal equation by the least-square method. (Art. 281.)

(*d*) The substitution back into the normal equation of the values found by elimination. (Art. 282.)

(*e*) Solution of the table of correlatives whereby the numerical values of the corrections to the angle and to the sides are obtained. (Art. 283.)

6. The substitution of the corrections to the angles and sides back into the angle and side equations (third and fourth columns, example, Art. 276, and fourth and fifth columns, example, Art. 279).

7. The determination of the final corrected spherical angles and sides as a result of the application of the side and angle corrections to the observed angles (last column of examples, Arts. 276 and 279).

275. Notation Used in Figure Adjustment.—An angle may be considered to be the difference in azimuth or direction of two lines bounding it. Azimuths are counted from the south through the west, north, and east. Therefore the angle 4.1.2 is equal to the azimuth of the line 2.1 minus that of 4.1, and may be written

$$4.1.2 \quad \text{or} \quad -\tfrac{4}{1}+\tfrac{2}{1}.$$

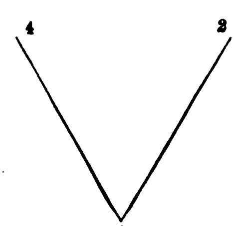


FIG. 176.—ANGLE AND SIDE NOTATION.

In the second form the number at the vertex of the angle is always written underneath the other numbers. If written in the first manner, 4.1.2, the numbers should be in such order that when the vertex of the angle is toward the observer the left-hand station number is written first. The angle 4.1.2, if written without signs, would then be read minus the side (or direction) 4.1 plus the side 2.1; that is, the angle would be $-4.1+2.1$.

276. Angle Equations.—The sums of the three measured angles of any triangle should $= 180° +$ its spherical excess. Each fails to do this, however, by a small amount, which is distributed as a correction to each angle. As a result each triangle furnishes *an equation of condition*, which is called the *angle equation. The number of angle equations in any figure is equal to the number of closed sides in the figure* $+ 1$ *and* $-$ *the number of stations.* Thus in a closed quadrilateral (Fig. 177) the number of angle equations is $6 + 1 - 4 = 3$. The corrections to the angles as found by solution are inserted in the fourth column of the following example of a figure adjustment. In this example the various angles are designated in the first column, in accordance with the notation just given; in the second column are written the plane angles resulting from the station adjustment (Art. 268) and the accompanying correction for reduction to center; in the last column are given the corrected spherical angles, the sum of which must equal $180° +$ spherical excess.

277. Spherical Excess.—The angles observed in the field are measured on a spherical surface, and the sum of the three measured angles of each triangle should, if exactly measured, equal 180° plus spherical excess. This quantity must be computed and subtracted from the sum of the angles only for the purpose of testing the accuracy of closure of the triangle, since in the final computation the angles are treated as plane angles.

Since the spherical excess amounts, between latitudes 25° and 45°, to about 1″ for an approximate area of 75.5 square miles, an *empirical formula* for approximately determining

EXAMPLE.—ANGLE EQUATIONS.

Triangle Sides.	Observed Angles.	Corrections to Sides.	Corrections for each Angle.	Corrected Spherical Angles.
	° ′ ″	″ ″	″	° ′ ″
1. 2. 3.	123 26 46.67	+ 1.447 + 0.962	+ 2.41	123 26 49.08
2. 3. 1.	34 09 11.89	+ 0.960 + 1.534	+ 2.49	34 09 14.38
3. 1. 2.	22 23 54.34	+ 1.538 + 1.448	+ 2.99	22 23 57.33
Sum =	179 59 52.90	+ 7.889	+ 7.89	180 00 00.79
	s. e. 0.79			
a......	Error, − 7.89			
2. 3. 4.	76 57 41.76	+ 0.960 − 0.574	+ 0.38	76 57 42.14
3. 4. 2.	45 15 50.41	− 0.575 − 0.487	− 1.06	46 15 49.35
4. 2. 3.	57 46 28.77	− 0.485 + 0.962	+ 0 48	57 46 29.25
Sum =	180 00 00.94	− 0.199	− 0.20	180 00 00.74
	s. e. 0.74			
b......	Error, + 0.20			
1. 3. 4.	42 48 29.87	− 1.534 − 0.574	− 2.11	42 48 27.76
3. 4. 1.	105 36 13.12	− 0.575 − 0.088	− 0.66	105 36 12.46
4. 1. 3.	31 35 22.54	− 0.090 − 1.538	− 1.63	31 35 20.91
Sum =	180 00 05.53	− 4.399	− 4.40	180 00 01.13
	s. e. 1.13			
c......	Error, + 4.40			
1. 2. 4.	65 40 17.90	+ 1.447 + 0.485	+ 1.93	65 40 19.83
2. 4. 1.	60 20 22.71	+ 0.487 − 0 088	+ 0.40	60 20 23.11
4. 1. 2.	53 59 16.88	− 0.090 + 1.448	+ 1.36	53 59 18 24
Sum =	179 59 57.49	+ 3.689	+ 3.69	1 0 00 01.18
	s. e. 1.18			
	Error, − 3.69			

spherical excess in triangles of less area than 500 square miles is

$$E \text{ (in seconds)} = \frac{\text{area sq. mi.}}{75.5} \text{ at lat. } 40^\circ. \quad . \quad (69)$$

For latitude 20° a constant divisor is 74.76, and for latitude 60° it is 76.42. The area of the triangle may be computed with sufficient accuracy by considering the angles as correct, and subtracting one-third of the excess of the angles above 180° from each angle.

When the area of a triangle is larger than 100 square miles the spherical excess in seconds should be determined by the equation

$$E = \frac{A}{r^2 \sin 1''} = \frac{ab \sin c}{2r^2 \sin 1''}, \quad . \quad . \quad . \quad . \quad (70)$$

in which A = area of triangle in square miles, and
r = radius of curvature of the earth in miles, and is a constant for a given latitude, or may be assumed as a constant in the latitudes included within the area of the United States.

The value of $A = \dfrac{ab \sin c}{2}$ may be determined by the empirical formula (69). The log mean radius of earth in miles, r = 3.5972790.

As the value of the divisor in formulas (70) is a constant for different latitudes, it may be expressed thus:

$$m = \frac{1}{2r^2 \sin 1''},$$

and we have

$$E = ab \sin C \times m. \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

TABLE XXXVI.

LOG *m* FOR DETERMINING SPHERICAL EXCESS.

FOR DISTANCES IN METERS.

(Computed for Clarke's Spheroid of 1866 from Appendix 7, U. S. Coast and Geodetic Survey Report for 1884.)

Latitude.	Log *m*.	Latitude.	Log *m*.	Latitude.	Log *m*.	Latitude.	Log *m*.
° ′		° ′		° ′		° ′	
20 00	1.40625	32 00	1.40528	44 00	1.40410	56 00	1.40290
20 30	622	32 30	524	44 30	405	56 30	285
21 00	619	33 00	519	45 00	400	57 00	280
21 30	615	33 30	514	45 30	395	57 30	276
22 00	612	34 00	509	46 00	390	58 00	271
22 30	608	34 30	505	46 30	385	58 30	266
23 00	604	35 00	500	47 00	380	59 00	262
23 30	601	35 30	495	47 30	375	59 30	257
24 00	597	36 00	491	48 00	369	60 00	253
24 30	592	36 30	486	48 30	364	60 30	249
25 00	588	37 00	481	49 00	359	61 00	244
25 30	584	37 30	476	49 30	354	61 30	240
26 00	580	38 00	471	50 00	349	62 00	236
26 30	576	38 30	466	50 30	344	62 30	231
27 00	572	39 00	461	51 00	339	63 00	227
27 30	568	39 30	456	51 30	334	63 30	223
28 00	564	40 00	451	52 00	329	64 00	219
28 30	559	40 30	446	52 30	324	64 30	215
29 00	555	41 00	441	53 00	319	65 00	211
29 30	551	41 30	436	53 30	314	65 30	207
30 00	547	42 00	431	54 00	309	66 00	203
30 30	542	42 30	426	54 30	304	66 30	200
31 00	537	43 00	420	55 00	299	67 00	196
31 30	1.40533	43 30	1.40415	55 30	1.40295	67 30	1.40192

EXAMPLE.—Let a and b be the lengths of the two sides, and C the included angle; m is a constant to be derived from Table XXXVI for distances in meters. For the mean latitude 30° 40′ of the example chosen, $m = 1.40540$. Then we have, solving by formula (71),

Triangles	1.2.3	2.3.4	1.3.4	1.2.4
Angles $C =$	123° 26′ 50″	76° 57′ 40″	42° 48′ 30″	65° 40′ 20″
Log (side) a	4.36885	4.20055	4.54093	4.36885
" " b	4.20055	4.27642	4.27642	4.33733
" sine C	9.92137	9.98866	9.83222	9.95962
" const. m	1.40540	1.40540	1.40540	1.40540
Log s.e. =	9.89617	9.87103	0.05497	0.07160
s.e. =	0″.79	0″.74	1″.13	1″.18

Then one-third 0″.79 is to be subtracted from each of the three angles of the triangle 1.2.3, etc.

278. Side Equations.—It is evident that the distribution of corrections to the three angles of a triangle to make their sum 180°, affects not only the angles, but as a consequence the sides, diminishing the lengths of the latter as the angles are diminished or increasing the lengths as the opposite angles are increased. Therefore the adjustments to the sides of the triangles must be made with the adjustments to the angles in order that the triangle shall not be distorted. The determination of the corrections to be applied to the sides is performed through the formation of side equations, which are best explained by reference to Fig. 177. The solution of these is performed at the same time with that of the angle equations in Articles 279 to 283.

Suppose 4.1.2.3 to represent the projection of a pyramid of which 1.2.3, the shaded side, is the base and 4 the apex.

A *geometric condition* of such a figure is that *the sums of the logarithmic sines of angles about the base taken in one direction*

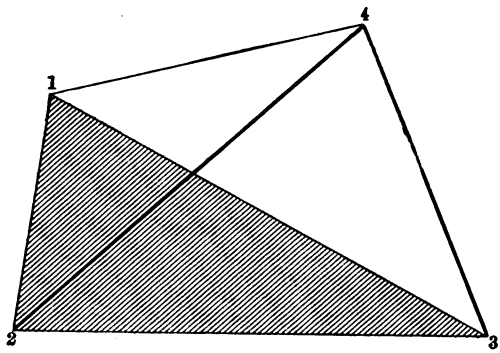


FIG. 177.—ANGLE AND SIDE EQUATIONS.

must equal similar sums taken in the other direction; that is, *the product of the sines must be equal.* In this case, therefore,

$$\log \text{sine } 4.1.2 + \log \text{sine } 4.2.3 + \log \text{sine } 1.3\ 4$$

should equal

$$\log \text{sine } 1.2.4 + \log \text{sine } 2.3.4 + \log \text{sine } 4.1.3.$$

The number of side equations which can be formed in any figure is equal to the number of lines in the figure plus three, minus twice the number of stations, or $1 + 3 - 2n$. In a quadrilateral, therefore, $6 + 3 - 8 = 1$; hence such a figure contains one side equation, or *equation of condition.* The numerical term in each side equation is the difference between the sums of the logarithmic sines taken in each direction. The coefficients for the unknown corrections are the differences for one second in the logarithmic sines of the angles.

Further examples of the method of arranging equations of condition and applying corrections to the angles may be best shown by a continuation of the example selected (Art. 276), which is the simplest, being that of a quadrilateral (Fig. 177).

The method of forming correlative and normal equations and their solution is similar to that for station adjustment (Arts. 271, 272). In the *equations of conditions and correlatives* the angles are designated by directions to which the corrections are finally applied.

279. Solution of Angle and Side Equations.—The corrections to the sides bounding an angle are empirically denoted by enclosing them in brackets and prefixing the proper signs. Thus the corrections to be found for the angle 3.4.1 may be denoted by its side corrections, $-(\frac{3}{4}) + (\frac{1}{4})$.

In the triangle 1.2.3 write each observed angle and indicate a correction, to be found, as above:

$$1.2.3 - [1.2] + [3.2] + 2.3.1 - [2.3] + [1.3] + 3.1.2 - [3.1] + [2.1] = 180 + \text{s.e.};$$

or, instead of the angle numbers, 1.2.3, etc., write their combined value:

$$179^\circ\ 59'\ 52''.9 - [1.2] + [3.2] - [2.3] + [1.3] - [3.1] + [2.1] = 180^\circ + \text{s. e.} = 180^\circ\ 00'\ 00''.79.$$

This equation reduced gives

$$-[1.2]+[3.2]-[2.3]+[1.3]-[3.1]+[2.1]-7.89 = 0. \quad (a)$$

In this manner equations are formed for the other triangles, 1.2.4 being assumed to be the dependent triangle, and we have:

$$-[2.3]+[4.3]-[3.4]+[2.4]-[4.2]+[3.2]+0.20 = 0;\ . \quad (b)$$

$$-[1.3]+[4.3]-[3.4]+[1.4]-[4.1]+[3.1]+4.40 = 0.\ . \quad (c)$$

Arrange the logarithmic sines as shown in the following tabular example by writing opposite each sine its logarithmic difference for one second as given in a table of seven-place logarithms. The logarithmic differences for angles greater than 90° will have a minus sign, for those less the sign will be plus. For a small correction to any of these angles the change in the logarithmic sine will be equal to the correction in seconds multiplied by the tabular difference for 1″. As the correction for the angle 3.4.1 is denoted by $-\left(\frac{3}{4}\right)+\left(\frac{1}{4}\right)$, applying the tabular difference for 1″, we have the change in the log. sine $-5.9\left(-\left(\frac{3}{4}\right)+\left(\frac{1}{4}\right)\right)$.

EXAMPLE.—SUMMATION OF SIDE AND ANGLE EQUATIONS.

Angles.	Log. Sines.	Difference, 1″.	Correc. to Angles.	Correc. to Sines.	Corrected Sines.
			″		
3, 4, 1	9.9836918.6	− 5.9	− 0.66	+ 3.9	9.9836922.5
3, 1, 2	9.5809762.8	+ 51.1	+ 2.99	+ 152.8	9.5809915.6
4, 2, 3	9.9273484.8	+ 13.2	+ 0.48	+ 6.3	9.9273491.1
Sum... +	9.4920166.2	....	....	+ 163.0	9.4920329.2
4, 1, 3	9.7191913.9	+ 34.2	− 1.63	− 55.7	9.7191858.2
1, 2, 3	9.9213757.3	− 13.9	+ 2.41	− 33.5	9.9213723.8
3, 4, 2	9.8514769.5	+ 20.8	− 1.06	− 22.0	9.8514747.5
Sum... −	9.4920440.7 9.4920166.2	....	....	− 111.2	9.4920329.5
	Error = − 274.5			274.2	

As shown in the beginning of this Article, the equation which must be satisfied in an adjusted quadrilateral to fix the relative length of the sides is

$$\log \sin 3.4.1 + \log \sin 3.1.2 + \log \sin 4.2.3$$
$$- \log \sin 4.1.3 - \log \sin 1.2.3 - \log \sin 3.4.2 = 0.$$

Substituting in this, with changed signs, corrections found as above, we have, after reducing, the following *side equation :*

$+ 5.9[3.4] - 5.9[1.4] - 51.1[3.1] + 51.1[2.1] - 13.2[4.2]$
$+ 13.2[3.2] + 34.2[4.1] - 34.2[3.1] - 13.9[1.2] + 13.9[3.2]$
$+ 20.8[3.4] - 20.8[2.4] - 274.5 = 0.$

This being a true algebraic equation, it may be divided by any number without altering its value. Dividing it through by some convenient multiple of 10, as 80, to give smaller coefficients, and combining algebraically the coefficients of [3.4], [3.2], [3.1], each of which appears twice, gives

$$+.334[3.4] - .074[1.4] - 1.066[3.1] + .639[2.1] - 1.65[4.2]$$
$$+.339[3.2] + .427[4.1] - .174[1.2] - .260[2.4] - 3.427 = 0. \quad (d)$$

Thus for example, for the coefficients of [3.4], we had

$$+ 5.9\,[3.4] + 20.8\,[3.4] = + 26.7\,[3.4] \div 80 = .334\,[3.4],$$

and so for the others.

280. Correlates and Normal Equations.—We now have the equations necessary for the complete adjustment of the quadrilateral, and from them values must be found by means of correlates, each equation of condition having a correlate, and each correction coefficient giving a correlate coefficient. The algebraic sum of the coefficients of each correlate will be zero.

The following tables of correlates can now be formed from the above, as was done in the station adjustment (Art. 271). The solution of this can only be made after the normal equation has been formed and solved by elimination (Art. 281).

The first of the tables is *formed from* the adopted *equations of condition* (*a*), (*b*), (*c*), and (*d*) (pages 625 and 627) by arranging corresponding columns *a*, *b*, etc. In these and on line with the various side numbers 2.1, 3.1, etc., are placed the coefficients of the latter with their signs from the equations (*a*), (*b*), etc. Thus on line 1 the side 2.1 occurs + 1 time in equation (*a*) and again + .639 times in equation (*d*), and so for the other sides.

EXAMPLE.

TABLE OF CORRELATES FORMED. TABLE OF CORRELATES SOLVED.

Line.	Sides.	*a.*	*b.*	*c.*	*d.*	A. + 1.446	B. − 0.486	C. − 0.088	D. + 0.004	Totals.
1	2.1	+ 1	....	......	+ .639	+ 1.446	....	......	+ 0.002	+ 1.448
2	3.1	− 1	....	+ 1	− 1.066	− 1.446	....	− 0.088	− 0.004	− 1.538
3	4.1	....	....	− 1	+ .427	.....	....	+ 0.088	+ 0.002	+ 0.090
4	1.2	− 1	....	....	− .174	− 1.446	....	......	− 0.001	− 1.447
5	3.2	+ 1	+ 1	.....	+ .339	+ 1.446	− 0.486	....	+ 0.002	+ 0.962
6	4.2	......	− 1	....	− .105	....	+ 0.486	.....	− 0.001	+ 0.485
7	1.3	+ 1	....	− 1	.....	+ 1.446	.....	+ 0.088	....	+ 1.534
8	2.3	− 1	− 1	....	......	− 1.446	+ 0.486	....	....	− 0.960
9	4.3	....	+ 1	+ 1	......	....	− 0.486	− 0.088	....	− 0.574
10	1.4	.....	....	+ 1	− .074	....	....	− 0.088	− 0.000	− 0.088
11	2.4	....	+ 1	....	− .260	....	− 0.486	....	− 0.001	− 0.487
12	3.4	......	− 1	− 1	+ .334	....	+ 0.486	+ 0.088	+ 0.001	+ 0.575
	Sums	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

The first of the tables of correlates as formed above may now be arranged as a *normal equation* by application of formula (68) as for station adjustment (Art. 271), resulting as follows:

EXAMPLE.—TABLE OF NORMAL EQUATIONS FORMED FROM ABOVE TABLES OF CORRELATES.

	(*a*)	(*b*)	(*c*)	(*d*)		(Residuals.)
a.	+ 6.000	+ 2.000	− 2.000	+ 2.218	− 7.890 = O	(.001)
b.	+ 2.000	+ 6.000	+ 2.000	− 0.090	+ 0.200	(.000)
c.	− 2.000	+ 2.000	+ 6.000	− 1.901	+ 4.400	(.000)
d.	+ 2.218	− 0.090	− 1.901	+ 2.084	− 3.427	(.000)

The symmetry of the normal equations as shown by the underscoring gives a partial check on their accuracy.

281. Algebraic Solution of Normal Equations.—The normal equation being now arranged, it may be *solved by elimination* in tabular manner as given on page 630.

The logarithm of each number in line 1 is placed in line 2. The logarithm (= 0.77815) of the left-hand number is then subtracted from each of the other logarithms. The remainders, the logarithms of quotients, are written in line 3. The number corresponding to the logarithm 0.11893, in the right-hand column, is placed in a parenthesis in line 5 with a sign opposite to that of the number above it in line 1.

The logarithms in line 3, columns (*b*), (*c*), and (*d*), are to be used as the logarithms of multipliers. The sign of each multiplier is the opposite to that of the number above it, and is written in line 4. The logarithms of multipliers are next placed on a slip of paper, and the logarithms of products found by adding the logarithms of the multipliers to the logarithms of numbers in line 1. For example, using (*b*) multiplier, line 3, column (*b*), we have

Log. of multiplier	= 9.52288 sign –	(Numbers);
Log. of product (*b*)(*b*)	= 9.82391 sign –	(– 0.667);
Log. of product (*b*)(*c*)	= 9.82391 sign +	(+ 0.667);
Log. of product (*b*)(*d*)	= 9.86884 sign –	(– 0.739);
((*b*) by absolute term)	= 0.41996 sign +	(+ 2.630).

Write the numbers corresponding to these products in line 7, equation (*b*). Products belong to the equations having the same letter as the multiplier and in the same columns with the multiplicands.

The algebraic sums of the numbers in lines 6 and 7 are written in line 8. In this manner form the products with the multipliers in (*c*) and (*d*) columns and add them to *c* and *d* equations respectively.

The above process is to be repeated for the numbers in line 8. The products, line 16, are added to the numbers in line 15. Proceed as before with the numbers in line 17. This line has but one multiplier. The logarithm of + 0.995 (line 28) is subtracted from logarithm of – 0.004; the number corresponding to quotient is + 0.004 (line 32), which is the value of *d* correlate.

The logarithm of this number (7.60206 sign +) is added to each of the logarithms of multipliers in column (*d*). Thus, commencing at the bottom:

EXAMPLE.—SOLUTION OF NORMAL EQUATIONS.

	(*a*)	(*b*)	(*c*)	(*d*)	Absolute Terms.	Line No.
a	+ 6.000	+ 2.000	− 2.000	+ 2.218	− 7.890	1
	0.77815	0.30103	0.30103	0.34596	0.89708	2
	0.00000	9.52288	9.52288	9.56781	0.11893	3
	−	−	+	−	+	4
		(+ 0.162)	(− 0.029)	(− 0.002)	(+ 1.315)	5
b		+ 6.000	+ 2.000	− 0.090	+ 0.200	6
		− 0.667	+ 0.667	− 0.739	+ 2.630	7
		+ 5.333	+ 2.667	− 0.829	+ 2.830	8
		0.72700	0.42600	9.91855	0.45179	9
		0.00000	9.69900	9.19155	9.72479	10
		−	−	+		11
			(+ 0.044)	(+ 0.001)	(− 0.531)	12
c			+ 6.000	− 1.901	+ 4.400	13
			− 0.667	+ 0.739	− 2.630	14
			+ 5.333	− 1.167	+ 1.770	15
			− 1.333	+ 0.415	− 1.415	16
			+ 4.000	− 0.747	+ 0.355	17
			0.60206	9.87332	9.55023	18
			0.00000	9.27126	8.94817	19
			−	+		20
				(+ 0.001)	(− 0.089)	21
d				+ 2.084	− 3.427	22
				− 0.820	+ 2.917	23
				+ 1.264	− 0.510	24
				− 0.129	+ 0.440	25
				+ 1.135	− 0.070	26
				− 0.140	+ 0.066	27
				+ 0.995	− 0.004	28
				9.99782	7.60206	29
				0.00000	7.60424	30
				−		31
					(+ 0.004)	32

Correlates.

$a = + 1.446$

$b = - .486$

$c = - .088$

$d = + .004$

Log. of correlate d $= 7.60206$ sign $+$

$\left.\begin{array}{l}\text{added to log. in line 19,}\\ \text{column } (d)\text{, gives the log.}\\ \text{of product } d \text{ multiplier}\\ \text{by } (d) \text{ log.}\end{array}\right\} = 6.87332 \text{ sign } + \text{ No.} = +.001$

Line 10, d multiplier by (a) log. $= 6.79361$ sign $+$ No. $= +.001$

Line 3, d multiplier by d log. $= 7.16987$ sign $-$, Number $= -.002$. Each number is written in parenthesis under the log. of its multiplicand.

Any line of multipliers corresponds to an equation. Take, for example, line 19: the numbers corresponding to each log. taken with the letter of the column give

$$-1c + 0.187d - 0.089 = 0.$$

The product of $+0.187$ (log. 9.27126) by the value of d $(+.004) = +.001$ (nearest unit in third place of decimals);

hence $$-c + .001 - .089 = 0; \text{ and}$$
$$c = -.088.$$

Take the log. of c and proceed as with log. of d, obtaining the numbers $+0.044$, line 12, and -0.029, line 5, for products of c correlate by c column multipliers.

Add algebraically the numbers on line 12 and we get the value of $b = -0.486$. Find its product with b multiplier line 3, and write it on line 5. Add the numbers on line 5 for the value of a.

The values of d, c, b, a can also be found from lines 28, 17, 8, 1.

For example, $+4.00c - 0.747d + 0.355 = 0$ (line 17): substituting the value of d $(+.004)$ and combining gives

$$+4.00c + 0.352 = 0; \text{ and}$$
$$c = -0.088, \text{ as before.}$$

The same operation as is illustrated on page 630 can be more simply and quickly performed by the method of solution by *reciprocals* and *Crelle's tables*, instead of by use of logarithms, where the former are available; see Appendix 8, pages 26 to 28, U. S. Coast and Geodetic Survey Report for 1878.

282. Substitution in Normal Equations.—The values found for the correlates a, b, etc., must now be substituted in the *normal equations* (Art. 280) to test the accuracy of the solution.

For equation d (p. 629), commencing at the left:

$$\begin{aligned} +2.218 \times (+1.446 = a) &= +3.207 \\ -0.090 \times (-0.486 = b) &= +0.045 \\ -1.901 \times (-.088 = c) &= +0.167 \\ +2.084 \times (+.004 = d) &= +0.008 \\ \text{absolute term} &= -3.427 \\ \hline \text{Sum} &= 0.000 \end{aligned}$$

This proves the equation correctly solved. In the same way substitute in a, b, and c equations. In equation a, as solved above, there is an error amounting to .001, but as only corrections to the nearest hundredth are desired, this small residual may be neglected.

283. Substitution in Table of Correlates.—The values of the correlatives are next placed at the head of columns A, B, C, and D (p. 628, Table of Correlates Solved), and products by corresponding coefficient in column a, b, c, or d, of the adjoining table are found and written on the proper lines in A, B, C, and D columns.

The sums of the products on each horizontal line are placed in the column of totals. As a check on this part of the work see that the sum of the numbers in each column = 0.

On each line in the column of totals is the correction for the side adjacent to an angle, the numbers for which are given in the column of sides.

For the angle 1.2.3. = (− 1.2 + 3.2) the correction is as follows:

For the side 1.2 (line 4) the correction is − 1.447.
For the side 3.2 (line 5) the correction is + .0962.
Hence for the sides − 1.2 and + 3.2 we have

$$
\begin{array}{l}
+\ 1.447 = -\ \text{correction for } 1.2 \\
+\ 0.962 = +\ \text{correction for } 3.2 \\
\hline
+\ 2.409 = \text{correction for angle } 1.2.3
\end{array}
$$

These are the corrections which are written in the third and fourth columns of the *angle equation* (Art. 276, p. 620) as side corrections and angle corrections respectively. From their application result the *corrected spherical angles*, column five.

The correction for each log. sine is the product of its angle correction by its difference for 1″. For example, sine of angle 3.4.1 (Art. 279, p. 626), Dif. for 1″ = − 5.9, column three, correction for angle = − 0″.66, column four.

$$- 5.9 \times - 0.66 = + 3.9 = \text{cor. to sine, column five.}$$

284. Weighted Observations.—Where a number of observations differ somewhat from each other and the causes are believed to be known for such difference, it occasionally becomes desirable to give greater value to one observation than to another; thus one may be given two or three times the value of another. This operation is called *weighting* (Art. 264). To find the weighted mean of a number of observations which have been given unequal weights, each is multiplied by its proper weight, and the sum of the product is divided by the sum of the weights, the quotient being the *weighted mean.*

EXAMPLE.	$38° - 54' - 55''.0$	$\times$ Wt. $1 =$	55
	54	$\times$ Wt. $1 =$	54
	56	$\times$ Wt. $2 =$	112
	53	$\times$ Wt. $1 =$	53
	57	$\times$ Wt. $2 =$	114
	Sum		7)388
			55.4

Weighted mean = $38° \; 54' \; 55''.4$

Weights are used in a least-square adjustment in the following manner: the adjustment is carried forward as above described till the table of correlates is reached; then opposite each angle number in a station adjustment, or opposite the side numbers in a figure adjustment, place the weight of the angle or side in a separate column. Every product formed in the table of correlates must be divided by the weight written on the horizontal line with the multiplicand. *The weight is used only as a divisor.*

The following example from a station adjustment will illustrate the method of using weights in station or figure adjustment.

Equation a, at the bottom of page 635, is formed from the left half of the table on the same page, thus:

$$\begin{aligned} -1 \times -1 \div 2 &= +0.50 \\ +1 \times +1 \div 1 &= +1.00 \\ -1 \times -1 \div 2 &= +0.50 \end{aligned}$$

$$\text{Sum} = +2.00 = \text{term } (a), \text{ equation } a.$$

Term b, same equation is from second line.

$$+1 \times -1 \div 1 = -1.$$

Term $c = 0$. The absolute term -3.00 is the same that it would have been for an unweighted equation.

Equation c is formed thus:

$$
\begin{aligned}
&\text{Term } a = 0. \\
&\text{Term } b = +1 \times -1 \div 2 = -0.50 \\
&\text{Term } c = -1 \times -1 \div 1 = +1.0 \\
&\qquad\quad +1 \times +1 \div 2 = +0.50 \\
&\qquad\quad -1 \times -1 \div \tfrac{1}{3} = +3.00 \\
&\qquad\qquad\qquad \text{Sum } +4.50 = \text{term } c.
\end{aligned}
$$

EXAMPLE.—TABLE OF WEIGHTED CORRELATES.

No. of Correction.	Weight.	a	b	c	A $+2.779$	B $+2.558$	C -0.605	Totals.	Totals divided by Weights.
									″
1	2	− 1	....	....	− 2.779	....	....	− 2.779	− 1.39
2	1	+ 1	− 1	....	+ 2.779	− 2.558	....	+ 0.221	+ 0.22
3	2	− 1	....	....	− 2.779	....	....	− 2.779	− 1.39
4	1	....	....	− 1	....	......	+ 0.605	+ 0.605	+ 0.60
5	2	....	− 1	+ 1	....	− 2.558	− 0.605	− 3.163	− 1.58
6	$\frac{1}{3}$	....	....	− 1	....	....	+ 0.605	+ 0.605	+ 1.81
7	4	....	− 1	....	....	− 2.558	....	− 2.558	− 0.64
360°	∞	....	+ 1	....	....	......	....	....	0

EXAMPLE.—EQUATIONS FORMED FROM ABOVE CORRELATES.

	(a)	(b)	(c)	(d)
a.	+ 2.000	− 1.000	....	− 3.00 = 0
b.	− 1.000	+ 1.750	− 0.500	− 2.00 = 0
c.	....	− 0.500	+ 4.500	+ 4.00 = 0

The equations are solved in the ordinary way. The values for the correlates are multiplied by their respective coefficients, the products being written in columns A, B, C, no attention being paid to the weights until after the totals for each horizontal line are found and written in the column for totals. *These totals must be divided by the weights.* The quotients, written in the right-hand column, are the weighted corrections for the angles, the numbers of which are in the left-hand column.

CHAPTER XXIX.

COMPUTATION OF DISTANCES AND COORDINATES.

285. Geodetic Coordinates.—The position of a point on the surface of the earth is determined by its altitude and geodetic latitude and longitude. These may be called its *geographic coordinates*. In order to extend and compute a system of triangulation from such a point, it is necessary to determine also its *polar coordinates*, which are the distances and directions or azimuths between it and various other points. A complete statement of what are designated the *geodetic coordinates* of a point includes, therefore, its geographic and polar coordinates.

As understood in geodetic computation the *azimuth of a line* (Art. 288) is the angle which defines its direction with relation to the true meridian. This angle is always measured from the south towards the west, north and east, in the direction of the hands of a watch. The *zero of azimuth* is the south, and a true westerly direction is 90°, a northerly direction 180°, and an easterly direction 270°. *Astronomic azimuths* (Chap. XXXIII) and *latitudes* (Chap. XXXIV) are to be determined by observations on stars, and longitudes by the same supplemented by telegraphic exchanges of time (Chap. XXXV). Distance is obtained by direct measure (Chap. XXI) reduced to sea-level, of the length of the line considered.

The *computation of geodetic coordinates* consists of two distinct operations. The first is the computation of the lengths of the sides of the triangles (Art. 286) by which all the parts of the triangle are solved. The second operation consists in starting out in any figure, or in the simplest figure, a triangle, with the lengths of the sides and dimensions of the angles known, as well as with the latitude and longitude of one, or, preferably, for purposes of check, of two apices of the triangle known. Also, the azimuth of known side joining the two known positions. With these quantities given it is possible to compute the latitudes and longitudes of the other apices or stations and the azimuths of the lines joining them (Art. 288).

286. Computation of Distances.—The figure adjustment having been completed and the spherical excess in each triangle computed (Arts. 273 and 277), the next operation is the computation of the distances or lengths of each of the sides forming the various triangles. In each triangle there is a known base, that is, the length of one side is known and the three angles are known. The remaining sides are computed on the *principle of proportion of sides to sines of opposite angles ;* expressed mathematically this is

$$a = \frac{b \text{ sine } A}{\text{sine } B}. \quad . \quad . \quad . \quad . \quad . \quad (72)$$

In this computation the distances are expressed in logs. of meters because the tables used in the after-computation are prepared for metric computations. The solution of the above formula is best explained graphically by the following example (see also Fig. 178).

EXAMPLE.

Triangle.	Stations.	Spherical Angle.	ς. ε.	Plane Angles.	Log. Sines.
			Log. sine angle at McKenzie...		9.999 9963
			Log. distance Chuska—Zuñi...		4.928 0539
1	McKenzie	89° 45′ 51″.692	− 2″.899	89° 45′ 48″.793	0.000 0037 a. c.
	Chuska	35 56 55.884	− 2.900	35 56 52.984	9.768 6762
	Zuni	54 17 21.122	− 2.899	54 17 18.223	9.909 5375
		180 80 08.698	8.698		
			Log. side McKenzie—Zuñi.....		4.696 7338
			Log. side McKenzie—Chuska..		4.837 5951

In the above, under the *column of stations*, is placed first the station from which the distance is to be determined, and then follow those stations to which distances are to be determined and between which the distances are already known. In the *column of spherical angles* are written the final adjusted angles resulting from the figure adjustment (Art. 276). In the *column of spherical excess* is written opposite each angle one-third of the total spherical excess of the triangle (Art. 277). In column of *plane angles* are written the angles resulting from the subtraction of spherical excess from the adjusted spherical angles.

In the *column of log. sines* are written logarithmic sines of the plane angles as obtained from a table of logarithms. Above this column is written opposite "log. of dist." the length of the known side Chuska-Zuñi, and above it, for reference, the log. sine of the angle at the known station, McKenzie. On the line McKenzie, in column of log. sines, is then written the arithmetical complement, a. c., of the sine of its angle. Opposite the other two angles, Chuska and Zuñi, are written the logarithms of their sines.

The *quantities sought*, viz., distances McKenzie—Zuñi and McKenzie—Chuska, are found by adding the a. c. log. sine of the angle at McKenzie and the log. sine of the angle at Chuska in the first case, and in the second the a. c. log. sine of the angle at McKenzie and the log. sine angle at Zuñi to the log. dist. Chuska—Zuñi.

287. Formulas for Computing Geodetic Coordinates. The last operation in the computation of a system of primary triangulation is the determination of the geodetic coordinates of each station. Having now the log. dist. and accordingly the actual lengths of the sides and the dimensions of each plane angle, there remains only to determine the latitude and longitude of each station and the azimuth of each direction. To do this the latitude and longitude of one station must be known, and the azimuth from it to one of the other stations.

The formulas for computing new latitudes, longitudes, and azimuth from the known positions consist in determining the differences of latitude, longitude, and azimuth, $\Delta\phi$, $\Delta\lambda$, and $\Delta\alpha$, and adding or subtracting these to or from the known positions.

$$-\Delta\phi = S\cos\alpha . B + S^2 \sin^2\alpha . C + (\Delta\phi)^2 . D - h . \sin^2\alpha . E. \qquad (73)$$

The above is simple of application by use of the log. factors B, C, D, E, etc., Table XXXVII.

$$\Delta\lambda = \frac{S \sin\alpha}{\cos\phi'} A', \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (74)$$

$$-\Delta\alpha = \Delta\lambda \frac{\sin \frac{1}{2}(\phi + \phi')}{\cos \frac{1}{2}(\Delta\phi)} + \Delta\lambda^3 F, \quad . \; . \qquad (75)$$

and

$$\alpha' = \alpha + 180^\circ + \Delta\alpha. \quad . \; . \; . \; . \; . \; . \qquad (76)$$

The constants from Table XXXVII have the following algebraic values, and the notation used is as given below:

$$B = \frac{1}{R \text{ arc } 1''} = \frac{(1 - e^2 \sin^2\phi)^{\frac{3}{2}}}{\alpha(1 - \alpha^2) \text{ arc } 1''}; \quad . \; . \; . \; . \; . \; . \qquad (77)$$

$$C = \frac{\tan\phi}{2NR \text{ arc } 1''} = \frac{(1 - e^2 \sin^2\phi)^2 \tan\phi}{2\alpha^2(1 - e^2) \text{ arc } 1''}; \quad . \; . \; . \; . \; . \qquad (78)$$

$$D = \frac{\frac{3}{2}e^2 \sin\phi \cos\phi \text{ arc } 1''}{(1 - e^2 \sin^2\phi)^{\frac{3}{2}}}; \quad . \; . \; . \; . \; . \qquad (79)$$

$$E = \frac{1 + 3\tan^2\phi}{6N^2} = \frac{(1 + 3\tan^2\phi)(1 - e^2 \sin^2\phi)}{6\alpha^2}; \quad . \; . \; . \qquad (80)$$

$$A' = \frac{1}{N \text{ arc } 1''} = \frac{(1 - e^2 \sin^2\phi)^{\frac{1}{2}}}{\alpha \text{ arc } 1''}; \text{ (referred to new position;)} \qquad (81)$$

in which

$$N = \frac{a}{(1 - e^2 \sin^2 \phi)^{\frac{1}{2}}} = \text{normal ending at minor axis;}$$

a = equatorial radius (Art. 292);

$$R = \frac{N^3}{a^2}(1 - e^2) = \text{radius of curvature of the meridian:}$$

$\rho = N \cos \phi$ = radius of curvature of the parallel:

e = eccentricity (Art. 292); and

$$\log F = \tfrac{1}{12} \sin \phi \cos^2 \phi \operatorname{arc}^2 1'', \quad . \quad . \quad . \quad . \quad (82)$$

$$h = sB \cos \alpha \text{ or first term.}$$

Also ϕ = latitude of known station, + if north;

λ = longitude of known station, + if west;

α = azimuth to second station from first, with careful regard of algebraic signs,

ϕ', λ', and α' = same for new or required position.

For *distances less than twenty-five miles* omit the quantities depending on the constants D and E, which give the logs. (III) and (IV) in example on page 644. Also omit the small log. cor. to $\Delta\lambda$ depending on log. (V); the small correction to $\Delta\alpha$ depending on log. F: and that to log. (VI) derived from log. sec. $\left(\frac{\Delta\phi}{2}\right)$.

For *distances greater than one hundred miles* the following formulas should be employed:

$$\tan \tfrac{1}{2}(\alpha' + \zeta - \Delta\lambda) = \frac{\sin \frac{1}{2}(\gamma - \theta)}{\sin \frac{1}{2}(\gamma + \theta)} \cot \frac{\alpha}{2}: \quad . \quad (83)$$

$$\tan \tfrac{1}{2}(\alpha' + \zeta + \Delta\lambda) = \frac{\cos \frac{1}{2}(\gamma - \theta)}{\cos \frac{1}{2}(\gamma + \theta)} \cot \frac{\alpha}{2}; \quad . \quad (84)$$

$$\phi'-\phi=\frac{s}{\rho \sin 1''}\cdot\frac{\sin\frac{1}{2}(\alpha'+\zeta-\alpha)}{\sin\frac{1}{2}(\alpha'+\zeta+\alpha)}\left[1+\frac{\theta^2\sin^2 1''}{12}\cos^2\tfrac{1}{2}(\alpha'-\alpha)\right]: \quad (85)$$

in which γ = colatitude of old point;

ϕ^m = mean latitude of old and new points,

$$\rho=\frac{a(1-e^2)}{(1-e^2\sin^2\phi^m)^{\frac{3}{2}}};$$

$$\theta=\frac{s}{r\sin 1''};$$

$$r=\frac{a}{(1-e^2\sin^2\phi)^{\frac{1}{2}}}.$$

$$\zeta=\frac{e^2\theta^2\sin 1''}{4(1-e^2)}\cdot\cos^2\phi\sin 2\alpha;$$

which are constants for the particular case.

In terms of the coordinates of rectangular axes referred to one of the points of the triangulation, the latitude and longitude of which are known,—y being the ordinate in the direction of the meridian, and x the ordinate perpendicular to it,—the values may be expressed:

$$\phi'=\phi\pm\frac{y}{R\sin 1''}-\tfrac{1}{2}\sin 1''\left(\frac{x}{N\sin 1''}\right)^2\times\tan\left(\phi\pm\frac{y}{R\sin 1''}\right); \quad (86)$$

$$\lambda'=\lambda\pm\left(\frac{x}{N\sin 1''}\right)\times\frac{1}{\cos\phi'}; \quad . \ . \ . \ . \ . \ . \ . \ . \quad (87)$$

$$\alpha'=(180^\circ+\alpha)\pm\frac{x}{N\sin 1''}\tan\phi'. \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (88)$$

The *convergence of meridians*, or the amount by which the azimuth at one end of a line exceeds the azimuth at the other, is expressed by the quantity

$$\frac{u''\sin\alpha}{\cos\phi'}\sin\tfrac{1}{2}(\phi+\phi') \quad \text{or} \quad (\lambda'-\lambda)\sin\tfrac{1}{2}(\phi+\phi'). \ . \ . \quad (89)$$

288. Computation of Geodetic Coordinates: Example. —From Fig. 178, platted roughly in proper relation to the points of the compass, we are able to ascertain the mode of *determining the azimuths* between the various stations. The known side, Chuska—Zuñi, is drawn heavier than the others, and in the following computations the geodetic coordinates of

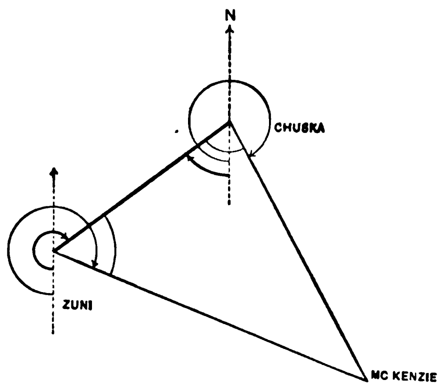


FIG. 178.—COMPUTATION OF AZIMUTHS.

both Chuska and Zuñi are supposed to have been determined previously, the coordinates of McKenzie being desired. Drawing north and south lines through Chuska and Zuñi, it is evident that to obtain the azimuth Chuska—McKenzie 360° must be added to the azimuth Chuska—Zuñi and the spherical angle at Chuska be subtracted from this, the result being the azimuth Chuska—McKenzie. Likewise, knowing the azimuth of the line Zuñi—Chuska, the azimuth of the line Zuñi—McKenzie is obtained by adding to the former the spherical angle at Zuñi.

On pages 644 and 645 is an example of the method of computing geodetic coordinates. The order of computation consists—

First, in *determining* the *new azimuths*, as just described, and as illustrated at the top of the pages of example.

Second, the *latitude of* the *new point* is obtained as shown in the left-hand column of either page. The latitude of the known point, as Chuska, is written opposite ϕ, then a difference of latitude, $\Delta\phi$, is obtained through the process of the entire computation shown in the left-hand columns. This amount, which is found at the bottom of the left side of the page, is then written under the latitude of Chuska with its proper algebraic sign, and subtracting it, in this case, the latitude of the unknown point, McKenzie, is found.

Third, the *longitude of* the *new point* is obtained as shown in the columns on the right-hand side of the pages. These are arranged in manner similar to the latitude computation by writing the longitude of the known point, λ, and then determining a difference of longitude, $\Delta\lambda$, which in the example is minus and is therefore subtracted from the longitude of Chuska to obtain the longitude of the unknown point, McKenzie. The signs in both of the above cases can be verified by a diagram showing the relative positions of the points. Such a diagram (Fig. 178) shows clearly that McKenzie is south of Chuska and its latitude less, and that it is also east of Chuska and its longitude therefore less.

Fourth, the *azimuth computation* is performed as shown in the lower part of the right-hand columns. This consists in *determining* the *back* or the reverse *azimuth* from McKenzie to Chuska or Zuñi. This is done by determining the difference of azimuths, $\Delta\alpha$, which is written at the top of the right-hand column with its proper sign. The latter can be verified again by reference to the diagrammatic figure.

Finally, at the extreme bottom of the right-hand column is a *test of* the *azimuth computation.* This check is had by subtracting the back azimuths one from the other and noting if the result is equal to the spherical angle at McKenzie.

Latitude, longitude, and azimuth, or ϕ, λ, and α, of the known points, must have been previously determined by astronomic observations (Chaps. XXXIII to XXXV) or from

Azimuth α.	*Chuska — Zuñi*	364° 36′ 23″.224
Spherical Angle:	*at Chuska*	35 56 55 .884
Azimuth α':	*Chuska — McKenzie*	328 39 27 .340
$\Delta\alpha + 180°$		180 13 39 .838
Azimuth (α):	*McKenzie — Chuska*	508 53 07 .178
		−360
		148

Geodetic Coordinates.

LATITUDE.			LONGITUDE.
ϕ: 35° 41′ 38″.911	*Chuska*	λ:	100° 19′ 42″.314
$\Delta\phi$ 31 49 .170−		$\Delta\lambda$:	23 34 .255−
ϕ' 35 09 49 .741	*McKenzie*	λ':	99 56 08 .059

Computation for latitude:

log. s	4.8375951
" B	8.5112082
" cos α'	9.9314956+
log. (I)	3.2802989+
log s^2	9.67519
" C	1.26412
" $\sin^2 \alpha'$	9.43226
log. (II)	0.36887
log. D	2.3631
" $[I+II]^2$	6.5617
log (III)	8.9248
log E	6.0178
" $s^2 \sin^2 \alpha'$	9.1074
" (I)	3.2803+
−log. (IV)+	8.4055−

log. s = − 85
cor. log. $\Delta\lambda$ = +35
log. cor. = − 50

cor. $\Delta\phi^2$ = 9.451
log. F = 7.871
log. cor. = 7.328
cor = 0″.002

ϕ =	35°	41′	38″	.911
ϕ' =	35	09	49	.741
$\phi+\phi'$ =	70	51	28	.652
$\frac{\phi+\phi'}{2}$ =	35	25	44	.326

Computation for longitude:

log. s	4.8375951
" sin α'	9.7161297−
" A′	8.5093005
" sec. ϕ'	0.0875075
log. (V)	3.1505328−
log. cor.	−50
	278

$\Delta\lambda$ 1414″.255−

Computation of azimuth:

log. (V)	3.1505278−
" sin. $\left(\frac{\phi+\phi'}{2}\right)$	9.7631983
" sec. $\left(\frac{\Delta\phi}{2}\right)$	0.0000055
log. (VI)	2.9137316−
$-\Delta\alpha$	−819″.836+
	13′ 39″.836+
	.002
	13′ 39″.838+

(I)	1906″.773+
(II)	2 .338+
(III)	0 .084+
(IV)	0 .025−
$-\Delta\phi$+	1909 .170−
	31′ 49 .170−

[I + II]	1909.111
log "	3.28083
	2
" $[I + II]^2$	6.56166

Check:
Spher. angle
at

Azimuth check.

Azimuth α	*Zuñi — Chuska*	184° 33' 48".211
Spherical Angle:	*at Zuñi*	54 17 21 .122
Azimuth α'.	*Zuñi — McKenzie*	238 51 09 .333
$\Delta\alpha + 180°$		180 16 06 .161
Azimuth (α):	*McKenzie — Zuñi*	419 07 15 .494
		−360
		59

Geodetic Coordinates.

LATITUDE.			LONGITUDE.	
ϕ:	34° 55' 57".982	*Zuñi*	λ	100° 24' 10".479
$\Delta\phi$	13 51 .758+		$\Delta\lambda$	28 02 .240−
ϕ'	35 09 49 .740	*McKenzie*	λ'	99 56 08 .069

Computation for latitude:

log. s	4.6967338
" B	8.5112628
" cos α'	9.7136935−
log. (I)	2.9216901−
log. s²	9.39347
" C	1.24926
" sin² α'	9.86478
log. (II)	0.50751
log. D	2.3591
" [I + II]²	5.8400
" (III)	8.1991
log. E	6.0032
" s² sin² α'	9.2582
" (I)	2.9217−
−log. (IV)	− 8.1831+

log. s = − 45
cor. log. $\Delta\lambda$ = + 48
log. cor. = + 03

cor. $\Delta\lambda^2$ = 9.678
log. F = 7.877
log cor. = 7.555
cor. = 0".003

ϕ = 34° 55' 57".982
ϕ' = 35 09 49 .740
$\phi + \phi'$ = 70 05 47 .722
$\frac{\phi+\phi'}{2}$ = 35 02 53 .861

Computation for longitude:

log s	4.6967338
" sin α'	9.9323922−
" A'	8.5093005
" sec. ϕ	0.0875075
log (V)	3.2259340−
	+ 03
	43

$\Delta\lambda$ 1682".420

Computation of azimuth:

log. (V)	3.2259343−
" sin $\left(\frac{\phi+\phi'}{2}\right)$	9.7591136
" sec. $\left(\frac{\Delta\phi}{2}\right)$	0.0000011
log. (VI)	2.9850490−
$-\Delta\alpha$	− 966".158+
	16' 06".158+
	.003
	16' 06".161+

(I)	835".007−
(II)	3 .217+
(III)	0 .017+
(IV)	0 .015+
$-\Delta\phi$	−831 .758+
	13' 51 .758+

	[I+II]	831".790
log.	"	2 .9200
		2
"	[I+II]²	5 .8400

Azimuth check.

McK. — *Chus*	148° 53' 07".18
" — *Zuñi*	59 07 15 .49
Check:	89 45 51 .69
Spher angle at *McKenzie*	89 45 51 .69

computation; the log. s, or length of the known side, has been obtained from previous computation (Art. 286). The quantities B, C, D, and E are obtained from Table XXXVII by using the known latitude, ϕ, as an argument. The quantity A' comes from the same table by using the new latitude, ϕ', as an argument.

The small correction dif. log. s is obtained from Table XXXVIII by using log. s as an argument. The cor. $\Delta\lambda$ is obtained from the same table by using the argument log. (V), which is log. $\Delta\lambda$. In applying these corrections signs must be carefully watched. The resulting log. cor. is applied, with attention to signs, as a correction to log. (V).

The log. F, which is added to cor. $\Delta\lambda^2$, is obtained from Table XL, using latitude ϕ as an argument. This correction is very small and is only made in cases of very long distances, as in the illustration used.

$$\text{The sec. of } \phi' = \text{a. c. log. cos } \phi' = \frac{1}{\text{cosin } \phi'}.$$

The sec. $\dfrac{\Delta\phi}{2}$ is obtained from Table XXXIX.

289. Knowing Latitudes and Longitudes of Two Points, to Compute Azimuths and Distances.—This is the inverse problem of that considered in the preceding article. It not infrequently occurs when the latitudes and longitudes of two positions are known that it is desired to find the distance between them and their mutual azimuths. This problem may prove useful in exploratory surveying when the latitudes of two intervisible mountain peaks which differ but little in longitude can be determined by sextant or transit, and longitudes by flashing signals or by chronometer. Then by this problem a base for triangulation may be obtained which will enable the explorer to rapidly extend the area of his survey. Providing the initial points are at a considerable distance apart, the station error will be inappreciable for small-scale mapping. Again, in a system of triangulation it may be desired to ascertain whether two stations hidden by trees, haze, etc., are intervisible. Providing their latitudes and longitudes are known or can be computed from other triangulation points, their initial azimuths may be found from this problem, when it will be possible to set up an instrument at one station and lay off the exact azimuth to the other for guidance in clearing timber or in heliotroping.

This problem can be readily solved in tabular manner by arranging the form of solution used on pp. 644 and 645.

To do this divide $\Delta\lambda = \sin\alpha \,.\, A \sec\phi'$ by the first term for $\Delta\phi$, $\qquad h = s\cos\alpha \,.\, B$, whence we get

$$\tan\alpha = \frac{\Delta\lambda \,.\, B}{A \sec\phi' h}. \quad . \quad . \quad . \quad . \quad . \quad (90)$$

If h were known, this would give the azimuth at once, since $\Delta\lambda$ is given.

The following example shows the method of performing the operation. The northernmost point should be used as the initial position, then all signs for (I), (II), and (III) are $+$, and for (IV) $-$. The value of $\Delta\lambda$ may be either $+$ or $-$, but this sign need only be used in determining in which quadrant the azimuth angle α falls, i.e., the sign of $\tan\alpha$ (12). An inspection of a rough plot of the position will also determine this. The correction to $\Delta\lambda$ is found from a distance scaled off from the plot, and need not be very close. In (8) the term $(\mathrm{I} + \mathrm{II})^2$ is the square of the difference of latitude $\Delta\phi$ in seconds. Since (IV) is always small, log (I) in (ϕ) may be taken as log of $\Delta\lambda$ from (10). If $\cos a$ is smaller than $\sin a$, find s from $\log s \cos\alpha$ in (11). As a check on the work compute the second position, using distance and azimuth found as above. The order of solution is shown by figures in parentheses. The cosines of latitudes are proportional to the intercepted parallels.

The results obtained from this problem should be checked by computing latitudes and longitudes by the direct method as shown in the example on pages 644 and 645.

Latitude.

$\phi = 38° \; 23' \; 27''.0$

$\phi = 37 \;\; 45 \;\; 09 \; .3$

$\Delta\phi = \quad 38' \; 17''.7 = \phi = \phi'$

$= \quad 2297''.7 \qquad (1)$

$\log \Delta\phi = \quad 3.3612933$

$\log C \quad 1.30360$

$\log s^2 \sin^2 \alpha \quad 8.75770$

$0.06130 \quad (7)$

$(II) = \; + 1''.152$

$\log D \quad 2.3812$

$\log (I + II)^2 \quad 6.7226$

$9.1038 \quad (8)$

$(III) = + 0''.13$

$\log E \quad 6.0711$

$\log s^2 \sin^2 \alpha \quad 8.7577 \quad (9)$

$\log (I) \quad 3.3613$

8.1901

$(IV) = - \quad 0''.02$

$(II) = + 1''.15$

$(III) = + \quad .13$

$(IV) = - \quad .02 \quad (10)$

$\text{Sum} = + \; 1''.26$

$\Delta\phi \quad 2297.7$

$(I) = \; 2296''.4 = \text{difference}$

Longitude.

$104° \; 32' \; 48''.2 = \lambda$

$104 \;\; 49 \;\; 05 \; .5 = \lambda'$

$\Delta\lambda = 16' \; 17''.3 +$

$= \quad 977''.3 + \qquad (2)$

$\log \Delta\lambda = 2.9900279 +$

correction to $\Delta\lambda \quad 83 +$

$\Delta\lambda' \; 2.9900362 \qquad (4)$

cor. $\Delta\lambda \quad 17 +$

$\Delta s \; 100 -$

$83 - \quad (3)$

$\log A' = 8.5091750 \qquad (5)$

$\sec s' = 0.1020092$

$8.6111842 \; (-)$

$\log \Delta\lambda' = 2.9900362 \; (+)$

$\log s \sin \alpha = 4.3788520 \; (+) \qquad (6)$

$\log s \cos \alpha = 4.8500742 \; (-)$

$\dfrac{\sin \alpha}{\cos \alpha} = \tan \alpha = 9.5287778 \qquad (12)$

$\log (I) = 3.3610475$

$\log B = 8.5109733$

$\log s \cos \alpha = 4.8500742 \qquad (11)$

$\text{Azimuth} = \alpha = 18° \; 40' \; 10''.8 \qquad (13)$

(or 180 + this angle)

$\log s \sin \alpha = 4.3788520$

$\log \sin \alpha = 9.5053013 \qquad (14)$

Distance (log) $= \log s = 4.8735507$

TABLE XXXVII.

FACTORS FOR COMPUTATION OF GEODETIC LATITUDES, LONGITUDES, AND AZIMUTHS.

(From Appendix No. 9, Report of U. S. Coast and Geodetic Survey, 1894.)

LATITUDE 30°.

Lat.	log A diff. 1″ = − 0.06	log B diff. 1″ = − 0.19	log C diff. 1″ = + 0.48	log D diff. 1″ = + 0.02	log E diff. 1″ = + 0.05
° ′					
30 00	8.509 3588	8.511 5729	1.16692	2.3298	5.9127
1	84	18	721	99	30
2	81	06	750	2.3301	33
3	77	8.511 5695	778	02	36
4	73	84	807	04	39
05	69	73	836	05	42
6	66	62	865	06	45
7	62	51	894	08	48
8	58	40	923	09	51
9	55	28	952	11	54
10	8.509 3551	8.511 5617	1.16981	2.3312	5.9157
11	47	06	1.17010	14	59
12	43	8.511 5595	039	15	62
13	40	84	068	17	65
14	36	73	097	18	68
15	32	61	126	19	71
16	29	50	155	21	74
17	25	39	184	22	77
18	21	28	212	24	80
19	17	17	241	25	83
20	8.509 3514	8.511 5505	1.17270	2.3327	5.9186
21	10	8.511 5494	299	28	89
22	06	83	328	30	92
23	02	72	357	31	95
24	8.509 3499	61	385	32	98
25	95	49	414	34	5.9200
26	91	38	443	35	03
27	88	27	472	37	06
28	84	16	500	38	09
29	80	04	529	39	12
30	8.509 3476	8.511 5393	1.17558	2.3341	5.9215
31	72	82	587	42	18
32	69	71	615	44	21
33	65	59	644	45	24
34	61	48	673	47	27
35	57	37	701	48	30
36	54	26	730	49	33
37	50	14	759	51	36
38	46	03	788	52	39
39	42	8.511 5292	816	54	42
40	8.509 3439	8.511 5281	1.17845	2.3355	5.9245
41	35	69	874	56	48
42	31	58	902	58	51
43	27	47	931	59	53
44	24	35	959	60	56
45	20	24	988	62	59
46	16	13	1.18017	63	62
47	12	02	045	65	65
48	09	8.511 5190	074	66	68
49	05	79	102	67	71
50	8.509 3401	8.511 5168	1.18131	2.3368	5.9274
51	8.509 3397	56	160	70	77
52	94	45	188	71	80
53	90	34	217	73	83
54	86	22	245	74	86
55	82	11	274	76	89
56	78	00	302	77	92
57	75	8.511 5088	331	78	95
58	71	77	359	80	98
59	67	66	388	81	5.9301
60	8.509 3363	8.511 5054	1.18416	2.3382	5.9304

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 31°.

Lat.	log A diff. 1″ = −0.06	log B diff. 1″ = −0.19	log C diff. 1″ = +0.47	log D diff. 1″ = +0.02	log E diff. 1″ = +0.05
° ′					
31 00	8.509 3363	8.511 5054	1.18416	2.3382	5.9304
1	60	43	445	84	07
2	56	32	473	85	10
3	52	20	501	86	13
4	48	09	530	88	16
05	44	8.511 4998	558	89	19
6	41	86	587	90	22
7	37	75	615	92	25
8	33	64	643	93	28
9	29	52	672	95	31
10	8.509 3325	8.511 4941	1.18700	2.3396	5.9334
11	22	29	729	97	37
12	18	18	757	99	39
13	14	07	785	2.3400	42
14	10	8.511 4895	813	01	45
15	06	84	842	02	48
16	03	72	870	04	51
17	8.509 3299	61	898	05	54
18	95	50	927	06	57
19	91	38	955	08	60
20	8.509 3287	8.511 4827	1.18983	2.3409	5.9363
21	84	15	1.19012	10	66
22	80	04	040	12	69
23	76	8.511 4793	068	13	72
24	72	81	096	14	75
25	68	70	125	16	78
26	65	58	153	17	81
27	61	47	181	18	84
28	57	35	209	20	87
29	53	24	238	21	90
30	8.509 3249	8.511 4713	1.19266	2.3422	5.9393
31	46	01	294	23	96
32	42	8.511 4690	322	25	99
33	38	78	351	26	5.9402
34	34	67	379	27	05
35	30	55	407	29	08
36	26	44	435	30	11
37	23	32	463	31	14
38	19	21	491	33	17
39	15	09	520	34	20
40	8.509 3211	8.511 4598	1.19548	2.3435	5.9423
41	07	86	576	36	26
42	03	75	604	38	29
43	00	63	632	39	32
44	8.509 3196	52	660	40	35
45	92	40	688	41	38
46	88	29	716	43	41
47	84	17	744	44	44
48	81	06	772	45	47
49	77	8.511 4494	800	47	50
50	8.509 3173	8.511 4483	1.19828	2.3448	5.9453
51	69	71	856	49	56
52	65	60	884	50	59
53	61	48	912	52	62
54	57	37	940	53	65
55	54	25	968	54	68
56	50	14	996	55	72
57	46	02	1.20024	57	75
58	42	8.511 4391	052	58	78
59	38	79	080	59	81
60	8.509 3134	8.511 4368	1.20108	2.3460	5.9484

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 32°.

Lat.	log *A* diff. 1″ = − 0.06	log *B* diff. 1″ = − 0.19	log *C* diff. 1″ = + 0.46	log *D* diff. 1″ = + 0.02	log *E* diff. 1″ = + 0.05
° ′					
32 00	8.509 3134	8.511 4368	1.20108	2.3460	5.9484
1	31	56	136	62	87
2	27	44	164	63	90
3	23	33	192	64	93
4	19	21	220	65	96
05	15	10	248	67	99
6	11	8.511 4298	276	68	5.9502
7	07	87	304	69	05
8	04	75	332	70	08
9	00	63	360	71	11
10	8.509 3096	8.511 4252	1.20387	2.3473	5.9514
11	92	40	415	74	17
12	88	29	443	75	20
13	84	17	471	76	23
14	80	05	499	78	26
15	76	8.511 4194	527	79	29
16	73	82	555	80	32
17	67	71	582	81	35
18	65	59	610	82	38
19	61	47	638	84	41
20	8.509 3057	8.511 4136	1.20666	2.3485	5.9544
21	53	24	694	86	47
22	49	13	722	87	50
23	46	01	749	88	53
24	42	8.511 4089	777	90	56
25	38	78	805	91	60
26	34	66	833	92	63
27	30	54	860	93	66
28	26	43	888	94	69
29	22	31	916	96	72
30	8.509 3018	8.511 4020	1.20944	2.3497	5.9575
31	15	08	971	98	78
32	11	8.511 3996	999	99	81
33	07	85	1.21027	2.3500	84
34	03	73	054	02	87
35	8.509 2999	61	082	03	90
36	95	50	110	04	93
37	91	38	137	05	96
38	87	26	165	06	99
39	83	15	193	07	5.9602
40	8.509 2980	8.511 3903	1.21220	2.3509	5.9605
41	76	8.511 3891	248	10	08
42	72	79	276	11	11
43	68	68	303	12	15
44	64	56	331	13	18
45	60	44	358	14	21
46	56	33	386	16	24
47	52	21	414	17	27
48	48	09	441	18	30
49	44	8.511 3798	469	19	33
50	8.509 2940	8.511 3786	1.21496	2.3520	5.9636
51	37	74	524	21	39
52	33	63	551	23	42
53	29	51	579	24	45
54	25	39	607	25	48
55	21	27	634	26	51
56	17	16	662	27	54
57	13	04	689	28	58
58	09	8.511 3692	717	29	61
59	05	80	744	31	64
60	8.509 2901	8.511 3669	1.21772	2.3532	5.9667

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 33°

Lat.	log *A* diff. 1″ = − 0.07	log *B* diff. 1″ = − 0.20	log *C* diff. 1″ = + 0.45	log *D* diff. 1″ = + 0.02	log *E* diff. 1″ = + 0.05
° ′					
33 00	8.509 2901	8.511 3669	1.21772	2.3532	5.9667
1	8.509 2897	57	799	23	70
2	94	45	827	34	73
3	90	33	854	35	76
4	86	22	882	36	79
05	82	10	909	37	82
6	78	8.511 3598	937	38	85
7	74	86	964	40	88
8	70	75	992	41	92
9	66	63	1.22019	42	95
10	8.509 2862	8.511 3551	1.22047	2.3543	5.9698
11	58	39	074	44	5.9701
12	54	28	101	45	04
13	51	16	129	46	07
14	47	04	156	47	10
15	43	8.511 3492	184	49	13
16	39	80	211	50	16
17	35	69	238	51	19
18	31	57	266	52	22
19	27	45	293	53	26
20	8.509 2823	8.511 3433	1.22321	2.3554	5.9729
21	19	21	348	55	32
22	15	10	375	56	35
23	11	8.511 3398	403	57	38
24	07	86	430	58	41
25	03	74	457	60	44
26	8.509 2799	62	485	61	47
27	95	51	512	62	50
28	91	39	539	63	53
29	88	27	567	64	57
30	8.509 2784	8.511 3315	1.22594	2.3565	5.9760
31	80	03	621	66	63
32	76	8.511 3291	648	67	66
33	72	80	676	68	69
34	68	68	703	69	72
35	64	56	730	70	75
36	60	44	757	71	78
37	56	32	785	73	81
38	52	20	812	74	85
39	48	09	839	75	88
40	8.509 2744	8.511 3197	1.22866	2.3576	5.9791
41	40	85	893	77	94
42	36	73	921	78	97
43	32	61	948	79	5.9800
44	28	49	975	80	03
45	24	37	1.23002	81	06
46	20	25	029	82	10
47	16	13	057	83	13
48	12	02	084	84	16
49	08	8.511 3090	111	85	19
50	8.509 2704	8.511 3078	1.23138	2.3586	5.9822
51	01	66	165	87	25
52	8.509 2697	54	192	88	28
53	93	42	220	89	31
54	89	30	247	91	35
55	85	18	274	92	38
56	81	06	301	93	41
57	77	8.511 2995	328	94	44
58	73	83	355	95	47
59	69	71	382	96	50
60	8.509 2665	8.511 2959	1.23409	2.3597	5.9853

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 34°

Lat.	log *A* diff. 1″ = − 0.07	log *B* diff. 1″ = − 0.20	log *C* diff. 1″ = + 0.45	log *D* diff. 1″ = + 0.02	log *E* diff. 1″ = + 0.05
° ′					
34 00	8.509 2665	8.511 2959	1.23409	2.3597	5.9853
1	61	47	437	98	57
2	57	35	464	99	60
3	53	23	491	2.3600	63
4	49	11	518	01	66
05	45	8.511 2899	545	02	69
6	41	87	572	03	72
7	37	75	599	04	75
8	33	63	626	05	79
9	30	51	653	06	82
10	8.509 2625	8.511 2840	1.23680	2.3607	5.9885
11	21	28	707	08	88
12	17	16	734	09	91
13	13	04	761	10	94
14	09	8.511 2792	788	11	97
15	05	80	815	12	5.9901
16	01	68	842	13	04
17	8.509 2597	56	869	14	07
18	93	44	896	15	10
19	89	32	923	16	13
20	8.509 2585	8.511 2720	1.23950	2.3617	5.9916
21	81	08	977	18	19
22	77	8.511 2696	1.24004	19	23
23	73	84	031	20	26
24	69	72	058	21	29
25	65	60	085	22	32
26	61	48	112	23	35
27	57	36	139	24	38
28	53	24	165	25	42
29	49	12	192	26	45
30	8.509 2545	8.511 2600	1.24219	2.3627	5.9948
31	41	8.511 2588	246	28	51
32	37	76	273	29	54
33	33	64	300	30	57
34	29	52	327	31	61
35	25	40	354	32	64
36	21	28	381	33	67
37	17	16	408	34	70
38	13	04	434	35	73
39	09	8.511 2492	461	36	76
40	8.509 2505	8.511 2480	1.24488	2.3637	5.9980
41	01	68	515	38	83
42	8.509 2497	56	542	39	86
43	93	44	569	40	89
44	89	32	595	41	92
45	85	20	622	42	96
46	81	08	649	43	99
47	77	8.511 2396	676	44	6.0002
48	73	84	703	44	05
49	69	72	729	45	08
50	8.509 2465	8.511 2360	1.24756	2.3646	6.0011
51	61	48	783	47	15
52	57	35	810	48	18
53	53	23	837	49	21
54	49	11	863	50	24
55	45	8.511 2299	890	51	27
56	41	87	917	52	31
57	37	75	944	53	34
58	33	63	970	54	37
59	29	51	997	55	40
60	8.509 2425	8.511 2239	1.25024	2.3656	6.0043

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 35°.

Lat.	log A diff. 1″ = −0.07	log B diff. 1″ = −0.20	log C diff. 1″ = −0.44	log D diff. 1″ = +0.01	log E diff. 1″ = +0.05
° ′					
35 00	8.509 2425	8.511 2239	1.25024	2.3656	6.0043
1	21	27	050	57	47
2	17	15	077	58	50
3	13	03	104	59	53
4	09	8.511 2191	131	59	56
05	05	78	157	60	59
6	01	66	184	61	63
7	8.509 2396	54	211	62	66
8	93	42	237	63	69
9	88	30	264	64	72
10	8.509 2384	8.511 2118	1.25291	2.3665	6.0075
11	80	06	317	66	79
12	76	8.511 2094	344	67	82
13	72	82	371	68	85
14	68	70	397	69	88
15	64	57	424	70	91
16	60	45	451	70	95
17	56	33	477	71	98
18	52	21	504	72	6.0101
19	48	09	531	73	04
20	8.509 2344	8.511 1997	1.25557	2.3674	6.0107
21	40	85	584	75	11
22	36	72	610	76	14
23	32	60	637	77	17
24	28	48	664	78	20
25	24	36	690	79	22
26	20	24	717	79	27
27	16	12	743	80	30
28	12	00	770	81	33
29	08	8.511 1887	796	82	36
30	8.509 2304	8.511 1875	1.25823	2.3683	6.0140
31	00	63	850	84	43
32	8.509 2296	51	876	85	46
33	92	39	903	86	49
34	87	27	929	86	52
35	83	15	956	87	56
36	79	02	982	88	59
37	75	8.511 1790	1.26009	89	62
38	71	78	035	90	65
39	67	66	062	91	69
40	8.509 2263	8.511 1754	1.26088	2.3692	6.0172
41	59	41	115	93	75
42	55	29	141	93	78
43	51	17	168	94	81
44	47	05	194	95	85
45	43	8.511 1693	221	96	88
46	39	80	247	97	91
47	35	68	274	98	94
48	31	56	300	99	98
49	27	44	327	99	6.0201
50	8.500 2222	8.511 1632	1.26353	2.3700	6.0204
51	18	20	380	01	07
52	14	07	406	02	11
53	10	8.511 1595	432	03	14
54	06	83	459	04	17
55	02	71	485	05	20
56	8.509 2198	58	512	05	24
57	94	46	538	06	27
58	90	34	565	07	30
59	86	22	591	08	33
60	8.509 2182	8.511 1510	1.26617	2.3709	6.0237

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 36°.

Lat.	log *A* diff. 1″ = − 0.07	log *B* diff. 1″ = − 0.20	log *C* diff. 1″ = + 0.44	log *D* diff. 1″ = + 0.01	log *E* diff. 1″ = + 0.05
° ′					
36 00	8.509 2182	8.511 1510	1.26617	2.3709	6.0237
1	78	8.511 1497	644	10	40
2	74	85	670	10	43
3	70	73	697	11	46
4	65	61	723	12	50
05	61	48	749	13	53
6	57	36	776	14	56
7	53	24	802	14	59
8	49	12	828	15	63
9	45	8.511 1399	855	16	66
10	8.509 2141	8.511 1387	1.26881	2.3717	6.0269
11	37	75	908	18	72
12	33	63	934	19	76
13	29	50	960	19	79
14	25	38	987	20	82
15	21	26	1.27013	21	85
16	16	14	039	22	89
17	12	01	066	23	92
18	08	8.511 1289	092	23	95
19	04	77	118	24	99
20	8.509 2100	8.511 1265	1.27145	2.3725	6.0302
21	8.509 2096	52	171	26	05
22	92	40	197	27	08
23	88	28	223	27	12
24	84	15	250	28	15
25	80	03	276	29	18
26	75	8.511 1191	302	30	21
27	71	79	329	31	25
28	67	66	355	31	28
29	63	54	381	32	31
30	8.509 2059	8.511 1142	1.27407	2.3733	6.0334
31	55	29	434	34	38
32	51	17	460	35	41
33	47	05	486	35	44
34	43	8.511 1092	512	36	48
35	39	80	539	37	51
36	35	68	565	38	54
37	30	56	591	38	57
38	26	43	617	39	61
39	22	31	644	40	64
40	8.509 2018	8.511 1019	1.27670	2.3741	6.0367
41	14	06	696	41	71
42	10	8.511 0994	722	42	74
43	06	82	748	43	77
44	02	69	775	44	80
45	8.509 1998	57	801	45	84
46	93	45	827	45	87
47	89	32	853	46	90
48	85	20	879	47	94
49	81	08	905	48	97
50	8.509 1977	8.511 0895	1.27932	2.3748	6.0400
51	73	83	958	49	03
52	69	71	984	50	07
53	65	58	1.28010	51	10
54	61	46	036	51	13
55	56	34	062	52	17
56	52	21	088	53	20
57	48	09	114	54	23
58	44	8.511 0797	141	54	27
59	40	84	167	55	30
60	8.509 1936	8.511 0772	1.28193	2.3756	6.0433

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 37°.

Lat.	log A diff. 1″ = −0.07	log B diff. 1″ = −0.21	log C diff. 1″ = +0.43	log D diff. 1″ = +0.01	log E diff. 1″ = +0.06
° ′					
37 00	8.509 1936	8.511 0772	1.28193	2.3756	6.0433
1	32	60	219	56	37
2	28	47	245	57	40
3	23	35	271	58	43
4	19	22	297	59	46
05	15	10	324	59	50
6	11	8.511 0698	350	60	53
7	07	85	376	61	56
8	03	73	402	62	60
9	8.509 1899	61	428	62	63
10	8.509 1895	8.511 0648	1.28454	2.3763	6.0466
11	90	36	480	64	70
12	86	23	506	65	73
13	82	11	532	65	76
14	78	8.511 0599	558	66	80
15	74	86	584	67	83
16	70	74	610	67	86
17	66	61	636	68	89
18	62	49	662	69	93
19	57	37	688	69	96
20	8.509 1853	8.511 0524	1.28715	2.3770	6.0499
21	49	12	741	71	6.0503
22	45	00	767	72	06
23	41	8.511 0487	793	72	09
24	37	75	819	73	13
25	33	62	845	74	16
26	28	50	871	74	19
27	24	37	897	75	23
28	20	25	923	76	26
29	16	13	949	76	29
30	8.509 1812	8.511 0400	1.28975	2.3777	6.0533
31	08	8.511 0388	1.29001	78	36
32	04	75	027	79	39
33	00	63	053	79	43
34	8.509 1795	51	079	80	46
35	91	38	104	81	49
36	87	26	130	81	53
37	83	13	156	82	56
38	79	01	182	83	59
39	75	8.511 0288	208	83	63
40	8.509 1771	8.511 0276	1.29234	2.3784	6.0566
41	66	64	260	85	69
42	62	51	286	85	73
43	58	39	312	86	76
44	54	26	338	87	79
45	50	14	364	87	83
46	46	01	390	88	86
47	41	8.511 0189	416	89	89
48	37	76	442	89	93
49	33	64	468	90	96
50	8.509 1729	8.511 0151	1.29494	2.3791	6.0600
51	25	39	520	91	03
52	21	26	546	92	06
53	16	14	571	93	10
54	12	02	597	93	13
55	08	8.511 0089	623	94	16
56	04	77	649	95	20
57	00	64	675	95	23
58	8.509 1696	52	701	96	26
59	92	39	727	96	30
60	8.509 1687	8.511 0027	1.29753	2.3797	6.0633

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 38°.

Lat.	log *A* diff. 1″ = −0.07	log *B* diff. 1″ = −0.21	log *C* diff. 1″ = +0.43	log *D* diff. 1″ = +0.01	log *E* diff. 1″ = +0.06
° ′					
38 00	8.509 1687	8.511 0027	1.29753	2.3797	6.0633
1	83	14	778	98	36
2	79	02	804	98	40
3	75	8.510 9989	830	99	43
4	71	77	856	2.3800	47
05	67	64	882	00	50
6	62	52	908	01	53
7	58	39	934	02	57
8	54	27	959	02	60
9	50	14	985	03	63
10	8.509 1646	8.510 9902	1.30011	2.3803	6.0667
11	42	8.510 9889	037	04	70
12	37	77	063	05	73
13	33	64	089	05	77
14	29	52	114	06	80
15	25	39	140	07	84
16	21	27	166	07	87
17	17	14	192	08	90
18	12	02	218	08	94
19	08	8.510 9789	243	09	97
20	8.509 1604	8.510 9777	1.30269	2.3810	6.0701
21	00	64	295	10	04
22	8.509 1596	52	321	11	07
23	92	39	347	12	11
24	87	27	372	12	14
25	83	14	398	13	17
26	79	01	424	13	21
27	75	8.510 9689	450	14	24
28	71	77	476	15	28
29	66	64	501	15	31
30	8.509 1562	8.510 9652	1.30527	2.3816	6.0734
31	58	39	553	16	38
32	54	27	579	17	41
33	50	14	604	17	44
34	46	01	630	18	48
35	41	8.510 9589	656	19	51
36	37	76	682	19	55
37	33	64	707	20	58
38	29	51	733	20	61
39	25	39	759	21	65
40	8.509 1521	8.510 9526	1.30785	2.3822	6.0768
41	16	14	810	22	72
42	12	01	836	23	75
43	08	8.510 9488	862	23	78
44	04	76	887	24	82
45	00	63	913	24	85
46	8.509 1495	51	939	25	89
47	91	38	965	26	92
48	87	26	990	26	95
49	83	13	1.31016	27	99
50	8.509 1479	8.510 9401	1.31042	2.3827	6.0802
51	75	8.510 9388	067	28	06
52	70	76	093	28	09
53	66	63	119	29	13
54	62	50	144	30	16
55	58	38	170	30	19
56	53	25	196	31	23
57	49	13	221	31	26
58	45	00	247	32	30
59	41	8.510 9287	273	32	33
60	8.509 1437	8.510 9275	1.31299	2.3833	6.0836

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 39°.

Lat.	log A diff. 1″ = − 0.07	log B diff. 1″ = − 0.21	log C diff. 1″ = + 0.43	log D diff. 1″ = + 0.01	log E diff. 1″ = + 0.06
° ′					
39 00	8.509 1437	8.510 9275	1.31299	2.3833	6.0836
1	33	62	324	33	40
2	28	50	350	34	43
3	24	37	375	35	47
4	20	25	401	35	50
05	16	12	427	36	53
6	12	8.510 9199	452	36	57
7	07	87	478	37	60
8	03	74	504	37	64
9	8.509 1399	62	529	38	67
10	8.509 1395	8.510 9149	1.31555	2.3838	6.0871
11	91	36	581	39	74
12	86	24	606	39	77
13	82	11	632	2.3840	81
14	78	8.510 9098	658	40	84
15	74	86	683	41	88
16	70	73	709	41	91
17	65	61	734	42	95
18	61	48	760	43	98
19	57	36	786	43	6.0902
20	8.509 1353	8.510 9023	1.31811	2.3844	6.0905
21	49	10	837	44	08
22	44	8.510 8998	862	45	12
23	40	85	888	45	15
24	36	73	913	46	19
25	32	60	939	46	22
26	28	47	965	47	26
27	23	35	990	47	29
28	19	22	1.32016	48	32
29	15	09	041	48	36
30	8.509 1311	8.510 8897	1.32067	2.3849	6.0939
31	07	84	092	49	43
32	02	72	118	2.3850	46
33	8.509 1298	59	144	50	50
34	94	46	169	51	53
35	90	34	195	51	57
36	86	21	220	52	60
37	81	08	246	52	63
38	77	8.510 8796	271	53	67
39	73	83	297	53	70
40	8.509 1269	8.510 8771	1.32323	2.3854	6.0974
41	64	58	348	54	77
42	60	45	374	55	81
43	56	33	399	55	84
44	52	20	425	56	88
45	48	07	450	56	91
46	43	8.510 8695	476	57	95
47	39	82	501	57	98
48	35	69	527	57	6.1002
49	31	57	552	58	05
50	8.509 1227	8.510 8644	1.32578	2.3858	6.1008
51	22	31	603	59	12
52	18	19	629	59	15
53	14	06	654	2.3860	19
54	10	8.510 8593	680	60	22
55	06	81	705	61	26
56	01	68	731	61	29
57	8.509 1197	55	756	62	33
58	93	43	782	62	36
59	89	30	807	63	40
60	8.509 1184	8.510 8517	1.32833	2.3863	6.1043

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 40°.

Lat.	log *A* diff. 1″ = − 0.07	log *B* diff. 1″ = − 0.21	log *C* diff. 1″ = + 0.42	log *D* diff. 1″ = + 0.01	log *E* diff. 1″ = + 0.06
° ′					
40 00	8.509 1184	8.510 8517	1.32833	2.3863	6.1043
1	80	05	858	64	47
2	76	8.510 8492	884	64	50
3	72	79	909	64	54
4	67	67	935	65	57
05	63	54	960	65	61
6	59	41	986	66	64
7	55	29	1.33011	66	67
8	50	16	037	67	71
9	46	03	062	67	74
10	8.509 1142	8.510 8391	1.33088	2.3868	6.1078
11	38	78	113	68	81
12	34	65	139	68	85
13	29	53	164	69	88
14	25	40	189	69	92
15	21	27	215	2.3870	95
16	17	15	240	70	99
17	12	02	266	71	6.1102
18	08	8.510 8289	291	71	06
19	04	77	317	72	09
20	8.509 1100	8.510 8264	1.33342	2.3872	6.1113
21	8.509 1096	51	368	72	16
22	91	38	393	73	20
23	87	26	418	73	23
24	83	13	444	74	27
25	79	00	469	74	30
26	74	8.510 8188	495	74	34
27	70	75	520	75	37
28	66	62	546	75	41
29	62	50	571	76	44
30	8.509 1057	8.510 8137	1.33596	2.3876	6.1148
31	53	24	622	77	51
32	49	11	647	77	55
33	45	8.510 8099	673	77	58
34	41	86	698	78	62
35	36	73	723	78	65
36	32	61	749	79	69
37	28	48	774	79	72
38	24	35	800	79	76
39	19	23	825	2.3880	79
40	8.509 1015	8.510 8010	1.33850	2.3880	6.1183
41	11	8.510 7997	876	81	86
42	07	84	901	81	90
43	02	72	926	81	93
44	8.509 0998	59	952	82	97
45	94	46	977	82	6.1200
46	90	33	1.34003	83	04
47	85	21	028	83	07
48	81	08	053	83	11
49	77	8.510 7895	079	84	15
50	8.509 0973	8.510 7883	1.34104	2.3884	6.1218
51	68	70	129	84	22
52	64	57	155	85	25
53	60	44	180	85	29
54	56	32	206	86	32
55	52	19	231	86	36
56	47	06	256	86	39
57	43	8.510 7793	282	87	43
58	39	81	307	87	46
59	34	68	332	87	50
60	8.509 0930	8.510 7755	1.34358	2.3888	6.1253

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 41°.

Lat.	log A diff. 1″ = −0.07	log B diff. 1″ = −0.21	log C diff. 1″ = +0.42	log D diff. 1″ = +0.01	log E diff. 1″ = +0.06
° ′					
41 00	8.509 0930	8.510 7755	1.34358	2.3888	6.1253
1	26	42	383	88	57
2	22	30	408	89	60
3	18	17	434	89	64
4	13	04	459	89	67
05	09	8.510 7691	484	90	71
6	05	79	510	90	75
7	00	66	535	90	78
8	8.509 0896	53	560	91	82
9	92	40	586	91	85
10	8.509 0888	8.510 7628	1.34611	2.3891	6.1289
11	83	15	636	92	92
12	79	02	662	92	96
13	75	8.510 7590	687	93	99
14	71	77	712	93	6.1303
15	67	64	738	93	06
16	62	51	763	94	10
17	58	39	788	94	14
18	54	26	814	94	17
19	49	13	839	95	21
20	8.509 0845	8.510 7500	1.34864	2.3895	6.1324
21	41	8.510 7488	890	95	28
22	37	75	915	96	31
23	32	62	940	96	35
24	28	49	965	96	38
25	24	36	991	97	42
26	20	24	1.35016	97	46
27	15	11	041	97	49
28	11	8.510 7398	066	98	53
29	07	85	092	98	56
30	8.509 0803	8.510 7373	1.35117	2.3898	6.1360
31	8.509 0798	60	142	99	63
32	94	47	168	99	67
33	90	34	193	99	70
34	86	22	218	2.3900	74
35	81	09	243	00	78
36	77	8.510 7296	269	00	81
37	73	83	294	00	85
38	69	70	319	01	88
39	64	58	345	01	92
40	8.509 0760	8.510 7245	1.35370	2.3901	6.1395
41	56	32	395	02	99
42	52	19	420	02	6.1403
43	47	07	446	02	06
44	43	8.510 7194	471	03	10
45	39	81	496	03	13
46	35	68	522	03	17
47	30	55	547	03	20
48	26	43	572	04	24
49	22	30	597	04	28
50	8.509 0718	8.510 7117	1.35623	2.3904	6.1431
51	13	04	648	05	35
52	09	8.510 7091	673	05	38
53	05	79	698	05	42
54	00	66	723	05	46
55	8.509 0696	53	749	06	49
56	92	40	774	06	53
57	88	27	799	06	56
58	83	15	824	07	60
59	79	02	850	07	63
60	8.509 0675	8.510 6989	1.35875	2.3907	6.1467

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 42°.

Lat.	log *A* diff. 1″ = − 0.07	log *B* diff. 1″ = − 0.21	log *C* diff. 1″ = + 0.42	log *D* diff. 1″ = + 0.00	log *E* diff. 1″ = + 0.06
° ′					
42 00	8.509 0675	8.510 6989	1.35875	2.3907	6.1467
1	71	76	900	08	71
2	66	64	925	08	74
3	62	51	951	08	78
4	58	38	976	08	81
05	54	25	1.36001	09	85
6	49	12	026	09	89
7	45	00	052	09	92
8	41	8.510 6887	077	09	96
9	36	74	102	10	99
10	8.509 0632	8.510 6861	1.36127	2.3910	6.1503
11	28	48	152	10	07
12	24	36	178	10	10
13	19	23	203	11	14
14	15	10	228	11	17
15	11	8.510 6797	253	11	21
16	07	84	278	12	25
17	02	72	304	12	28
18	8.509 0598	59	329	12	32
19	94	46	354	12	35
20	8.509 6590	8.510 6733	1.36379	2.3913	6.1539
21	85	20	404	13	43
22	81	07	430	13	46
23	77	8.510 6695	455	13	50
24	72	82	480	13	54
25	68	69	505	14	57
26	64	56	530	14	61
27	60	43	556	14	64
28	55	31	581	14	68
29	51	18	606	15	72
30	8.509 0547	8.510 6605	1.36631	2.3915	6.1575
31	43	8.510 6592	656	15	79
32	38	79	682	15	83
33	34	66	707	16	86
34	30	54	732	16	90
35	25	41	757	16	93
36	21	28	782	16	97
37	17	15	808	17	6.1601
38	13	02	833	17	04
39	08	8.510 6490	858	17	08
40	8.509 0504	8.510 6477	1.36883	2.3917	6.1612
41	00	64	908	17	15
42	8.509 0496	51	934	18	19
43	91	38	959	18	22
44	87	25	984	18	26
45	83	13	1.37009	18	30
46	78	00	034	19	33
47	74	8.510 6387	059	19	37
48	70	74	085	19	41
49	66	61	110	19	44
50	8.509 0461	8.510 6348	1.37135	2.3919	6.1648
51	57	36	160	20	52
52	53	23	185	20	55
53	48	10	210	20	59
54	44	8.510 6297	235	20	63
55	40	84	261	20	66
56	36	71	286	21	70
57	31	59	311	21	73
58	27	46	336	21	77
59	23	33	361	21	81
60	8.509 0419	8.510 6220	1.37386	2.3921	6.1684

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 43°.

Lat.	log *A* diff. 1″ = − 0.07	log *B* diff. 1″ = − 0.21	log *C* diff. 1″ = + 0.42	log *D* diff. 1″ = + 0.00	log *E* diff. 1″ = + 0.06
° ′					
43 00	8.509 0419	8.510 6220	1.37386	2.3921	6.1684
1	14	07	412	22	88
2	10	8.510 6195	437	22	92
3	06	82	462	22	95
4	01	69	487	22	99
05	8.509 0397	56	512	22	6.1703
6	93	43	537	22	06
7	89	30	563	23	10
8	84	17	588	23	14
9	80	05	613	23	17
10	8.509 0376	8.510 6092	1.37638	2.3923	6.1721
11	71	79	663	23	25
12	67	66	688	24	28
13	63	53	713	24	32
14	59	40	739	24	36
15	54	28	764	24	39
16	50	15	789	24	43
17	46	02	814	24	47
18	41	8.510 5989	839	25	50
19	37	76	864	25	54
20	8.509 0333	8.510 5963	1.37889	2.3925	6.1758
21	29	50	915	25	61
22	24	38	940	25	65
23	20	25	965	25	69
24	16	12	990	25	72
25	12	8.510 5899	1.38015	26	76
26	07	86	040	26	80
27	03	73	065	26	83
28	8.509 0299	60	091	26	87
29	94	48	116	26	91
30	8.509 0290	8.510 5835	1.38141	2.3926	6.1795
31	86	22	166	27	98
32	82	09	191	27	6.1802
33	77	8.510 5796	216	27	06
34	73	83	241	27	09
35	69	71	266	27	13
36	64	58	292	27	17
37	60	45	317	27	20
38	56	32	342	27	24
39	52	19	367	28	28
40	8.509 0247	8.510 5706	1.38392	2.3928	6.1831
41	43	8.510 5693	417	28	35
42	39	81	442	28	39
43	34	68	467	28	42
44	30	55	492	28	46
45	26	42	518	28	50
46	22	29	543	28	53
47	17	16	568	29	57
48	13	03	593	29	61
49	09	8.510 5591	618	29	65
50	8.509 0204	8.510 5578	1.38643	2.3929	6.1868
51	00	65	668	29	72
52	8.509 0196	52	693	29	76
53	92	39	719	29	79
54	87	26	744	29	83
55	83	13	769	30	87
56	79	01	794	30	91
57	74	8.510 5488	819	30	94
58	70	75	844	30	98
59	66	62	869	30	6.1902
60	8.509 0162	8.510 5449	1.38894	2.3930	6.1905

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 44°.

Lat.	log A diff. 1″ = −0.07	log B diff. 1″ = −0.21	log C diff. 1″ = +0.42	log D diff. 1″ = +0.00	log E diff. 1″ = +0.06
° ′					
44 00	8.509 0162	8.510 5449	1.38894	2.3930	6.1905
1	57	36	919	30	09
2	53	23	945	30	13
3	49	01	970	30	17
4	44	8.510 5388	995	30	20
05	40	75	1.39020	31	24
6	36	62	045	31	28
7	31	49	070	31	31
8	27	36	095	31	35
9	23	23	120	31	39
10	8.509 0119	8.510 5311	1.39145	2.3931	6.1943
11	14	07	171	31	46
12	10	8.510 5295	196	31	50
13	06	82	221	31	54
14	02	69	246	31	58
15	8.509 0097	56	271	31	61
16	93	43	296	31	65
17	89	30	321	32	69
18	84	18	346	32	72
19	80	05	371	32	76
20	8.509 0076	8.510 5192	1.39396	2.3932	6.1980
21	72	79	422	32	84
22	67	66	447	32	87
23	63	53	472	32	91
24	59	40	497	32	95
25	54	28	522	32	99
26	50	15	547	32	6.2002
27	46	02	572	32	06
28	42	8.510 5089	597	32	10
29	37	76	623	32	14
30	8.509 0033	8.510 5063	1.39648	2.3932	6.2017
31	29	50	673	32	21
32	24	37	698	32	25
33	20	25	723	33	29
34	16	12	748	33	32
35	11	8.510 4999	773	33	36
36	07	86	798	33	40
37	03	73	823	33	44
38	8.508 9999	60	848	33	47
39	94	47	873	33	51
40	8.508 9990	8.510 4935	1.39898	2.3933	6.2055
41	86	22	924	33	59
42	81	09	949	33	62
43	77	8.510 4896	974	33	66
44	73	83	999	33	70
45	69	70	1.40024	33	74
46	64	57	049	33	77
47	60	44	074	33	81
48	56	32	099	33	85
49	51	19	124	33	89
50	8.508 9947	8.510 4806	1.40149	2.3933	6.2092
51	43	8.510 4793	174	33	96
52	39	80	200	33	6.2100
53	34	67	225	33	04
54	30	54	250	33	08
55	26	41	275	33	11
56	21	29	300	33	15
57	17	16	325	33	19
58	13	03	350	33	23
59	09	8.510 4690	375	33	27
60	8.508 9904	8.510 4677	1.40400	2.3933	6.2130

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 45°

Lat.	log *A* diff. 1″ = − 0.07	log *B* diff. 1″ = − 0.21	log *C* diff. 1″ = + 0.42	log *D* diff. 1″ = ± 0.00	log *E* diff. 1″ = + 0.06
° ′					
45 00	8.508 9904	8.510 4677	1.40400	2.3933	6.2130
1	00	64	425	33	34
2	8.508 9896	51	450	33	38
3	91	39	475	34	42
4	87	26	501	34	46
05	83	13	526	34	49
6	78	00	551	34	53
7	74	8.510 4587	576	34	57
8	70	74	601	34	61
9	66	61	626	34	64
10	8.508 9861	8.510 4548	1.40651	2.3934	6.2168
11	57	36	676	34	72
12	53	23	701	34	76
13	48	10	727	34	80
14	44	8.510 4497	752	34	83
15	40	84	777	33	87
16	36	71	802	33	91
17	31	59	827	33	95
18	27	46	852	33	99
19	23	33	877	33	02
20	8.508 9818	8.510 4420	1.40902	2.3933	6.2206
21	14	07	927	33	10
22	10	8.510 4394	952	33	14
23	06	81	978	33	18
24	01	68	1.41003	33	21
25	8.508 9797	56	028	33	25
26	93	43	053	33	29
27	88	30	078	33	33
28	84	17	103	33	37
29	80	04	128	33	40
30	8.508 9776	8.510 4291	1.41153	2.3933	6.2244
31	71	78	178	33	48
32	67	65	203	33	52
33	63	52	229	33	56
34	58	40	254	33	60
35	54	27	279	33	63
36	50	14	304	33	67
37	46	01	329	33	71
38	41	8.510 4188	354	33	75
39	37	75	379	33	79
40	8.508 9733	8.510 4162	1.41404	2.3933	6.2283
41	28	49	429	33	86
42	24	37	454	33	90
43	20	24	479	33	94
44	16	11	505	33	98
45	11	8.510 4098	530	33	6.2302
46	07	85	555	32	06
47	03	72	580	32	09
48	8.508 9698	60	605	32	13
49	94	47	630	32	17
50	8.508 9690	8.510 4034	1.41655	2.3932	6.2321
51	86	21	680	32	25
52	82	08	705	32	29
53	78	8.510 3995	731	32	32
54	74	82	756	32	36
55	68	69	781	32	40
56	64	57	806	32	44
57	60	44	831	32	48
58	55	31	856	32	52
59	51	18	881	32	55
60	8.508 9647	8.510 3905	1.41906	2.3932	6.2359

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 46°

Lat.	log A diff. 1″ = − 0.07	log B diff. 1″ = − 0.21	log C diff. 1″ = + 0.42	log D diff. 1″ = − 0.00	log E diff. 1″ = + 0.06
° ′					
46 00	8.508 9647	8.510 3905	1.41906	2.3932	6.2359
1	43	8.510 3892	931	32	63
2	38	79	957	31	67
3	34	67	982	31	71
4	30	54	1.42007	31	75
05	25	41	032	31	79
6	21	28	057	31	82
7	17	15	082	31	86
8	13	02	107	31	90
9	08	8.510 3789	132	31	94
10	8.508 9604	8.510 3776	1.42157	2.3931	6.2398
11	00	64	183	31	6.2402
12	8.508 9595	51	208	31	06
13	91	38	233	30	09
14	87	25	258	30	13
15	83	12	283	30	17
16	78	8.510 3699	308	30	21
17	74	86	333	30	25
18	70	74	358	30	29
19	65	61	384	30	33
20	8.508 9561	8.510 3648	1.42409	2.3930	6.2436
21	57	35	434	30	40
22	53	22	459	30	44
23	48	09	484	29	48
24	44	8.510 3596	509	29	52
25	40	84	534	29	56
26	35	71	559	29	60
27	31	58	584	29	64
28	27	45	610	29	67
29	23	32	635	29	71
30	8.509 9518	8.510 3519	1.42660	2.3929	6.2475
31	14	06	685	29	79
32	10	8.510 3494	710	28	83
33	05	81	735	28	87
34	01	68	760	28	91
35	8.508 9497	55	786	28	95
36	93	42	811	28	99
37	88	29	836	28	6.2502
38	84	17	861	28	06
39	80	04	886	28	10
40	8.508 9475	8.510 3391	1.42911	2.3927	6.2514
41	71	78	936	27	18
42	67	65	961	27	22
43	63	52	987	27	26
44	58	39	1.43012	27	30
45	54	27	037	27	34
46	50	14	062	27	38
47	45	01	087	26	41
48	41	8.510 3288	112	26	45
49	37	75	137	26	49
50	8.508 9433	8.510 3262	1.43163	2.3926	6.2553
51	28	49	188	26	57
52	24	37	213	26	61
53	20	24	238	26	65
54	16	11	263	25	69
55	11	8.510 3198	288	25	73
56	07	85	314	25	77
57	03	72	339	25	81
58	8.508 9398	60	364	25	84
59	94	47	389	25	88
60	8.508 9390	8.510 3134	1.43414	2.3924	6.2592

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 47°.

Lat.	log A diff. 1″ = − 0.07	log B diff. 1″ = − 0.21	log C diff. 1″ = + 0.42	log D diff. 1″ = + 0.00	log E diff. 1″ = + 0.07
° ′					
47 00	8.508 9390	8.510 3134	1.43414	2.3924	6.2592
1	86	21	439	24	96
2	81	08	465	24	6.2600
3	77	8.510 3095	490	24	04
4	73	82	515	24	08
05	68	70	540	24	12
6	64	57	565	23	16
7	60	44	590	23	20
8	56	31	615	23	24
9	51	18	641	23	28
10	8.508 9347	8.510 3005	1.43666	2.3923	6.2632
11	43	8.510 2993	691	23	35
12	38	80	716	22	39
13	34	67	741	22	43
14	30	54	766	22	47
15	26	41	792	22	51
16	21	28	817	22	55
17	17	16	842	21	59
18	13	03	867	21	63
19	09	8.510 2890	892	21	67
20	8.508 9304	8.510 2877	1.43917	2.3921	6.2671
21	00	64	943	21	75
22	8.508 9296	51	968	20	79
23	91	39	993	20	83
24	87	26	1.44018	20	87
25	83	13	043	20	91
26	79	00	069	20	95
27	74	8.510 2787	094	19	99
28	70	74	119	19	6.2702
29	66	62	144	19	06
30	8.508 9261	8.510 2749	1.44169	2.3919	6.2710
31	57	36	195	19	14
32	53	23	220	18	18
33	49	10	245	18	22
34	44	8.510 2698	270	18	26
35	40	85	295	18	30
36	36	72	321	18	34
37	32	59	346	17	38
38	27	46	371	17	42
39	23	33	396	17	46
40	8.508 9219	8.510 2621	1.44421	2.3917	6.2750
41	14	08	447	16	54
42	10	8.510 2595	472	16	58
43	06	82	497	16	62
44	02	69	522	16	66
45	8.508 9197	57	547	16	70
46	93	44	573	15	74
47	89	31	598	15	78
48	84	18	623	15	82
49	80	05	648	15	86
50	8.508 9176	8.510 2493	1.44673	2.3914	6.2790
51	72	80	699	14	94
52	67	67	724	14	98
53	63	43	749	14	6.2802
54	59	41	774	13	06
55	55	28	800	13	10
56	50	16	825	13	14
57	49	03	850	13	18
58	42	8.510 2390	875	12	22
59	38	77	900	12	26
60	8.508 9133	8.510 2364	1.44926	2.3912	6.2830

Table XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 48°.

Lat.	log *A* diff. 1″ = −0.07	log *B* diff. 1″ = −0.21	log *C* diff. 1″ = +0.42	log *D* diff. 1″ = −0.00	log *E* diff. 1″ = +0.07
° ′					
48 00	8.508 9133	8.510 2364	1.44926	2.3912	6.2830
1	29	52	951	12	34
2	25	39	976	11	38
3	20	26	1.45001	11	42
4	16	13	027	11	46
05	12	00	052	11	50
6	08	8.510 2288	077	10	54
7	03	75	102	10	58
8	8.508 9099	62	128	10	62
9	95	49	153	10	66
10	8.508 9091	8.510 2236	1.45178	2.3909	6.2870
11	86	24	203	09	74
12	82	11	229	09	78
13	78	8.510 2198	254	08	82
14	74	85	279	08	86
15	69	72	304	08	90
16	65	60	330	08	94
17	61	47	355	07	98
18	57	34	380	07	6.2902
19	52	21	406	07	06
20	8.508 9048	8.510 2108	1.45431	2.3907	6.2910
21	44	8.510 2096	456	06	14
22	39	83	481	06	18
23	35	70	507	06	22
24	31	57	532	05	26
25	27	45	557	05	30
26	22	32	582	05	34
27	18	19	608	05	38
28	14	06	633	04	42
29	10	8.510 1993	658	04	46
30	8.508 9005	8.510 1981	1.45683	2.3904	6.2950
31	01	68	709	03	54
32	8.508 8997	55	734	03	58
33	93	42	759	03	62
34	88	30	785	02	66
35	84	17	810	02	70
36	80	04	835	02	74
37	76	8.510 1891	861	02	78
38	71	78	886	01	82
39	67	66	911	01	86
40	8.508 8963	8.510 1853	1.45937	2.3901	6.2990
41	59	40	962	00	94
42	54	27	987	00	98
43	50	15	1.46012	00	6.3002
44	46	02	038	2.3899	06
45	41	8.510 1789	063	99	10
46	37	76	088	99	15
47	33	64	114	98	19
48	29	51	139	98	23
49	24	38	164	98	27
50	8.509 8920	8.510 1725	1.46190	2.3897	6.3031
51	16	13	215	97	35
52	12	00	240	97	39
53	08	8.510 1687	266	96	43
54	03	74	291	96	47
55	8.508 8899	62	316	96	51
56	95	49	342	95	55
57	90	36	367	95	59
58	86	23	392	95	63
59	82	10	418	94	67
60	8.508 8878	8.510 1598	1.46443	2.3894	6.3071

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 49°.

Lat.	log A diff. 1″ = − 0.07	log B diff. 1″ = − 0.21	log C diff. 1″ = + 0.42	log D diff. 1″ = − 0.01	log E diff. 1″ = + 0.07
° ′					
49 00	8.508 8878	8.510 1598	1.46443	2.3894	6.3071
1	73	85	468	94	75
2	69	72	494	93	79
3	65	59	519	93	84
4	61	47	544	93	88
05	57	34	570	92	92
6	52	21	595	92	96
7	48	08	621	92	6.3100
8	44	8.510 1496	646	91	04
9	39	83	671	91	08
10	8.508 8835	8.510 1470	1.46696	2.3891	6.3112
11	31	58	722	90	16
12	27	45	747	90	20
13	23	32	773	2.3889	24
14	18	19	798	89	28
15	14	07	824	89	32
16	10	8.510 1394	849	88	37
17	06	81	874	88	41
18	01	68	899	88	45
19	8.508 8797	56	925	87	49
20	8.508 8793	8.510 1343	1.46950	2.3887	6.3153
21	89	30	976	87	57
22	84	17	1.47001	86	61
23	80	05	026	86	65
24	76	8.510 1292	052	85	69
25	72	79	077	85	73
26	67	67	103	85	78
27	63	54	128	84	82
28	59	41	153	84	86
29	55	28	179	83	90
30	8.508 8750	8.510 1216	1.47204	2.3883	6.3194
31	46	03	230	83	98
32	42	8.510 1190	255	82	6.3202
33	38	78	281	82	06
34	33	65	306	82	10
35	29	52	331	81	15
36	25	39	357	81	19
37	21	27	382	80	23
38	16	14	408	80	27
39	12	01	433	80	31
40	8.508 8708	8.510 1088	1.47459	2.3879	6.3235
41	04	77	484	79	39
42	00	63	509	78	43
43	8.508 8695	50	535	78	47
44	91	38	560	78	52
45	87	25	586	77	56
46	83	12	611	77	60
47	78	00	637	76	64
48	74	8.510 0987	662	76	68
49	70	74	688	75	72
50	8.508 8666	8.510 0962	1.47713	2.3875	6.3276
51	61	49	738	75	81
52	57	36	764	74	85
53	53	23	789	74	89
54	49	11	815	73	93
55	45	8.510 0898	840	73	97
56	40	85	866	73	6.3301
57	36	73	891	72	05
58	32	60	917	72	09
59	28	48	942	71	14
60	8.508 8623	8.510 0835	1.47968	2.3871	6.3318

TABLE XXXVII.

FACTORS USED IN GEODETIC COMPUTATIONS.

LATITUDE 50°.

Lat.	log *A* diff. 1″ = − 0.07	log *B* diff. 1″ = − 0.21	log *C* diff. 1″ = − 0.43	log *D* diff. 1″ = − 0.01	log *E* diff. 1″ = + 0.07
° ′					
50 00	8.508 8623	8.510 0835	1.47968	2.3871	6.3318
1	19	22	993	70	22
2	15	09	1.48019	70	26
3	11	8.510 0797	044	70	30
4	06	84	070	69	34
05	02	71	095	69	39
6	8.508 8598	59	121	68	43
7	94	46	146	68	47
8	90	33	172	67	51
9	85	21	197	67	55
10	8.508 8581	8.510 0708	1.48223	2.3866	6.3359
11	77	8.510 0695	248	66	63
12	75	83	274	66	68
13	68	70	299	65	72
14	64	57	325	65	76
15	60	45	350	64	80
16	56	32	376	64	84
17	52	19	401	63	88
18	47	07	427	63	93
19	43	8.510 0594	452	62	97
20	8.508 8539	8.510 0581	1.48478	2.3862	6.3401
21	35	69	504	61	05
22	30	56	529	61	09
23	26	43	555	60	14
24	22	31	580	60	18
25	18	18	606	60	22
26	14	05	631	59	26
27	09	8.510 0493	657	59	30
28	05	80	682	58	34
29	01	67	708	58	39
30	8.508 8497	8.510 0455	1.48734	2.3857	6.3443
31	93	42	759	57	47
32	88	29	785	56	51
33	84	17	810	56	55
34	80	04	836	55	60
35	76	8.510 0392	861	55	64
36	71	79	887	54	68
37	67	66	913	54	72
38	63	54	938	53	76
39	59	41	964	53	81
40	8.508 8455	8.510 0328	1.48989	2.3852	6.3485
41	50	16	1.49015	52	89
42	46	03	041	51	93
43	42	8.510 0291	066	51	97
44	38	78	092	50	6.3502
45	34	65	117	50	06
46	29	53	143	49	10
47	25	40	169	49	14
48	21	27	194	48	18
49	17	15	220	48	23
50	8.508 8413	8.510 0202	1.49246	2.3847	6.3527
51	08	8.510 0190	271	47	31
52	04	77	297	46	35
53	00	64	322	46	40
54	8.508 8396	52	348	45	44
55	92	39	374	45	48
56	87	27	399	44	52
57	83	14	425	44	56
58	79	01	451	43	61
59	75	8.510 0089	476	43	65
60	8.508 8371	8.510 0076	1.49502	2.3842	6.3569

Table XXXVIII.

CORRECTIONS TO LONGITUDE FOR DIFFERENCE IN ARC AND SINE.

(From Appendix No. 9, Report of U. S. Coast and Geodetic Survey, 1894.)

Log S (−)	Log Difference.	Log $\Delta\lambda$ (+)	Log S (−)	Log Difference.	Log $\Delta\lambda$ (+)
3.876	0.000 0001	2.385	5.010	0.000 0186	3.519
4.026	02	2.535	5.017	192	3.526
4.114	03	2.623	5.025	199	3.534
4.177	04	2.686	5.033	206	3.542
4.225	05	2.734	5.040	213	3.549
4.265	06	2.774	5.047	221	3.556
4.298	07	2.807	5.054	228	3.563
4.327	08	2.836	5.062	236	3.571
4.353	09	2.862	3.068	243	3 577
4.376	10	2.885	5.075	251	3.584
4.396	11	2.905	5.082	259	3.591
4.415	12	2.924	5.088	267	3.597
4.433	13	2.942	5.095	275	3.604
4.449	14	2.958	5.102	284	3.611
4.464	15	2.973	5.108	292	3.617
4.478	16	2.987	5.114	300	3.623
4.491	17	3.000	5.120	309	3.629
4.503	18	3.012	5.126	318	3.635
4.526	20	3.035	5.132	327	3.641
4.548	23	3.057	5.138	336	3.647
4.570	25	3.079	5.144	345	3.653
4.591	27	3.100	5.150	354	3.659
4.612	30	3.121	5.156	364	3.665
4.631	33	3.140	5.161	373	3.670
4.649	36	3.158	5.167	383	3.676
4.667	39	3.176	5.172	392	3.681
4.684	42	3.193	5.178	402	3.687
4.701	45	3.210	5.183	412	3.692
4.716	48	3.225	5.188	422	3.697
4.732	52	3.241	5.193	433	3.702
4.746	56	3.255	5.199	443	3.708
4.761	59	3.270	5.204	453	3.713
4.774	63	3.283	5.209	464	3.718
4.788	67	3.297	5.214	474	3.723
4.801	71	3.310	5.219	486	3.728
4.813	75	3.322	5.223	497	3.732
4.825	80	3.334	5.228	508	3.737
4.834	84	3.343	5.233	519	3.742
4.849	89	3.358	5.238	530	3.747
4.860	94	3.369	5.242	541	3.751
4.871	98	3.380	5.247	553	3.756
4.882	103	3.391	5.251	565	3.760
4.892	108	3.401	5.256	577	3.765
4.903	114	3.412	5.260	588	3.769
4.913	119	3.422	5.265	600	3.774
4.922	124	3.431	5.269	613	3.778
4.932	130	3.441	5.273	625	3.782
4.941	136	3.450	5.278	637	3.787
4.950	142	3.459	5.282	650	3.791
4.959	147	3.468	5.286	663	3.795
4.968	153	3.477			
4.976	160	3.485			
4.985	166	3.494			
4.993	172	3.502			
5.002	179	3.511			

TABLE XXXIX.

VALUES OF LOG $\frac{1}{\cos \frac{1}{2}d\phi}$.

(From Appendix No. 9, Report of U. S. Coast and Geodetic Survey, 1894.)

$\Delta\phi$.	log. sec. $\left(\frac{\Delta\phi}{2}\right)$.	$\Delta\phi$.	log. sec. $\left(\frac{\Delta\phi}{2}\right)$.	$\Delta\phi$.	log. sec. $\left(\frac{\Delta\phi}{2}\right)$.
10′	0.000 000	40′	0.000 007	70	0.000 022
11	1	41	8	71	23
12	1	42	8	72	24
13	1	43	8	73	24
14	1	44	9	74	25
15	1	45	9	75	26
16	1	46	10	76	26
17	1	47	10	77	27
18	1	48	11	78	28
19	2	49	11	79	29
20	2	50	11	80	29
21	2	51	12	81	30
22	2	52	12	82	31
23	2	53	13	83	32
24	3	54	13	84	32
25	3	55	14	85	33
26	3	56	14	86	34
27	3	57	15	87	35
28	4	58	15	88	36
29	4	59	16	89	36
30	4	60	16	90	37
31	4	61	17	91	38
32	5	62	18	92	39
33	5	63	18	93	40
34	5	64	19	94	41
35	6	65	19	95	41
36	6	66	20	96	42
37	6	67	21	97	43
38	7	68	21	98	44
39	7	69	22	99	45

TABLE XL.

LOG *F*.

(From Appendix No. 9, Report of U. S. Coast and Geodetic Survey, 1894.)

Lat.	Log *F*.	Lat.	Log *F*.	Lat.	Log. *F*.	Lat.	Log *F*.
23°	7.812	34°	7.877	45°	7.840	56°	7.706
24	23	35	77	46	32	57	7.688
25	32	36	77	47	24	58	69
26	41	37	76	48	14	59	49
27	49	38	74	49	04	60	27
28	55	39	72	50	7.792	61	05
29	61	40	69	51	80	62	7.581
30	66	41	64	52	67	63	56
31	70	42	60	53	53	64	29
32	73	43	54	54	38	65	01
33	75	44	48	55	23	66	7.471

CHAPTER XXX.

GEODETIC CONSTANTS AND REDUCTION TABLES.

290. Constants Depending on Spheroidal Figure of Earth.—The following are based on Clarke's spheroid of 1886:

Equatorial semi-axis,	$a =$ 20926062. feet;		log $=$ 7.3206875;
" "	$a =$ 3963.3 miles;		" $=$ 3.5980536;
Polar "	$b =$ 20855121. feet;		" $=$ 7.3192127;
" "	$b =$ 3949.8 miles;		" $=$ 3.5965788;
Equatorial radius,	$a =$ 6378206.4 meters;		" $=$ 6.8046986;
Polar "	$b =$ 6356583.8 "		" $=$ 6.8032238;

Equatorial R. : Polar R. or $a : b$ :: 294.98 : 293.98 ; $\dfrac{b}{a} = \dfrac{293.98}{294.98}$.

Circumference of equator $=$ 24,901.96 miles.
Area surface of earth $=$ 196,940,400 square miles.

$e^2 = 2E =$ Eccentricity $= .0067687$ meters; log $= \bar{7}.8305028$;

$\dfrac{e^2}{2} = E =$ Ellipticity $= \dfrac{a-b}{a} = \dfrac{1}{294.98}$; " $= 7.5294689$;

$1 - e^2 = .9932313$; " $= 9.9970503$;

$\dfrac{1}{a \text{ arc } 1''}$; " $= \bar{8}.5097266$.

291. Numerical Constants.—Circumference of circle, diameter unity,

$$= \pi = 3.14159265 = \log 0.4971499;$$
$$2\pi = 6.2831853 = \text{"}\ 0.7981799;$$
$$\pi^2 = 9.8696044 = \text{"}\ 0.9942997.$$

Length of an arc, a, with radius, r,

$$= \frac{a\pi r}{180^\circ} = \text{nearly} \sqrt{\frac{1}{4}c^2 + \text{ver sin}^2},$$

c being the chord of the arc a.

log. sine 1″.......... = 4.6855748668;
log. ½ sine 1″........ = 4.3845448711;
a. c. log. sine 1″ ..... = 5.3144251;
1″ for radius = 1 mile = 0.3072 inches;
1′ " " " = 18.431 "
nat. sine or tang. 1″ = 0.00000485.

TABLE XLI.

INTERCONVERSION OF ENGLISH LINEAR MEASURES.

(From Smithsonian Geographical Tables.)

Unit of linear measure is the yard.

Inches.	Feet.	Yards.	Rods.	Furlongs.	Miles.
1	0.083	0.028	0 00505	0.00012626	0.0000157828
12	1.	0.333	0.06060	0.00151515	0.00018939
36	3.	1.	0.1818	0.004545	0.00056818
198	16.5	5.5	1.	0.025	0.003125
7920	660.	220.	40.	1.	0.125
63360	5280.	1760.	320.	8.	1.

1 acre = 209 feet square ....Error + 1 : 720
1 " = 43,560 square feet.
1 mile = 1760 yards = 5280 feet = 63,360 inches.

To change log. miles to log. yards add 3.2455127;
" " log. yards " log. miles " 6.7544873.

Log. 3 = 0.4771212547;
Log. 12 = 1.0791812460;
Log. 5280 = 3.72263339225;
Log. 1760 = 3.2455127.

Other measures are the—

Surveyor's or Gunter's chain = 4 rods = 66 feet = 100 links of 7.92 inches each.

Fathom = 6 feet; Cable length = 120 fathoms.

Hand = 4 inches; Palm = 3 inches; Span = 9 inches.

TABLE XLII.

INTERCONVERSION OF ENGLISH SQUARE MEASURES.

(From Smithsonian Geographical Tables.)

Unit of square measure is the square yard.

Sq. Feet.	Sq. Yards.	Sq. Rods.	Roods.	Acres.	Sq. Mi.
1.	0.1111	0.00367309	0.000091827	0.000022957	
9.	1.	0.0330579	0.000826448	0.000206612	
272.25	30.25	1.	0.025	0.00625	
10890.	1210.	40.	1.	0.25	
43560.	4840.	160.	4.	1.	
27878400.	3097600.	102400.	2560.	640.	1.

292. Length of the Meter in Inches.—According to various authorities 1 meter = in inches:

39.370790	Kater, 1818.
39.38092	Hassler, 1832.
39.368505	Coast Survey, 1851–1858 (Hassler corrected).
39.370432	Clarke, 1866–1873.
39.36985	Lake Survey, 1885.
39.3704316	Chief of Engineers, U. S. A., (letter) 1895.
39.377786	Theoretic ten-millionth of quadrant (Clarke).
39.37	By Act of Congress, 1866.
39.37	U. S. Coast and Geodetic Survey, adopted 1891.

293. Interconversion of English and Metric Measures.—The units of measure of the two systems are the yard and the meter. The standard meter has its normal length at 32° F. = 0° C.; the yard at +62° F. Their relative values are

$$1 \text{ yard} = \tfrac{3600}{3937} \text{ of the meter.}$$

TABLE XLIII.

TO CONVERT METRIC TO ENGLISH MEASURES.

(From Appendix No. 6, U. S. Coast and Geodetic Survey Report for 1893.)

Meters to Inches.	Meters to Feet.	Meters to Yards.	Miles. Kilometers.
1 = 39.3700	1 = 3.28083	1 = 1.093611	0.62137 = 1
2 = 78.7400	2 = 6.56167	2 = 2.187222	1.24274 = 2
3 = 118.1100	3 = 9.84250	3 = 3.280833	1.86411 = 3
4 = 157.4800	4 = 13.12333	4 = 4.374444	2.48548 = 4
5 = 196.8500	5 = 16.40417	5 = 5.468056	3.10685 = 5
6 = 236.2200	6 = 19.68500	6 = 6.561667	3.72822 = 6
7 = 275.5900	7 = 22.96583	7 = 7.655278	4.34959 = 7
8 = 314.9600	8 = 26.24667	8 = 8.748889	4.97096 = 8
9 = 354.3300	9 = 29.52750	9 = 9.842500	5.59233 = 9

TABLE XLIV.

TO CONVERT ENGLISH TO METRIC MEASURES.

(From Appendix No. 6, U. S. Coast and Geodetic Survey Report for 1893.)

Inches to Millimeters.	Feet to Meters.	Yards to Meters.	Kilometers. Miles.
1 = 25.4001	1 = 0.304801	1 = 0.914402	1.60935 = 1
2 = 50.8001	2 = 0.609601	2 = 1.828804	3.21869 = 2
3 = 76.2002	3 = 0.914402	3 = 2.743205	4.82804 = 3
4 = 101.6002	4 = 1.219202	4 = 3.657607	6.43739 = 4
5 = 127.0003	5 = 1.524003	5 = 4.572009	8.04674 = 5
6 = 152.4003	6 = 1.828804	6 = 5.486411	9.65608 = 6
7 = 177.8004	7 = 2.133604	7 = 6.400813	11.26543 = 7
8 = 203.2004	8 = 2.438405	8 = 7.315215	12.87478 = 8
9 = 228.6005	9 = 2.743205	9 = 8.229616	14.48412 = 9

TABLE XLV.

TO CONVERT METERS INTO STATUTE AND NAUTICAL MILES.

Meter.	Miles.	Meter.	Miles.
1	0.00062137	6	0.00372822
2	0.00124274	7	0.00434959
3	0.00186411	8	0.00497096
4	0.00248548	9	0.00559233
5	0.00310685	10	0.00621370

Meters.	Statute Miles.	Nautical Miles.	Meters.	Statute Miles.	Nautical Miles.	Meters.	Statute Miles.	Nautical Miles.
10	0.006	0.005	100	0.062	0.054	1,000	0.621	0.540
20	0.012	0.011	200	0.124	0.108	2,000	1.243	1.079
30	0.019	0.016	300	0.186	0.162	3,000	1.864	1.619
40	0.025	0.022	400	0.249	0.216	4,000	2.486	2.158
50	0.031	0.027	500	0.311	0.270	5,000	3.107	2.698
60	0.037	0.032	600	0.373	0.324	6,000	3.728	3.238
70	0.043	0.038	700	0.435	0.378	7,000	4.350	3.777
80	0.050	0.043	800	0.497	0.432	8,000	4.971	4.317
90	0.056	0.049	900	0.559	0.486	9,000	5.592	4.856

294. Logarithms and Factors for Conversion of English and Metric Measures.—

TABLE XLVI.

LOGARITHMIC CONSTANTS FOR INTERCONVERSION OF METRIC AND COMMON MEASURES.

To change log. of meters to log. of miles	add 6.7933502;
" " log. of meters to log. of yards	" 0.0388629;
" " log. of meters to log. of feet	" 0.5159842;
" " log. of meters to log. of inches	" 1.5951654;
" " log. of miles to log. of meters	" 3.2066498;
" " log. of yards to log. of meters	" 9.9611371;
" " log. of feet to log. of meters	" 9.4840158;
" " log. of inches to log. of meters	" 8.4048346.

Table XLVII.

METRIC TO COMMON SYSTEM, WITH FACTORS AND LOGARITHMS.

Units Compared.				Logarithm of Factor.	Reciprocal of Factor.	Log. Rec. of Factor.
Centimeters	×	0.3937	= inches	1.595165	2.54	0.404835
Meters	×	3.2808333	= feet..............	0.515984	0.304801	$\bar{1}$.4840158
Kilometers	×	0.62137	= miles..........	9.79335	1.60935	0.20665
Square centimeters	×	0.15500	= square inches.....	1.190331	6.45163	0.809669
Square meters	×	10.7639	= square feet.......	1.031968	0.0929034	2.968032
Hectares	×	2.47104	= acres	0.39288	0.404687	$\bar{1}$.60712
Cubic centimeters	×	0.0610234	= cubic inches	8.785594	16.3872	1.214504
Cubic meters	×	35.3145	= cubic feet..	1.54795	0.028317	$\bar{2}$.452047
Cubic meters	×	264.17	= U. S. gallons.....	2.421884	0.0037854	$\bar{3}$.578116

Millimeters × .03937 = inches.
Millimeters ÷ 25.4 = inches.
Centimeters × .3937 = inches.
Centimeters ÷ 2.54 = inches.
Meters × 39.37 = inches. (Act of Congress.)
Meters × 3.281 = feet.
Meters × 1.094 = yards.
Kilometers × .621 = miles.
Kilometers ÷ 1.6093 = miles.
Kilometers × 3280.8 = feet.
Square millimeters × .0155 = square inches.
Square millimeters ÷ 645.1 = square inches.
Square centimeters × .155 = square inches.
Square centimeters ÷ 6.451 = square inches.
Square meters × 10.764 = square feet.
Square kilometers × 247.1 = acres.
Hectares × 2.471 = acres.
Hectares × 259 = square miles.

Table XLVIII.

MISCELLANEOUS METRIC EQUIVALENTS.

1 millimeter = $\frac{1}{25}$ inch.................. ....Error + 1 : 62
1 centimeter = $\frac{3}{8}$ inch.................... " + 1 : 21
1 meter = 3 feet $3\frac{3}{8}$ inches................ " + 1 : 8600
1 kilometer = $\frac{5}{8}$ mile.................. .. " − 1 : 180
1 gram = 15.4 grains....... .... " − 1 : 480
1 kilogram = $2\frac{1}{5}$ lbs. (avoirdupois)........ " − 1 : 480
1 liter = 1 quart......................... " — 1 : 18
1 foot = $\frac{3}{10}$ meters = 0.304801 meters
1 fathom = $1\frac{8}{10}$ meters = 1.829 "
1 Gunter's chain = $20\frac{1}{10}$ meters = 20.1168 "

PART VI.

GEODETIC ASTRONOMY.

CHAPTER XXXI.

ASTRONOMIC METHODS.

295. Method of Treatment.—In the following pages only such outline of the subject is given as is indispensable as a guide to practical field operations. The mathematics of astronomy is extended and complex, and is omitted excepting the more practical working formulas, as volumes would be required for a complete exposition of this subject alone. The effort here has been to give directions for observing, examples of reduction and computation, and the essential field tables only. For more detailed information the student is referred to Doolittle's or Chauvenet's Practical Astronomies, Hayford's Geodetic Astronomy, to the American Ephemeris, and to special tables and star catalogues.

The arrangement of the following is similar to that of the rest of this book. The simpler and more approximate methods of determining azimuths, latitudes, and longitudes are given first, as they would be used in exploratory or rough geographic surveying. (Chap. IV.) Following these are given the more refined methods of determining the same quantities, as initial positions for the extension of geodetic triangulation. (Art. 285.)

296. Geodetic Astronomy.—The topographer should distinguish in the beginning between an *astronomic* latitude and longitude and a *geodetic* latitude and longitude; in addition neither of these should be confused with *celestial* latitude and longitude. *Astronomic latitudes* and *longitudes* are referred to the action line of gravity at the station of observation. *Geodetic latitudes* and *longitudes* are referred to the gravity line which has been corrected for local deflection or station error. On the other hand *celestial latitudes* and *longitudes* refer to a system of spherical coordinates and, though much used by the astronomer, are rarely employed in topographic or geodetic operations. In geodetic astronomy the initial points of measurement are the equator and vernal equinox for the measure of declination and right ascension, whereas in celestial astronomy the ecliptic and vernal equinox furnish corresponding initial points.

The *field-work of the geodetic astronomer* is of the most practical kind and has for its objects:

1. To determine the astronomic latitude of the station;

2. To determine the true local time at the instant of observation, or the true astronomic longitude of the station;

3. To determine the azimuth of a line joining the observation station with some other terrestrial point.

Finally, as one of the operations performed in the above determinations consists in the finding of the horizon line, he consequently determines the zenith distance of some terrestrial object as a reference point for vertical triangulation. The zenith and a celestial object are therefore the two points on the celestial sphere always observed. The right ascension and declination of the object observed become known independently of the observations.

297. Definitions of Astronomic Terms.—For some of the following definitions and explanations of the operations of geodetic astronomy I am indebted to the admirable text-book on this subject recently published by Mr. John F. Hayford of the U. S. Coast and Geodetic Survey, to which the reader is referred for more detailed information upon this subject, as well as to Doolittle's and Chauvenet's Practical Astronomies.

The *astronomic latitude* of a point on the surface of the earth is the angle between the line of action of gravity at that station and the plane of the equator. It is measured on the celestial sphere along the meridian from the equator to the zenith.

The *astronomic longitude* of a point on the surface of the earth is the angle between the meridian plane of that point and some arbitrarily chosen meridian plane. The meridian of Greenwich, England, is accepted universally in the United States as this initial meridian for geodetic operations, and is generally accepted throughout the world in most nautical and geodetic work. The meridian of Washington, D. C., is sometimes used in the United States chiefly in connection with public land lines.

Geodetic latitudes and longitudes are defined by applying the distinguishing explanation already made to the above definitions.

In the operations of geodetic astronomy the *bodies considered* are the sun, the moon, the stars, and the planets, including the earth and moon; also the satellites of the planets. As seen from the point of the observer these appear to move about within the range of vision, and their apparent motions appear quite complicated. To clearly orient himself upon the earth by observation upon these heavenly bodies he must have an accurate conception of their apparent motions. In the *apparent motion* of each of the heavenly bodies he sees not only its actual motion, but also the actual motion of the seemingly solid and movable earth upon which he stands, since both are in motion. As admirably expressed by Mr. Hayford, he is like a passenger upon a train at night looking out upon the moving lights of a town. He sees the lights apparently all in motion. In some cases the apparent motion of a light may be entirely due to his own motion. In other cases the lights upon which he looks may be those of a wagon or of another moving train, and their apparent motions are often due to their actual motions and those of himself.

The *horizon* is the intersection with the celestial sphere of a plane passing through the eye of an observer perpendicular to the plumb-line or the line of action of gravity. The *zenith* is the point in which the action line of gravity produced upward intersects the celestial sphere, and is at right angles to the horizon of an observer, and opposite on the celestial sphere to the *nadir*.

The *plane of the equator* is a plane of a great circle of the celestial sphere passing through the center of the earth and perpendicular to the axis of its rotation. The *plane of the ecliptic* is the plane of a great circle of the celestial sphere and is the plane of the orbit of the earth. The *ecliptic* itself is the intersection of the plane of the ecliptic with the celestial sphere. These two planes are the *principal reference planes of astronomy*.

The *equinoxes* are the two points in which the equator and ecliptic intersect each other, the angle of their intersection being about 23° 27′. The *vernal* equinox is that at which the sun is found in the spring, and the *autumnal* that at which it is found in the fall.

An *hour-circle* is the intersection of a plane passing through the axis of the earth with the celestial sphere, and all hour-circles are great circles passing through the poles. The *hour-angle* of a star is the angle measured along the equator between the meridian and the hour-circle passing through it.

Right ascension of a celestial body is the angle measured along the equator between the hour-circles which pass through the star and the vernal equinox respectively. As right ascension is reckoned from west to east, opposite to the apparent motion of the stars, the sidereal time at the instant of a transit of a star is therefore the same as its right ascension. Right ascension may also be expressed as the sidereal time elapsed between the passage of the vernal equinox and the star across the meridian. It is usually expressed in hours, minutes, and seconds.

The *declination* of a celestial object is the angle between the line joining the center of the earth to the star or planet and the plane of the equator. It is also expressed as the angular distance of the heavenly body north or south of the equator, and is + when north and — when south.

The *culmination or transit* of a celestial object across the meridian of the observer is the passage of that star across such meridian. As the meridian is a great circle, any star has two transits if considered for a complete revolution of the earth upon its axis; the first of these, called the upper transit or culmination, being that over the half of the meridian which includes the zenith. The second, called the lower transit or culmination, includes the transit over that half of the meridian which passes through the nadir.

A *sidereal day* is the interval between two successive transits of the vernal equinox across the same meridian. *Sidereal time* at the station of observation and at a fixed instant of time is the right ascension of the meridian, which is the same as the hour-angle of the vernal equinox counted in the direction of the apparent motion of the stars. Sidereal time is zero hours, minutes, and seconds at the instant when the vernal equinox transits across the meridian. It includes 24 hours, numbered consecutively from zero.

An *apparent solar day* is the interval between two successive transits of the sun across the meridian. The *apparent solar time* for any station of observer and any instant is the hour-angle of the real sun at that instant for that meridian.

The *mean solar day* is the interval between successive transits of a fictitious mean sun over the same meridian.

Mean solar time, usually called *mean time* for any station of observer and instant, is the hour-angle of mean sun at that instant from that meridian.

The *standard time* of any place is the mean solar time of the nearest fifteen degree meridian of longitude west of Greenwich. To reduce local mean solar time to standard time apply as a correction the difference of longitude of the place and its standard meridian.

The *equation of time* is the correction to be applied to the apparent time to reduce it to mean time. It is given in the American Ephemeris.

The *civil day* commences and ends at midnight. Its hours are counted from zero to 12 between midnight and noon, and from zero to 12 between noon and midnight. The *astronomic day* commences at noon of the civil day of the same date, and its hours are numbered from zero to 24, from noon of one day to noon of the next. Civil time is local mean solar time based on the civil day.

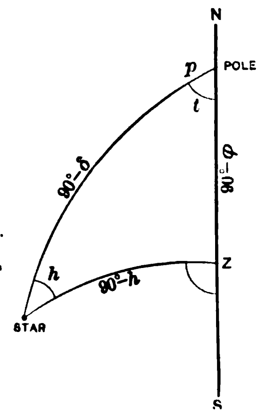


FIG. 179.—LATITUDE, DECLINATION, AND ALTITUDE.

298. Astronomic Notation. — The following is the notation employed in astronomic formulas and computations:

T = civil time at any place;

T_s = sidereal time corresponding to T = right ascension of the meridian of the place;

T_m = astronomic mean time corresponding to T_s;

I = interval of mean solar time;

C_s = correction to convert interval I into mean time (Ephemeris, Table III);

I' = interval of sidereal time corresponding to I;

α_s = R. A. mean sun for next preceding mean noon for place, P, and date, D_t,

= sidereal time of mean noon for place and date;

E = equinox;
t = hour-angle = difference between sidereal and mean time;
A = azimuth of star or other celestial object;
h = altitude of same;
α = R. A. = right ascension of celestial object;
ϕ = latitude of place = angle of pole above horizon;
δ = declination of celestial object;
z = its observed zenith distance = $90° - h$;
z_m = its observed meridional zenith distance;
ζ = its true meridional zenith distance;
p = its polar distance = $90° - \delta$;
q = its parallactic angle or angle at star between pole and zenith.

299. Fundamental Astronomic Formulas.—*To find altitude, given latitude, declination, and hour-angle:*

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t. \quad . \quad . \quad (96)$$

By means of logarithms and an auxiliary angle M, where

$$\sin \delta = m \sin M \quad \text{and} \quad \cos \delta \cos t = m \cos M,$$

we have

$$\sin h = m \cos (\phi - M). \quad . \quad . \quad . \quad (97)$$

To find azimuth, given latitude, declination, and hour-angle:

$$\tan A = \frac{\tan t \cos M}{\sin (\phi - M)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

and

$$\tan A = - \frac{\sin t}{\cos \phi \tan \delta (1 - \tan \phi \cos \delta \cos t)}. \quad . \quad (99)$$

Where a series of observations are made, this formula is simplified into the following working form:

$$\tan A = -\frac{a \sin t}{1 - b \cos t},$$

where $a = \sec \phi \cot \delta$, and $b = \tan \phi \cot \delta$.

To find declination, given altitude and azimuth:

$$\sin \delta = \sin \phi \sin h - \cos \phi \cos h \cos A; \quad . \quad (100)$$

or, for logarithmic computation,

$$\sin \delta = m \sin (\phi - M); \quad . \quad . \quad . \quad . \quad (101)$$

or, having t,

$$\tan \delta = \tan (\phi - M) \cos t.$$

To find hour-angle, given altitude and azimuth:

$$\tan t = \frac{\tan A \sin M}{\cos (\phi - M)}. \quad . \quad . \quad . \quad . \quad . \quad (102)$$

To find hour-angle and azimuth in terms of zenith distance:

$$\cos t = \frac{\cos z - \sin \phi \sin \delta}{\cos \phi \cos \delta}; \quad . \quad . \quad . \quad (103)$$

$$\cos A = \frac{\sin \phi \cos z - \sin \delta}{\cos \phi \sin z}. \quad . \quad . \quad . \quad (104)$$

To find hour-angle and zenith distance of a star at elongation:

$$\cos t = \frac{\tan \phi}{\tan \delta}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (105)$$

$$\sin A = \frac{\cos \delta}{\cos \phi}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (106)$$

$$\cos z = \frac{\sin \phi}{\sin \delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (107)$$

To find hour-angle and azimuth of a star in the horizon or at time of rising or setting:

$$\cos t = -\tan \phi \tan \delta; \quad . \quad . \quad . \quad . \quad .. \quad (108)$$

$$\cos A = -\frac{\sin \delta}{\cos \phi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (109)$$

To find hour-angle, zenith distance, and parallactic angle for transit of a star across prime vertical:

$$\cos t = \frac{\tan \delta}{\tan \phi}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

$$\cos z = \frac{\sin \delta}{\sin \phi}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (111)$$

$$\sin q = \frac{\cos \phi}{\cos \delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (112)$$

300. **Finding the Stars.**—The following brief statement of the positions of the more prominent stars as referred, one to the other, is derived from Lieutenant Qualtrough's "Sailor's Manual." The most conspicuous stars have been designated by names, and the stars in each constellation are distinguished, for reference, by letters and numbers. The letters used for this purpose are the small letters of the Greek alphabet.

In finding any star in the heavens, it is customary to refer to some one star or constellation as known: The *Great Bear*, called also by the Latin name of *Ursa Major*, in the northern part of the heavens and consisting of seven principal stars, is the most convenient for the purpose.

The two stars α and β point nearly to *Polaris*, or the Pole Star, and are hence called the *Pointers*.

A line from Polaris through η, the last of the tail, passes at 31° beyond η through *Arcturus*, a very bright star.

A line from Polaris perpendicular to the line of the Pointers and on the opposite side to the Great Bear passes at 48° distance through *Capella*, one of the brightest stars.

In the same line, about the same distance on the opposite side of the Pole, is *α Lyræ*, also called *Vega* and *Lyra*, a large white star in the Harp.

At one third of the distance from *Arcturus* to *α* Lyræ is *Alphacca*, the brightest star of a semicircular group called the *Northern Crown*.

About 23° to the eastward of *α* Lyræ, and about the same distance as this star is from Polaris, is *μ Cygni*, the bright star in the Swan.

A line from Polaris passing between this last and *α* Lyræ, and produced to an equal distance between them, passes through *α Aquilæ*, or *Altair*, a bright star between two small ones.

A line from Polaris drawn between *Capella* and a star close to the eastward of it passes to the westward of the constellation Orion. The two northern stars of the four at the corners are the shoulders, the northernmost of which is *α Orionis*. The brightest of the two southern stars, the feet, is called *Rigel*. In the middle are three stars forming the *Belt*, the northernmost of which is nearly on the equator.

About 25° to the northwestward of the Belt, and not far out of its line, is *Aldebaran*, which may be known by its red color.

A line from *Aldebaran* through the Belt passes at about 20° on the other side through *Sirius*, the brighest star in the heavens.

Sirius, the eastern shoulder, and *Procyon*, to the eastward of Orion and northward of Sirius, form an equilateral triangle.

Midway between the Great Bear and Orion are the Twins, *Castor* and *Pollux*, the latter the southern and brighter, about

4° apart. The line from Polaris to Procyon passes between them.

A line from Rigel through Procyon passes at an equal distance beyond to the northward of *Regulus.*

A line from Polaris through Ursa Majoris passes at 70° distance through *Spica Virginis.*

A line from Regulus through Spica passes at 45° distance through *Antares*, a bright reddish star.

The line from the Pointers carried through the Pole to about 75° beyond it, passes through *Marcab* or *α Pegasi.*

A line from Polaris through Marcab passes at 45° distance through *Fomalhaut*, a very bright star.

Achernar, *Fomalhaut*, and *Canopas* are in a line and nearly equidistant, being about 40° apart.

The *Southern Cross* is about as far from the South Pole as the Great Bear is from the North Pole—γ is the head, and α the foot.

When *some stars* are *known*, the *rest* are *easily found* by the times of their meridian passages and their declinations. A star may also be identified by means of its altitude or azimuth, computed approximately.

301. Parallax.—The word parallax is generally used to designate the *apparent displacement* due to the change in the position of the observer. As used in referring to the sun or other celestial object, it is employed to indicate the *difference of direction of such object* as seen from the center of the earth and from a station on the surface of the earth. The *horizontal parallax*, or that of the object in the horizon of the observer, is the angle subtended at the sun by the radius of the earth.

The *horizontal parallax* may be represented by the formula

$$P = \frac{r}{d \sin 1''} = 9'' \text{ (approximate)}, \quad . \quad . \quad (113)$$

in which $P =$ horizontal parallax of the sun in seconds of arc;
$r =$ radius of the earth; and
$d =$ distance between the center of the earth and the sun.

If it is desired to know the exact value of the *equatorial parallax*, which is the parallax of a celestial object as seen by an observer at the equator, this may be found in the American Ephemeris.

The *parallax of the sun* at any position above the horizon may be determined by the formula

$$p = P \cos A, \quad . \quad . \quad . \quad . \quad . \quad (114)$$

in which $p =$ parallax of the sun at any position, and
$A =$ angle at the earth's surface between the object observed and the horizon.

The following table (XLIX), from Hayford, gives the parallax of the sun for any date and altitude. As the distance of the sun is nearly the same for the same date in different years, this table may be used for any year.

302. **Refraction.**—A ray of light from any celestial object encounters, as it approaches the earth, successive strata of air, each more dense than the upper. In passing through these the ray is continually bent out of the straight line so as to cause the portion of its path through the atmosphere of the earth to be a curve. This is *refraction.*

Refraction acts according to the following *general laws:*

1. *When a ray passes from a lighter to a denser medium it is refracted towards the normal to the separating surface by an amount which is a function of the angle between the ray and the normal, and of the densities of the two media.*

2. *A plane containing a normal and an original ray also contains a refracted ray.*

The *effect of refraction* is noted directly in measuring altitudes, refraction always making the observed altitude too great (Arts. 166 and 239). Refraction has no apparent effect on azimuth. As the theory and computations of refraction are complicated, the topographer is referred for the application of the effects of refraction to the following tables, derived from Hayford.

Table L gives the mean refraction, R_m, under a barometric pressure of 29.9 inches and temperature of 50 degrees Fahrenheit. As mean refraction is a function of the altitude, it must be multiplied by a factor, C_B, derived from Table LI, if the barometric reading is not 29.9 inches. Finally, the mean refraction must be multiplied by the factor C_D (Table LII) where the temperature of the observation station differs from 50 degrees. Refraction R, as computed by these tables, is

$$R = R_m C_B C_D C_A. \qquad (115)$$

TABLE XLIX.

PARALLAX OF SUN (p) FOR FIRST DAY OF EACH MONTH.

(From Hayford's Geodetic Astronomy.)

Altitude.	Jan. 1st.	Feb. 1st. Dec. 1st.	Mar. 1st. Nov. 1st.	April 1st. Oct. 1st.	May 1st. Sept. 1st.	June 1st. Aug. 1st.	July 1st.	Zenith Distance.
0°	9″.0	9″.0	8″.9	8″.9	8″.8	8″.7	8″.7	90°
3	9 .0	9 .0	8 .9	8 .8	8 .8	8 .7	8 .7	87
6	9 .0	8 .9	8 .9	8 .8	8 .7	8 .7	8 .7	84
9	8 .9	8 .9	8 .8	8 .8	8 .7	8 .6	8 .6	81
12	8 .8	8 .8	8 .7	8 .7	8 .6	8 .5	8 .5	78
15	8 .7	8 .7	8 .6	8 .6	8 .5	8 .4	8 .4	75
18	8 .6	8 .6	8 .5	8 .4	8 .4	8 .3	8 .3	72
21	8 .4	8 .4	8 .3	8 .3	8 .2	8 .2	8 .1	69
24	8 .2	8 .2	8 .2	8 .1	8 .0	8 .0	8 .0	66
27	8 .0	8 .0	8 .0	7 .9	7 .8	7 .8	7 .8	63
30	7 .8	7 .8	7 .7	7 .7	7 .6	7 .6	7 .6	60
33	7 .6	7 .5	7 .5	7 .4	7 .4	7 .3	7 .3	57
36	7 .3	7 .3	7 .2	7 .2	7 .1	7 .1	7 .0	54
39	7 .0	7 .0	6 .9	6 .9	6 .8	6 .8	6 .8	51
42	6 .7	6 .7	6 .6	6 .6	6 .5	6 .5	6 .5	48
44	6 .5	6 .5	6 .4	6 .4	6 .3	6 .3	6 .3	46
46	6 .3	6 .2	6 .2	6 .2	6 .1	6 .1	6 .0	44
48	6 .0	6 .0	6 .0	5 .9	5 .9	5 .8	5 .8	42
50	5 .8	5 .8	5 .7	5 .7	5 .6	5 .6	5 .6	40
52	5 .6	5 .5	5 .5	5 .4	5 .4	5 .4	5 .4	38
54	5 .3	5 .3	5 .2	5 .2	5 .2	5 .1	5 .1	36
56	5 .0	5 .0	5 .0	5 .0	4 .9	4 .9	4 .9	34
58	4 .8	4 .8	4 .7	4 .7	4 .7	4 .6	4 .6	32
60	4 .5	4 .5	4 .5	4 .4	4 .4	4 .4	4 .4	30
62	4 .2	4 .2	4 .2	4 .2	4 .1	4 .1	4 .1	28
64	4 .0	3 .9	3 .9	3 .9	3 .8	3 .8	3 .8	26
66	3 .7	3 .7	3 .6	3 .6	3 .6	3 .6	3 .5	24
68	3 .4	3 .4	3 .4	3 .3	3 .3	3 .3	3 .3	22
70	3 .1	3 .1	3 .1	3 .0	3 .0	3 .0	3 .0	20
72	2 .8	2 .8	2 .8	2 .7	2 .7	2 .7	2 .7	18
74	2 .5	2 .5	2 .5	2 .4	2 .4	2 .4	2 .4	16
76	2 .2	2 .2	2 .2	2 .1	2 .1	2 .1	2 .1	14
78	1 .9	1 .9	1 .9	1 .8	1 .8	1 .8	1 .8	12
80	1 .6	1 .6	1 .6	1 .5	1 .5	1 .5	1 .5	10
82	1 .2	1 .2	1 .2	1 .2	1 .2	1 .2	1 .2	8
84	0 .9	0 .9	0 .9	0 .9	0 .9	0 .9	0 .9	6
86	0 .6	0 .6	0 .6	0 .6	0 .6	0 .6	0 .6	4
88	0 .3	0 .3	0 .3	0 .3	0 .3	0 .3	0 .3	2
90	0 .0	0 .0	0 .0	0 .0	0 .0	0 .0	0 .0	0

Table L.

MEAN REFRACTION (R_M). BAROMETER 29.9 INCHES, TEMP. 50° F.

(From Hayford's Geodetic Astronomy.)

Altitude.	Mean Refraction.	Change per Minute.	Altitude.	Mean Refraction.	Change per Minute.	Altitude.	Mean Refraction.	Change per Minute.	Altitude.	Mean Refraction.	Change per Minute.
0° 00'	34' 08".6	11".66	7° 00'	7' 24".2	0".95	19° 00'	2' 47".6	0".16	33° 00'	1' 29".4	0".06
10	32 15 .9	10 .88	10	7 14 .9	0 .91	20	2 44 .6	0 .15	20	1 28 .2	0 .06
20	30 31 .1	10 .10	20	7 06 .0	0 .88	40	2 41 .6	0 .15	40	1 27 .1	0 .05
30	28 53 .9	9 .64	30	6 57 .4	0 .84	20 00	2 38 .7	0 .14	34 00	1 26 .1	0 .05
40	27 18 .2	9 .20	40	6 49 .1	0 .81	20	2 35 .9	0 .14	20	1 25 .0	0 .05
50	25 49 .8	8 .50	50	6 41 .2	0 .78	40	2 33 .2	0 .13	40	1 24 .0	0 .05
1 00	24 28 .3	7 .82	8 00	6 33 .5	0 .76	21 00	2 30 .6	0 .13	35 00	1 23 .0	0 .05
10	23 13 .5	7 .17	10	6 26 .0	0 .73	20	2 28 .1	0 .13	20	1 22 .0	0 .05
20	22 04 .9	6 .58	20	6 18 .9	0 .70	40	2 25 .6	0 .12	40	1 21 .0	0 .05
30	21 01 .8	6 .06	30	6 12 .0	0 .68	22 00	2 23 .2	0 .12	36 00	1 20 .0	0 .05
40	20 03 .7	5 .60	40	6 05 .3	0 .66	20	2 20 .9	0 .12	30	1 18 .5	0 .05
50	19 09 .8	5 .20	50	5 58 .9	0 .63	40	2 18 .6	0 .11	37 00	1 17 .1	0 .04
2 00	18 19 .7	4 .84	9 00	5 52 .7	0 .61	23 00	2 16 .4	0 .11	30	1 15 .7	0 .04
10	17 33 .1	4 .50	20	5 40 .8	0 .58	20	2 14 .2	0 .11	38 00	1 14 .4	0 .04
20	16 49 .7	4 .18	40	5 29 .7	0 .54	40	2 12 .1	0 .10	30	1 13 .1	0 .04
30	16 09 .5	3 .88	10 00	5 19 .2	0 .51	24 00	2 10 .1	0 .10	39 00	1 11 .8	0 .04
40	15 32 .1	3 .62	20	5 09 .4	0 .48	20	2 08 .1	0 .10	30	1 10 .5	0 04
50	14 57 .1	3 .39	40	5 00 .1	0 .46	40	2 06 .1	0 .10	40 00	1 09 .3	0 .04
3 00	14 24 .3	3 .18	11 00	4 51 .2	0 .43	25 00	2 04 .2	0 .09	30	1 08 .1	0 .04
10	13 53 .6	2 .98	20	4 42 .8	0 .40	20	2 02 .4	0 .09	41 00	1 06 9	0 .04
20	13 24 .8	2 .79	40	4 35 .0	0 .38	40	2 00 .6	0 .09	30	1 05 .7	0 .04
30	12 57 .8	2 .61	12 00	4 27 .5	0 .37	26 00	1 58 .8	0 .09	42 00	1 04 .6	0 .04
40	12 32 .5	2 .46	20	4 20 .3	0 .35	20	1 57 1	0 .09	30	1 03 .5	0 .04
50	12 08 .7	2 .33	40	4 13 .5	0 .33	40	1 55 .4	0 .08	43 00	1 02 .4	0 04
4 00	11 46 .0	2 .20	13 00	4 07 .1	0 .32	27 00	1 53 .8	0 .08	30	1 01 .3	0 .04
10	11 24 .6	2 .09	20	4 00 .9	0 .30	20	1 52 .2	0 .08	44 00	1 00 .2	0 .03
20	11 04 .2	1 .98	40	3 55 .1	0 .28	40	1 50 .6	0 .08	30	1 59 .2	0 .03
30	10 44 .9	1 .88	14 00	3 49 .5	0 .27	28 00	1 49 .1	0 .08	45 00	0 58 .2	0 .03
40	10 26 .5	1 .79	20	3 44 .2	0 .26	20	1 47 .6	0 .07	30	0 57 .2	0 .03
50	10 09 .1	1 .70	40	3 39 .1	0 .25	40	1 46 .1	0 .07	46 00	0 56 .2	0 .03
5 00	9 52 .6	1 .61	15 00	3 34 .1	0 .24	29 00	1 44 .6	0 .07	30	0 55 .2	0 .03
10	9 36 .9	1 .54	20	3 29 .4	0 .23	20	1 43 .2	0 .07	47 00	0 54 .2	0 .03
20	9 21 .9	1 .46	40	3 24 .8	0 .23	40	1 41 .8	0 .07	30	0 53 .3	0 .03
30	9 07 .6	1 .40	16 00	3 20 .4	0 .22	30 00	1 40 .5	0 .07	48 00	0 52 .5	0 .03
40	8 54 .0	1 .33	20	3 16 .1	0 .21	20	1 39 .1	0 .07	30	0 51 .6	0 .03
50	8 41 .0	1 .27	40	3 12 .0	0 .20	40	1 37 .8	0 .06	49 00	0 50 .7	0 .03
6 00	8 28 .6	1 .22	17 00	3 08 .2	0 .19	31 00	1 36 .6	0 .06	30	0 49 .8	0 .03
10	8 16 .7	1 .16	20	3 04 .5	0 .19	20	1 35 .3	0 .06	50 00	0 48 .9	0 .03
20	8 05 .3	1 .12	40	3 00 .9	0 .18	40	1 34 .1	0 06	30	0 48 .0	0 .03
30	7 54 .3	1 .07	18 00	2 57 .4	0 .17	32 00	1 32 .0	0 .06	51 00	0 47 .2	0 .03
40	7 43 .9	1 .02	20	2 54 .0	0 .17	20	1 31 .8	0 .06	30	0 46 .3	0 .03
50	7 33 .9	0 .98	40	2 50 .7	0 .16	40	1 30 .6	0 .06	52 00	0 45 .5	0 .03

MEAN REFRACTION (R_M).—*Cont.*

Altitude.	Mean Refraction.	Change per Minute.
52° 30′	0′ 44″.7	0″.03
53 00	0 43 .9	0 .03
30	0 43 .1	0 .03
54 00	0 42 .3	0 .03
30	0 41 .6	0 .03
55 00	0 40 .8	0 .03
30	0 40 .0	0 .03
56 00	0 39 .3	0 .025
57 00	0 37 .8	0 .024
58 00	0 36 .4	0 .023
59 00	0 35 .0	0 .023
60 00	0 33 .6	0 .022
61 00	0 32 .3	0 .022
62 00	0 31 .0	0 .022
63 00	0 29 .7	0 .022
64 00	0 28 .4	0 .021
65 00	0 27 .2	0 .021
66 00	0 25 .9	0 .021
67 00	0 24 .7	0 .020
68 00	0 23 .6	0 .020
69 00	0 22 .4	0 .020
70 00	0 21 .2	0 .019
71 00	0 20 .1	0 .019
72 00	0 18 .9	0 .019
73 00	0 17 .8	0 .018
74 00	0 16 .7	0 .018
75 00	0 15 .6	0 .018
76 00	0 14 .5	0 .018
77 00	0 13 .5	0 .018
78 00	0 12 .4	0 .018
79 00	0 11 .3	0 .018
80 00	0 10 .3	0 .018
81 00	0 09 .2	0 .018
82 00	0 08 .2	0 .018
83 00	0 07 .2	0 .018
84 00	0 06 .1	0 .018
85 00	0 05 .1	0 .018
86 00	0 04 .1	0 .017
87 00	0 03 .1	0 .017
88 00	0 02 .0	0 .017
89 00	0 01 .0	0 .017
90 00	0 00 .0	0 .017

TABLE LI.

CORRECTON (C_B) TO MEAN REFRACTION DEPENDING UPON READING OF BAROMETER.

$$R = (R_M)(C_B)(C_D)(C_A).$$

(From Hayford's Geodetic Astronomy.)

Barometer. Inches.	Barometer. Millimeters.	C_B	Barometer. Inches.	Barometer. Millimeters.	C_B	Barometer. Inches.	Barometer. Millimeters.	C_B
20.0	508	0.670	24.2	615	0.809	28.4	721	0.949
20.1	511	0.673	24.3	617	0.813	28.5	724	0.953
20.2	513	0.676	24.4	620	0.816	28.6	726	0.956
20.3	516	0.679	24.5	622	0.820	28.7	729	0.959
20.4	518	0.682	24.6	625	0.823	28.8	732	0.963
20.5	521	0.685	24.7	627	0.826	28.9	734	0.966
20.6	523	0.688	24.8	630	0.829	29.0	737	0.970
20.7	526	0.692	24.9	632	0.832	29.1	739	0.973
20.8	528	0.696	25.0	635	0.835	29.2	742	0.976
20.9	531	0.699	25.1	637	0.838	29.3	744	0.979
21.0	533	0.703	25.2	640	0.842	29.4	747	0.983
21.1	536	0.706	25.3	643	0.846	29.5	749	0.986
21.2	538	0.709	25.4	645	0.849	29.6	752	0.989
21.3	541	0.712	25.5	648	0.853	29.7	754	0.992
21.4	544	0.716	25.6	650	0.856	29.8	757	0.996
21.5	546	0.719	25.7	653	0.859	29.9	759	0.999
21.6	549	0.722	25.8	655	0.862	30.0	762	1.003
21.7	551	0.725	25.9	658	0.866	30.1	765	1.007
21.8	554	0.729	26.0	660	0.869	30.2	767	1.010
21.9	556	0.732	26.1	663	0.872	30.3	770	1.013
22.0	559	0.735	26.2	665	0.875	30.4	772	1.016
22.1	561	0.739	26.3	668	0.879	30.5	775	1.020
22.2	564	0.742	26.4	671	0.882	30.6	777	1.023
22.3	566	0.746	26.5	673	0.885	30.7	780	1.026
22.4	569	0.749	26.6	676	0.889	30.8	782	1.029
22.5	572	0.752	26.7	678	0.892	30.9	785	1.033
22.6	574	0.755	26.8	681	0.896	31.0	787	1.036
22.7	576	0.759	26.9	683	0.899			
22.8	579	0.762	27.0	686	0.902			
22.9	582	0.766	27.1	688	0.905			
23.0	584	0.770	27.2	691	0.909			
23.1	587	0.773	27.3	693	0.912			
23.2	589	0.776	27.4	696	0.916			
23.3	592	0.779	27.5	690	0.920			
23.4	594	0.783	27.6	701	0.923			
23.5	597	0.786	27.7	704	0.926			
23.6	599	0.789	27.8	706	0.929			
23.7	602	0.792	27.9	709	0.933			
23.8	605	0.796	28.0	711	0.936			
23.9	607	0.799	28.1	714	0.939			
24.0	610	0.803	28.2	716	0.942			
24.1	612	0.806	28.3	719	0.946			

Table LII.

CORRECTION (C_D) TO MEAN REFRACTION DEPENDING UPON READING OF DETACHED THERMOMETER.

$$R = (R_M)(C_B)(C_D)(C_A).$$

(From Hayford's Geodetic Astronomy.)

Temp. Fahr.	Temp. Cent.	C_D	Temp. Fahr.	Temp. Cent.	C_D	Temp. Fahr.	Temp. Cent.	C_D	Temp. Fahr.	Temp. Cent.	C_D
−25°	−31°.7	1.172	20°	−6°.7	1.062	65°	18°.3	0.972	110°	43°.3	0.895
−24	−31 .1	1.169	21	−6 .1	1.060	66	18 .9	0.970	111	43 .9	0.894
−23	−30 .6	1.166	22	−5 .6	1.058	67	19 .4	0.968	112	44 .4	0.892
−22	−30 .0	1.164	23	−5 .0	1.056	68	20 .0	0.966	113	45 .0	0.891
−21	−29 .4	1.161	24	−4 .4	1.054	69	20 .6	0.964	114	45 .6	0.890
−20	−28 .9	1.158	25	−3 .9	1.051	70	21 .1	0.962	115	46 .1	0.888
−19	−28 .3	1.156	26	−3 .3	1.049	71	21 .7	0.961	116	46 .7	0.886
−18	−27 .8	1.153	27	−2 .8	1.047	72	22 .2	0.959	117	47 .2	0.885
−17	−27 .2	1.151	28	−2 .2	1.045	73	22 .8	0.957	118	47 .8	0.884
−16	−26 .7	1.148	29	−1 .7	1.043	74	23 .3	0.955	119	48 .3	0.882
−15	−26 .1	1.145	30	−1 .1	1.041	75	23 .9	0.953	120	48 .9	0.881
−14	−25 .6	1.143	31	−0 .6	1.039	76	24 .4	0.952	121	49 .4	0.880
−13	−25 .0	1.140	32	0 .0	1.036	77	25 .0	0.950	122	50 .0	0.878
−12	−24 .4	1.138	33	+0 .6	1.034	78	25 .6	0.948	123	50 .6	0.877
−11	−23 .9	1.135	34	1 .1	1.032	79	26 .1	0.946	124	51 .1	0.876
−10	−23 .3	1.133	35	1 .7	1.030	80	26 .7	0.945	125	51 .7	0.874
− 9	−22 .8	1.130	36	2 .2	1.028	81	27 .2	0.943	126	52 .2	0.873
− 8	−22 .2	1.128	37	2 .8	1.026	82	27 .8	0.941	127	52 .8	0.871
− 7	−21 .7	1.125	38	3 .3	1.024	83	28 .3	0.939	128	53 .3	0.870
− 6	−21 .1	1.123	39	3 .9	1.022	84	28 .9	0.938	129	53 .9	0.868
− 5	−20 .6	1.120	40	4 .4	1.020	85	29 .4	0.936	130	54 .4	0.867
− 4	−20 .0	1.118	41	5 .0	1.018	86	30 .0	0.934			
− 3	−19 .4	1.115	42	5 .6	1.016	87	30 .6	0.933			
− 2	−18 .9	1.113	43	6 .1	1.014	88	31 .1	0.931			
− 1	−18 .3	1.111	44	6 .7	1.012	89	31 .7	0.929			
0	−17 .8	1.108	45	7 .2	1.010	90	32 .2	0.928			
+ 1	−17 .2	1.106	46	7 .8	1.008	91	32 .8	0.926			
2	−16 .7	1.103	47	8 .3	1.006	92	33 .3	0.924			
3	−16 .1	1.101	48	8 .9	1.004	93	33 .9	0.923			
4	−15 .6	1.099	49	9 .4	1.002	94	34 .4	0.921			
5	−15 .0	1.096	50	10 .0	1.000	95	35 .0	0.919			
6	−14 .4	1.094	51	10 .6	0.998	96	35 .6	0.917			
7	−13 .9	1.092	52	11 .1	0.996	97	36 .1	0.916			
8	−13 .3	1.089	53	11 .7	0.994	98	36 .7	0.914			
9	−12 .8	1.087	54	12 .2	0.992	99	37 .2	0.912			
10	−12 .2	1.085	55	12 .8	0.990	100	37 .8	0.911			
11	−11 .7	1.082	56	13 .3	0.988	101	38 .3	0.909			
12	−11 .1	1.080	57	13 .9	0.986	102	38 .9	0.908			
13	−10 .6	1.078	58	14 .4	0.985	103	39 .4	0.906			
14	−10 .0	1.076	59	15 .0	0.983	104	40 .0	0.905			
15	− 9 .4	1.073	60	15 .6	0.981	105	40 .6	0.903			
16	− 8 .9	1.071	61	16 .1	0.979	106	41 .1	0.902			
17	− 8 .3	1.069	62	16 .7	0.977	107	41 .7	0.900			
18	− 7 .8	1.067	63	17 .2	0.975	108	42 .2	0.899			
19	− 7 .2	1.064	64	17 .8	0.973	109	42 .8	0.897			

Table LIII.

Correction (C_A) to Mean Refraction Depending upon Reading of Attached Thermometer.

$$R = (R_M)(C_B)(C_D)(C_A).$$

(From Hayford's Geodetic Astronomy.)

Temp. Fahr.	Temp. Cent.	C_A
−30°	−34°.4	1.007
−20	−28 .9	1.006
−10	−23 .3	1.005
0	−17 .8	1.005
+10	−12 .2	1.004
20	− 6 .7	1.003
30	− 1 .1	1.002
40	+ 4 .4	1.001
50	10 .0	1.000
60	15 .6	0.999
70	21 .1	0.998
80	26 .7	0.997
90	32 .2	0.996
100	37 .8	0.996
110	43 .3	0.995
120	48 .9	0.994
130	54 .4	0.993

CHAPTER XXXII.

TIME.

303. Interconversion of Time.—Sidereal time is referred to a fixed star, mean time to the sun. There is one more sidereal than solar day in the year.

366.24 sidereal days = 365.24 mean solar days.

24 hrs. sidereal time = 23 hrs. 56 min. 04.091 sec. mean solar time.

24 hrs. mean time = 24 hrs. 03 min. 56.555 sec. sidereal time.

Relations of Sidereal, Civil, and Mean Solar Time.

$$T_s - \alpha_s = I \text{ when } T_s > \alpha_s; \quad (116)$$

$$24^h + T_s - \alpha_s = I \text{ when } T_s < \alpha_s; \quad (117)$$

$$T_s = \alpha_s + T_m; \quad (118)$$

$$T_m = I - C_s \text{ for } D_t; \quad (119)$$

$$T_m = T \text{ for } D_t \text{ if } T_m < 12^h; \quad (120)$$

$$T_m = T_s - \alpha_s; \quad (121)$$

$$T = T_m - 12^h \text{ for } D_t + I \text{ if } T_m > 12^h. \quad (122)$$

For example see Article 315.

Relation of Sidereal Time to Right Ascension and Hour-angle of a Star.

$$T_s = \alpha + t \quad (123)$$

and

$$t = T_s - \alpha \quad (124)$$

Relations of Sidereal and Mean Solar Intervals of Time.—Let r = ratio of tropical year, expressed in sidereal day to tropical year expressed in mean solar day; then

TABLE LIV.

CONVERSION OF MEAN TIME INTO SIDEREAL TIME.

(From Smithsonian Geographical Tables.)

s	m 0 (h m s)	m 1 (h m s)	m 2 (h m s)	m 3 (h m s)
0	0 0 0	6 5 15	12 10 29	18 15 44
1	0 6 5	6 11 20	12 16 34	18 21 49
2	0 12 10	6 17 25	12 22 40	18 27 54
3	0 18 16	6 23 30	12 28 45	18 33 59
4	0 24 21	6 29 36	12 34 50	18 40 5
5	0 30 26	6 35 41	12 40 55	18 46 10
6	0 36 31	6 41 46	12 47 1	18 52 15
7	0 42 37	6 47 51	12 53 6	18 58 20
8	0 48 42	6 53 56	12 59 11	19 4 26
9	0 54 47	6 0 2	13 5 16	19 10 21
10	1 0 52	7 6 7	13 11 21	19 16 36
11	1 6 58	7 12 12	13 17 27	19 22 41
12	1 13 3	7 18 17	13 23 32	19 28 47
13	1 19 8	7 24 23	13 29 37	19 34 52
14	1 25 13	7 30 28	13 35 42	19 40 57
15	1 31 19	7 36 33	13 41 48	19 47 2
16	1 37 24	7 42 38	13 47 53	19 53 7
17	1 43 29	7 48 44	13 53 58	19 59 13
18	1 49 34	7 54 49	14 0 3	20 5 18
19	1 55 40	8 0 54	14 6 9	20 11 23
20	2 1 45	8 6 59	14 12 14	20 17 28
21	2 7 50	8 13 5	14 18 19	20 23 34
22	2 13 55	8 19 10	14 24 24	20 29 39
23	2 20 1	8 25 15	14 30 30	20 35 44
24	2 26 6	8 31 20	14 36 35	20 41 49
25	2 32 11	8 37 26	14 42 40	20 47 55
26	2 38 16	8 43 31	14 48 45	20 54 0
27	2 44 22	8 49 36	14 54 51	21 0 5
28	2 50 27	8 55 41	15 0 56	21 6 10
29	2 56 32	9 1 47	15 7 1	21 12 16
30	3 2 37	9 7 52	15 13 6	21 18 21
31	3 8 43	9 13 57	15 19 12	21 24 26
32	3 14 48	9 20 2	15 25 17	21 30 31
33	3 20 53	9 26 8	15 31 22	21 36 37
34	3 26 58	9 32 13	15 37 27	21 42 42
35	3 33 3	9 38 18	15 43 33	21 48 47
36	3 39 9	9 44 23	15 49 38	21 54 52
37	3 45 14	9 50 28	15 55 43	22 0 58
38	3 51 19	9 56 34	16 1 48	22 7 3
39	3 57 24	10 2 39	16 7 54	22 13 8
40	4 3 30	10 8 44	16 13 59	22 19 13
41	4 9 35	10 14 49	16 20 4	22 25 19
42	4 15 40	10 20 55	16 26 9	22 31 24
43	4 21 45	10 27 0	16 32 14	22 37 29
44	4 27 51	10 33 5	16 38 20	22 43 34
45	4 33 56	10 39 10	16 44 25	22 49 39
46	4 40 1	10 45 16	16 50 30	22 55 45
47	4 46 6	10 51 21	16 56 35	23 1 50
48	4 52 12	10 57 26	17 2 41	23 7 55
49	4 58 17	11 3 31	17 8 46	23 14 0
50	5 4 22	11 9 37	17 14 51	23 20 6
51	5 10 27	11 15 42	17 20 56	23 26 11
52	5 16 33	11 21 47	17 27 2	23 32 16
53	5 22 38	11 27 52	17 33 7	23 38 21
54	5 28 43	11 33 58	17 39 12	23 44 27
55	5 34 48	11 40 3	17 45 17	23 50 32
56	5 40 54	11 46 8	17 51 23	23 56 37
57	5 46 59	11 52 13	17 57 28	24 2 42
58	5 53 4	11 58 19	18 3 33	24 8 48
59	5 59 9	12 4 24	18 9 38	24 14 53
60	6 5 15	12 10 29	18 15 44	24 20 58

s	m s	s	m s
0.00	0 0	0.50	3 3
0.01	0 4	0.51	3 6
0.02	0 7	0.52	3 10
0.03	0 11	0.53	3 14
0.04	0 15	0.54	3 17
0.05	0 18	0.55	3 21
0.06	0 22	0.56	3 25
0.07	0 26	0.57	3 28
0.08	0 29	0.58	3 32
0.09	0 33	0.59	3 35
0.10	0 37	0.60	3 39
0.11	0 40	0.61	3 43
0.12	0 44	0.62	3 46
0.13	0 47	0.63	3 50
0.14	0 51	0.64	3 54
0.15	0 55	0.65	3 57
0.16	0 58	0.66	4 1
0.17	1 2	0.67	4 5
0.18	1 6	0.68	4 8
0.19	1 9	0.69	4 12
0.20	1 13	0.70	4 16
0.21	1 17	0.71	4 19
0.22	1 20	0.72	4 23
0.23	1 24	0.73	4 27
0.24	1 28	0.74	4 30
0.25	1 31	0.75	4 34
0.26	1 35	0.76	4 38
0.27	1 39	0.77	4 41
0.28	1 42	0.78	4 45
0.29	1 46	0.79	4 49
0.30	1 50	0.80	4 52
0.31	1 53	0.81	4 56
0.32	1 57	0.82	4 59
0.33	2 1	0.83	5 3
0.34	2 4	0.84	5 7
0.35	2 8	0.85	5 10
0.36	2 11	0.86	5 14
0.37	2 15	0.87	5 18
0.38	2 19	0.88	5 21
0.39	2 22	0.89	5 25
0.40	2 26	0.90	5 29
0.41	2 30	0.91	5 32
0.42	2 33	0.92	5 36
0.43	2 37	0.93	5 40
0.44	2 41	0.94	5 43
0.45	2 44	0.95	5 47
0.46	2 48	0.96	5 51
0.47	2 52	0.97	5 54
0.48	2 55	0.98	5 58
0.49	2 58	0.99	6 2
0.50	3 3	1.00	6 5

Example.—Let the given mean time be $14^h\ 54^m\ 32^s.56$.

The table gives

first for $14^h\ 57^m\ 51^s$ $\quad 2^m\ 27^s$

then for $\quad 2\ 41 \quad\quad 0.44$

$\quad\quad\quad\quad 2\ 27.44$

The sum

$14^h\ 57^m\ 32^s.56 + 2^m\ 27^s.44 = 15^h\ 0^m\ 0^s$

is the required sidereal time.

TABLE LV.

CONVERSION OF SIDEREAL TIME INTO MEAN TIME.

(From Smithsonian Geographical Tables.)

s	m 0 (h m s)	m 1 (h m s)	m 2 (h m s)	m 3 (h m s)
0	0 0 0	6 6 15	12 12 29	18 18 44
1	0 6 6	6 12 21	12 18 35	18 24 50
2	0 12 12	6 18 27	12 24 42	18 30 56
3	0 18 19	6 24 33	12 30 48	18 37 2
4	0 24 25	6 30 40	12 36 54	18 43 9
5	0 30 31	6 36 46	12 43 0	18 49 15
6	0 36 37	6 42 52	12 49 7	18 55 21
7	0 42 44	6 48 58	12 55 13	19 1 27
8	0 48 50	6 55 4	13 1 19	19 7 34
9	0 54 56	7 1 11	13 7 25	19 13 40
10	1 1 2	7 7 17	13 13 31	19 19 46
11	1 7 9	7 13 23	13 19 38	19 25 52
12	1 13 15	7 19 29	13 25 44	19 31 59
13	1 19 21	7 25 36	13 31 50	19 38 5
14	1 25 27	7 31 42	13 37 56	19 44 11
15	1 31 34	7 37 48	13 44 3	19 50 17
16	1 37 40	7 43 54	13 50 9	19 56 23
17	1 43 46	7 50 1	13 56 15	20 2 30
18	1 49 52	7 56 7	14 2 21	20 8 36
19	1 55 59	8 2 13	14 8 28	20 14 42
20	2 2 5	8 8 19	14 14 34	20 20 48
21	2 8 11	8 14 26	14 20 40	20 26 55
22	2 14 17	8 20 32	14 26 46	20 33 1
23	2 20 24	8 26 38	14 32 53	20 39 7
24	2 26 30	8 32 44	14 38 59	20 45 13
25	2 32 36	8 38 51	14 45 5	20 51 20
26	2 38 42	8 44 57	14 51 11	20 57 26
27	2 44 49	8 51 3	14 57 18	21 3 32
28	2 50 55	8 57 9	15 3 24	21 9 38
29	2 57 1	9 3 16	15 9 30	21 15 45
30	3 3 7	9 9 22	15 15 36	21 21 51
31	3 9 14	9 15 28	15 21 43	21 27 57
32	3 15 20	9 21 34	15 27 49	21 34 3
33	3 21 26	9 27 41	15 33 55	21 40 10
34	3 27 32	9 33 47	15 40 1	21 46 16
35	3 33 38	9 39 53	15 46 8	21 52 22
36	3 39 45	9 45 59	15 52 14	21 58 28
37	3 45 51	9 52 5	15 58 20	22 4 35
38	3 51 57	9 58 12	16 4 26	22 10 41
39	3 58 3	10 4 18	16 10 33	22 16 47
40	4 4 10	10 10 24	16 16 39	22 22 53
41	4 10 16	10 16 30	16 22 45	22 29 0
42	4 16 22	10 22 37	16 28 51	22 35 6
43	4 22 28	10 28 43	16 34 57	22 41 12
44	4 28 35	10 34 49	16 41 4	22 47 18
45	4 34 41	10 40 55	16 47 10	22 53 24
46	4 40 47	10 47 2	16 53 16	22 59 31
47	4 46 53	10 53 8	16 59 22	23 5 37
48	4 53 0	10 59 14	17 5 29	23 11 43
49	4 59 6	11 5 20	17 11 35	23 17 49
50	5 5 12	11 11 27	17 17 41	23 23 56
51	5 11 18	11 17 33	17 23 47	23 30 2
52	5 17 25	11 23 39	17 29 54	23 36 8
53	5 23 31	11 29 45	17 36 0	23 42 14
54	5 29 37	11 35 52	17 42 6	23 48 21
55	5 35 43	11 41 58	17 48 12	23 54 27
56	5 41 50	11 48 4	17 54 19	24 0 33
57	5 47 56	11 54 10	18 0 25	24 6 39
58	5 54 2	12 0 17	18 6 31	24 12 46
59	6 0 8	12 6 23	18 12 37	24 18 52
60	6 6 15	12 12 29	18 18 44	24 24 58

s	m s	s	m s
0.00	0 0	0.50	3 3
0.01	0 4	0.51	3 7
0.02	0 7	0.52	3 10
0.03	0 11	0.53	3 14
0.04	0 15	0.54	3 18
0.05	0 18	0.55	3 21
0.06	0 22	0.56	3 25
0.07	0 26	0.57	3 29
0.08	0 29	0.58	3 32
0.09	0 33	0.59	3 36
0.10	0 37	0.60	3 40
0.11	0 40	0.61	3 43
0.12	0 44	0.62	3 47
0.13	0 48	0.63	3 51
0.14	0 51	0.64	3 54
0.15	0 55	0.65	3 58
0.16	0 59	0.66	4 2
0.17	1 2	0.67	4 5
0.18	1 6	0.68	4 9
0.19	1 10	0.69	4 13
0.20	1 13	0.70	4 16
0.21	1 17	0.71	4 20
0.22	1 21	0.72	4 24
0.23	1 24	0.73	4 27
0.24	1 28	0.74	4 31
0.25	1 32	0.75	4 35
0.26	1 35	0.76	4 38
0.27	1 39	0.77	4 42
0.28	1 43	0.78	4 46
0.29	1 46	0.79	4 49
0.30	1 50	0.80	4 53
0.31	1 54	0.81	4 57
0.32	1 57	0.82	5 0
0.33	2 1	0.83	5 4
0.34	2 5	0.84	5 8
0.35	2 8	0.85	5 11
0.36	2 12	0.86	5 15
0.37	2 16	0.87	5 19
0.38	2 19	0.88	5 22
0.39	2 23	0.89	5 26
0.40	2 26	0.90	5 30
0.41	2 30	0.91	5 33
0.42	2 34	0.92	5 37
0.43	2 37	0.93	5 41
0.44	2 41	0.94	5 44
0.45	2 45	0.95	5 48
0.46	2 48	0.96	5 52
0.47	2 52	0.97	5 55
0.48	2 56	0.98	5 59
0.49	2 59	0.99	6 3
0.50	3 3	1.00	6 6

Example.—Given $15^h 0^m 0^s$.
The table gives

first for	$14^h 57^m 18^s$	$2^m 27^s$
then for	2 42	0.44
	15 0 0	2 27.44

The difference
$15^h 0^m 0^s - 2^m 27^s.44 = 14^h 57^m 32^s.56$
is the required mean time

$$r = \frac{366.2422}{365.2422} = 1.002738;$$

$$I' = rI = I + (r - 1)I = I + 0.002738I; \quad . \quad . \quad (125)$$

$$I = r^{-1}I' = I' - (1 - r^{-1})I' = I' - 0.002730I'. \quad (126)$$

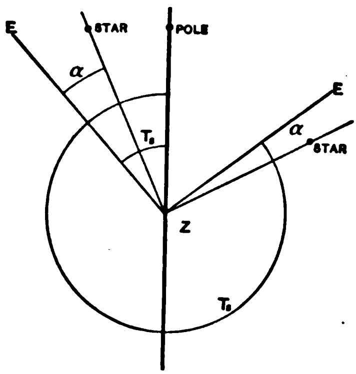


FIG. 180.—CONVERSION OF TIME.

These conversions are made by means of Tables LIV and LV, in which examples are shown.

304. Interconversion of Time and Arc.—In the performance of geodetic operations it frequently becomes necessary to convert time into longitude expressed in degrees of arc, and *vice versa.* The following tables facilitate this and similar operations and are for the interconversion of sidereal time and parts of the equator in degrees of arc, or of sidereal time and terrestrial longitude in arc.

TABLE LVI.

CONSTANTS FOR THE INTERCONVERSION OF TIME AND ARC.

			Logarithms.
12 hours, expressed in seconds	=	43200.	4.6354837
Complement to the same	=	.00002315	5.3645163
24 hours, expressed in seconds	=	86400.	4.9365137
Complement to the same	=	.00001157	5.0634863
360 degrees, expressed in seconds..........	=	1296000.	6.1126050
To convert sidereal time into mean solar time............			9.9988126

TABLE LVII.

CONVERSION OF TIME INTO ARC OR TERRESTRIAL LONGITUDE.

(From Lee's Tables.)

Hours.		Minutes.				Seconds.			
Time.	Arc.	Time.	Arc.	Time.	Arc.	Time.	Arc.	Time.	Arc.
h.	°	m.	° ′	m.	° ′	s.	′ ″	s.	′ ″
1	15	1	0 15	31	7 45	1	0 15	31	7 45
2	30	2	0 30	32	8 00	2	0 30	32	8 00
3	45	3	0 45	33	8 15	3	0 45	33	8 15
4	60	4	1 00	34	8 30	4	1 00	34	8 30
5	75	5	1 15	35	8 45	5	1 15	35	8 45
6	90	6	1 30	36	9 00	6	1 30	36	9 00
7	105	7	1 45	37	9 15	7	1 45	37	9 15
8	120	8	2 00	38	9 30	8	2 00	38	9 30
9	135	9	2 15	39	9 45	9	2 15	39	9 45
10	150	10	2 30	40	10 00	10	2 30	40	10 00
11	165	11	2 45	41	10 15	11	2 45	41	10 15
12	180	12	3 00	42	10 30	12	3 00	42	10 30
13	195	13	3 15	43	10 45	13	3 15	43	10 45
14	210	14	3 30	44	11 00	14	3 30	44	11 00
15	225	15	3 45	45	11 15	15	3 45	45	11 15
16	240	16	4 00	46	11 30	16	4 00	46	11 30
17	255	17	4 15	47	11 45	17	4 15	47	11 45
18	270	18	4 30	48	12 00	18	4 30	48	12 00
19	285	19	4 45	49	12 15	19	4 45	49	12 15
20	300	20	5 00	50	12 30	20	5 00	50	12 30
21	315	21	5 15	51	12 45	21	5 15	51	12 45
22	330	22	5 30	52	13 00	22	5 30	52	13 00
23	345	23	5 45	53	13 15	23	5 45	53	13 15
24	360	24	6 00	54	13 30	24	6 00	54	13 30
		25	6 15	55	13 45	25	6 15	55	13 45
		26	6 30	56	14 00	26	6 30	56	14 00
		27	6 45	57	14 15	27	6 45	57	14 15
		28	7 00	58	14 30	28	7 00	58	14 30
		29	7 15	59	14 45	29	7 15	59	14 45
		30	7 30	60	15 00	30	7 30	60	15 00

TABLE LVIII.

CONVERSION OF TIME INTO ARC, ETC.—(*Continued.*)

(From Lee's Tables.)

Tenths of Seconds.										Thousandths of Seconds of Time.	Arc.
Time.	Arc.	Time.	Arc.	Time.	Arc.	Time.	Arc.	Time.	Arc.		
s.	″	s.	″	s.	″	s.	″	s.	″	s.	″
0.01	0.15	0.21	3.15	0.41	6.15	0.61	9.15	0.81	12.15	0.001	0.015
0.02	0.30	0.22	3.30	0.42	6.30	0.62	9.30	0.82	12.30	0.002	0.030
0.03	0.45	0.23	3.45	0.43	6.45	0.63	9.45	0.83	12.45	0.003	0.045
0.04	0.60	0.24	3.60	0.44	6.60	0.64	9.60	0.84	12.60	0.004	0.060
0.05	0.75	0.25	3.75	0.45	6.75	0.65	9.75	0.85	12.75	0.005	0.075
0.06	0.90	0.26	3.90	0.46	6.90	0.66	9.90	0.86	12.90	0.006	0.090
0.07	1.05	0.27	4.05	0.47	7.05	0.67	10.05	0.87	13.05	0.007	0.105
0.08	1.20	0.28	4.20	0.48	7.20	0.68	10.20	0.88	13.20	0.008	0.120
0.09	1.35	0.29	4.35	0.49	7.35	0.69	10.35	0.89	13.35	0.009	0.135
0.10	1.50	0.30	4.50	0.50	7.50	0.70	10.50	0.90	13.50	0.010	0.150
0.11	1.65	0.31	4.65	0.51	7.65	0.71	10.65	0.91	13.65		
0.12	1.80	0.32	4.80	0.52	7.80	0.72	10.80	0.92	13.80		
0.13	1.95	0.33	4.95	0.53	7.95	0.73	10.95	0.93	13.95		
0.14	2.10	0.34	5.10	0.54	8.10	0.74	11.10	0.94	14.10		
0.15	2.25	0.35	5.25	0.55	8.25	0.75	11.25	0.95	14.25		
0.16	2.40	0.36	5.40	0.56	8.40	0.76	11.40	0.96	14.40		
0.17	2.55	0.37	5.55	0.57	8.55	0.77	11.55	0.97	14.55		
0.18	2.70	0.38	5.70	0.58	8.70	0.78	11.70	0.98	14.70		
0.19	2.85	0.39	5.85	0.59	8.85	0.79	11.85	0.99	14.85		
0.20	3.00	0.40	6.00	0.60	9.00	0.80	12.00	1.00	15.00		

305. Determination of Time.—This consists in finding the correction to the clock, watch, or chronometer used to record time.

Let T = clock time of any event at any place;
ΔT = clock correction; and
T_0 = required corresponding true time. Then

$$T_0 = T + \Delta T. \quad . \quad . \quad . \quad . \quad . \quad (127)$$

Clock correction may be found by several methods, three of the best and simplest being those given by Prof. R. S.

TABLE LIX.—CONVERSION OF ARC INTO TIME.

(From Smithsonian Geographical Tables.)

°	h. m.	°	h. m.	°	h. m.	°	h. m.	°	h. m.	°	h. m.	'	m. s.	"	s.
0	0 0	**60**	4 0	**120**	8 0	**180**	12 0	**240**	16 0	**300**	20 0	**0**	0 0	**0**	0.000
1	0 4	61	4 4	121	8 4	181	12 4	241	16 4	301	20 4	1	0 4	1	0.067
2	0 8	62	4 8	122	8 8	182	12 8	242	16 8	302	20 8	2	0 8	2	0.133
3	0 12	63	4 12	123	8 12	183	12 12	243	16 12	303	20 12	3	0 12	3	0.200
4	0 16	64	4 16	124	8 16	184	12 16	244	16 16	304	20 16	4	0 16	4	0.267
5	0 20	**65**	4 20	**125**	8 20	**185**	12 20	**245**	16 20	**305**	20 20	**5**	0 20	**5**	0.333
6	0 24	66	4 24	126	8 24	186	12 24	246	16 24	306	20 24	6	0 24	6	0.400
7	0 28	67	4 28	127	8 28	187	12 28	247	16 28	307	20 28	7	0 28	7	0.467
8	0 32	68	4 32	128	8 32	188	12 32	248	16 32	308	20 32	8	0 32	8	0.533
9	0 36	69	4 36	129	8 36	189	12 36	249	16 36	309	20 36	9	0 36	9	0.600
10	0 40	**70**	4 40	**130**	8 40	**190**	12 40	**250**	16 40	**310**	20 40	**10**	0 40	**10**	0.667
11	0 44	71	4 44	131	8 44	191	12 44	251	16 44	311	20 44	11	0 44	11	0.733
12	0 48	72	4 48	132	8 48	192	12 48	252	16 48	312	20 48	12	0 48	12	0.800
13	0 52	73	4 52	133	8 52	193	12 52	253	16 52	313	20 52	13	0 52	13	0.867
14	0 56	74	4 56	134	8 56	194	12 56	254	16 56	314	20 56	14	0 56	14	0.933
15	1 0	**75**	5 0	**135**	9 0	**195**	13 0	**255**	17 0	**315**	21 0	**15**	1 0	**15**	1.000
16	1 4	76	5 4	136	9 4	196	13 4	256	17 4	316	21 4	16	1 4	16	1.067
17	1 8	77	5 8	137	9 8	197	13 8	257	17 8	317	21 8	17	1 8	17	1.133
18	1 12	78	5 12	138	9 12	198	13 12	258	17 12	318	21 12	18	1 12	18	1.200
19	1 16	79	5 16	139	9 16	199	13 16	259	17 16	319	21 16	19	1 16	19	1.267
20	1 20	**80**	5 20	**140**	9 20	**200**	13 20	**260**	17 20	**320**	21 20	**20**	1 20	**20**	1.333
21	1 24	81	5 24	141	9 24	201	13 24	261	17 24	321	21 24	21	1 24	21	1.400
22	1 28	82	5 28	142	9 28	202	13 28	262	17 28	322	21 28	22	1 28	22	1.467
23	1 32	83	5 32	143	9 32	203	13 32	263	17 32	323	21 32	23	1 32	23	1.533
24	1 36	84	5 36	144	9 36	204	13 36	264	17 36	324	21 36	24	1 36	24	1.600
25	1 40	**85**	5 40	**145**	9 40	**205**	13 40	**265**	17 40	**325**	21 40	**25**	1 40	**25**	1.667
26	1 44	86	5 44	146	9 44	206	13 44	266	17 44	326	21 44	26	1 44	26	1.733
27	1 48	87	5 48	147	9 48	207	13 48	267	17 48	327	21 48	27	1 48	27	1.800
28	1 52	88	5 52	148	9 52	208	13 52	268	17 52	328	21 52	28	1 52	28	1.867
29	1 56	89	5 56	149	9 56	209	13 56	269	17 56	329	21 56	29	1 56	29	1.933
30	2 0	**90**	6 0	**150**	10 0	**210**	14 0	**270**	18 0	**330**	22 0	**30**	2 0	**30**	2.000
31	2 4	91	6 4	151	10 4	211	14 4	271	18 4	331	22 4	31	2 4	31	2.067
32	2 8	92	6 8	152	10 8	212	14 8	272	18 8	332	22 8	32	2 8	32	2.133
33	2 12	93	6 12	153	10 12	213	14 12	273	18 12	333	22 12	33	2 12	33	2.200
34	2 16	94	6 16	154	10 16	214	14 16	274	18 16	334	22 16	34	2 16	34	2.267
35	2 20	**95**	6 20	**155**	10 20	**215**	14 20	**275**	18 20	**335**	22 20	**35**	2 20	**35**	2.333
36	2 24	96	6 24	156	10 24	216	14 24	276	18 24	336	22 24	36	2 24	36	2.400
37	2 28	97	6 28	157	10 28	217	14 28	277	18 28	337	22 28	37	2 28	37	2.467
38	2 32	98	6 32	158	10 32	218	14 32	278	18 32	338	22 32	38	2 32	38	2.533
39	2 36	99	6 36	159	10 36	219	14 36	279	18 36	339	22 36	39	2 36	39	2.600
40	2 40	**100**	6 40	**160**	10 40	**220**	14 40	**280**	18 40	**340**	22 40	**40**	2 40	**40**	2.667
41	2 44	101	6 44	161	10 44	221	14 44	281	18 44	341	22 44	41	2 44	41	2.733
42	2 48	102	6 48	162	10 48	222	14 48	282	18 48	342	22 48	42	2 48	42	2.800
43	2 52	103	6 52	163	10 52	223	14 52	283	18 52	343	22 52	43	2 52	43	2.867
44	2 56	104	6 56	164	10 56	224	14 56	284	18 56	344	22 56	44	2 56	44	2.933
45	3 0	**105**	7 0	**165**	11 0	**225**	15 0	**285**	19 0	**345**	23 0	**45**	3 0	**45**	3.000
46	3 4	106	7 4	166	11 4	226	15 4	286	19 4	346	23 4	46	3 4	46	3.067
47	3 8	107	7 8	167	11 8	227	15 8	287	19 8	347	23 8	47	3 8	47	3.133
48	3 12	108	7 12	168	11 12	228	15 12	288	19 12	348	23 12	48	3 12	48	3.200
49	3 16	109	7 16	169	11 16	229	15 16	289	19 16	349	23 16	49	3 16	49	3.267
50	3 20	**110**	7 20	**170**	11 20	**230**	15 20	**290**	19 20	**350**	23 20	**50**	3 20	**50**	3.333
51	3 24	111	7 24	171	11 24	231	15 24	291	19 24	351	23 24	51	3 24	51	3.400
52	3 28	112	7 28	172	11 28	232	15 28	292	19 28	352	23 28	52	3 28	52	3.467
53	3 32	113	7 32	173	11 32	233	15 32	293	19 32	353	23 32	53	3 32	53	3.533
54	3 36	114	7 36	174	11 36	234	15 36	294	19 36	354	23 36	54	3 36	54	3.600
55	3 40	**115**	7 40	**175**	11 40	**235**	15 40	**295**	19 40	**355**	23 40	**55**	3 40	**55**	3.667
56	3 44	116	7 44	176	11 44	236	15 44	296	19 44	356	23 44	56	3 44	56	3.733
57	3 48	117	7 48	177	11 48	237	15 48	297	19 48	357	23 48	57	3 48	57	3.800
58	3 52	118	7 52	178	11 52	238	15 52	298	19 52	358	23 52	58	3 52	58	3.867
59	3 56	119	7 56	179	11 56	239	15 56	299	19 56	359	23 56	59	3 56	59	3.933
60	4 0	**120**	8 0	**180**	12 0	**240**	16 0	**300**	20 0	**360**	24 0	**60**	4 0	**60**	4.000

Woodward in Smithsonian Geographical Tables, and they are reproduced here from that work. They are·

1. By observing the transit of a star, whose right ascension is known, across the meridian.

2. By a single observed altitude of a star, which gives a fair approximate measure of time for geographic purposes.

3. By equal altitudes of a star.

The first is the most accurate and is that used in refined work. It is fully explained in Article 308 and in connection with determination of longitude (Chap. XXXV). The second method, elaborated in Article 306, requires a knowledge of the latitude of the place, which may be approximately measured from a good map or obtained by simple observations (Art. 316).

The third method is an extension of the preceding. A mean of the times when a star has the same altitude east and west of the meridian, is the time of meridian transit. With an engineer's transit with telescope clamped, this method gives a good approximation to the time correction, freed of constant instrumental errors. The same method can be satisfactorily applied to the sun, using either engineer's transit or sextant (Arts. 85 and 336). This is done by making measurements about two hours before and after noon of a series of altitudes before and after passage of the meridian, and taking a mean of the half-sums for time of meridian transit (Art. 308).

306. Time by a Single Observed Altitude of a Star.—An approximate determination of time, often sufficient for the purposes of the geographer, may be had by observing the altitude or zenith distance of a known star. The method requires also a knowledge of the latitude of the place.

Let z_1 = the observed zenith distance of the star;
z = the true zenith distance of the star
$= z_1 + R$.

Then the hour-angle t may be computed by the formula

$$\tan^2 \tfrac{1}{2}t = \frac{\sin(\sigma - \phi)\cos(\sigma - \delta)}{\cos\sigma\cos(\sigma - z)} \quad . \quad . \quad (128)$$

in which $\sigma = \frac{1}{2}(\phi + \delta + z)$.

307. Approximate Time from Sun.—To find approximate time or watch correction, observe the sun shortly before noon and, when it reaches its highest point, note the watch time of observed greatest altitude. This is a quick method, giving time within 10 m.

Example of Computation, April 16, 1898.

		h.	m.	s.	
The instant of sun's greatest altitude occurs at apparent noon........................		12	00	00	
Equation of time, April 16...............				− 17	
True local mean time..............		11	59	43	
Watch time of instant of sun's greatest altitude..............................		1	15	30	P.M.
Watch correction..........................	−	1	15	47	

Watch is therefore 1 h. 16 m. fast of local mean time, which is about what it would be if "Pacific standard" time were used in longitude 136°.

308. Time by Meridian Transits.—The time of transit across the meridian being observed of a star whose right ascension is known, we have

$$\Delta T = \alpha - T. \quad . \quad . \quad . \quad . \quad . \quad (129)$$

Meridian transits of stars may be observed by means of a theodolite or transit instrument mounted so that its telescope describes the meridian when rotated about its horizontal axis. The meridian transit instrument is specially designed for this purpose, and affords the most precise method of determining time. (Fig. 181.)

Since it is impossible to place the telescope of such an instrument exactly in the meridian, it is essential in precise work to determine certain constants, which define this defect of adjustment, along with the clock correction. These constants are the azimuth of the telescope when in the horizon, the inclination of the horizontal axis of the telescope, and the error of collimation of the telescope.

Let a = azimuth constant;
b = inclination or level constant;
c = collimation constant.

a is considered plus when the instrument points east of south; b is plus when the west end of the rotation axis is the higher; and c is intrinsically plus when the star observed crosses the thread too soon from lack of collimation.

Also let

$$A = \frac{\sin(\phi - \delta)}{\cos\delta} = \text{the ``azimuth factor''}; \qquad (130)$$

$$B = \frac{\cos(\phi - \delta)}{\cos\delta} = \text{the ``level factor''}; \quad . \qquad (131)$$

$$C = \frac{1}{\cos\delta} = \text{the ``collimation factor.''} \quad . \quad . \qquad (132)$$

Then, when a, b, c are not greater than 10^s each and preferably as small as 1^s each,

$$T + \Delta T + Aa + Bb + Cc + r(T - T_0) = \alpha. \quad . \qquad (133)$$

This is known as *Mayer's formula* for the computation of time from star transits.

The quantity Bb is generally observed directly with a striding-level. Assuming it to be known and combined with T, the above equation gives

$$\Delta T + Aa + Cc + r(T - T_0) = \alpha - T. \quad . \quad . \qquad (134)$$

This equation involves *four unknown quantities*, $\varDelta T$, a, c, and r; so that in general it will be essential to observe at least four different stars in order to get the objective quantity $\varDelta T$. Where great precision is not needed, the effect of the rate, for short intervals of time, may be ignored, and the collimation c may be rendered insignificant by adjustment. Then the equation (134) is simplified into

$$\varDelta T + Aa = \alpha - T. \quad . \quad . \quad . \quad . \quad (135)$$

This shows that observations of two stars of different declinations will suffice to give $\varDelta T$. Since the factor A is plus for stars south of the zenith in north latitude and minus for stars north of the zenith, if stars be so chosen as to make the two values of A equal numerically but of opposite signs, $\varDelta T$ will result from the mean of two equations of the form (135). With good instrumental adjustments, i.e. b and c small, this simple form of observation with a theodolite will give $\varDelta T$ to the nearest second.

A still better plan for *approximate determination of time* is to observe a pair of north and south stars as above, and then reverse the telescope and observe another pair similarly situated, since the remaining error of collimation will be partly if not wholly eliminated. Indeed, a well-selected and well-observed set of four stars will give the *error of the timepiece* used *within a half second* or less. This method is especially available to geographers who may desire such an approximate value of the timepiece correction for use in determining azimuth. It will suffice in the application of the method to set up the theodolite or transit in the vertical plane of Polaris, which is always close enough to the meridian. The determination will then proceed according to the following programme:

1. Observe time of transit of a star south of zenith;
2. Observe time of transit of a star north of zenith.

Reverse the telescope and

3. Observe time of transit of a star south of zenith;

4. Observe time of transit of a star north of zenith.

Each star observation will give an equation of the form (134), and the mean of the four resulting equations is

$$\Delta T + a\frac{\Sigma A}{4} + c\frac{\Sigma C}{4} + r\frac{\Sigma(T - T_0)}{4} = \frac{\Sigma(\alpha - T)}{4}. \quad (136)$$

Now the coefficient of r in this equation may be always made zero by taking for the epoch T_0 the mean of the observed times T. Likewise, ΣA and ΣC may be made small by suitably selected stars, since two of the A's and C's are positive and two negative. The value $\frac{1}{4}\Sigma(\alpha - T)$ is thus always a close approximation to ΔT for the epoch $T_0 = \frac{1}{4}\Sigma T$, when ΣA and ΣC approximate to zero. But if these sums are not small, approximate values of a and c may be found from the four equations of the form (134), neglecting the rate, and these substituted in the above formula will give all needful precision.

For refined work, as in determining differences of longitude, several groups of stars are observed, half of them with the telescope in one position and half in the reverse position, and the quantities ΔT, a, c, and r are computed by the method of least squares (Art. 264). In such work it is always advantageous to select the stars with a view to making the sums of the azimuth and collimation coefficients approximate to zero, since this gives the highest precision and entails the simplest computations.

CHAPTER XXXIII.

AZIMUTH.

309. Determination of Azimuth.—The azimuth of a line is the angle which it makes with a true north and south line; this angle is measured from the south around towards the west. It gives the initial direction from which the directions of other lines in a trigonometric or traverse survey are derived. Azimuth is obtained by means of astronomic observations by more or less approximate methods. For primary triangulation or traverse such observations are made by the most accurate methods, and at intervals not greater than 50 to 75 miles in primary triangulation, and not exceeding 10 miles in primary traverse.

The determination of the azimuth of a terrestrial line consists in the measurement of an angle between two vertical planes, one passing through a terrestrial mark called the *azimuth mark* and the center of the instrument, and the other through an observed star and the center of the instrument. The exact time at which pointing is made upon the star must be noted by a chronometer, the error of which is known, because the angle of the star between these two points is continually changing. With the aid of the recorded time, the hour-angle of the star and its azimuth as seen from the station may be computed. The measured hour-angle at the station between the star and the azimuth mark added to or subtracted from the computed azimuth of the star gives the azimuth of the terrestrial mark from the station.

310. Observing for Azimuth.—The instrument used in making azimuth observations is a theodolite similar to that used in primary triangulation or traverse (Art. 241). Azimuth observations may be made, however, for secondary purposes, as for the reduction of transit traverse lines by means of latitudes and departures with the instrument used in running the traverse (Arts. 90 and 85). In this case the method employed is similar to that hereafter described, but is more simple because the instruments employed are less accurate and call, therefore, for less care in their use (Art. 311). The observation for azimuth by astronomic methods, as those required in the determination of primary azimuths of a base line or in a belt of triangulation, consists in the measurement of the horizontal angle between some close circumpolar star, usually Polaris, and a terrestrial mark. The latter is generally a bull's-eye lantern set at a distance of at least half to one mile from the observer's station.

Since the star is at a much higher angle than the terrestrial mark, it is necessary to measure the error of level and to correct for it in addition to carefully leveling the instrument. As a result the value of a division of the level-bubble must be accurately known. Observations for azimuth may be made at any time of night, preferably near the time of elongation, since the star is then moving most slowly in azimuth and any error in time has the least effect in the result. The error of the watch must be obtained by comparison with a standard of time and corrected for the difference in longitude between the observing station and the meridian of such standard of time.

311. Approximate Solar Azimuth.—This observation may be made to obtain meridians in public-land surveys and for similar approximate work, the only instrument required being an engineer's transit in good adjustment, thus doing away with the solar (Art. 339) or other special attachments.

Observations should be made in the morning and afternoon, the routine being as follows:

1. Point on azimuth mark and read horizontal circle;
2. Point on sun with telescope direct;

3. Point on sun with telescope inverted;
4. Point on azimuth mark and read horizontal circle.

To eliminate errors of collimation and of verticality and horizontality of cross-hairs, two pointings on the sun are made thus:

In the morning observations the sun is passing to the right and rising. Observe in upper left and lower right quadrants, the contacts being on lower and first limb. This operation is so performed as to facilitate manipulation of tangent motions of the instrument and in order that only one slow motion shall be used during the observation. The horizontal thread in the first case should be set above the limb of the sun so that, as the latter rises, it may be closely watched, and just as the sun moves off contact the vertical cross-hair must be made tangent to the first limb by the slow motion. In the second case the vertical cross-hair may be set just to the right of the limb and slow motion made with the vertical tangent screw. In the afternoon the routine is to observe in the upper right and lower left quadrants, the method of manipulation being similar to the above.

Let $s =$ sun's diameter;
$c =$ collimation error;
$o =$ position of azimuth mark;
$a' = A' - s - c =$ horizontal angle between some fixed point, o, and the sun's center in the first observation with telescope;
$a'' = A'' + s - c =$ horizontal angle between some point, o, and the sun's center on the second observation with the telescope inverted;
$A' =$ circle reading when telescope is pointed at sun's first limb;
$A'' =$ circle reading when telescope is pointed at sun's second limb.

Then

$$\frac{(\delta' + \delta'')}{2} = \tfrac{1}{2}(A' + A'). \quad . \quad . \quad . \quad (137)$$

$$H' = h' + s + c;$$
$$H'' = h'' - s - c;$$
$$\frac{H + H''}{2} = \frac{h' + h''}{2}.$$

Let δ = declination = $\frac{1}{2}(h + \phi + p)$;
h = altitude corrected for refraction;
ϕ = latitude;
p = sun's or star's polar distance;
α = azimuth counted from the north.

Then

$$\tan \frac{1}{2}\alpha = \frac{\sin (s - h) \sin (s - \phi)}{\cos s \,.\, \cos (s - p)}. \quad . \quad . \quad (138)$$

EXAMPLE.

Azimuth Mark. Ver. A.	Ver. B.	Sun, Horizontal Circle. Ver. A.	Ver. B.
25° 24′	205° 23′	23° 02′	203° 01′
25 24	205 25	23 14	203 15
Mean, 25° 24′		23° 08′	

	25° 24′
	23 08
Angle between mark and mean place of sun	2° 16′

	Sun, Vertical Circle.	Index.
	8° 37′	+ 3
	12 47	− 1
	10° 42′	+ 1
	+ 1	
Altitude	10° 41′	
Refraction	− 5	always negative
Corrected altitude..	10° 36′	

More pointings than two are desirable, and these should be equally divided between the two limbs so as to eliminate the semi-diameter of sun.

First find sun's polar distance corresponding to approximate time of observation.

Date, April 16, 1898.

Approximate time of observation by watch.....	7 h. 15 m. A.M.	
Watch correction (Art. 303).....................	− 1 15	
	6 00	
Map longitude from Greenwich 136° = $\frac{136}{15}$ hrs.=	+ 8 48 always positive	
Greenwich time, approximate..................	15 hours after midnight	
Subtract	12 "	
Greenwich time after noon, because declination is given for noon.........................	3 P.M.	
Hourly change in declination, April 16.........	+ 53″	
3 h. × 53″ =	+ 159″ = + 2′ 39″	
Sun's declination, April 16, Greenwich noon (Almanac)................................		+ 10 13 49
Sun's declination, April 16, 3 hours later.......		10 16 28
Sun's polar distance 90° − sun's declination....		79 44
Sun's polar distance....................	79° 44′	
Latitude..............................	64 23	
Observed altitude.....................	10 36	
Sum..............................	154 43	
½ sum..............................	77 21	log. sec. = 0.6596
½ sum − altitude......................	66 45	" sin. = 9.9632
½ sum − latitude......................	12 58	" sin. = 9 3510
½ sum − polar distance...............	2 23	" sec. = 0.0004
Check sum.......................	154 41	2)9.9742
Log. tan. ½ azimuth...........................		9.9871
½ azimuth	44° 09′	[from north.
Azimuth......................	88 18	always measured
Angle between sun and mark	2 16	
Azimuth of mark (east of north).............	90° 34′	

Another and quite simple formula for determining the meridian by a single solar observation is given by Mr. W. Newbrough and is as follows:

$$\text{Cos } \tfrac{1}{2}\alpha = \sqrt{\frac{\cos S \cos (S - p)}{\cos \phi \cos h}}, \quad . \quad . \quad . \quad (139)$$

where α = sun's azimuth measured from north;

p = sun's polar distance or codeclination;

h = sun's altitude minus correction for refraction;

S = half the sum of polar distance, latitude, and true azimuth; and

ϕ = latitude of place of observation.

312. Azimuths of Secondary Accuracy.—In observing an azimuth on a primary traverse, less care and accuracy are required than in observing a primary azimuth in triangulation. The following procedure illustrates the observing and computing of such a secondary azimuth. The instrument is centered over any station on the traverse line, and the azimuth mark placed at the next station. This should be at least 1500 feet from the instrument, and may be a narrow slit in a box containing a light, or the small colored light on the side of a bicycle-lamp carefully centered over the station. The error of a good watch should be known by comparison with telegraph time signal, which is sent over all Western Union lines once every day.

The angle is then measured between the azimuth mark and Polaris, making at least six pointings at both mark and star, three with telescope direct and three with telescope reversed, and at each star pointing the time is noted to nearest second. Then the reduction is made in the following manner:

EXAMPLE.

Latitude 34° 32′ (to nearest minute); longitude 92° 35′.

(GEO. T. HAWKINS, Observer and Computer.)

Azimuth observation at Benton, Ark., October 8, 1898. Between instrument traverse stations 106 and 107. Instrument at sta. 107, mark at sta. 106. 90° meridian time by watch, which was compared with Western Union time at 10 o'clock to-day and found to be 1.0 minute slow (Arts. 230 and 234).

Time by watch	7^h	10^m	49^s	= mean time of 6 pointings on Polaris.	
Correction of watch	+	01	00		
	7	11	49	= 90° meridian time of observation.	
Correction for longitude	—	10	20		
	7	01	29	= astr. local mean time observation.	
Upper culmination, subtract	— 12	14	06		
Hour-angle Polaris at observation	18	47	23	U. C. Polaris Oct. 1 (Table LXIII)	12 37.7
Subtract from	23	56	06	Reduction to Oct. 7 (Table LXIV)	— 23.6
Time argument	5	08	43	U. C. Polaris, Oct. 7	12 14.1

Azimuth of Polaris at observation (Table LXV)	1° 28′ 17″ or 181° 28′ 17″
Angle at sta. 106 bet. sta. 107 and Polaris (mean of 6 readings)	+ 43 02 00
Azimuth from sta. 107 to sta. 106	= 224° 30′ 17″
	— 180°
Azimuth from sta. 106 to sta. 107	44° 30′ 17″

TABLE LX.

APPROXIMATE LOCAL MEAN ASTRONOMIC TIMES OF THE CULMINATIONS AND ELONGATIONS OF POLARIS FOR THE YEAR 1900.

Latitude 40° north ; longitude 6^h west of Greenwich.

Date.	Eastern Elongation.	Upper Culmination.	Western Elongation.	Lower Culmination.
1900.	h m	h m	h m	h m
Jan. 1	0 41.5	6 36.3	12 31.1	18 34.3
15	23 42.3	5 41.0	11 35.8	17 39.0
Feb. 1	22 35.1	4 33.9	10 28.7	16 31.9
15	21 39.9	3 38.6	9 33.5	15 36.6
Mar. 1	20 44.7	2 43.4	8 38.2	14 41.4
15	19 49.5	1 48.2	7 43.0	13 46.3
Apr. 1	18 42.6	0 41.3	6 36.1	12 39.4
15	17 47.6	23 42.4	5 41.1	11 44.4
May 1	16 44.8	22 39.5	4 38.3	10 41.5
15	15 49.9	21 44.6	3 43.4	9 46.6
June 1	14 43.2	20 38.0	2 36.7	8 40.0
15	13 48.4	19 43.2	1 41.9	7 45.2
July 1	12 45.7	18 40.5	0 39.2	6 42.5
15	11 50.9	17 45.7	23 40.5	5 47.7
Aug. 1	10 44.3	16 39.1	22 33.9	4 41.1
15	9 49.5	15 44.3	21 39.1	3 46.3
Sept. 1	8 42.8	14 37.6	20 32.4	2 39.6
15	7 47.9	13 42.7	19 37.5	1 44.7
Oct. 1	6 45.1	12 39.9	18 34.7	0 41.9
15	5 50.1	11 44.9	17 39.7	23 43.0
Nov. 1	4 43.3	10 38.1	16 32.9	22 36.1
15	3 48.1	9 42.9	15 37.7	21 40.9
Dec. 1	2 45.1	8 39.9	14 34.7	20 37.9
15	1 49.9	7 44.7	13 39.5	19 42.7

To refer to any calendar day other than the first and fifteenth of each month, subtract 3.94^m from every day between it and the preceding tabular day, or add 3.94^m for every day between it and the succeeding tabular day.

It will be noticed that for the tabular year two eastern elongations occur on January 10, and two western elongations on July 9; there are also two upper culminations on April 10 and two lower culminations on October 10.

The lower culmination either follows or precedes the upper culmination at an interval of 11^h 58.0^m. Also east elongation either follows west elongation at an interval of 12^h 06.5^m or precedes it at an interval of 11^h 49.6^m.

To refer the tabular times to any year subsequent to the year 1898, add 0.2^m (nearly) for every additional year.

Table LXI.

AZIMUTHS OF POLARIS AT ELONGATION

Between 1900 and 1910 and Latitudes 25° and 75° North.

(From U. S. Land Survey Manual.)

Latitude.	1900.	1901.	1902.	1903.	1904.	1905.	1906.	1907.	1908.	1909.	1910.
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
30	1 24.9	1 24.6	1 24.2	1 23.9	1 23.5	1 23.1	1 22.8	1 22.4	1 22.1	1 21.7	1 21.3
31	25.8	25.5	25.1	24.7	24.4	24.0	23.6	23.2	22.9	22.5	22.2
32	26.7	26.4	26.0	25.6	25.3	24.9	24.5	24.1	23.8	23.4	23.1
33	27.7	27.3	27.0	26.6	26.2	25.9	25.5	25.1	24.7	24.3	24.0
34	28.7	28.4	28.0	27.6	27.2	26.9	26.5	26.1	25.7	25.3	25.0
35	1 29.8	1 29.4	1 29.0	1 28.7	1 28.3	1 27.9	1 27.5	1 27.1	1 26.8	1 26.4	1 26.0
36	30.9	30.5	30.1	29.8	29.4	29.0	28.6	28.2	27.9	27.5	27.1
37	32.1	31.7	31.3	30.9	30.5	30.1	29.7	29.3	29.0	28.6	28.2
38	33.4	33.0	32.6	32.2	31.8	31.4	31.0	30.6	30.2	29.8	29.4
39	34.7	34.3	33.9	33.5	33.1	32.7	32.3	31.8	31.4	31.0	30.6
40	1 36.0	1 35.6	1 35.2	1 34.8	1 34.4	1 34.0	1 33.6	1 33.2	1 32.8	1 32.4	1 32.0
41	37.5	37.1	36.7	36.2	35.8	35.4	35.0	34.6	34.2	33.8	33.4
42	39.0	38.6	38.2	37.7	37.3	36.9	36.5	36.0	35.6	35.2	34.8
43	40.6	40.2	39.8	39.3	38.9	38.5	38.1	37.6	37.2	36.8	36.3
44	42.3	41.8	41.4	41.0	40.5	40.1	39.7	39.2	38.8	38.4	37.9
45	1 44.0	1 43.6	1 43.2	1 42.7	1 42.3	1 41.8	1 41.4	1 40.9	1 40.5	1 40.1	1 39.6
46	45.9	45.5	45.0	44.6	44.2	43.7	43.2	42.7	42.3	41.9	41.4
47	47.9	47.4	46.9	46.5	46.0	45.6	45.1	44.6	44.2	43.7	43.3
48	49.9	49.5	49.0	48.6	48.1	47.7	47.2	46.7	46.3	45.8	45.3
49	52.1	51.7	51.2	50.7	50.2	49.8	49.3	48.8	48.4	47.9	47.4
50	1 54.4	1 54.0	1 53.5	1 53.0	1 52.5	1 52.0	1 51.5	1 51.0	1 50.6	1 50.1	1 49.6

TABLE LXII.

CORRECTION TO AZIMUTHS OF POLARIS FOR EACH MONTH.

(From U. S. Land Survey Manual.)

For middle of—	Latitude. 25°.	40°.	55°.	For middle of—	Latitude. 25°.	40°.	55°.
	′	′	′		′	′	′
January....	− 0.3	− 0.4	− 0.5	July........	+ 0.2	+ 0.3	+ 0.4
February...	− 0.3	− 0.3	− 0.4	August.....	+ 0.1	+ 0.1	+ 0.2
March......	− 0.1	− 0.2	− 0.2	September..	0.0	− 0.1	− 0.1
April.......	0.0	0.0	0.0	October....	− 0.2	− 0.3	− 0.4
May........	+ 0.2	+ 0.2	+ 0.2	November..	− 0.5	− 0.6	− 0.7
June........	+ 0.2	+ 0.3	+ 0.4	December...	− 0.6	− 0.8	− 0.9

TABLE LXIII.

LOCAL MEAN TIME OF UPPER CULMINATION OF POLARIS.

Computed for Longitude 6 hours or 90° W. of Greenwich.

(From U. S. Land Survey Manual.)

Date.	1900	1901.	1902.	1903.	1904.	1905.	Diff. for 1 Day.
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	m.
Jan. 1	6 36.3	6 37.4	6 38.5	6 39.6	6 40.7	6 41.8	3.95
15	5 41.0	5 42.1	5 43.2	5 44.3	5 45.4	5 46.5	3.95
Feb. 1	4 33.9	4 35.0	4 36.1	4 37.2	4 38.3	4 39.4	3.95
15	3 38.6	3 39.7	3 40.8	3 41.9	3 43.0	3 44.1	3.95
Mar. 1	2 43.4	2 44.5	2 45.6	2 46.7	2 47.8	2 48.9	3.94
15	1 48.2	1 49.3	1 50.4	1 51.5	1 52.6	1 53.7	3.94
Apr. 1	0 41.3	0 42.4	0 43.5	0 44.6	0 45.7	0 46.8	3.94
15	23 42.4	23 43.5	23 44.6	23 45.7	23 46.8	23 47.9	3.93
May 1	22 39.5	22 40.6	22 41.7	22 42.8	22 43.9	22 44.0	3.93
15	21 44.6	21 45.7	21 46.8	21 47.9	21 49.0	21 50.1	3.92
June 1	20 38.0	20 39.1	20 40.2	20 41.3	20 42.4	20 43.5	3.92
15	19 43.2	19 44.3	19 45.4	19 46.5	19 47.6	19 48.7	3.92
July 1	18 40.5	18 41.6	18 42.7	18 43.8	18 44.9	18 46.0	3.92
15	17 45.7	17 46.8	17 47.9	17 49.0	17 50.1	17 51.2	3.92
Aug. 1	16 39.1	16 40.2	16 41.3	16 42.4	16 43.5	16 44.6	3.91
15	15 44.3	15 45.4	15 46.5	15 47.6	15 48.7	15 49.8	3.92
Sept. 1	14 37.6	14 38.7	14 39.8	14 40.9	14 42.0	14 43.1	3.92
15	13 42.7	13 43.8	13 44.9	13 46.0	13 47.1	13 48.2	3.92
Oct. 1	12 39.9	12 41.0	12 42.1	12 43.2	12 44.3	12 45.4	3.93
15	11 44.9	11 46.0	11 47.1	11 48.2	11 49.3	11 50.4	3.93
Nov. 1	10 38.1	10 39.2	10 40.3	10 41.4	10 42.5	10 43.6	3.93
15	9 42.9	9 44.0	9 45.1	9 46.2	9 47.3	9 48.4	3.94
Dec. 1	8 39.9	8 41.0	8 42.1	8 43.2	8 44.3	8 45.4	3.94
15	7 44.7	7 45.8	7 46.9	7 48.0	7 49.1	7 50.2	3.94

TABLE LXIV.

AZIMUTHS OF POLARIS.

(From U. S. Land Survey Manual.)

STAR AND AZIMUTH.

W. of N. when hour-angle is *less* than $11^h\ 58^m$; E. of N. when hour-angle is *greater* that $11^h\ 58^m$.
Time argument, the star's hour-angle (or $23^h\ 56^m.1$ *minus* the star's hour-angle).

POLARIS *above* THE POLE. To determine the true meridian, the azimuth will be laid off to the *east* when the hour-angle is *less* than $11^h\ 58^m$, and to the *west* when *greater* than $11^h\ 58^m$.

For the Year 1900.

POLARIS *below* THE POLE. To determine the true meridian, the azimuth will be laid off to the *east* when the hour-angle is *less* than $11^h\ 58^m$, and to the *west* when *greater* than $11^h\ 58^m$.

Above: 30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°	m.	h.	h.	m.	Below: 30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
° ′	° ′	° ′	°	° ′	° ′	° ′	° ′	° ′	° ′	° ′					° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2	4	0	11	54	0 1	0 1	0 1	0 2	0 2	0 2	0 2	0 2	0 2	0 2	0 2
3	3	3	3	3	4	4	4	4	4	4	8			50	3	3	3	3	3	3	3	4	4	4	4
5	5	5	5	5	5	6	6	6	6	6	12			45	5	5	5	5	5	5	5	5	6	6	6
6	6	6	7	7	7	7	8	8	8	8	16			41	6	6	6	6	7	7	7	7	8	8	8
8	8	8	8	9	9	9	9	10	10	11	21			37	8	8	8	8	8	8	9	9	9	10	10
9	10	10	10	10	11	11	11	12	12	13	25			33	9	9	9	10	10	10	11	11	11	12	12
11	11	11	12	12	12	13	13	14	14	15	29			29	11	11	11	11	12	12	12	13	13	14	14
12	13	13	13	14	14	15	15	16	16	17	33			25	12	12	13	13	13	14	14	14	15	15	16
14	14	15	15	15	16	16	17	18	18	19	37			21	14	14	14	15	15	15	16	16	17	17	18
15	16	16	17	17	18	18	19	20	20	21	41			16	15	15	16	16	17	17	18	18	19	19	20
17	17	18	18	19	19	20	21	21	23	23	46			12	17	17	17	18	18	19	19	20	21	21	22
19	19	19	20	21	22	22	23	23	24	25	50			8	18	18	19	19	20	20	21	22	22	23	25
20	21	21	22	22	23	24	24	25	26	27	54			4	20	20	20	21	22	22	23	23	24	25	26
22	22	23	23	24	25	25	26	27	28	29	58	0	11	0	21	21	22	23	23	24	24	25	26	27	28
23	24	24	25	26	26	27	28	29	30	32	2	1	10	55	23	23	24	24	25	25	26	27	28	29	30
25	26	26	27	28	28	29	30	31	33	34	7			50	24	25	25	26	27	27	28	29	30	31	32
27	27	28	29	30	31	32	33	34	35	37	13			45	26	27	27	28	29	29	30	31	32	34	35
29	29	30	31	32	33	34	35	36	38	39	18			40	28	29	29	30	31	31	32	33	35	36	37
31	31	32	33	34	35	36	37	38	40	42	23			35	30	30	31	32	33	34	35	36	37	38	40
32	33	34	35	36	37	38	39	41	42	44	28			29	32	32	33	34	35	36	37	38	39	41	42
34	35	36	37	38	39	40	42	43	45	47	34			24	33	34	35	36	37	38	39	40	41	43	44
36	37	38	39	40	41	42	44	45	47	49	39			19	35	36	37	37	39	40	41	42	43	45	47
38	39	40	41	42	43	44	46	47	49	51	44			14	37	38	39	39	40	41	43	44	46	47	49
39	40	41	42	44	45	46	48	50	52	54	49			8	39	39	40	41	42	43	45	46	48	49	51

Azimuths for Latitude.											For the Year 1900.				Azimuths for Latitude.										
30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°					30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	m.	h.	h.	m.	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0 41	0 42	0 43	0 44	0 46	0 47	0 48	0 50	0 52	0 54	0 56	55	1	10	3	0 40	0 41	0 42	0 43	0 44	0 45	0 47	0 48	0 50	0 52	0 54
43	44	45	46	47	49	50	52	54	56	0 59	0	2	9	58	42	43	44	45	46	47	49	50	52	54	56
45	46	47	48	49	51	52	54	56	58	1 1	5			53	44	45	46	47	48	49	51	52	54	56	58
46	47	49	50	51	53	54	56	58	1	3	10			47	45	46	47	48	50	51	53	54	56	0 58	1 0
48	49	50	51	53	54	56	58	1 0	3	5	16			42	47	48	49	50	51	53	54	56	58	1 0	2
50	51	52	53	55	56	0 58	1 0	2	5	8	21			37	49	50	51	52	53	55	56	0 58	1 0	2	5
51	52	54	55	57	58	1 0	2	4	7	10	26			31	50	51	52	53	55	56	0 58	1 0	2	4	7
53	54	55	57	58	1 0	2	4	6	9	12	32			26	51	53	54	55	57	58	1 0	2	4	6	9
54	55	57	58	1 0	2	4	6	8	11	14	37			21	53	54	56	57	58	1 0	2	4	6	8	11
56	57	58	1 0	2	3	6	8	10	13	16	42			15	55	56	57	58	1 0	2	4	5	8	10	13
57	59	1 0	2	3	5	7	9	12	15	18	48			10	56	57	59	1 0	2	3	5	7	10	12	15
59	1 0	2	3	5	7	9	11	14	17	20	53		9	5	58	0 59	1 0	2	3	5	7	9	11	14	17
1 0	2	3	5	7	8	11	13	16	19	22	58	2	8	59	59	1 0	2	3	5	7	9	11	13	16	19
2	3	5	6	8	10	12	15	17	20	24	4	3		54	1 0	2	3	5	6	8	10	12	15	18	21
3	4	6	8	10	12	14	16	19	22	26	9			49	2	3	5	6	8	10	12	14	17	20	23
5	6	8	10	12	13	16	18	21	24	28	16			42	3	5	6	8	10	12	14	16	19	22	25
6	8	9	11	13	15	18	20	23	27	30	23			35	5	6	8	9	11	13	16	18	21	24	27
8	9	11	13	15	17	19	22	25	28	32	29			29	7	8	10	11	13	15	17	20	22	26	29
9	11	13	14	16	19	21	24	27	30	34	36			23	8	9	11	13	15	17	19	22	24	28	31
11	13	14	16	18	20	23	25	29	32	36	43			15	10	11	13	14	16	18	21	23	26	29	33
12	14	16	17	20	22	25	27	31	34	38	51		8	7	11	13	14	16	18	20	23	25	28	31	35
14	15	17	19	21	24	26	29	32	36	40	59	3	7	59	13	14	16	18	20	22	25	27	30	33	37
15	17	19	21	23	25	28	31	34	38	42	7	4		51	14	16	18	19	21	24	26	29	32	35	39
17	19	21	23	25	27	30	33	36	40	44	17			40	16	18	19	21	24	26	28	31	34	38	42
19	21	23	24	27	29	32	35	39	43	47	30			28	18	19	21	23	26	28	31	33	37	40	44
20	22	24	26	29	31	34	37	41	45	49	43			15	20	21	23	25	27	30	33	35	39	43	47
22	24	26	28	30	33	36	39	42	47	51	57	4	7	1	21	23	25	27	29	32	35	37	41	45	49
24	26	28	30	33	35	38	41	45	49	54	23	5		35	23	25	27	29	32	34	37	40	43	47	52
26	27	30	32	34	37	40	43	47	51	56	30			24	25	27	29	31	34	36	39	42	46	50	54
27	29	31	33	36	39	42	45	49	53	58	41		6	10	27	29	31	33	36	38	41	44	48	52	57
1 29	1 30	1 32	1 35	1 37	1 40	1 43	1 47	1 50	1 55	1 59	55	5	5	......	1 29	1 30	1 32	1 35	1 37	1 40	1 43	1 47	1 50	1 55	1 59

Table LXI was computed with the mean place (declination) of Polaris for each year. A closer result will be had by applying to the tabular results the correction from Table LXII, which depends upon the difference of the mean and the apparent declinations of the star.

Table LXV.

REDUCTION OF TIMES IN TABLE LXIII TO INTERMEDIATE DATES.

(From U. S. Land Survey Manual.)

Subtract the reduction when computing from a *preceding*, or add it when working from a *following*, date.

Day of the Month.	Reduction. Arg.—" Difference for 1 Day."					No. of Days elapsed.
	m. 3.91.	m. 3.92.	m. 3.93.	m. 3.94.	m. 3.95.	
	m.	m.	m.	m.	m.	
2 or 16	3.9	3.9	3.9	3.9	3.9	1
3 or 17	7.8	7.8	7.9	7.9	7.9	2
4 or 18	11.7	11.8	11.8	11.8	11.8	3
5 or 19	15.6	15.7	15.7	15.8	15.8	4
6 or 20	19.5	19.6	19.6	19.7	19.7	5
7 or 21	23.5	23.5	23.6	23.6	23.7	6
8 or 22	27.4	27.4	27.5	27.6	27.6	7
9 or 23	31.3	31.4	31.4	31.5	31.6	8
10 or 24	35.2	35.3	35.4	35.5	35.5	9
11 or 25	39.1	39.2	39.3	39.4	39.5	10
12 or 26	43.0	43.1	43.2	43.3	43.4	11
13 or 27	47.0	47.0	47.2	47.3	47.4	12
14 or 28	50.8	51.0	51.1	51.2	51.3	13
29	54.7	54.9	55.0	55.2	55.3	14
30	58.6	58.8	58.9	59.1	59.2	15
31	62.6	62.7	62.9	63.0	63.2	16

313. Primary Azimuths.—

EXAMPLE OF RECORD OF AZIMUTH OBSERVATION AT ANY POSITION OF STAR.

(Station: West base, near Little Rock, Ark. Fauth 8″, theod. No. 300. December 27, 1888. 1 div. micr. = 2″. 1 div. level = 3″.)

Object.	Time P.M.	Level. West end.	Level. East end.	Micrometer. A.	Micrometer. B.	Mean.	Angle.
				Telescope direct.			
	h. m. s.	Div.	Div.	° ′ Div.	° ′ Div.	° ′ ″	° ′ ″
Polaris.	11 00 18	13.9	47.1	346 00 14.8	165 58 25.1	345 59 39.9	
		50.5	10.2				
		64.4	57.3				115 32 30.0
		+7.1					
E. base (mark).	....	....	...	101 32 18.1	281 31 21.8	101 32 09.9	
E. base (mark).	....	....	....	101 32 19.8	281 31 19.7	101 32 09.5	115 34 16.1
Polaris.........	11 09 20	50.4	10.3	345 58 22.0	165 57 01.4	345 57 53.4	
		13.8	46.5				
		64.2	56.8				
		+7.4					
				Telescope reverse.			
Polaris.........	11 17 14	50.5	10.1	211 28 29.0	31 27 23.4	211 28 22.4	
		12 9	46.6				
		63.4	56.7				115 35 53.8
		+6.7					
E. base (mark).	....	....	....	327 05 06.7	147 03 09.5	327 04 16.2	
E. base (mark).	....	....	....	327 04 26.3	147 03 00.6	327 03 56.9	115 37 08.8
Polaris	11 26 22	14.3	46.3	211 27 10.7	31 26 07.4	211 26 48.1	
		50.1	10.5				
		64.4	56.8				
		+7.6					

SUMMARY OF RESULTS.

(Station: West base, Arkansas. December 27, 1888.)

Individual Results.		Combined Results.	Individual Results.		Combined Results.
	° ′ ″ ″	° ′ ″		° ′ ″ ″	° ′ ″
First set....	294 10 34.2, 36.3 } 35.25 D.		Second set....	294 10 42.4, 45.4 } 43.90 D.	
	49.9, 34.8 } 42.35 R.	38.80		26.4, 40.8 } 33.60 R.	38.75
	35.9, 46.3 } 41.10 R.			49.1, 45.0 } 47.05 R.	
	41.8, 33.5 } 37.65 D.	39.38		40.3, 26.0 } 33.15 D.	40.10
			Grand mean................		294 10 39.26

314. Reduction of Azimuth Observations.—The time of observation of a star is first to be corrected for the difference in longitude, assuming that standard time has been used, and for the error of the watch. It is then reduced from mean to sidereal time. From the sidereal time of observation is to be subtracted the right ascension of Polaris, if that star is used, which is given in the Nautical Almanac, the result being the hour-angle or the sidereal time which has elapsed since it passed the meridian of the place of observation, given in hours, minutes, and seconds. This result is to be converted into degrees, minutes, and seconds. Then

$$\tan A = -\frac{a \sin t}{1 - b \cos t}, \quad . \quad . \quad . \quad . \quad (140)$$

where $a = \sec \phi \cot \delta$;

$$b = \frac{\tan \phi}{\tan \delta};$$

A = angle between true north and the star.

The angle between the star and the mark is to be corrected for level as follows:

$$\text{level corr.} = -\frac{d}{4}\Big[(w + w') - (e + e')\Big] \tan h. \qquad (141)$$

where d = value of a division of the level;

$w + w'$ = readings of west end of level-bubble;

$e + e'$ = readings of east end of level-bubble;

h = the angular elevation of pole-star.

EXAMPLE OF REDUCTION.

(Station: West base; December 27, 1888. Observer, S. S. G.)
Latitude = 34° 45′ 26″.8. Longitude 92° 13′ 31″.5.

Time of observation $= T_w$	=	11^h	00^m	18^s
Correction; ninetieth meridian time to 92°.215 .	= −		8	54
Watch slow; ninetieth meridian time	+			02
Local mean time, T_m	=	10	51	26
Correction; mean to sidereal time	=		+1	47
Right ascension mean sun		18	26	36
Sidereal time of observation	=	29	19	49
R. A. Polaris	−	1	18	25
Hour-angle, t	=	28	01	24
	−	24		
t (time)	=	4^h	01^m	24^s
t (arc)	=	60°	21′	00″

$$\tan A = -\frac{a \sin t}{1 - b \cos t}, \text{ where } a = \sec\phi \cot\delta,\ b = \frac{\tan\phi}{\tan\delta}$$

$\phi = 34° 45' 26''.8$	log sec = 0.0853539		log tan	= 9.8413076
$\delta = 88\ 43\ 11.9$	log cot = 8.3491690		log tan	= 1.6508310
log a	= 8.4345229	log b		= 8.1904766
log sint t 60° 21′ 00″	= 9 9390515	log cos t		= 9.6943423
log a sin t	= 8.3735744	log − .0076704		= 7.8848189
log (1 − b cos t)	= 9.9966559	+ 1.0000000		
log tan A 178° 38′ 08″.0	= 8.3769185	0.9923296		$= 1 - b\cos t$
angle to mark +115 32 30. 0				
Level corr. −3. 8				
Az. of mark = 294° 10′ 34″.2				

$$= -\frac{d}{4}\{w + w') - (e + e')\} \tan h.$$

$$= \frac{3''.1}{4} \times \frac{\text{Div.}}{7.1} \times .694 = -3''.8$$

315. Azimuth at Elongation.—When observations for azimuth are to be made at elongation, it is necessary to know the mean time of elongation. This is computed by obtaining the hour-angle at elongation from the following equation:

$$\cos t_e = \tan\phi \cot\delta. \quad . \quad . \quad . \quad . \quad . \quad (142)$$

The hour-angle plus the right ascension of the star gives the sidereal time of its western elongation, which, reduced to mean time, gives the local mean time in question.

The azimuth of a pole-star at elongation is determined by the use of the equation

$$\sin A = \sec \phi \cos \delta. \quad . \quad . \quad . \quad . \quad . \qquad (143)$$

EXAMPLE OF COMPUTATION OF THE AZIMUTH AT ELONGATION, AND THE LOCAL MEAN TIMES OF BOTH ELONGATIONS OF POLARIS.

(Latitude $= \phi = 40°$. Meridian of Washington. November 28, 1891.)

Sine, Azimuth at elongation $=$ sec. $\phi \cos \delta$.

log sec 40°		$=$ 0.1157460
log cos δ	88° 44′ 05″.5	$=$ 8.3439803
log sine A	1 39 05. 8	$=$ 8.4597263

cos hour-angle at elongation, t_e, $= \tan \phi \cot \delta$

log tan 40°		$=$ 9.9238135
log cot δ	88° 44′ 05″.5	$=$ 8.3440862
log cos t_e	88 56 17. 5	$=$ 8.2678997

$t_e = 5^h\ 55^m\ 45^s.2$.

Sidereal time western elongation, $T_s =$ R. A. Polaris $+ t_e$.

R. A. Polaris .... $=$	1^h	19^m	35.2^s
$t_e =$	5	55	45.2
Sidereal time western elongation, $T_s =$	7	15	20.4
R. A. mean sun, a_s .... $=$	16	29	14.4
Sidereal interval before noon, I..... $=$	9	13	54.0
Correction sidereal to mean interval $=$	$-$	1	30.7
Mean interval before noon........	9	12	23.3 Nov. 28.
Local mean time, western elongation $=$	2	47	36.7 A.M., Nov. 28.
Sidereal time E. elongation $= 24^h + a - t_e =$	19^h	23^m	50.0^s
$a_s =$	16	29	14.4
Sidereal interval after noon, I.......... $=$	2	54	35.6
Correction sidereal to mean interval.... $=$	$-$	0	28.6
Local mean time eastern elongation..... $=$	2	54	07.0 P.M., Nov. 28.
Local mean time western elongation .... $=$	2	47	36.7 A.M., Nov. 28.

For longitudes west of Washington decrease times of elongation $0^s.66$ for each degree.

CHAPTER XXXIV.

LATITUDE.

316. Methods of Determining Latitude.—1. The most precise method known for determination of a terrestrial latitude is by measuring small *differences of zenith distances of two stars* with zenith telescope. (Art. 319.)

2. The simplest method is by measuring the *meridian zenith distance* or *altitude of a known star*, though the result is relatively approximate only. It is only essential to follow a star near meridian until its altitude is greatest. The formula is

$$z = z_1 + R,$$

and

$$\phi = \delta \pm z, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (144)$$

sign of z depending on whether the star is north or south of the zenith.

3. If the *time* be *known*, latitude may be determined by a *single measured altitude of the sun* or *a star*. (Art. 318.) This method gives fairly approximate results when time is known by a chronometer or watch to within two or three seconds, and is very useful in exploratory work.

4. *Time* being *known*, latitude may be simply and quite

accurately determined by *measuring circummeridian altitudes of Polaris;* this consists in applying the third method to Polaris. Then

$$\phi = h - p \cos t + \tfrac{1}{2}p^2 \sin 1''. \sin^2 t . \tan h, \quad . \quad (145)$$

in which p = polar distance of Polaris or complement of δ in seconds, which is about 5400″. Tables for finding p and $\frac{1}{2}p^2 \sin 1''$ are given in the American Ephemeris. The best time of observation is when the star is at one of the culminations. This method is especially adapted to the instruments available to the topographer, namely, a good theodolite or engineer's transit and a good timepiece.

5. Approximate latitude may be determined from an *observation on the sun at noon.* (Art. 317.) This is a very useful method for the explorer or land surveyor.

317. Approximate Solar Latitude.—The following is a method of obtaining the approximate latitude from an observation on the sun at noon:

Measure two altitudes, one of the upper and the other of the lower limb of the sun, commencing before noon and watching until the sun has reached its highest altitude. In order to eliminate errors of collimation, these two observations should be made on each limb with the telescope direct and inverted.

Let r = refraction;
h = altitude of sun's center;
ϕ = latitude;
δ = sun's declination at time of observation.

The declination is taken from the Nautical Almanac for the date of observation, and increased or diminished by the hourly difference multiplied by the longitude from the locus of the almanac, expressed in hours. Then

$$\phi = 90° - (h - r - \delta). \quad . \quad . \quad . \quad . \quad (146)$$

EXAMPLE.

April 16, 1898. Approximate longitude 136° 20′, measured from map.

Vertical circle reads, when pointing at sun's upper limb.........		36° 55′
Vertical circle reads, when pointing at sun's lower limb.........		36 23
Mean..		36° 39′
Mean correction for index error..................................	−	1′
Apparent altitude..		36° 38′
Refraction, always negative, enter table with arguments apparently negative..	−	1′
Altitude of sun's center..................................		36° 37′
Sun's declination, April 16—Greenwich noon; from almanac....	−	10° 14′
Hourly change from almanac = 53″, multiply by longitude and divide by 15: $\frac{53'' \times 136}{15} = 473''$.................. =	−	8′
Altitude of celestial equator..............................		26° 15′
Subtract from 90° gives latitude..........................		63° 45′

A result to be relied upon within 1′ or 2′, supposing the vertical circle and collimation correct to within the same amount.

318. Latitude from an Observed Altitude.—Latitude may be determined at sea or on an exploratory survey by measuring the altitude of a star or of the sun with a sextant, theodolite, or altazimuth. For this operation the time must be known, though the object observed may be in any position. The formula applicable is

$$\tan D = \tan \delta \sec t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (147)$$

$$\cos(\phi - D) = \sin h \sin D \operatorname{cosec} \delta, \quad . \quad . \quad . \quad (148)$$

in which δ = declination of star;

t = hour angle of star;

D = auxiliary angle taken to simplify computation—it should be less than 90° and + or − according to algebraic sign of the tangent;

h = altitude resulting from measurement after applying all corrections.

Although $\phi - D$ may be positive or negative, the latitude of the place ϕ is generally known with sufficient accuracy to decide this.

The altitude h must be corrected for instrumental errors (Arts. 323 and 324), refraction (Arts. 322 and 325), and, in the case of the sun, for parallax and semi-diameter (Art. 301).

319. Astronomic Transit and Zenith Telescope.—For the determinations of time and latitude separate transit instruments and zenith telescopes are sometimes employed. The *astronomic transit* is designed primarily for the determination of time when the telescope is in the plane of the meridian. Its essential parts are a telescope, an axis of revolution at right angles to the telescope, the supports for both, and a striding-level for the determination of the inclination of the axis. A *zenith telescope* is a somewhat differently constructed instrument provided with a large vertical circle and delicate level, and with a horizontal circle which turns with the upper part of the instrument much as does a theodolite.

The most compact and useful instrument for determination of both latitude and time is a *combination transit and zenith telescope*, such as is used by the U. S. Geological Survey (Fig. 181). This embodies the latest improvements in both instruments. It consists of a circular base resting upon three leveling screws, and upon this base the whole instrument may revolve when in use as a zenith telescope. About the base is a large graduated circle, provided with micrometer screw for slow motion to be used in setting the instrument and in adjusting it in azimuth. The telescope of the above instrument has a focal distance of 27 inches, a clear aperture of 2.5 inches, and its magnifying power with diagonal eyepiece is 74 diameters. For use as a zenith telescope there is attached a vertical circle reading by vernier to 20″, to which is fastened a delicate level. In the focus of the object-glass is a thread movable by means of a micrometer screw for the measurement of differences of zenith distances.

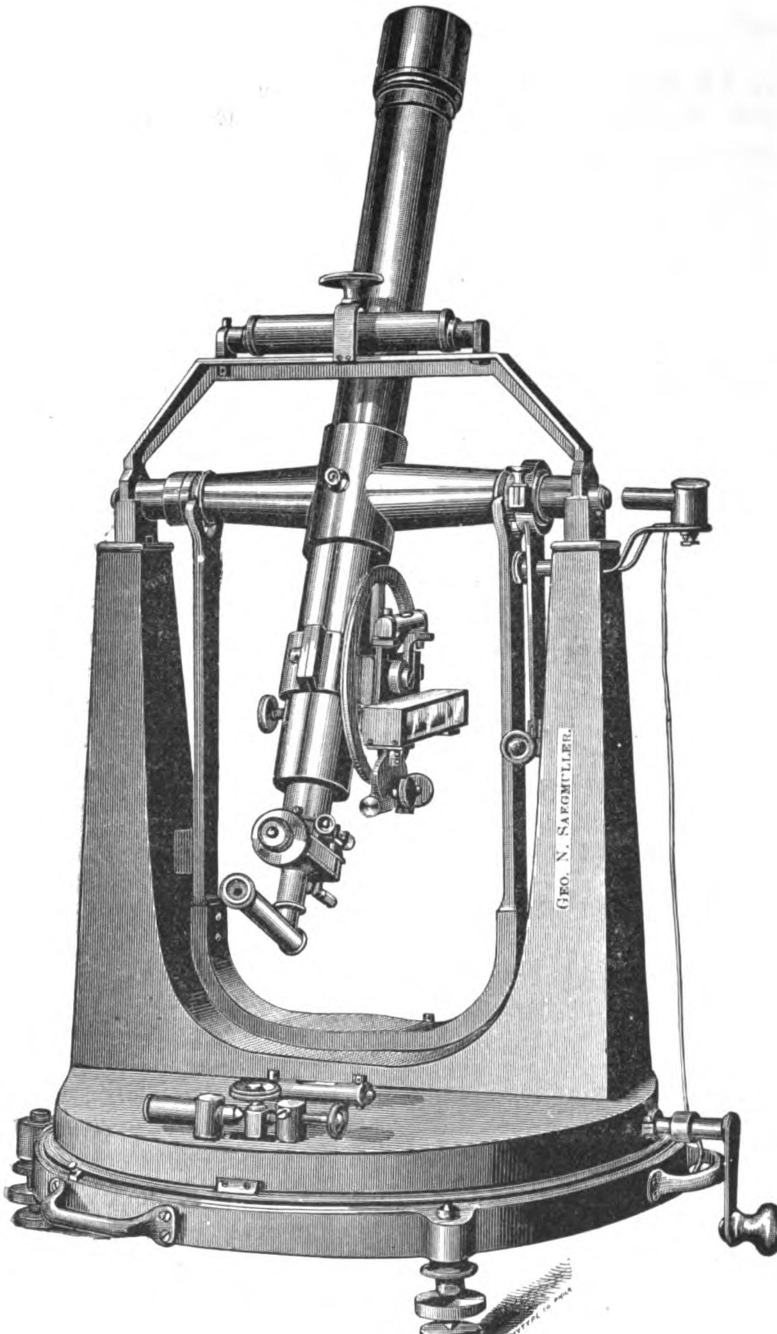


FIG. 181.—ASTRONOMIC TRANSIT AND ZENITH TELESCOPE.

For use as a transit the telescope is provided with a delicate striding-level for measurement of inclination of the axis, and a reversing apparatus for turning the telescope in the wyes. The stationary reticule in the focus of the instrument consists of five threads for observing as many transits of the star. The reticule is illuminated by lamps, the light of which enters the hollow axis of the telescope and is reflected by a mirror into the eye.

320. Latitude by Differences of Zenith Distances of Two Stars.—The *zenith distance* of a star on the meridian is the difference between the latitude of the station of observation and the declination of the star; therefore the measurement of the meridional zenith distance of a known star furnishes a determination of the latitude. The most accurate method of determining the latitude of a place, and that generally employed in geodetic operations, is that known as the Horrebow-Talcott method. In this, instead of the measurement of the absolute zenith distance of the star, the *small difference of zenith distances* of two stars culminating at about the same time on opposite sides of the zenith is measured. Then

$$z = \phi - \delta \quad \text{and}$$

$$z' = \delta' - \phi, \quad \text{hence}$$

$$\phi = \tfrac{1}{2}(\delta + \delta') + \tfrac{1}{2}(z - z'). \quad . \quad . \quad . \quad (149)$$

This method therefore requires that the difference $(z - z')$ be measured. The stars must be so chosen that $(z - z')$ may be measured by means of the micrometer in the telescope. A measurement of the latitude within $5''$ is possible by this method with a theodolite having full vertical circle. If now z' refer to the northern star, $(z - z')$ in terms of the observed micrometer readings becomes $(M - M')r$, in which r is the angular value of one turn of the micrometer screw.

321. Errors and Precision of Latitude Determinations.—Latitude determinations by zenith telescope or transit-zenith telescope are subject to errors of three general kinds:

1. External errors;
2. Instrumental errors; and
3. Observer's errors.

No attempt will be made here to fully discuss these errors, as the kind of work which this volume is intended to explain is not of such high quality as either to warrant their correction or a reduction of the results by least-square methods. These are fully explained in the more extended treatises already referred to.

The *external errors* are those due to abnormal refraction, star places, and to defective declinations. The latter have probable errors sufficiently large to account for more than half of the error in the final result. The errors in the computed differential refractions are probably very small. *Observer's errors* are those made in bisecting a star and in reading the level and micrometer. They are of the kind known as personal errors or those due to *personal equation*. *Instrumental errors* include those due to inclination of the micrometer line to the horizontal, to an erroneous level value, to inclination of the horizontal axis, to erroneous placing of the azimuth stops, to error of collimation, to an erroneous mean value of the micrometer screw, and to instability of relative positions of different parts of the instrument. The errors of the first and second sources are small, but must be carefully guarded against. In the first instance the observer should study to make the bisection in the middle of the field. If the error from using an erroneous level is small, the level corrections will be small.

In *planning a series of observations* the observer must determine the quality of the result desired which will fix for him as to how many observations shall be made and how many separate pairs observed. Increasing either of these increases the cost of field- and office-work. The ratio of observation to pairs should be such as to give a maximum accuracy for a given expenditure. Extremes of practice are given by Hayford as 210 observations on 30 pairs each observed on seven nights, and 100 observations on 100 pairs each observed but once. The first is the practice of the U. S. Coast and Geodetic Survey. The practice of the U. S. Geological Survey is 100 observations on 20 pairs on each of five nights.

322. **Field-work of Observing Latitude.**—The following description and example of the field-work of observing latitude is taken chiefly from Gannett's "Manual of the Topographic Methods of the U. S. Geological Survey." Before commencing the field-work a *list of pairs of stars* must be prepared, each pair of which shall have such zenith distances that they will culminate at nearly equal distance, one north and the other south of the zenith. Lists of such stars are published in the British Association Catalogue, various Greenwich catalogues, Safford's Catalogue of the Wheeler Survey, and in various miscellaneous publications giving star lists prepared for special surveys. To prepare such a star list it is necessary to know approximately the latitude of the station and the right ascensions and declinations of the stars. When the declination of a star is known, the zenith distance is obtained by subtracting the latitude of the place from its declination. The stars selected are such as culminate within a few minutes of one another and should be observed consecutively. In selecting them by pairs, therefore, only sufficient interval of time should be left between pairs to allow of the setting of the instrument.

At the beginning of the observation the *instrument* should be placed *in* the *line of* the *meridian* and *carefully collimated.* At the approach to the meridian of the first star of the pair, the instrument should be set for it by the vertical circle, the spirit-level upon that circle being made as nearly level as possible. As the star traverses the field of the telescope, the movable *thread in the reticule* is kept upon it by means of a micrometer screw until it crosses the middle vertical thread, then the micrometer and the divisions of the level-bubble are read. Immediately, without disturbing the setting of the telescope, the entire *instrument is revolved* through 180° on its horizontal axis, when it will point to the other side of the zenith at the same angle as before and will then be set for the opposite star. As this approaches culmination the same operation is performed as before, reading the micrometer and the level again.

For the determination of a latitude at least 20 such pairs of stars should be observed each evening, and the same pairs, if possible, should be observed upon several other evenings. The following example is taken from the observations at Rapid, South Dakota.

EXAMPLE.—LIST OF STARS, FOR OBSERVATION WITH ZENITH TELESCOPE.

(Station: Rapid, South Dakota. Approximate Latitude: 44° 05'.)
(From Gannett's Manual.)

Name or Number. Safford's Catalogue.	Mag.	Class.	R. A.	Dec.	Zen. Dist.	Setting.
			h. m.	° '	° '	° '
7 Lacertæ...	4.0	A A	22 27	49 43	5 38 N.	5 37 N.
10 Lacertæ..	5.0	A A	22 34	38 29	5 36 S.	
1539........	6.5	B	22 41	45 37	1 32 N.	1 27 N.
1551........	6.5	A	22 47	42 42	1 23 S.	
1565........	6.5	C	22 52	38 42	5 23 S.	5 22 S.
1579	5.0	A	22 59	49 26	5 21 N.	
1600........	6.0	A	23 08	56 34	12 29 N.	12 19 N.
1633........	6.7	B	23 18	31 56	12 09 S.	
1676.......	5.6	A	23 42	67 12	23 07 N.	23 05 N.
1686.......	6.5	A	23 47	21 03	23 02 S.	
1702........	4.5	A	23 52	24 32	19 33 S.	19 31 S.
1722........	6.5	B	24 00	63 35	19 30 N.	

EXAMPLE.—RECORD OF OBSERVATION.

(Station: Rapid, South Dakota. Date: November 9, 1890. Instrument: Fauth combined transit and zenith telescope No. 534. Observer: S. S. G. Recorder: A. F. D.)
(From Gannett's Manual.)

Star Name or Number.	N. or S.	Micrometer Reading.	Diff.	Level. N.	Level. S.	(N + S) −(N′+S′)	Remarks.
		Rev.	*Rev.*	*Div.*	*Div.*	*Div.*	
7 Lacertæ...	N.	26.256		39.9	16.7	+56.6	
10 Lacertæ..	S.	24.052	− 2.204	26.5	49.7	−76.2	
						−19.6	
1539........	N.	30.432		42.0	18.7	+60.7	
1551........	S.	20.095	− 10.337	21.9	45.0	−66.9	
						− 6.2	
1565........	S.	25.164		14.1	37.6	−51.7	Faint.
1579........	N.	26.703	+ 1.539	38.1	15.0	+53.1	Distinct.
						+ 1.4	
1600........	N.	32.214		37.5	14.1	+51.6	
1633........	S.	16.033	− 16.181	19.9	43.1	−63.0	Faint.
						−11.4	
1676........	N.	26.656		51.0	28.0	+79.0	
1686........	S.	17.684	− 8.972	17.0	39.6	−56.6	
						−22.4	
1702........	S.	25.345		18.0	40.9	−58.9	
1722........	N.	23.722	+ 1.623	36.0	13.2	+49.2	
						− 9.7	

323. Determination of Level and Micrometer Constants.—Before proceeding with the reduction of latitude observations, it is necessary to investigate the constants of the instrument, to ascertain the value of a division of the latitude level, and of a division of the head of the micrometer screw.

The *value of a division of* the head of the *micrometer screw* is measured by observing the transits of some close circumpolar star, when near elongation, across the movable thread; setting the thread repeatedly at regular intervals in advance of the star, and taking the time of its passage, with the reading of the micrometer. The precaution should be taken to read the latitude level occasionally and correct for it if necessary. This correction, which is to be applied to the observed time, is equal to one division of the level, in seconds of time, divided by the cosine of the declination of the star and multiplied by the level error, the average level reading being taken as the standard.

The *time from elongation of* the *star* requires a correction in order to reduce the curve in which the star apparently travels to a vertical line. The *hour-angle of* the *star* is first obtained from the equation

$$\cos t_0 = \cot \delta \tan \phi, \quad . \quad . \quad . \quad . \quad (150)$$

δ being the star's declination, and ϕ the latitude.

The *chronometer time of elongation*,

$$T_0 = \alpha - t_0 - \delta t, \quad . \quad . \quad . \quad . \quad . \quad . \quad (151)$$

in which α is the right ascension of the star obtained from the American Ephemeris, and δt the error of the chronometer.

Having thus obtained the chronometric time of elongation, the *correction* in question is *obtained from* the *observed interval of time* of each observation before or after elongation, from tables in Appendix No. 14, U. S. Coast and Geodetic Survey Report for 1880, pp. 58 and 59, and in part in the following articles (Tables LXVII to LXX). A discussion of this subject will be found in the appendix above referred to, in Hayford's Geodesy, pp. 174 to 181, and in Chauvenet's Astronomy, vol. II. pp. 360 to 364.

The *times of observation thus corrected* for level and distance from elongation, are then grouped in pairs, selected as being a certain number of revolutions of the micrometer apart, and the time intervals between the members of each pair obtained. The mean of these, divided by the sum of revolutions which separate the members of each pair, is yet to be *corrected for differential refraction*, which is derived from the following equation:

$$R = 57''.7 \sin r \sec^{\cdot} Z, \quad . \quad . \quad . \quad . \quad (152)$$

EXAMPLE.—DETERMINATION OF VALUE OF ONE REVOLUTION OF MICROMETER BELONGING TO Z. T. No. 534, by Observations on 51 Cephei near Eastern Elongation, November 15, 1890, Rapid, South Dakota.

log. cot. δ....87° 12′ 50″.8	= 8.6871826	log 1 div....	= 1.33 = 0.12385		h. m. s.		h. m. s.
log. tan. ϕ....44 04 45	= 9.9860380	a. c. log. cos. δ........	= 1.31334	t_0 =	5 49 11.8	T_0 =	1 00 08.6
log. cos. t_0...87 17 57	= 8.6732206	a. c. log. 15................	= 8.82391	a =	6 49 20.4	Δt =	−8.0
		log. 1ˢ.82....................	= 0.26110	T_0 =	1 00 08.6	chron. time elong. =	1 00 00.6

(From Gannett's Manual.)

Time of Observation (recorded on chronograph sheet.)	Micrometer Revolutions.	Level. N.	Level. S.	Level. N. + S.	Reduction to Mean Level.	Corr. for Change of Level.	Time from Elongation = t.	Correction for t.	Reduced Time.	Revolutions.	Time for Nine Revolutions.
h. m. s.		Div.	Div.	Div.	Div. s.	s.	m. s.	s.	h. m. s.		m. s.
00 40 19.7	31.0	37.0	16.0	53.0	+0.8×1.82=	+1.5	−19 40 9	+1.4	00 40 22.6	31.0 and 22.0	15 37.7
41 12.3	30.5					+1.3	18 48.3	+1.3	41 14.9	30 5 21.5	36.6
(lost on chronograph sheet.)	30.0	37 1	16.1	53.2	+0.6	+1.1				29.5 20.5	34 4
42 53.7	29.5					+0.8	17 06.9	+0.9	42 55.4	29.0 20.0	39.2
43 44.6	29 0					+0.5	16 16.0	+0 8	43 45.9	28.5 19.5	40.2
44 35.9	28.5					+0.3	15 24.7	+0.7	44 36.9	28.0 19.0	36.9
45 31.9	28 0					+0.0	14 28 7	+0.5	45 32.4	26.5 17.5	36.8
(missed)	27 5					−0.3				26.0 17 0	37.5
(missed)	27 0					−0.5				25.5 16.5	36.2
48 08.0	26.5					−0.8	11 52.6	+0.3	48 07.5	25.0 16.0	29 6
48 57.1	26.0	38.3	17.5	55.8	−0 6	−1.1	11 03.5	+0.2	48 56 2	24.5 15.5	33.1
49 50.2	25.5					−1.1	10 10.4	+0.2	49 49.3	23.5 14 5	36.5
50 47.0	25 0					−1.0	9 13 6	+0.1	50 46.1	23 0 14.0	31.3
51 42.3	24 5					−1.0	8 18 3	+0.1	51 41.4	22.5 13.5	35.2
(missed)	24.0					−0.9					15 35.80 = mean
53 22.4	23.5	38.3	17.3	55.6	−0.5	−0.9	6 38.2	+0.1	53 21.6		
54 18 8	23.0					−0.9	5 41.8	+0.0	54 17.9		
55 07.1	22.5					−0.9	4 53.5	0.0	55 06.2		
56 01.2	22.0					−0.9	3 59.4	0.0	56 00.3		
56 52.4	21.5	38.3	17.3	55.6	−0.5	−0.9	3 08.5	0 0	56 51.5		
57 41.2	21.0					−0 9	2 19.4	0.0	57 40.3		
58 30.7	20.5					−0.9	1 29.9	0.0	58 29.8		
59 25.9	20.0					−0.8	−0 34.7	+0.0	59 25.1		
1 00 17 9	19.5					−0.8	+0 17.3	−0.0	1 00 17 1		
01 10.0	19.0					−0 7	1 09.4	0.0	01 09.3		
(missed)	18.5					−0 6		0.0			
02 52.8	18.0					−0.6	2 52.2	0.0	02 52.2		
03 44.8	17 5	38.0	17.0	55.0	−0.3	−0 5	3 44.2	0.0	03 44.3		
04 34.2	17.0					−0.5	4 33.6	0.0	04 33.7		
05 25.9	16.5					−0.4	5 25.3	0.0	05 25.5		
06 16.0	16.0					−0.3	6 15.4	0.0	06 15.7		
07 14.8	15.5					−0.2	7 14 2	−0.1	07 14 5		
08 05.1	15.0					−0.1	8 04.5	−0 1	08 04 9		
08 58.2	14.5	37.8	16.8	54.6	−0.0	−0.0	8 57.6	−0.1	08 58.1		
09 49.1	14.0					+0.3	9 48.5	−0.2	09 49.2		
10 41.0	13.5					+0.6	10 40.4	−0.2	10 41.4		
11 35 0	13.0	37.2	16.2	53.4	+0.5	+0.9	11 34.4	−0 3	11 35.6		
Mean level....				=54.5							

	s.
log...............935.80	=2.97118
log. cos. δ....	8.68666
log. 15................	1.17659
a. c. log. 9........ ...	9.04576
log. R.........75″.804	=1.87969
Refraction....— .044	
One rev'lut'n=75.760	

DIFF. REFRACTION.

log ....57″.7	=1.7612
log. sin ....75 .8	=6.5652
log. sec. z	0.1568
log. sec. z	0.1568
Diff. refr.. ... .=″.044 log.	=8.6400

r being the value of a division of the micrometer, and Z the zenith distance of the star. Four-place logarithms (Tables V and VI) are sufficient for computing this correction, as it is small. On the preceding page is given an example of record and computation of the value of a revolution of the micrometer of combined instrument No. 534 of the Geological Survey.

If d be the value of one division of the latitude level, and n and s the north and south readings; then if the numbering of the level-tube graduation increases each way from the middle, the inclination of the vertical axis i is

$$i = \frac{d}{4}[(n + n') - (s + s')] \quad . \quad . \quad . \quad . \quad (153)$$

The *value of a division of the level* is commonly measured with a level-trier. The latitude level may, however, be easily measured by means of the micrometer, the value of a revolution of that being obtained by the following method:

Point the telescope upon some well-defined terrestrial mark and set the level at an extreme reading near one end of the tube. Set the movable thread upon the object and read the micrometer and the level. Now move the telescope and level until the bubble is near the other end of the tube. Again set the movable thread upon the object and again read both micrometer and level. It is evident that the micrometer and the level have measured the same angle, and that the ratio between these readings equals that between a revolution of the micrometer and a level division. (See Example next page.)

Secondly, when the star is observed off the line of collimation, the instrument remaining in the plane of the meridian, then

$$m = \frac{2\sin^2 \frac{1}{2}r}{\sin 1''} \sin \delta \cos \delta, \quad \text{or} \quad m = \frac{2\sin^2 \frac{1}{2}r}{\sin 1''} \cdot \tfrac{1}{2} \sin 2\delta, \quad (154)$$

and the correction to the latitude is half of this quantity.

EXAMPLE.—DETERMINATION OF VALUE OF ONE DIVISION OF LATITUDE LEVEL No. 584.

(By comparison with micrometer screw 534.)

(From Gannett's Manual.)

Micrometer.	Level.		Difference.		*aa.*	*ab.*
	N.	S.	Microm.	Level.		
r.	*d.*	*d.*	*b.* *d.*	*a.* *d.*		
8.025	47.3	29.2				
8.508	20.7	02.7	48.3	26.55	704.9	1283
8.509	18.9	01.0				
7.984	49.8	31.0	52.5	30.45	927.2	1599
8.511	18.5	00.6				
8.045	47.2	29.1	46.6	28.60	818.0	1333
9.076	18.7	00.8				
8.604	46.0	28.0	47.2	27.25	742.6	1286
9.442	23.7	06.0				
9.009	48.0	30.0	43.3	24.15	583.2	1046
10.055	21.8	04.0				
9.574	48.0	30.1	48.1	26.15	683.8	1258
10.661	24.0	06.1				
10.212	50.7	33.0	44.9	26.80	718.2	1203
11.771	18.3	00.7				
11.252	48.3	31.9	51.9	30.60	936.4	1588
12.328	20.0	02.3				
11.872	46.1	28.5	45.6	26.15	683.8	1192
12.869	22.2	04.6				
12.438	47.7	30.0	43.1	25.45	647.7	1097
13.468	23.0	05.3				
13.080	44.5	26.9	38.8	21.55	464.4	836
14.146	20.1	02.4				
13.702	45.4	27.8	44.4	25.35	642.6	1125
14.758	22.3	04.8				
14.282	48.6	31.0	47.6	26.25	689.1	1249
Sum..	....	....	....	....	9241.9	16095

log .... 16095. = 4.20669
a. c. log .... 9241.9 = 6.03424
log 1 div. micrometer .... = 9.87966

1 div. level .... = 1″.320 log = 0.12059

whether the star be north or south; and if the two stars forming a pair are observed off the line of collimation, two such corrections, separately computed, must be added to the latitude. If the stars should be south of the equator, the essential sign of the correction is negative. The value of m for every 5° of declination is given in the following table:

Table LXVI.

VALUES OF m FOR EVERY 5° δ.

	10s.	15s.	20s.	25s.	30s.	35s.	40s.	45s.	50s.	55s.	60s.	
δ	"	"	"	"	"	"	"	"	"	"	"	δ
5°	.00	.01	.02	.03	.04	.06	.08	.10	.12	.14	.17	85°
10	.01	.02	.04	.06	.08	.11	.15	.19	.23	.28	.34	80
15	.01	.03	.05	.09	.12	.17	.22	.28	.34	.41	.49	75
20	.02	.04	.07	.11	.16	.22	.28	.36	.44	.53	.63	70
25	.02	.05	.08	.13	.19	.26	.34	.42	.52	.63	.75	65
30	.02	.05	.09	.15	.21	.29	.38	.48	.59	.71	.85	60
35	.03	.06	.10	.16	.23	.31	.41	.53	.64	.77	.92	55
40	.03	.06	.11	.17	.24	.33	.43	.54	.67	.81	.97	50
45	.03	.06	.11	.17	.25	.33	.44	.55	.68	.82	.98	45

Reduction of Observations on Close Circumpolar Stars, Made in Determining the Value of a Revolution of the Micrometer.—Let t = difference of time of observation and elongation of the star expressed in seconds, and z'' = number of seconds of arc in the direction of the vertical from elongation, then

$$z'' = \frac{\cos\delta \sin t}{\sin 1''},$$

for which we can write

$$z'' = 15 \cos\delta\{t - \tfrac{1}{6}(15 \sin 1'')^2 t^3\}. \quad . \quad . \quad (155)$$

It is convenient to apply the term $\frac{1}{6}(15 \sin 1'')^2 t^3$ to the observed time of noting either elongation, additive to the observed time before, and subtractive after. The following table gives the value of $\frac{1}{6}(15 \sin 1'')^2 t^3$, also of the additional term $-\frac{1}{120}(15 \sin 1'')^4 t^5$ when sensible, for every minute of time from elongation to 65^m.

Table LXVII.

REDUCTION OF OBSERVATIONS ON CLOSE CIRCUMPOLAR STARS.

(From Appendix 14, U. S Coast and Geodetic Survey Report for 1880.)

t	Term.	t	Term.	t	Term.	t	Term.	t	Term.	t	Term.
m.	s.	m.	s.	m.	s.	m.	s.	m.	s.	m.	s.
6	0.0	16	0.8	26	3.3	36	8.9	46	18.5	56	33.3
7	0.1	17	0.9	27	3.7	37	9.6	47	19.7	57	35.1
8	0.1	18	1.1	28	4.2	38	10.4	48	21.0	58	37.0
9	0.1	19	1.3	29	4.6	39	11.3	49	22.3	59	39.0
10	0.2	20	1.5	30	5.1	40	12.2	50	23.7	60	41.0
11	0.2	21	1.8	31	5.7	41	13.1	51	25.2	61	43.1
12	0.3	22	2.0	32	6.2	42	14.1	52	26.7	62	45.2
13	0.4	23	2.3	33	6.8	43	15.1	53	28.3	63	47.4
14	0.5	24	2.6	34	7.5	44	16.2	54	29.9	64	49.7
15	0.6	25	3.0	35	8.2	45	17.3	55	31.6	65	52.1

324. Corrections to Observations for Latitude by Talcott's Method.—*Correction for Differential Refraction.*—The difference of refraction for any pair of stars is so small that we can neglect the variation in the state of the atmosphere at the time of the observation from that mean state supposed in the refraction tables. The refraction being nearly proportional to the tangent of the zenith distance, the difference of refraction for the two stars will be given by

$$R - R' = 57''.7 \sin(z - z') \sec^2 z; \quad . \quad . \quad (156)$$

and since the difference of zenith distances is measured by the micrometer, the following table of correction to the latitude for differential refraction has been prepared for the argument ½ difference of zenith distance, or ½ difference of micrometer reading, on the side, and the argument "zenith distance" on the top. The sign of the correction is the same as that of the micrometer difference.

TABLE LXVIII.

CORRECTION FOR DIFFERENTIAL REFRACTION.

(From Appendix 14, U. S. Coast and Geodetic Survey Report for 1880.)

½ Diff. in Zenith Distance.	Zenith Distance.					
	0°	10°	20°	25°	30°	35°
′	″	″	″	″	″	″
0	.00	.00	.00	.00	.00	.00
0.5	.01	.01	.01	.01	.01	.01
1	.02	.02	.02	.02	.02	.02
1.5	.02	.03	.03	.03	.03	.03
2	.03	.03	.04	.04	.04	.05
2.5	.04	.04	.05	.05	.05	.06
3	.05	.05	.06	.06	.07	.08
3.5	.06	.06	.07	.07	.08	.09
4	.07	.07	.08	.08	.09	.10
4.5	.08	.08	.09	.09	.10	.11
5	.08	.09	.10	.10	.11	.13
5.5	.09	.10	.10	.11	.12	.14
6	.10	.10	.11	.12	.13	.15
6.5	.11	.11	.12	.13	.14	.16
7	.12	.12	.13	.14	.15	.18
7.5	.13	.13	.14	.15	.16	.19
8	.13	.14	.15	.16	.18	.21
8.5	.14	.15	.16	.17	.19	.22
9	.15	.16	.17	.18	.20	.23
9.5	.16	.17	.18	.20	.21	.24
10	.17	.18	.19	.21	.23	.26
10.5	.18	.19	.20	.22	.24	.27
11	.18	.19	.21	.23	.25	.28
11.5	.19	.20	.22	.24	.26	.30
12	.20	.21	.23	.25	.27	.31

Reduction to the Meridian.—First, when the line of collimation of the telescope is off the meridian, the instrument having been revolved in azimuth and the star observed at the hour-angle τ, near the middle thread, then

$$m = \frac{2\sin^2 \frac{1}{2}\tau}{\sin 1''} \cdot \frac{\cos\phi \cos\delta}{\sin\zeta}, \quad . \quad . \quad . \quad (157)$$

and the correction to the latitude, if the two stars are observed off the meridian, is

$$\text{Cor. } \phi = \tfrac{1}{2}(m' - m). \quad . \quad . \quad . \quad . \quad (158)$$

The value of $\frac{2\sin^2 \frac{1}{2}\tau}{\sin 1''}$ for every second of time up to two minutes (a star being rarely observed at a greater distance than this from the meridian in zenith-telescope observations) is given in the following table:

TABLE LXIX.

VALUES OF $\frac{2\sin^2 \frac{1}{2}\tau}{\sin 1''}$.

τ	Term.	τ	Term.	τ	Term.	τ	Term.	τ	Term.	τ	Term.
s.	"	s.	"	s.	"	s.	"	s.	"	s.	"
1	0.00	21	0.24	41	0.91	61	2.03	81	3.58	101	5.56
2	0.00	22	0.26	42	0.96	62	2.10	82	3.67	102	5.67
3	0.00	23	0.28	43	1.01	63	2.16	83	3.76	103	5.78
4	0.01	24	0.31	44	1.06	64	2.23	24	3.85	104	5.90
5	0.01	25	0.34	45	1.10	65	2.31	85	3.94	105	6.01
6	0.02	26	0.37	46	1.15	66	2.38	86	4.03	106	6.13
7	0.02	27	0.40	47	1.20	67	2.45	87	4.12	107	6.24
8	0.03	28	0.43	48	1.26	68	2.52	88	4.22	108	6.36
9	0.04	29	0.46	49	1.31	69	2.60	89	4.32	109	6.48
10	0.05	30	0.49	50	1.36	70	2.67	90	4.42	110	6.60
11	0.06	31	0.52	51	1.42	71	2.75	91	4.52	111	6.72
12	0.08	32	0.56	52	1.48	72	2.83	92	4.62	112	6.84
13	0.09	33	0.59	53	1.53	73	2.91	93	4.72	113	6.96
14	0.11	34	0.63	54	1.59	74	2.99	94	4.82	114	7.09
15	0.12	35	0.67	55	1.65	75	3.07	95	4.92	115	7.21
16	0.14	36	0.71	56	1.71	76	3.15	96	5.03	116	7.34
17	0.16	37	0.75	57	1.77	77	3.23	97	5.13	117	7.46
18	0.18	38	0.80	58	1.83	78	3.32	98	5.24	118	7.60
19	0.20	39	0.83	59	1.89	79	3.40	99	5.34	119	7.72
20	0.22	40	0.87	60	1.96	80	3.49	100	5.45	120	7.85

The Determination of Apparent Declinations of Stars Used is the next step. Whenever possible these should be taken from the American Ephemeris, the Berliner Jahrbuch, or other reliable sources. The positions of stars are also given in Safford's Catalogue for the epoch of 1875, together with the annual precession and proper motion. The declinations given there should be revised by the aid of more recent catalogues, particularly with reference to stars of the class C. The annual precession and proper motion multiplied by the number of years which have elapsed, applied with the effect of secular variation in precession, give the declination at the beginning of the year. To reduce from mean place at the beginning of the year to apparent place at any date may, with the aid of the Ephemeris, be put in the following form for apparent right ascension and declination at a stated time:

$$\alpha = \alpha_0 + f + \tau\mu + \tfrac{1}{15}g \sin(G + \alpha_0) \tan \delta_0 + \tfrac{1}{15}h \sin(H + \alpha_0) \sec \delta_0 \ldots\text{in time}; \quad (159)$$

$$\delta = \delta^0 + \tau\mu' + g \cos(G + \alpha_0) + h \cos(H + \alpha_0) \sin \delta_0 + i \cos \delta_0 \ldots\text{in arc}; \quad (160)$$

in which α_0 and δ_0 = right ascension and declination at beginning of year;

τ = elapsed portion of fictitious year expressed in units of one year as given in the Ephemeris;

μ and μ' = annual proper motions in right ascension and declination;

H, G, f, g, h, and i = quantities called *independent star numbers* and are given in the Ephemeris.

EXAMPLE.—COMPUTATION OF APPARENT DECLINATION OF STAR 1539 (SAFFORD'S CATALOGUE)

AT ITS TRANSIT AT RAPID, SOUTH DAKOTA, NOV. 9, 1890.

(S. S. GANNETT, Computer.)

$$\delta_0 = 45^\circ\ 38'\ 11''.86\ ; \qquad \alpha_0 = 340^\circ\ 23' = 22^h\ 41^m\ 32^s.$$

Rapid is west of Washington	$1^h\ 45^m$
Rapid sidereal time of transit or α	22 $41\frac{1}{2}$
Washington sidereal time	24 $26\frac{1}{2}$
Sidereal time of mean midnight, Nov. 9 (Am. Ephemeris, p. 383).	27 17
Hence sidereal interval before Washington midnight for stated time is	2 $50\frac{1}{2}$
Equivalent to	0.12 day

The Ephemeris, p. 291, gives the following values Nov. 8, Washington mean midnight:

τ	G	H	log g	log h	log i
.86	316° 43′	40° 45′	+ 1.0096	+ 1.2938	+ 0.7460
Nov. 9					
.86	346 56	39 46	+ 1.0103	+ 1.2945	+ 0.7377

By interpolating the values at time of observation at Rapid .12 day before Washington midnight, Nov. 9, 1890, in accordance with formula (160), the computation of δ is:

G	H	log g	log h	log i
346° 55′	39° 53′	1.0102	1.2944	0.7387

log g	= 1.0102	δ_0 =	45° 38′ 11″.85
log cos $(G + \alpha_0)$ 327° 18′	= 9.9251	$\tau\mu' = .86 \times -0''.03$ =	− 0.03
log g cos $(G + \alpha_0)$	= 0.9353		
g cos $(G + \alpha_0)$	= 8″.62	=	+ 8.62
log h	= 1.2944		
log cos $(H + \alpha_0)$ 20° 16′	= 9.9722		
log sin δ_0	= 9.8543		
	1.1209		
h cos $(H + \alpha_0)$ sin δ_0	= 13″.21	=	+ 13.21
log i	0.7387		
log cos δ_0	9.8446		
	0.5833		
i cos δ	= 3″.83	=	+ 3.83

Apparent declination at time of observation Nov. 9, 1890. = 45° 38′ 37″.48

325. Reduction of Latitude Observations.—With all this preliminary work done, the final reduction of latitude observations is a comparatively simple matter. Grouping the observations by pairs, the *mean declination of each pair* is obtained, the corrections for difference of micrometer readings and levels are applied, with a small correction for differential refraction, and the result is the desired latitude.

Applying the foregoing corrections to formula (149), we have the following working formula for reduction of latitude observations:

$$\phi = \tfrac{1}{2}(\delta + \delta') + (M - M')\frac{r}{2} + \frac{d}{4}[(n + n') - (s + s')] + \tfrac{1}{2}(R - R') + \frac{m}{2} - \frac{m'}{2}. \quad (169)$$

EXAMPLE.—REDUCTION OF LATITUDE OBSERVATIONS.

(Station : Rapid, South Dakota. November 9, 1890. Half rev. micrometer = 37″.900. One div. level = 1″.33.)

Date.	Star Numbers.	δ_1	δ_2	$\frac{1}{2}(\delta_1 + \delta_2)$
		° ′ ″	° ′ ″	° ′ ″
Nov. 9....	7 Lacert and 10 Lacert.	49 42 87.33	38 29 04.60	44 06 15.97
	1539 1551	45 38 37.48	42 44 04.63	11 21.06
	1565 1579	38 43 39.78	49 27 41.04	05 40.41
	1600 1633	56 34 06.66	31 55 56.91	15 01.78
	1676 1686	67 12 10.93	21 03 54.02	08 02.48
	1702 1722	24 32 09.04	63 35 27.34	03 48.19

Star Numbers.	Corrections. Microm.	Corrections. Level.	Corrections. Refr.	Latitude n.	Weight p.	$p. n.$
	′ ″	″	″	° ′ ″		″
7 Lacert and 10 Lacert.	− 1 23.53	− 6.51	−.03	44 04 45.90	.98	5.78
1539 1551	− 6 31.77	− 2.06	−.11	47.12	.90	6.41
1565 1579	− 0 58.33	+ 0.46	−.03	42.51	.79	1.98
1600 1633	− 10 13.25	− 3.78	−.19	44.56	.90	4.10
1676 1686	− 3 08.43	− 7.44	−.07	46.54	.93	6.08
1702 1722	+ 1 01.51	− 3.22	+.02	46.50	.90	5.85
					5.40	30.20

November 9. Weighted mean = 44° 04′ 45″.59.

CHAPTER XXXV.

LONGITUDE.

326. Determination of Longitude.—Determining the longitude of a point on the surface of the earth consists in finding the angle between the two meridian planes passing through the station and a reference meridian. In the United States, Greenwich, England, is generally accepted as the zero of longitude. Time and arc are interchangeable (Art. 304), differences in longitude may be expressed in time or angle. Thus 24 hours equals 360°, 1 hour equals 15°, 1 minute of time equals 15′ and 1 second equals 15″ of arc (Tables LVI to LIX). Therefore the angle between the two meridian planes above described is the same as the differences of the local times of the two stations. Accordingly, to determine the longitude of a station is to determine the differences between the local time at Greenwich and the local time of that station (Art. 305), generally referred to some nearer station the longitude of which is already known.

327. Astronomic Positions: Cost, Speed, and Accuracy.—Practically the whole expense involved in determining the latitude, longitude, and azimuth of a station is included in the telegraphic exchange of signals and time observations for longitude, the additional observations required to determine latitude and azimuth being made in the meanwhile.

The U. S. Coast and Geodetic Survey determines longitudes of prime importance at an average cost of $1500 per station. The observations are made by transit instrument for time and telegraphic exchange of clock signals on five nights.

The observers then change stations and repeat the same observation on five additional nights, making a total of ten nights, requiring about six weeks of actual time. The probable error of a location is $\pm$ 0.01 second of time, equivalent to from 10 to 15 feet in distance.

The U. S. Geological Survey determines longitudes at a cost of about $500 per station. The method is by exchange of telegraphic signals, as in the Coast Survey, but on four nights only, the personal equation being determined on four other nights either preceding or following the field season. Accordingly, a determination of personal equation by the Geological Survey method serves for from three to four longitude determinations in a season, the average time per station, including observations for personal equation, being ten days to two weeks. The probable error of such a determination is $\pm$.03 seconds of time, equivalent to from 30 to 45 feet in distance.

328. Longitude by Chronometers.—When it is impracticable to determine longitude by telegraphic exchange of signals (Art. 330), the same principle may be employed between two intervisible stations, as points on a shore line or the summits of mountain peaks, by flashes of light.

The simpler and more usual way, however, of determining longitudes in the absence of the telegraph is by means of chronometers or chronometer watches carried from some point the longitude of which is known to that at which it is to be determined. This performs the same purpose as the telegraph by comparing local times at the two stations.

The mode of determining longitudes with chronometers is to *observe transits of stars* on as many nights as practicable, generally from 10 to 50, catching the transit by eye and the chronometer beat by ear. At the known station there should be from 2 to 4 chronometers, part set to sidereal and part to mean time, and these should remain stationary and protected from changes of temperature. At the new station there should be a similar number of stationary instruments. Finally, several chronometers, part set to mean and part set to sidereal time, should be carried back and forth between the two stations.

The *method of observing* is to compare the moving with the stationary chronometers, and these compared with the transit observations serve to determine the error of each chronometer. The moving chronometer must be handled with the greatest possible care, and the results cannot be satisfactory where they are carried on wagons, or on the backs of animals. They may be carried with fairly satisfactory results in the hand, however. Where the mode of travel is rough, chronometer watches will give as satisfactory results as can be attained by attempting to transport large chronometers.

The object of having chronometers set to both sidereal and mean time is similar to that of reading a vernier. The sidereal chronometer gains gradually on the mean-time chronometer, and about once in three minutes the two chronometers tick exactly together.

The mode of *computing chronometric longitudes* consists in applying to the time of a mean-time chronometer the correction to local mean time, the result being local mean solar time. This must then be reduced to sidereal interval to give sidereal interval from preceding mean noon. The time of sidereal preceding mean noon must then be applied, giving local sidereal time. This compared with the time of the sidereal chronometer gives the correction to the latter.

329. Longitude by Lunar Distances.—If the direct methods of determining longitude are unavailable, such as those by telegraphic exchange of time signals with the chronograph (Art. 330) or by means of chronometers (Art. 328), there remains but one other method of determining longitude, dependent upon the motion of the moon. The position of the moon has been determined frequently at fixed observatories. As a result its orbit and its various perturbations have been computed. Tables giving the right ascension and declination of the moon for every hour, and other tables defining its place, are to be found in the American Ephemeris. If the topographer wishes to determine longitude by the moon, he determines its position and notes the local time at which his observation was made. Then by consulting the Ephemeris and finding what interval by Greenwich time the moon was actually in the position in which he observed it, the difference between this time and the local time of his observation is longitude reckoned from Greenwich.

The various methods by which the position of the moon may be determined are all approximate, and the field-work connected with the making of these observations is laborious considering the inferior quality of the results. The attainment of accuracy by any method involving the moon is difficult, because the moon requires about 27⅓ days to make one complete circuit in its orbit about the earth. The apparent motion of the moon among the stars is accordingly $\frac{1}{27}$ as fast as the apparent motion of the stars relative to the observer's meridian, which furnishes his measure of time. Any error in determining the position of the moon is accordingly multiplied by at least 27 when converted into time. Moreover, the motion of the moon is so difficult to compute that its positions at various times as given in the Ephemeris are in error by amounts which become whole seconds when multiplied by 27. Finally, the limb or edge of the visible disk of the moon, which is the object really observed, is seen as a ragged outline which makes it difficult to use for purposes of measurement. The computations required for the determination of longitude by lunar observations are long and complicated, and the theories involved require much study for their mastery. Accordingly, no attempt will be made to explain here the methods of determining longitude by lunar observations, reference being made to Doolittle's Practical Astronomy, the American Ephemeris, and to Chauvenet's Astronomy.

Recently there has been devised by Captain E. H. Hills of the British Army a method of determining longitude by photographs of the moon and of one or more bright stars of approximately the same declination. In 1895 Captain Hills took advantage of the despatch of a surveying expedition to the Niger River in Africa to carry out a series of field experiments for the determination of photographic longitudes. The results obtained are reported as most encouraging. No difficulty was experienced in the field although the observer was quite new to the work. The only apparent disadvantage of this method, and one which is not serious, rests on the fact that the results are not at once available, although this is generally of small moment, as the office measurement and computation can be done at leisure and with access to accurate measuring micrometers. Underlying the principle of this method is that of obtaining a photograph of the moon and the traces of one or more bright stars. The position of the moon is determined at the time of taking the photograph by means of some angle-reading instrument attached to or separate from the camera. After the exposure has been made on the moon the time is noted which elapses between it and the passage of some bright star across the field of the camera as denoted by the cross-hairs of the finding instrument. When an exposure of some duration is made on a star so that it shall leave a trace on the plate, or, in fact, several exposures are made as explained hereafter, the declination of the moon and stars being known and the time which has elapsed, these quantities, with the micrometric measurement of the distance between the limb of the moon and the star trace, give all the data from which to compute the longitude.

In Chapter XXXVII are given a description and an example of the method of determining a photographic longitude, prepared by Mr. Wm. J. Peters of the U. S. Geological Survey.

330. Longitude by Chronograph.—As already explained, all methods of determining longitude are reduced to determining the differences of local times and converting these into differences in longitude (Art. 305). The most accurate method of *determining time differences* is by meridian transit observations (Art. 308) for time at two stations, and the comparison of the results by the exchange of telegraphic signals. The operation consists in the observation of stars for time with a transit instrument of the type described in Article 319. These stars are observed in sets by previous agreement between the observers at the station the longitude of which is known and that at which it is to be determined. At some time about the middle of the night's observations, between two sets of time observations, arbitrary signals are exchanged by telegraph between the two stations, and these serve to compare the chronometers and thus to compare the local times at the two places as determined during the star observations.

For the purpose of recording the time of transit of stars as observed with the transit an instrument called a *chronograph* is used. This consists of a drum upon which is wound a strip of paper kept in revolution by clockwork controlled by an escapement (Fig. 182). A pen carried by a car which travels slowly in a direction parallel to the axis of the cylinder traces a line on the drum. This pen is held in place by a magnet carried also upon the car, and as long as the current from the battery passes through the coil and thus holds the armature the pen traces an unbroken spiral line. If the current is suddenly broken or destroyed, as by a touch of the observing key, the armature is freed in an instant and a jog is made in the line. The batteries employed with this apparatus are the ordinary zinc, copper, and sulphate of copper apparatuses of four cells. Dry batteries are also used successfully.

As a part of this apparatus a *break-circuit chronometer* is used which differs from ordinary chronometers in that it is arranged to break an electric circuit temporarily at regular intervals. Those used in the U. S. Geological Survey break circuit every two seconds, the end of a minute being indicated by a break at the 59th as well as 60th second. One of these chronometers being connected with a battery and the chrono-

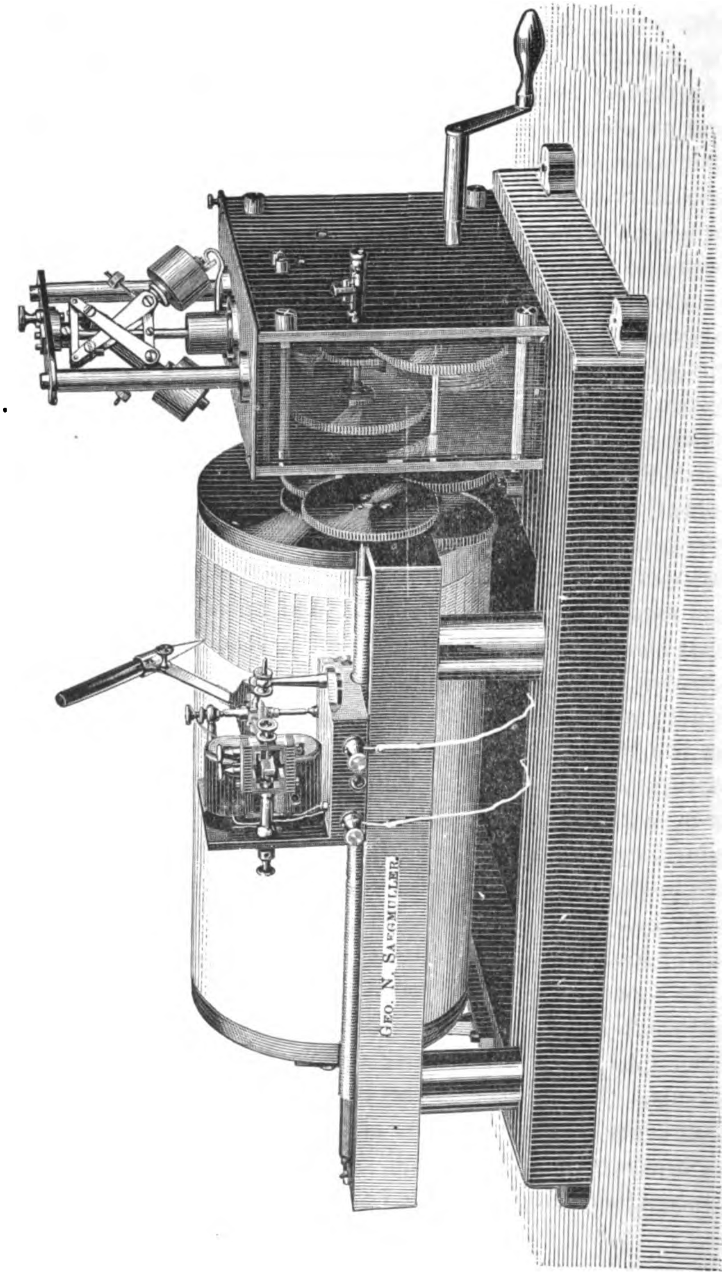


FIG. 182.—CHRONOGRAPH.

graph being introduced in the same circuit, the beginning of every second is recorded upon the chronograph battery by a jog, and the distance between any two jogs represents therefore 2 seconds. The observer at the transit watches a star near the meridian, and as it crosses a thread in the telescope he presses an observing-key which is in circuit with the chronograph, and thus records by a jog on the chronograph sheet the time of passage between the threads.

331. **Observing for Time.**—The transit being mounted, leveled, and adjusted in the meridian as described in Article 322, and the chronograph set up and running connected in a circuit with a battery, a chronometer, and a telegraph key, time observations are made in the following manner:

A *list of time stars* should be consulted, as that given in the Berliner Jahrbuch, this being one of the fullest lists which give day places. Stars are selected north and south of the zenith so that the azimuth errors will balance one another as nearly as possible. On the approach of the selected star to the meridian the telescope is set by means of the vertical circle for the *altitude of the star* above the horizon, as determined from the declination and latitude. As the star crosses each thread in the reticule the fact is recorded upon the chronograph sheet by the observer pressing the *observing key*. At least four time stars, as those between the equator and zenith, are designated, and one circumpolar star should be observed and the telescope be reversed in the wyes and a similar set be observed. Two such half-sets with the reversal of the telescope between gives an accurate determination of time. The same sets of stars are by previous agreement observed at each station.

Between observations upon any two stars the *striding-level* should be placed upon the pivots of the instrument and readings taken to ascertain the departure of the axis from a horizontal position. In order to avoid *unequal expansion of the pivots* from unequal heating, both bull's-eye lamps must be lighted and placed in their stands, in order that both pivots may be equally heated. After the comparison of chronometers at the two stations, to be hereafter described, a similar set of stars should be observed, thus giving rate of the chronometer.

332. Reduction of Time Observations.—Certain *constants of the transits* should be measured before proceeding with the reduction of time observations. The *value of a division of the striding-level* should be measured by means of a level-trier. The equatorial interval of time between each of the threads and the mean of all the threads should be obtained, as it is not infrequently needed in utilizing broken or imperfect observations. These can best be obtained from observations on slow-moving stars, but any stars may be used for the purpose. The intervals as observed are reduced to the equator by multiplying them by the cosine of the declination of the star observed. The object of these observations is specifically the determination of the *error of the chronometer*. This error equals the right ascension of a star minus its observed time of transit, corrected for certain instrumental errors. These errors are as follows:

The *correction for level error*, designated by b (Art. 308), is ascertained from the readings of the striding-level. The value of a division of the level in seconds of time must have been previously ascertained by means of a level-trier. The effect of the level error is greatest at the zenith and diminishes to zero at the horizon. The correction in seconds of time is given (see formula (131)) by the following equation:

$$bB = b \cos (\phi - \delta) \sec \delta. \quad . \quad . \quad . \quad . \quad (162)$$

When the declination is north, it is to be regarded as having a plus sign for upper and a minus sign for lower culmination. When south it is negative.

The correction for inequality of pivots can be made a part of the level correction.

Let $p =$ the inequality of pivots;
$B =$ inclination of axis given by level for clamp west;
$B' =$ inclination of axis given by level for clamp east;
$b =$ true inclination of axis for clamp west;
$b' =$ true inclination of axis for clamp east;—then

$$p = \frac{B' - B}{4}; \quad . \quad . \quad . \quad . \quad . \quad (163)$$

$$b = B + p \text{ for clamp west;}$$
$$b' = B' - p \text{ for clamp east.}$$

The correction for error of collimation, designated by c (Art. 308), is the departure of the mean of the threads from the optical axis of the telescope. For stars at upper culmination with clamp west it is plus when the mean of the threads is east of the axis, and minus when it is west of it. For stars at lower culmination the reverse is the case. The value of c is one-half the difference between the clock error indicated by stars observed before and after reversal of the instrument, divided by the mean secant of the declinations of the stars. This is slightly complicated with the azimuth, although the effect of that is largely eliminated by the proper selection of stars. Consequently it is to be obtained by approximations, in conjunction with the azimuth errors. The correction to be applied to each star is, from formula (132),

$$cC = c \sec \delta, \quad . \quad . \quad . \quad . \quad . \quad (164)$$

which is plus for a star at upper culmination, and minus for a star at lower culmination. It is least for equatorial stars and increases with the secant of the declination.

The correction for deviation in azimuth, designated by a (Art. 308), represents the error in the setting of the instrument in the meridian. Its effect is zero at the zenith and inincreases towards the horizon. Since the instrument is liable to be disturbed during the operation of reversal, it is necessary to determine the azimuth error separately, both before and after reversal. A comparison of the clock error, determined from observations upon north and south stars, will furnish the data necessary for the determination of azimuth. Practically, it is determined by elimination from equations involving the mean of all these stars observed in each of the two positions of the instrument, after correcting for level, and as it is slightly complicated with collimation it must be reached by two or more approximations. The errror is essentially positive when the telescope points east of south, and negative when west of south. The correction applicable to any star is expressed (see formula (130)) in the following equation:

$$aA = a \sin(\phi - \delta) \sec \delta. \quad . \quad . \quad . \quad . \quad (165)$$

It must be understood that the declination when north is positive for upper and negative for lower culmination, and that with south declination it is negative.

The right ascension of stars, as taken from the Star Catalogue, must be *corrected for diurnal aberration*, which equals $0^s.021 \cos \phi \sec \delta$. This correction is positive for upper and negative for lower culmination.

The foregoing corrections are summarized (see formula (135)) in the following equation:

$$\Delta T = \alpha - (T_0 + aA + bB + cC). \quad . \quad . \quad . \quad (166)$$

A, B, C are constants, depending upon the latitude of the place of observation and the declination of the star. Tables for these quantities will be found in an appendix to Annual Report U. S. Coast and Geodetic Survey for 1880. (Extract reprinted herewith, Table LXX.)

333. **Record of Time Observations.**—On pages 755 to 757 is an example of the form for record of observation and reduction of time observations, taken from a series made for the determination of position of Rapid, South Dakota.

EXAMPLE OF RECORD OF TIME OBSERVATIONS.

(Rapid, South Dakota, November 20, 1890. Fauth transit, No. 534. Sidereal chronometer: Bond & Sons, No. 187. 1 division of level = $0^s.118$. Hourly rate of chronometer = $0^s.133$.)
(From Gannett's Manual.)

Star	γ Cephei.	φ Pegasi.	ω Piscium.	33 Piscium.	α Androm.	
Clamp	W.	W.	W.	W.	W.	W.
Level	Telescope north.	Telescope south.	Telescope south.	Telescope south.	Telescope south.	Telescope north.
	W. Sum. E.	W. Sum. E.	W. Sum. E.	W. Sum. E.	W. Sum. E.	W. Sum. E.
	d d d	d d d	d d d	d d d	d d d	d d d
	19.8 −88.1 68.3	68.0 +87.1 19.1	20.0 −89.5 69.5	68.2 +86.9 18.7	19.8 −89.4 69.5	19.7 −89.5 69.8
	68.2 +87.6 19.4	20.2 −89.2 69.0	68.8 +87.2 18.4	19.9 −89.4 69.5	68.3 +86.8 18.5	68.8 +87.3 18.5
Difference	− 0.5	− 2.1	− 2.3	− 2.5	− 2.5	− 2.2
	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	
Thread I	23 34 52.25	23 47 24.00	23 54 10.89	00 00 13.33	00 03 12.00	
" II	35 11.40	28.55	14.88	17.96	16.83	
" III	29.41	32.72	19.22	21.94	21.32	
" IV	46.78	36.75	23.14	25.95	26.00	
" V	36 05.00	41.09	27.20	29.83	30.85	
Sum	4.84	3.11	5.33	9.01	7.00	
Mean	23 35 28.97	23 47 32.62	23 54 19.07	00 00 21.80	00 03 21.40	
Aberration	−.07	−.02	−.02	−.02	−.02	
Correction for level, bB	−.22	−.06	−.05	−.04	−.06	
Correction for rate	+.05	+.03	+.01	+.00	+.00	
Reduced transit, t	23 35 28.83	23 47 32.57	23 54 19.01	00 00 21.74	00 03 21.32	
Tabular R. A., α	23 34 53.13	46 55.67	53 41.98	23 59 44.61	00 02 44.42	
$\alpha - t =$	− 35.70	−36.90	−37.03	−37.13	−36.90	

$$\text{Mean of levels} = \frac{-\overset{\text{div.}}{2.02}}{4} \times \overset{s.}{.118} = -\overset{s.}{.0596} = b. \quad \text{Inequality of pivots} = \overset{s.}{.00}.$$

(☞ See also next page.)

EXAMPLE OF RECORD OF TIME OBSERVATIONS—*Continued.*

Star	γ Pegasi.	Br. 6.	ι Ceti.	44 Piscium	12 Ceti.
Clamp	E.	E.	E.	E.	E.
Level	Telescope south. W. Sum. E. 19.2 −88.3 69.1 68 9 +87.8 18.9 d − 0.5	Telescope south. W. Sum. E. 68.7 +87.3 18.6 19.4 −88.7 69.3 d − 1.4	Telescope south. W. Sum. E. 19.2 −88.4 69.2 68.5 +86.7 18.2 d − 1.7	Telescope north. W. Sum. E. 68.9 +87.8 18.9 18.9 −87.9 69.0 d − 0.1	
	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.
Thread V	00 08 05.25	00 10 05.00	00 14 20.70	00 20 17.35	00 24 56.85
" IV	09.30	22.81	24.68	20.84	25 00.73
" III	13.54	39.30	28.52	24.93	05.37
" II	17.65	56.90	32.90	29.16	09.15
" I	22.00	11 15.49	37.23	33.42	13.07
Sum	7.74	.. 9.50	4.03	5.70	5.17
Mean	00 08 13.55	39.90	28.81	25.14	05.03
Correction for aberration	−.02	−.06	−.02	−.02	−.02
Correction for level, bB	−.02	−.09	−.02	−.02	−.02
Correction for rate	−.02	−.02	−.03	−.04	−.05
Reduced transit, t	00 08 13.49	00 10 39.73	00 14 28.74	00 20 25.06	00 24 04.94
Tabular R. A., a	00 07 36.59	10 03.56	00 13 51.75	00 19 48.17	00 24 27.91
$a - t =$	−36.90	−36.17	−36.99	−36.89	−37.03

Mean of levels $= \frac{-\overset{\text{div.}}{.925}}{4} \times \overset{s.}{.118} = -\overset{s.}{.027} = b.$ Inequality of pivots $= \overset{s.}{.00}.$

334. Longitude Computation.—

EXAMPLE OF REDUCTION.

(Rapid, South Dakota, November 20, 1890. After exchange of clock signals.)

Clamp.	1.	2. Star.	3. A.	4. C.	5. $a-t$.	6. Cc.	7. $a-t$ Corr. for Cc.	8. Aa_w.	9. Corrected for Aa_w.	10. Cc'.	11.	12. Aa'_w.	13.	v.	t.
	Wt. Δt_0					$c = -^s.019$				$C' + ^s.015$					
W.	1	γ Cephei.....	−2.42	+4.45	− 35^s.70	− .08	−35^s.78	−1^s.00	−36^s.78	.07	−36^s.71	− .05	−36^s.76	^{s}.00	23^h 58
	1	φ Pegasi.....	+0.45	1.05	36 .90	− .02	36 .92	+0 .19	.73	.01	.73	+.01	.71	− .05	.78
	1	ω Piscium....	+0.62	1.01	37 .03	− .02	37 .05	+0 .26	.79	.02	.77	+.01	.76	.00	.89
	1	33 Piscium...	+0.77	1.01	37 .13	− .02	37 .15	+0 .32	.83	.01	.82	+.02	.80	+ .04	.99
	1	α Androm....	+0.30	+1.14	− 36 .90	− .02	−36 .92	+0 .12	−36 .80	+ .02	.78	+.01	.77	+ .01	24 .01
		Sum ..=	+2.14	....	....	....	....	....	....	....	....	...	3 .80		
		Sum . =	−2.42	+8.66	−183 .66	....	....	....	3 .93						
		(1) Mean .=	−0.06Aw	+1.73c	− 36 .732	....	....	....	−36 .786	....	....	......	−36 .760		
								Aa.				Aa'_w.			
E.	1	γ Pegasi.....	+0.51	−1.03	− 36^s.90	+^{s}.02	−36^s.88	+0^s.15	−36^s.73	−^{s}.01	−36^s.74	−.01	−36^s.75	−^{s}.01	24^s.13
	1	Bradley 6....	−2.26	4.23	36 .17	.08	36 .09	− .64	.73	.06	.79	+.03	.76	.00	.17
	1	ι Ceti.......	+0.81	1.01	36 .99	.02	36 .97	+ .23	.74	.02	.76	−.01	.77	+ .01	.23
	1	44 Piscium...	+0.68	1.00	36 .89	.02	36 .87	+ .19	.68	.01	.69	−.01	.70	− .06	.33
	1	12 Ceti......	+0.75	−1.00	− 37 .03	+ .02	−37 .01	+ .21	−36 .80	− .02	.82	−.01	.83	+ .07	.41
		Sum ..=	+2.75	....	....	....	....	....	....	....	......	......	3 .81		
		Sum ..=	−2.26	−8.27	−183 .98	....	....	....	3 .68	....	......	......	....	Mean	=24 .05
		(2) Mean..=	+0.10Ae	−1.65c	− 36 .796	....	......	......	−36 .736	....	......	.....	−36 .762		

Subtracting (2) from (1), ignoring azimuth terms which are small, we have:

$$+3.38c + ^s.064 = 0$$

Approx.......$c = -^s.019$

From below. $c' = +.015$ Adopted..$c = -^s.004$

$$\begin{array}{ll} -2.42Aw - 35^s.78 = 0 & -2.26Ae - 36^s.09 = 0 \\ \underline{+0.54 \qquad -37\ .01 = 0} & \underline{+0.69 \qquad -36\ .93 = 0} \\ +2.96Aw - 1^s.23 = 0 & +2.95Ae - 0^s.84 = 0 \\ Aw = +^s.415 & Ae = +^s.285. \end{array}$$

Forming equations to determine c' from columns 4 and 9:

$$\begin{array}{l} +1.73c' - 36^s.786 = 0 \\ \underline{-1.65 \qquad -36\ .736 = 0} \\ +3.38c' - 0\ .050 = 0. \\ c' = +^s.015 \end{array}$$

Forming equations from columns 3 and 11:

$$\begin{array}{ll} -2.42A'w - 36^s.710 = 0 & -2.26A'e - 36^s.790 = 0 \\ \underline{+0.54A'w - 36\ .772 = 0} & \underline{+0.69 \qquad -36\ .752 = 0} \\ +2.96A'w - 0^s.062 = 0 & +2.95A'e + 0^s.038 = 0 \\ A'w = +^s.021 & A'e = -^s.013 \\ Aw = +\ .415 & Ae = +\ .285 \\ \text{Adopted..}Aw = +\ .436 & \text{Adopted..}Ae = +^s.272 \\ \text{Adopted..}\ C = -\ .004 & \end{array}$$

Normal equation, formed from columns 1, 3, 4, and 5

$$10\Delta T - .28Aw + .49Ae + .39c + 367^s.64 = 0$$

$$\Delta T - .028Aw + .049Ae + .039c + 36^s\ 764 = 0$$

$$\Delta T = -36^s.76 \text{ at } 24^h.05$$

$$\begin{array}{rcl} & a - T = & -36^s.764 \\ -.028 \times +^s.436 = & & -.012 \\ +.049 \times +.272 = & & +.013 \\ +.039 \times -.004 = & & \underline{-.000} \\ & \Delta T = & -36^s.763 \end{array}$$

TABLE LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

(Extracted from Appendix 14, U. S. Coast and Geodetic Survey Report for 1880.)

To find A enter left-hand column with the zenith distance; its intersection with declination column given azimuth factor.

To find B enter right-hand column with the zenith distance; its intersection with declination column gives level factor.

C is given on last line of each section of the table.

Azimuth factor $A = \sin \zeta \sec \delta$. Star's declination $\pm \delta$. Inclination factor $B = \cos \zeta \sec \delta$.

ζ	0°	10°	15°	20°	22°	24°	26°	28°	30°	32°	34°	36°	ζ
1°	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	89°
2	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	88
3	.05	.05	.05	.06	.06	.06	.06	.06	.06	.06	.06	.06	87
4	.07	.07	.07	.07	.08	.08	.08	.08	.08	.08	.08	.09	86
5	.09	.09	.09	.09	.09	.10	.10	.10	.10	.10	.10	.11	85
6	.11	.11	.11	.11	.11	.11	.12	.12	.12	.12	.13	.13	84
7	.12	.12	.13	.13	.13	.13	.14	.14	.14	.14	.15	.15	83
8	.14	.14	.14	.15	.15	.15	.16	.16	.16	.16	.17	.17	82
9	.16	.16	.16	.17	.17	.17	.17	.18	.18	.18	.19	.19	81
10	.17	.18	.18	.19	.19	.19	.19	.20	.20	.21	.21	.21	**80**
11	.19	.19	.20	.20	.21	.21	.21	.22	.22	.23	.23	.24	79
12	.21	.21	.22	.22	.22	.23	.23	.24	.24	.25	.25	.26	78
13	.22	.23	.23	.24	.24	.25	.25	.26	.26	.27	.27	.28	77
14	.24	.25	.25	.26	.26	.27	.27	.27	.28	.29	.29	.30	76
15	.26	.26	.27	.28	.28	.28	.29	.29	.30	.31	.31	.32	75
16	.28	.28	.29	.29	.30	.30	.31	.31	.32	.33	.33	.34	74
17	.29	.30	.30	.31	.31	.32	.33	.33	.34	.34	.35	.36	73
18	.31	.31	.32	.33	.33	.33	.34	.35	.36	.36	.37	.38	72
19	.33	.33	.34	.35	.35	.36	.36	.37	.38	.38	.39	.40	71
20	.34	.35	.35	.36	.37	.37	.38	.39	.40	.40	.41	.42	**70**
21	.36	.36	.37	.38	.39	.39	.40	.41	.41	.42	.43	.44	69
22	.37	.38	.39	.40	.40	.41	.42	.42	.43	.44	.45	.46	68
23	.39	.40	.41	.42	.42	.43	.44	.44	.45	.46	.47	.48	67
24	.41	.41	.42	.43	.44	.45	.45	.46	.47	.48	.49	.50	66
25	.42	.43	.44	.45	.46	.46	.47	.48	.49	.50	.51	.52	65
26	.44	.45	.45	.47	.47	.48	.49	.50	.51	.52	.53	.54	64
27	.45	.46	.47	.48	.49	.50	.51	.51	.52	.54	.55	.56	63
28	.47	.48	.49	.50	.51	.51	.52	.53	.54	.55	.57	.58	62
29	.48	.49	.50	.52	.52	.53	.54	.55	.56	.57	.58	.60	61
30	.50	.51	.52	.53	.54	.55	.56	.57	.58	.59	.60	.62	**60**
31	.52	.52	.53	.55	.56	.56	.57	.58	.59	.61	.62	.64	59
32	.53	.54	.55	.56	.57	.58	.59	.60	.61	.63	.64	.65	58
33	.54	.55	.56	.58	.59	.60	.61	.62	.63	.64	.66	.67	57
34	.56	.57	.58	.59	.60	.61	.62	.63	.65	.66	.67	.69	56
35	.57	.58	.59	.61	.62	.63	.64	.65	.66	.68	.69	.71	55
36	.59	.60	.61	.63	.63	.64	.65	.67	.68	.69	.71	.73	54
37	.60	.61	.62	.64	.65	.65	.67	.68	.70	.71	.73	.74	53
38	.62	.63	.64	.66	.66	.67	.69	.70	.71	.73	.74	.76	52
39	.63	.64	.65	.67	.68	.69	.70	.71	.73	.74	.76	.78	51
40	.64	.65	.66	.68	.69	.70	.72	.73	.74	.76	.77	.79	**50**
41	.66	.67	.68	.70	.71	.72	.73	.74	.76	.77	.79	.81	49
42	.67	.68	.69	.71	.72	.73	.74	.76	.77	.79	.81	.83	48
43	.68	.69	.71	.73	.74	.75	.76	.77	.79	.80	.82	.84	47
44	.69	.71	.72	.74	.75	.76	.77	.79	.80	.82	.84	.86	46
45	.71	.72	.73	.75	.76	.77	.79	.80	.82	.83	.85	.87	45

TABLE LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	0°	10°	15°	20°	22°	24°	26°	28°	30°	32°	34°	36°	ζ
46°	.72	.73	.74	.77	.78	.79	.80	.82	.83	.85	.87	.89	44°
47	.73	.74	.76	.78	.79	.80	.81	.83	.84	.86	.88	.90	43
48	.74	.76	.77	.79	.80	.81	.83	.84	.86	.88	.90	.92	42
49	.75	.77	.78	.80	.81	.83	.84	.86	.87	.89	.91	.93	41
50	.77	.78	.79	.82	.83	.84	.85	.87	.89	.90	.92	.95	**40**
51	.78	.79	.80	.83	.84	.85	.87	.88	.90	.92	.94	.96	39
52	.79	.80	.82	.84	.85	.86	.88	.89	.91	.93	.95	.97	38
53	.80	.81	.83	.85	.86	.87	.89	.91	.92	.94	.96	.99	37
54	.81	.82	.84	.86	.87	.89	.90	.92	.93	.95	.98	1.00	36
55	.82	.83	.85	.87	.88	.90	.91	.93	.95	.97	.99	1.01	35
56	.83	.84	.86	.88	.89	.91	.92	.94	.96	.98	1.00	1.02	34
57	.84	.85	.87	.89	.90	.92	.93	.95	.97	.99	1.01	1.04	33
58	.85	.86	.88	.90	.91	.93	.94	.96	.98	1.00	1.02	1.05	32
59	.86	.87	.89	.91	.92	.94	.95	.97	.99	1.01	1.03	1.06	31
60	.87	.88	.90	.92	.93	.95	.96	.98	1.00	1.02	1.04	1.07	**30**
61	.87	.89	.91	.93	.94	.96	.97	.99	1.01	1.03	1.05	1.08	29
62	.88	.90	.91	.94	.95	.97	.98	1.00	1.02	1.04	1.06	1.09	28
63	.89	.91	.92	.95	.96	.98	.99	1.01	1.03	1.05	1.07	1.10	27
64	.90	.91	.93	.96	.97	.98	1.00	1.02	1.04	1.06	1.08	1.11	26
65	.91	.92	.94	.96	.98	.99	1.01	1.03	1.05	1.07	1.09	1.12	25
66	.91	.93	.95	.97	.99	1.00	1.02	1.04	1.06	1.08	1.10	1.13	24
67	.92	.94	.95	.98	.99	1.01	1.02	1.04	1.06	1.09	1.11	1.14	23
68	.93	.94	.96	.99	1.00	1.02	1.03	1.05	1.07	1.09	1.12	1.15	22
69	.93	.95	.97	.99	1.01	1.02	1.04	1.06	1.08	1.10	1.13	1.15	21
70	.94	.95	.97	1.00	1.01	1.03	1.05	1.06	1.09	1.11	1.13	1.16	**20**
71	.95	.96	.98	1.01	1.02	1.04	1.05	1.07	1.09	1.12	1.14	1.17	19
72	.95	.97	.98	1.01	1.03	1.04	1.06	1.08	1.10	1.12	1.15	1.17	18
73	.96	.97	.99	1.02	1.03	1.05	1.06	1.08	1.10	1.13	1.15	1.18	17
74	.96	.98	1.00	1.02	1.04	1.05	1.07	1.09	1.11	1.13	1.16	1.19	16
75	.97	.98	1.00	1.03	1.04	1.06	1.08	1.09	1.12	1.14	1.16	1.19	15
76	.97	.99	1.00	1.03	1.05	1.06	1.08	1.10	1.12	1.14	1.17	1.20	14
77	.97	.99	1.01	1.04	1.05	1.07	1.08	1.10	1.13	1.15	1.17	1.20	13
78	.98	.99	1.01	1.04	1.05	1.07	1.09	1.11	1.13	1.15	1.18	1.21	12
79	.98	1.00	1.02	1.04	1.06	1.08	1.09	1.11	1.13	1.16	1.18	1.21	11
80	.98	1.00	1.02	1.05	1.06	1.08	1.10	1.12	1.14	1.16	1.19	1.22	**10**
81	.99	1.00	1.02	1.05	1.07	1.08	1.10	1.12	1.14	1.17	1.19	1.22	9
82	.99	1.01	1.03	1.05	1.07	1.08	1.10	1.12	1.14	1.17	1.19	1.22	8
83	.99	1.01	1.03	1.06	1.07	1.09	1.10	1.12	1.15	1.17	1.20	1.23	7
84	.99	1.01	1.03	1.06	1.07	1.09	1.11	1.13	1.15	1.17	1.20	1.23	6
85	1.00	1.01	1.03	1.06	1.07	1.09	1.11	1.13	1.15	1.17	1.20	1.23	5
86	1.00	1.01	1.03	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.20	1.23	4
87	1.00	1.01	1.03	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.20	1.23	3
88	1.00	1.01	1.03	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.20	1.23	2
89	1.00	1.02	1.04	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.21	1.24	1
90	1.00	1.02	1.04	1.06	1.08	1.09	1.11	1.13	1.15	1.18	1.21	1.24	**0**

TABLE LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	38°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	ζ
1°	.02	.02	.02	.02	.02	.02	.02	.02	.03	.03	.03	.03	89°
2	.04	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	88
3	.07	.07	.07	.07	.07	.07	.07	.07	.08	.08	.08	.08	87
4	.09	.09	.09	.09	.10	.10	.10	.10	.10	.10	.11	.11	86
5	.11	.11	.11	.12	.12	.12	.12	.13	.13	.13	.13	.13	85
6	.13	.14	.14	.14	.14	.15	.15	.15	.15	.16	.16	.16	84
7	.15	.16	.16	.16	.17	.17	.17	.18	.18	.18	.19	.19	83
8	.18	.18	.18	.19	.19	.19	.20	.20	.20	.21	.21	.22	82
9	.20	.20	.21	.21	.21	.22	.22	.22	.23	.23	.24	.24	81
10	.22	.23	.23	.23	.24	.24	.25	.25	.25	.26	.26	.27	80
11	.24	.25	.25	.26	.26	.27	.27	.28	.28	.28	.29	.30	79
12	.26	.27	.27	.28	.28	.29	.29	.30	.30	.31	.32	.32	78
13	.29	.29	.30	.30	.31	.31	.32	.32	.33	.34	.34	.35	77
14	.31	.32	.32	.33	.33	.34	.34	.35	.35	.36	.37	.38	76
15	.33	.34	.34	.35	.35	.36	.37	.37	.38	.39	.39	.40	75
16	.35	.36	.37	.37	.38	.38	.39	.40	.40	.41	.42	.43	74
17	.37	.38	.39	.39	.40	.41	.41	.42	.43	.44	.45	.45	73
18	.39	.40	.41	.42	.42	.43	.44	.44	.45	.46	.47	.48	72
19	.41	.42	.43	.44	.45	.45	.46	.47	.48	.49	.50	.51	71
20	.43	.45	.45	.46	.47	.48	.48	.49	.50	.51	.52	.53	70
21	.45	.47	.47	.48	.49	.50	.51	.52	.52	.54	.55	.56	69
22	.48	.49	.50	.50	.51	.52	.53	.54	.55	.56	.57	.58	68
23	.50	.51	.52	.53	.53	.54	.55	.56	.57	.58	.60	.61	67
24	.52	.53	.54	.55	.56	.57	.58	.59	.60	.61	.62	.63	66
25	.54	.55	.56	.57	.58	.59	.60	.61	.62	.63	.64	.66	65
26	.56	.57	.58	.59	.60	.61	.62	.63	.64	.65	.67	.68	64
27	.58	.59	.60	.61	.62	.63	.64	.65	.67	.68	.69	.71	63
28	.60	.61	.62	.63	.64	.65	.66	.68	.69	.70	.72	.73	62
29	.61	.63	.64	.65	.66	.67	.69	.70	.71	.72	.74	.75	61
30	.63	.65	.66	.67	.68	.69	.71	.72	.73	.75	.76	.78	60
31	.65	.67	.68	.69	.70	.72	.73	.74	.75	.77	.78	.80	59
32	.67	.69	.70	.71	.72	.74	.75	.76	.78	.79	.81	.82	58
33	.69	.71	.72	.73	.74	.76	.77	.78	.80	.81	.83	.85	57
34	.71	.73	.74	.75	.76	.78	.79	.80	.82	.84	.85	.87	56
35	.73	75	.76	.77	.78	.80	.81	.83	.84	.86	.87	.89	55
36	.75	.77	.78	.79	.80	.82	.83	.85	.86	.88	.90	.91	54
37	.76	.79	.80	.81	.82	.84	.85	.87	.88	.90	.92	.94	53
38	.78	.80	.82	.83	.84	.86	.87	.89	.90	.92	.94	.96	52
39	.80	.82	.83	.85	.86	.87	.89	.91	.92	.94	.96	.98	51
40	.82	.84	.85	.86	.88	.89	.91	.93	.94	.96	.98	1.00	50
41	.83	.86	.87	.88	.90	.91	.93	.94	.96	.98	1.00	1.02	49
42	.85	.87	.89	.90	.91	.93	.95	.96	.98	1.00	1.02	1.04	48
43	.86	.89	.90	.92	.93	.95	.96	.98	1.00	1.02	1.04	1.06	47
44	.89	.90	.92	.93	.95	.96	.98	1.00	1.02	1.04	1.06	1.08	46
45	.90	.92	.94	.95	.97	.98	1.00	1.02	1.04	1.06	1.08	1.10	45

TABLE LXX.
FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	38°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	ζ
46°	.91	.94	.95	.97	.98	1.00	1.02	1.04	1.05	1.07	1.10	1.12	44°
47	.93	.95	.97	.98	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.14	43
48	.94	.97	.98	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.16	42
49	.96	.99	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.15	1.17	41
50	.97	1.00	1.01	1.03	1.05	1.06	1.08	1.10	1.12	1.14	1.17	1.19	**40**
51	.99	1.01	1.03	1.05	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.21	39
52	1.00	1.03	1.04	1.06	1.08	1.10	1.11	1.13	1.15	1.18	1.20	1.23	38
53	1.01	1.04	1.06	1.07	1.09	1.11	1.13	1.15	1.17	1.19	1.22	1.24	37
54	1.03	1.06	1.07	1.09	1.11	1.12	1.14	1.16	1.19	1.21	1.23	1.26	36
55	1.04	1.07	1.08	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.25	1.27	35
56	1.05	1.08	1.10	1.12	1.13	1.15	1.17	1.19	1.22	1.24	1.26	1.29	34
57	1.06	1.09	1.11	1.13	1.15	1.17	1.19	1.21	1.23	1.25	1.28	1.31	33
58	1.08	1.11	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.27	1.29	1.32	32
59	1.09	1.12	1.14	1.15	1.17	1.19	1.21	1.23	1.26	1.28	1.31	1.33	31
60	1.10	1.13	1.15	1.17	1.18	1.20	1.22	1.25	1.27	1.29	1.32	1.35	**30**
61	1.11	1.14	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.31	1.33	1.36	29
62	1.12	1.15	1.17	1.19	1.21	1.23	1.25	1.27	1.29	1.32	1.35	1.37	28
63	1.13	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.31	1.33	1.36	1.39	27
64	1.14	1.17	1.19	1.21	1.23	1.25	1.27	1.29	1.32	1.34	1.33	1.40	26
65	1.15	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.33	1.35	1.38	1.41	25
66	1.16	1.19	1.21	1.23	1.25	1.27	1.29	1.32	1.34	1.37	1.39	1.42	24
67	1.17	1.20	1.22	1.24	1.26	1.28	1.30	1.33	1.35	1.38	1.40	1.43	23
68	1.18	1.21	1.23	1.25	1.27	1.29	1.31	1.33	1.36	1.39	1.41	1.44	22
69	1.18	1.22	1.24	1.26	1.28	1.30	1.32	1.34	1.37	1.40	1.42	1.45	21
70	1.19	1.23	1.25	1.26	1.28	1.31	1.33	1.35	1.38	1.40	1.43	1.46	**20**
71	1.20	1.23	1.25	1.27	1.29	1.31	1.34	1.36	1.39	1.41	1.44	1.47	19
72	1.21	1.24	1.26	1.28	1.30	1.32	1.34	1.37	1.39	1.42	1.45	1.48	18
73	1.21	1.25	1.27	1.29	1.31	1.33	1.35	1.38	1.40	1.43	1.46	1.49	17
74	1.22	1.25	1.27	1.29	1.31	1.34	1.36	1.38	1.41	1.44	1.46	1.49	16
75	1.23	1.26	1.28	1.30	1.32	1.34	1.37	1.39	1.42	1.44	1.47	1.50	15
76	1.23	1.27	1.29	1.31	1.33	1.35	1.37	1.40	1.42	1.45	1.48	1.51	14
77	1.24	1.27	1.29	1.31	1.33	1.35	1.38	1.40	1.43	1.46	1.48	1.52	13
78	1.24	1.28	1.30	1.32	1.34	1.36	1.38	1.41	1.43	1.46	1.49	1.52	12
79	1.25	1.28	1.30	1.32	1.34	1.36	1.39	1.41	1.44	1.47	1.50	1.53	11
80	1.25	1.29	1.30	1.33	1.35	1.37	1.39	1.42	1.44	1.47	1.50	1.53	**10**
81	1.25	1.29	1.31	1.33	1.35	1.37	1.40	1.42	1.45	1.48	1.51	1.54	9
82	1.26	1.29	1.31	1.33	1.35	1.38	1.40	1.43	1.45	1.48	1.51	1.54	8
83	1.26	1.30	1.32	1.34	1.36	1.38	1.40	1.43	1.46	1.48	1.51	1.54	7
84	1.26	1.30	1.32	1.34	1.36	1.38	1.41	1.43	1.46	1.49	1.52	1.55	6
85	1.26	1.30	1.32	1.34	1.36	1.38	1.41	1.43	1.46	1.49	1.52	1.55	5
86	1.27	1.30	1.32	1.34	1.36	1.39	1.41	1.44	1.46	1.49	1.52	1.55	4
87	1.27	1.30	1.32	1.34	1.37	1.39	1.41	1.44	1.46	1.49	1.52	1.55	3
88	1.27	1.30	1.32	1.34	1.37	1.39	1.41	1.44	1.46	1.49	1.52	1.55	2
89	1.27	1.31	1.32	1.35	1.37	1.39	1.41	1.44	1.47	1.49	1.52	1.56	1
90	1.27	1.31	1.32	1.35	1.37	1.39	1.41	1.44	1.47	1.49	1.52	1.56	**0**

Table LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	60½°	61°	ζ
1°	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03	.04	.04	89°
2	.06	.06	.06	.06	.06	.06	.06	.07	.07	.07	.07	.07	88
3	.08	.08	.09	.09	.09	.08	.10	.10	.10	.10	.11	.11	87
4	.11	.11	.12	.12	.12	.12	.13	.13	.14	.14	.14	.14	86
5	.14	.14	.14	.15	.15	.16	.16	.16	.17	.17	.18	.18	85
6	.17	.17	.17	.18	.18	.19	.19	.20	.20	.21	.21	.22	84
7	.19	.20	.20	.21	.21	.22	.22	.23	.24	.24	.25	.25	83
8	.22	.23	.23	.24	.24	.25	.26	.26	.27	.28	.28	.29	82
9	.25	.25	.26	.26	.27	.28	.29	.29	.30	.31	.32	.32	81
10	.28	.28	.29	.30	.30	.31	.32	.33	.34	.35	.35	.36	80
11	.30	.31	.32	.32	.33	.34	.35	.36	.37	.38	.39	.39	79
12	.33	.34	.35	.35	.36	.37	.38	.39	.40	.42	.42	.43	78
13	.36	.36	.37	.38	.39	.40	.41	.42	.44	.45	.46	.46	77
14	.38	.39	.40	.41	.42	.43	.44	.46	.47	.48	49	.50	76
15	.41	.42	.43	.44	.45	.46	.48	.49	.50	.52	.53	.53	75
16	.44	.45	.46	.47	.48	.49	.51	.52	.54	.55	.56	.57	74
17	.46	.47	.49	.50	.51	.52	.54	.55	.57	.58	.59	.60	73
18	.49	.50	.51	.53	.54	.55	.57	.58	.60	.62	.63	.64	72
19	.52	.53	.54	.55	.57	.58	.60	.61	.63	.65	.66	.67	71
20	.54	.56	.57	.58	.60	.61	.63	.64	.66	.68	.69	.70	70
21	.57	.58	.59	.61	.62	.64	.66	.68	.70	.72	.73	.74	69
22	.60	.61	.62	.64	.65	.67	.69	.71	.73	.75	.76	.77	68
23	.62	.63	.65	.66	.68	.70	.72	.74	.76	.78	.79	.81	67
24	.65	.66	.68	.69	.71	.73	.75	.77	.79	.81	.83	.84	66
25	.67	.69	.70	.72	.74	.76	.78	.80	.82	.85	.86	.87	65
26	.70	.71	.73	.75	.76	.78	.80	.83	.85	.88	.89	.90	64
27	.72	.74	.75	.77	.79	.81	.83	.86	.88	.91	.92	.94	63
28	.75	.76	.78	.80	.82	.84	.86	.89	.91	.94	.95	.97	62
29	.77	.79	.81	.82	.84	.87	.89	.91	.94	.97	.98	1.00	61
30	.79	.81	.83	.85	.87	.89	.92	.94	.97	1.00	1.01	1.03	60
31	.82	.84	.86	.88	.90	.92	.95	.97	1.00	1.03	1.05	1.06	59
32	.84	.86	.88	.90	.92	.95	.97	1.00	1.03	1.06	1.08	1.09	58
33	.87	.88	.91	.93	.95	.97	1.00	1.03	1.06	1.09	1.11	1.12	57
34	.89	.91	.93	.95	.97	1.00	1.03	1.05	1.09	1.12	1.14	1.15	56
35	.91	.93	.95	.98	1.00	1.03	1.05	1.08	1.11	1.15	1.16	1.18	55
36	9.3	.95	.98	1 00	1.03	1.05	1.08	1.11	1.14	1.18	1.19	1.21	54
37	.96	.98	1.00	1.02	1.05	1.08	1.10	1.14	1.17	1.20	1.22	1.24	53
38	.98	1.00	1.02	1.05	1.07	1.10	1.13	1.16	1.20	1.23	1.25	1.27	52
39	1.00	1.02	1.05	1.07	1.10	1.12	1.15	1.19	1.22	1.26	1.28	1.30	51
40	1.02	1.04	1.07	1.09	1.12	1.15	1.18	1.21	1.25	1.29	1.31	1.33	50
41	1.04	1.07	1.09	1.12	1.14	1.17	1.20	1.24	1.27	1.31	1.33	1.35	49
42	1.06	1.09	1.11	1.14	1.17	1.20	1.23	1.26	1.30	1.34	1.36	1.38	48
43	1.08	1.11	1.13	1.16	1.19	1.22	1.25	1.29	1.32	1.36	1.39	1.41	47
44	1.10	1.13	1.15	1.18	1.21	1.24	1.28	1.31	1.35	1.39	1.41	1.43	46
45	1.12	1.15	1.17	1.20	1.23	1.26	1.30	1.33	1.37	1.41	1.44	1.46	45

TABLE LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	60½°	61°	ζ
46°	1.14	1.17	1.19	1.22	1.25	1.29	1.32	1.36	1.40	1.44	1.46	1.48	44°
47	1.16	1.19	1.21	1.24	1.27	1.31	1.34	1.38	1.42	1.46	1.49	1.51	43
48	1.18	1.21	1.23	1.26	1.30	1.33	1.36	1.40	1.44	1.48	1.50	1.53	42
49	1.20	1.23	1.25	1.28	1.32	1.35	1.39	1.42	1.47	1.51	1.53	1.56	41
50	1.22	1.24	1.27	1.30	1.34	1.37	1.41	1.44	1.49	1.53	1.56	1.58	**40**
51	1.23	1.26	1.29	1.32	1.35	1.39	1.43	1.47	1.51	1.55	1.58	1.60	39
52	1.25	1.28	1.31	1.34	1.37	1.41	1.45	1.49	1.53	1.58	1.60	1.63	38
53	1.27	1.30	1.33	1.36	1.39	1.43	1.47	1.51	1.55	1.60	1.62	1.65	37
54	1.29	1.31	1.34	1.38	1.41	1.45	1.49	1.53	1.57	1.62	1.64	1.67	36
55	1.30	1.33	1.36	1.39	1.43	1.46	1.50	1.55	1.59	1.64	1.66	1.69	35
56	1.32	1.35	1.38	1.41	1.45	1.48	1.52	1.56	1.61	1.66	1.68	1.71	34
57	1.33	1.36	1.39	1.43	1.46	1.50	1.54	1.58	1.63	1.68	1.70	1.73	33
58	1.35	1.38	1.41	1.44	1.48	1.52	1.56	1.60	1.65	1.70	1.72	1.75	32
59	1.36	1.39	1.42	1.46	1.49	1.53	1.57	1.62	1.66	1.71	1.74	1.77	31
60	1.38	1.41	1.44	1.47	1.51	1.55	1.59	1.63	1.68	1.73	1.76	1.79	**30**
61	1.39	1.42	1.45	1.49	1.53	1.56	1.61	1.65	1.70	1.75	1.78	1.80	29
62	1.40	1.43	1.47	1.50	1.54	1.58	1.62	1.67	1.71	1.77	1.79	1.82	28
63	1.42	1.45	1.49	1.52	1.55	1.59	1.64	1.68	1.73	1.78	1.81	1.84	27
64	1.43	1.46	1.49	1.53	1.57	1.61	1.65	1.70	1.75	1.80	1.83	1.85	26
65	1.44	1.47	1.51	1.54	1.58	1.62	1.66	1.71	1.76	1.81	1.84	1.87	25
66	1.45	1.48	1.52	1.55	1.59	1.63	1.68	1.72	1.77	1.83	1.85	1.88	24
67	1.46	1.50	1.53	1.57	1.60	1.65	1.69	1.74	1.79	1.84	1.87	1.90	23
68	1.47	1.51	1.54	1.58	1.62	1.66	1.70	1.75	1.80	1.85	1.88	1.91	22
69	1.48	1.52	1.55	1.59	1.63	1.67	1.71	1.76	1.81	1.87	1.90	1.93	21
70	1.49	1.53	1.56	1.60	1.64	1.68	1.73	1.77	1.82	1.88	1.91	1.94	**20**
71	1.50	1.54	1.57	1.61	1.65	1.69	1.74	1.78	1.84	1.89	1.92	1.95	19
72	1.51	1.54	1.58	1.62	1.66	1.70	1.75	1.80	1.85	1.90	1.93	1.96	18
73	1.52	1.55	1.59	1.63	1.67	1.71	1.76	1.80	1.86	1.91	1.94	1.97	17
74	1.53	1.56	1.60	1.63	1.68	1.72	1.76	1.81	1.87	1.92	1.95	1.98	16
75	1.53	1.57	1.60	1.64	1.68	1.73	1.77	1.82	1.88	1.93	1.96	1.99	15
76	1.54	1.58	1.61	1.65	1.69	1.73	1.78	1.83	1.88	1.94	1.97	2.00	14
77	1.55	1.58	1.62	1.66	1.70	1.74	1.79	1.84	1.89	1.95	1.98	2.01	13
78	1.55	1.59	1.62	1.66	1.70	1.75	1.80	1.85	1.90	1.96	1.99	2.02	12
79	1.56	1.59	1.63	1.67	1.71	1.76	1.80	1.85	1.91	1.96	1.99	2.02	11
80	1.56	1.60	1.64	1.67	1.72	1.76	1.81	1.86	1.91	1.97	2.00	2.03	**10**
81	1.57	1.60	1.64	1.68	1.72	1.77	1.81	1.86	1.92	1.98	2.01	2.04	9
82	1.57	1.61	1.64	1.68	1.73	1.77	1.82	1.87	1.92	1.98	2.01	2.04	8
83	1.58	1.61	1.65	1.69	1.73	1.77	1.82	1.87	1.93	1.99	2.02	2.05	7
84	1.58	1.62	1.65	1.69	1.73	1.78	1.83	1.88	1.93	1.99	2.02	2.05	6
85	1.58	1.62	1.65	1.69	1.74	1.78	1.83	1.88	1.93	1.99	2.02	2.05	5
86	1.59	1.62	1.66	1.70	1.74	1.78	1.83	1.88	1.94	2.00	2.03	2.06	4
87	1.59	1.62	1.66	1.70	1.74	1.79	1.83	1.88	1.94	2.00	2.03	2.06	3
88	1.59	1.62	1.66	1.70	1.74	1.79	1.83	1.89	1.94	2.00	2.03	2.06	2
89	1.59	1.62	1.66	1.70	1.74	1.79	1.84	1.89	1.94	2.00	2.03	2.06	1
90	1.59	1.62	1.66	1.70	1.74	1.79	1.84	1.89	1.94	2.00	2.03	2.06	**0**

Table LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	61½°	62°	62½°	63°	63½°	64°	64½°	65°	65½°	66°	66½°	67°	ζ
1°	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	.04	89°
2	.07	.07	.08	.08	.08	.08	.08	.08	.08	.09	.09	.09	88
3	.11	.11	.11	.12	.12	.12	.12	.12	.13	.13	.13	.13	87
4	.15	.15	.15	.15	.16	.16	.16	.17	.17	.17	.18	.18	86
5	.18	.19	.19	.19	.20	.20	.20	.21	.21	.21	.22	.22	85
6	.22	.22	.23	.23	.23	.24	.24	.25	.25	.26	.26	.27	84
7	.26	.26	.26	.27	.27	.28	.28	.29	.29	.30	.31	.31	83
8	.29	.30	.30	.31	.31	.32	.32	.33	.34	.34	.35	.36	82
9	.33	.33	.34	.35	.35	.36	.36	.37	.38	.39	.39	.40	81
10	.36	.37	.38	.38	.39	.40	.40	.41	.42	.43	.43	.44	**80**
11	.40	.41	.41	.42	.43	.44	.44	.45	.46	.47	.48	.49	79
12	.44	.44	.45	.46	.47	.47	.48	.49	.50	.51	.52	.53	78
13	.47	.48	.49	.50	.50	.51	.52	.53	.54	.55	.56	.58	77
14	.51	.52	.52	.53	.54	.55	.56	.57	.58	.59	.61	.62	76
15	.54	.55	.56	.57	.58	.59	.60	.61	.62	.64	.65	.66	75
16	.58	.59	.60	.61	.62	.63	.64	.65	.66	.68	.69	.71	74
17	.61	.62	.63	.64	.66	.67	.68	.69	.70	.72	.73	.75	73
18	.65	.66	.67	.68	.69	.70	.72	.73	.74	.76	.77	.79	72
19	.68	.69	.70	.72	.73	.74	.76	.77	.78	.80	.82	.83	71
20	.72	.73	.74	.75	.77	.79	.79	.81	.83	.84	.86	.88	**70**
21	.75	.76	.78	.79	.80	.82	.83	.85	.86	.88	.90	.92	69
22	.78	.80	.81	.82	.84	.85	.87	.89	.90	.92	.94	.96	68
23	.82	.83	.85	.86	.88	.89	.91	.92	.94	.96	.98	1.00	67
24	.85	.87	.88	.90	.91	.93	.94	.96	.98	1.00	1.02	1.04	66
25	.89	.90	.92	.93	.95	.96	.98	1.00	1.02	1.04	1.06	1.08	65
26	.92	.93	.95	.97	.98	1.00	1.02	1.04	1.06	1.08	1.10	1.12	64
27	.95	.97	.98	1.00	1.02	1.04	1.05	1.07	1.09	1.12	1.14	1.16	63
28	.98	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.15	1.18	1.20	62
29	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.15	1.17	1.19	1.22	1.24	61
30	1.05	1.07	1.08	1.10	1.12	1.14	1.16	1.18	1.21	1.23	1.25	1.28	**60**
31	1.08	1.10	1.11	1.13	1.15	1.17	1.20	1.22	1.24	1.27	1.29	1.32	59
32	1.11	1.13	1.15	1.17	1.19	1.21	1.23	1.25	1.28	1.30	1.33	1.36	58
33	1.14	1.16	1.18	1.20	1.22	1.24	1.26	1.29	1.31	1.34	1.37	1.39	57
34	1.17	1.19	1.21	1.23	1.25	1.27	1.30	1.32	1.35	1.37	1.40	1.43	56
35	1.20	1.22	1.24	1.26	1.29	1.31	1.33	1.36	1.38	1.41	1.44	1.47	55
36	1.23	1.25	1.27	1.30	1.32	1.34	1.37	1.39	1.42	1.45	1.47	1.51	54
37	1.26	1.28	1.30	1.33	1.35	1.37	1.40	1.42	1.45	1.48	1.51	1.54	53
38	1.29	1.31	1.33	1.36	1.38	1.40	1.43	1.46	1.48	1.51	1.54	1.58	52
39	1.32	1.34	1.36	1.39	1.41	1.43	1.46	1.49	1.52	1.55	1.58	1.61	51
40	1.35	1.37	1.39	1.42	1.44	1.47	1.49	1.52	1.55	1.58	1.61	1.65	**50**
41	1.37	1.40	1.42	1.45	1.47	1.50	1.53	1.55	1.58	1.61	1.64	1.68	49
42	1.40	1.42	1.45	1.47	1.50	1.53	1.55	1.58	1.61	1.64	1.68	1.71	48
43	1.43	1.45	1.48	1.50	1.53	1.56	1.58	1.61	1.64	1.68	1.71	1.75	47
44	1.46	1.48	1.50	1.53	1.56	1.58	1.61	1.64	1.67	1.71	1.74	1.78	46
45	1.48	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.70	1.74	1.77	1.81	45

TABLE LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	61½°	62°	62½°	63°	63½°	64°	64½°	65°	65½°	66°	66½°	67°	ζ
46°	1.51	1.53	1.56	1.58	1.61	1.64	1.67	1.70	1.74	1.77	1.80	1.84	44°
47	1.53	1.55	1.58	1.61	1.64	1.67	1.70	1.73	1.76	1.80	1.83	1.87	43
48	1.55	1.58	1.60	1.63	1.66	1.69	1.72	1.75	1.79	1.82	1.86	1.90	42
49	1.58	1.61	1.63	1.66	1.69	1.72	1.75	1.79	1.82	1.86	1.89	1.93	41
50	1.60	1.63	1.66	1.69	1.72	1.75	1.78	1.81	1.85	1.88	1.92	1.96	**40**
51	1.63	1.66	1.68	1.71	1.74	1.77	1.80	1.84	1.87	1.91	1.95	1.99	39
52	1.65	1.68	1.71	1.74	1.77	1.80	1.83	1.86	1.90	1.94	1.98	2.02	38
53	1.67	1.70	1.73	1.76	1.79	1.82	1.85	1.89	1.93	1.96	2.00	2.04	37
54	1.69	1.72	1.75	1.78	1.81	1.85	1.88	1.91	1.95	1.99	2.03	2.07	36
55	1.72	1.74	1.77	1.80	1.84	1.87	1.90	1.94	1.98	2.01	2.05	2.10	35
56	1.74	1.77	1.80	1.83	1.86	1.89	1.93	1.96	2.00	2.04	2.08	2.12	34
57	1.76	1.79	1.82	1.85	1.88	1.91	1.95	1.98	2.02	2.06	2.10	2.15	33
58	1.78	1.81	1.84	1.87	1.90	1.93	1.97	2.01	2.05	2.08	2.13	2.17	32
59	1.80	1.83	1.86	1.89	1.92	1.95	1.99	2.03	2.07	2.11	2.15	2.19	31
60	1.81	1.84	1.88	1.91	1.94	1.97	2.01	2.05	2.09	2.13	2.17	2.22	**30**
61	1.83	1.86	1.89	1.93	1.96	2.00	2.03	2.07	2.11	2.15	2.19	2.24	29
62	1.85	1.88	1.91	1.94	1.98	2.01	2.05	2.09	2.13	2.17	2.21	2.26	28
63	1.87	1.90	1.93	1.96	2.00	2.03	2.07	2.11	2.15	2.19	2.23	2.28	27
64	1.88	1.91	1.95	1.98	2.02	2.05	2.09	2.13	2.17	2.21	2.25	2.30	26
65	1.90	1.93	1.96	2.00	2.03	2.07	2.11	2.14	2.19	2.23	2.27	2.32	25
66	1.91	1.95	1.98	2.01	2.05	2.08	2.12	2.16	2.20	2.25	2.29	2.34	24
67	1.93	1.96	1.99	2.03	2.06	2.10	2.14	2.18	2.22	2.26	2.31	2.36	23
68	1.94	1.97	2.01	2.04	2.08	2.11	2.15	2.19	2.24	2.28	2.32	2.37	22
69	1.96	1.99	2.02	2.06	2.09	2.13	2.17	2.21	2.25	2.30	2.34	2.39	21
70	1.97	2.00	2.03	2.07	2.11	2.14	2.18	2.22	2.27	2.31	2.36	2.40	**20**
71	1.98	2.01	2.05	2.08	2.12	2.16	2.20	2.24	2.28	2.32	2.37	2.42	19
72	1.99	2.03	2.06	2.09	2.13	2.17	2.21	2.25	2.29	2.34	2.38	2.43	18
73	2.00	2.04	2.07	2.11	2.14	2.18	2.22	2.26	2.31	2.35	2.40	2.45	17
74	2.01	2.05	2.08	2.12	2.15	2.19	2.23	2.27	2.32	2.36	2.41	2.46	16
75	2.02	2.06	2.09	2.13	2.16	2.20	2.24	2.29	2.33	2.37	2.42	2.47	15
76	2.03	2.07	2.10	2.14	2.17	2.21	2.25	2.30	2.34	2.39	2.43	2.48	14
77	2.04	2.07	2.11	2.15	2.18	2.22	2.26	2.31	2.35	2.40	2.44	2.49	13
78	2.05	2.08	2.12	2.15	2.19	2.23	2.27	2.31	2.36	2.40	2.45	2.50	12
79	2.06	2.09	2.13	2.16	2.20	2.24	2.28	2.32	2.37	2.41	2.46	2.51	11
80	2.06	2.10	2.13	2.17	2.21	2.25	2.29	2.33	2.38	2.42	2.47	2.52	**10**
81	2.07	2.10	2.14	2.18	2.21	2.25	2.29	2.34	2.38	2.43	2.48	2.53	9
82	2.08	2.11	2.15	2.18	2.22	2.26	2.30	2.34	2.39	2.43	2.48	2.53	8
83	2.08	2.12	2.15	2.19	2.22	2.26	2.31	2.35	2.39	2.44	2.49	2.54	7
84	2.08	2.12	2.15	2.19	2.23	2.27	2.31	2.35	2.40	2.45	2.49	2.55	6
85	2.09	2.12	2.16	2.19	2.23	2.27	2.31	2.36	2.40	2.45	2.50	2.55	5
86	2.09	2.13	2.16	2.20	2.24	2.28	2.32	2.36	2.41	2.45	2.50	2.55	4
87	2.09	2.13	2.16	2.20	2.24	2.28	2.32	2.36	2.41	2.46	2.50	2.56	3
88	2.09	2.13	2.16	2.20	2.24	2.28	2.32	2.36	2.41	2.46	2.51	2.56	2
89	2.10	2.13	2.17	2.20	2.24	2.28	2.32	2.37	2.41	2.46	2.51	2.56	1
90	2.10	2.13	2.17	2.20	2.24	2.28	2.32	2.37	2.41	2.46	2.51	2.56	**0**

TABLE LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	67½°	68°	68½°	69°	69½°	70°	70¼°	70½°	70¾°	71°	71¼°	71½°	ζ
1°	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	89°
2	.09	.09	.10	.10	.10	.10	.10	.10	.11	.11	.11	.11	88
3	.14	.14	.14	.15	.15	.15	.15	.16	.16	.16	.16	.16	87
4	.18	.19	.19	.20	.20	.20	.21	.21	.21	.21	.22	.22	86
5	.23	.23	.24	.24	.25	.25	.26	.26	.26	.27	.27	.27	85
6	.27	.28	.28	.29	.30	.31	.31	.31	.32	.32	.33	.33	84
7	.32	.33	.33	.34	.35	.36	.36	.37	.37	.37	.38	.38	83
8	.36	.37	.38	.39	.40	.41	.41	.42	.42	.43	.43	.44	82
9	.41	.42	.43	.44	.45	.46	.46	.47	.47	.48	.49	.49	81
10	.45	.46	.47	.49	.50	.51	.51	.52	.53	.53	.54	.55	**80**
11	.50	.51	.52	.53	.54	.56	.56	.57	.58	.59	.59	.60	79
12	.54	.56	.57	.58	.59	.61	.62	.62	.63	.64	.65	.66	78
13	.59	.60	.61	.63	.64	.66	.67	.67	.68	.69	.70	.71	77
14	.63	.65	.66	.68	.69	.71	.72	.72	.73	.74	.75	.76	76
15	.68	.69	.71	.72	.74	.76	.77	.78	.78	.79	.80	.81	75
16	.72	.74	.75	.77	.79	.81	.82	.83	.84	.85	.86	.87	74
17	.76	.78	.80	.81	.83	.85	.86	.88	.89	.90	.91	.92	73
18	.81	.83	.84	.86	.88	.90	.91	.93	.94	.95	.96	.97	72
19	.85	.87	.89	.91	.93	.95	.96	.98	.99	1.00	1.01	1.03	71
20	.89	.91	.93	.95	.98	1.00	1.01	1.02	1.04	1.05	1.06	1.08	**70**
21	.94	.96	.98	1.00	1.02	1.05	1.06	1.07	1.09	1.10	1.11	1.13	69
22	.98	1.00	1.02	1.05	1.07	1.09	1.11	1.12	1.14	1.15	1.17	1.18	68
23	1.02	1.04	1.07	1.09	1.12	1.14	1.16	1.17	1.19	1.20	1.21	1.23	67
24	1.06	1.09	1.11	1.14	1.16	1.19	1.20	1.22	1.23	1.25	1.27	1.28	66
25	1.10	1.13	1.15	1.18	1.21	1.24	1.25	1.27	1.28	1.30	1.31	1.33	65
26	1.15	1.17	1.20	1.22	1.25	1.28	1.30	1.31	1.33	1.35	1.36	1.38	64
27	1.19	1.21	1.24	1.27	1.30	1.33	1.34	1.36	1.38	1.39	1.41	1.43	63
28	1.23	1.25	1.28	1.31	1.34	1.37	1.39	1.41	1.42	1.44	1.46	1.48	62
29	1.27	1.29	1.32	1.35	1.38	1.42	1.43	1.45	1.47	1.49	1.51	1.53	61
30	1.31	1.33	1.36	1.39	1.43	1.46	1.48	1.50	1.52	1.54	1.56	1.58	**60**
31	1.35	1.38	1.40	1.44	1.47	1.51	1.52	1.54	1.56	1.58	1.60	1.62	59
32	1.39	1.42	1.45	1.48	1.51	1.55	1.57	1.59	1.61	1.63	1.65	1.67	58
33	1.42	1.45	1.49	1.52	1.55	1.59	1.61	1.63	1.65	1.67	1.69	1.72	57
34	1.46	1.49	1.53	1.56	1.60	1.63	1.65	1.68	1.70	1.72	1.74	1.76	56
35	1.50	1.53	1.56	1.60	1.64	1.68	1.70	1.72	1.74	1.76	1.78	1.81	55
36	1.54	1.57	1.60	1.64	1.68	1.72	1.74	1.76	1.78	1.80	1.83	1.85	54
37	1.57	1.61	1.64	1.68	1.72	1.76	1.78	1.80	1.83	1.85	1.87	1.90	53
38	1.61	1.64	1.68	1.72	1.76	1.80	1.82	1.84	1.87	1.89	1.91	1.94	52
39	1.65	1.68	1.72	1.75	1.80	1.84	1.86	1.88	1.91	1.93	1.96	1.98	51
40	1.68	1.72	1.75	1.79	1.84	1.88	1.90	1.93	1.95	1.97	2.00	2.03	**50**
41	1.71	1.75	1.79	1.83	1.87	1.92	1.94	1.96	1.99	2.01	2.04	2.07	49
42	1.75	1.79	1.83	1.87	1.91	1.96	1.98	2.00	2.03	2.05	2.08	2.11	48
43	1.78	1.82	1.86	1.90	1.95	1.99	2.02	2.04	2.07	2.09	2.12	2.15	47
44	1.82	1.85	1.90	1.94	1.98	2.03	2.06	2.08	2.11	2.13	2.16	2.19	46
45	1.85	1.89	1.93	1.97	2.02	2.07	2.09	2.12	2.14	2.17	2.20	2.23	45

TABLE LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	67½°	68°	68½°	69°	69½°	70°	70¼°	70½°	70¾°	71°	71¼°	71½°	ζ
46°	1.88	1.92	1.96	2.01	2.05	2.10	2.13	2.15	2.18	2.21	2.24	2.27	44°
47	1.91	1.95	2.00	2.04	2.09	2.14	2.16	2.19	2.22	2.25	2.27	2.30	43
48	1.94	1.98	2.02	2.07	2.12	2.17	2.19	2.22	2.25	2.28	2.31	2.34	42
49	1.97	2.01	2.06	2.11	2.16	2.21	2.23	2.26	2.29	2.32	2.35	2.38	41
50	2.00	2.04	2.09	2.14	2.19	2.24	2.27	2.29	2.32	2.35	2.38	2.41	**40**
51	2.03	2.07	2.12	2.17	2.22	2.27	2.30	2.33	2.36	2.39	2.42	2.45	39
52	2.06	2.10	2.15	2.20	2.25	2.30	2.33	2.36	2.39	2.42	2.45	2.48	38
53	2.09	2.13	2.18	2.23	2.28	2.33	2.36	2.39	2.42	2.45	2.48	2.52	37
54	2.11	2.16	2.21	2.26	2.31	2.37	2.39	2.42	2.45	2.48	2.52	2.55	36
55	2.14	2.19	2.23	2.29	2.34	2.40	2.42	2.45	2.48	2.52	2.55	2 58	35
56	2.17	2.21	2.26	2.31	2.37	2.42	2.45	2.48	2.51	2.55	2.58	2.61	34
57	2.19	2.24	2.29	2.34	2.39	2.45	2.48	2.51	2.54	2.58	2.61	2.64	33
58	2.22	2.26	2.32	2.37	2.42	2.48	2.51	2.54	2.57	2.61	2.64	2.67	32
59	2.24	2.29	2.34	2.39	2.45	2.51	2.54	2.57	2.60	2.63	2.67	2.70	31
60	2.26	2.31	2.36	2.42	2.47	2.53	2.56	2.59	2.63	2.66	2.69	2.73	**30**
61	2.29	2.33	2.39	2.44	2.50	2.56	2.59	2.62	2.65	2.69	2.72	2.76	29
62	2.31	2.36	2.41	2.46	2.52	2.58	2.61	2.64	2.68	2.71	2.75	2.78	28
63	2.33	2.38	2.43	2.49	2.54	2.60	2.64	2.67	2.70	2.74	2.77	2.81	27
64	2.35	2.40	2.45	2.51	2.57	2.63	2.66	2.69	2.73	2.76	2.80	2.83	26
65	2.37	2.42	2.47	2.53	2.59	2.65	2.68	2.71	2.75	2.78	2.82	2.86	25
66	2.39	2.44	2.49	2.55	2.61	2.67	2.70	2.74	2.77	2.81	2.84	2.88	24
67	2.41	2.46	2.51	2.57	2.63	2.69	2.72	2.76	2.79	2.83	2.86	2.90	23
68	2.42	2.47	2.53	2.59	2.65	2.71	2.74	2.78	2.81	2.85	2.88	2.92	22
69	2.44	2.49	2.55	2.61	2.67	2.73	2.76	2.80	2.83	2.87	2.90	2.94	21
70	2.46	2.51	2.56	2.62	2.68	2.75	2.78	2.81	2.85	2.89	2.92	2.96	**20**
71	2.47	2.52	2.58	2.64	2.70	2.77	2.80	2.83	2.87	2.90	2.94	2.98	19
72	2.49	2.54	2.59	2.65	2.72	2 78	2.81	2.85	2.88	2.92	2.96	3.00	18
73	2.50	2.55	2.61	2.67	2.73	2.80	2.83	2.86	2.90	2.94	2.97	3.01	17
74	2.51	2.57	2.62	2.68	2.74	2.81	2.84	2.88	2.92	2.95	2.99	3.03	16
75	2.52	2.58	2.64	2.70	2.76	2.82	2.86	2.89	2.93	2.97	3.00	3.04	15
76	2.54	2.59	2.65	2.71	2.77	2.84	2.87	2.91	2.95	2.99	3.02	3.06	14
77	2.55	2.60	2.66	2.72	2.78	2.85	2.88	2.92	2.95	2.99	3.03	3.07	13
78	2.56	2.61	2.67	2.73	2.79	2.86	2.89	2.93	2.97	3.00	3.04	3.08	12
79	2.57	2.62	2.68	2.74	2.80	2.87	2.91	2.94	2.98	3.02	3.05	3.09	11
80	2.57	2.63	2.69	2.75	2.81	2.88	2.91	2.95	2.99	3.02	3.06	3.10	**10**
81	2.58	2.64	2.69	2.76	2.82	2.89	2.92	2.96	3.00	3.03	3.07	3.11	9
82	2.59	2.64	2.70	2.76	2.83	2.90	2.93	2.97	3.00	3.04	3.08	3.12	8
83	2.59	2.65	2.71	2.77	2.83	2.90	2.94	2.97	3.01	3.05	3.09	3.13	7
84	2.60	2.66	2.71	2.78	2.84	2.91	2.94	2.98	3.02	3.06	3.09	3.13	6
85	2.60	2.66	2.72	2.78	2.84	2.91	2.95	2.98	3.02	3.06	3.10	3.14	5
86	2.61	2.66	2.72	2.78	2.85	2.92	2.95	2.99	3.03	3.06	3.10	3.14	4
87	2.61	2.67	2.72	2.79	2.85	2.92	2.95	2.99	3.03	3.07	3.11	3.15	3
88	2.61	2.67	2.73	2.79	2.85	2.92	2.96	2.99	3.03	3.07	3.11	3.15	2
89	2.61	2.67	2.73	2.79	2.86	2.92	2.96	3.00	3.03	3.07	3.11	3.15	1
90	2.61	2.67	2.73	2.79	2.86	2.92	2.96	3.00	3.03	3.07	3.11	3.15	**0**

Table LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	71¾°	72°	72¼°	72½°	72¾°	73°	73¼°	73½°	73¾°	74°	74½°	ζ
1°	.05	.06	.06	.06	.06	.06	.06	.06	.06	.06	.06	89°
2	.11	.11	.11	.12	.12	.12	.12	.12	.12	.13	.13	88
3	.17	.17	.17	.17	.18	.18	.18	.18	.19	.19	.19	87
4	.22	.23	.23	.23	.23	.24	.24	.24	.25	.25	.26	86
5	.28	.28	.29	.29	.29	.30	.30	.31	.31	.32	.32	85
6	.33	.34	.34	.35	.35	.36	.36	.37	.37	.38	.39	84
7	.39	.39	.40	.41	.41	.42	.42	.43	.44	.44	.45	83
8	.44	.45	.46	.46	.47	.48	.48	.49	.50	.50	.51	82
9	.50	.51	.51	.52	.53	.53	.54	.55	.56	.57	.58	81
10	.55	.56	.57	.58	.59	.60	.60	.61	.62	.63	.64	**80**
11	.61	.62	.63	.63	.64	.65	.66	.67	.68	.69	.70	79
12	.66	.67	.68	.69	.70	.71	.72	.73	.74	.75	.77	78
13	.72	.73	.74	.75	.76	.77	.78	.79	.80	.82	.83	77
14	.77	.78	.79	.80	.82	.83	.84	.85	.87	.88	.89	76
15	.83	.84	.85	.86	.87	.89	.90	.91	.93	.94	.95	75
16	.88	.89	.91	.92	.93	.94	.96	.97	.99	1.00	1.02	74
17	.93	.95	.96	.97	.99	1.00	1.01	1.03	1.05	1.06	1.08	73
18	.99	1.00	1.01	1.03	1.04	1.06	1.07	1.09	1.10	1.12	1.14	72
19	1.04	1.05	1.07	1.08	1.10	1.11	1.13	1.15	1.16	1.18	1.20	71
20	1.09	1.11	1.12	1.14	1.15	1.17	1.19	1.20	1.22	1.24	1.26	**70**
21	1.14	1.16	1.17	1.19	1.21	1.22	1.24	1.26	1.28	1.30	1.32	69
22	1.20	1.21	1.23	1.25	1.26	1.28	1.30	1.32	1.34	1.36	1.38	68
23	1.25	1.26	1.20	1.30	1.32	1.34	1.36	1.38	1.40	1.42	1.44	67
24	1.30	1.32	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.48	1.50	66
25	1.35	1.37	1.39	1.41	1.42	1.45	1.47	1.49	1.51	1.53	1.56	65
26	1.40	1.42	1.44	1.46	1.48	1.50	1.52	1.54	1.57	1.59	1.61	64
27	1.45	1.47	1.49	1.51	1.53	1.55	1.58	1.60	1.62	1.65	1.67	63
28	1.50	1.52	1.54	1.56	1.58	1.60	1.63	1.65	1.68	1.70	1.73	62
29	1.55	1.57	1.59	1.61	1.63	1.66	1.68	1.71	1.73	1.76	1.79	61
30	1.60	1.62	1.64	1.66	1.69	1.71	1.73	1.76	1.79	1.81	1.84	**60**
31	1.64	1.67	1.69	1.71	1.74	1.76	1.79	1.81	1.84	1.87	1.90	59
32	1.69	1.71	1.74	1.76	1.79	1.81	1.84	1.87	1.89	1.92	1.95	58
33	1.74	1.76	1.79	1.81	1.84	1.86	1.89	1.92	1.95	1.98	2.01	57
34	1.79	1.81	1.83	1.86	1.89	1.91	1.94	1.97	2.00	2.03	2.06	56
35	1.83	1.86	1.88	1.91	1.93	1.96	1.99	2.02	2.05	2.08	2.11	55
36	1.88	1.90	1.93	1.95	1.98	2.01	2.04	2.07	2.10	2.13	2.16	54
37	1.92	1.95	1.97	2.00	2.03	2.06	2.09	2.12	2.15	2.18	2.22	53
38	1.97	1.99	2.02	2.05	2.08	2.11	2.14	2.17	2.20	2.23	2.27	52
39	2.01	2.04	2.06	2.09	2.12	2.15	2.18	2.22	2.25	2.28	2.32	51
40	2.05	2.08	2.11	2.14	2.17	2.20	2.23	2.26	2.30	2.33	2.37	**50**
41	2.09	2.12	2.15	2.18	2.21	2.24	2.28	2.31	2.34	2.38	2.42	49
42	2.14	2.16	2.19	2.22	2.26	2.29	2.32	2.36	2.39	2.43	2.46	48
43	2.18	2.21	2.24	2.27	2.30	2.33	2.37	2.40	2.44	2.47	2.51	47
44	2.22	2.25	2.28	2.31	2.34	2.38	2.41	2.45	2.48	2.52	2.56	46
45	2.26	2.29	2.32	2.35	2.38	2.42	2.45	2.49	2.53	2.56	2.60	45

TABLE LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	71¾°	72°	72¼°	72½°	72¾°	73°	73¼°	73½°	73¾°	74°	74¼°	ζ
46°	2.30	2.33	2.36	2.39	2.42	2.46	2.49	2.53	2.57	2.61	2.65	44°
47	2.33	2.37	2.40	2.43	2.47	2.50	2.54	2.57	2.61	2.65	2.69	43
48	2.37	2.40	2.44	2.47	2.51	2.54	2.58	2.62	2.66	2.70	2.74	42
49	2.41	2.44	2.48	2.51	2.55	2.58	2.62	2.66	2.70	2.74	2.78	41
50	2.45	2.48	2.51	2.55	2.58	2.62	2.66	2.70	2.74	2.78	2.82	**40**
51	2.48	2.51	2.55	2.58	2.62	2.66	2.70	2.74	2.78	2.82	2.86	39
52	2.52	2.55	2.58	2 62	2.66	2.69	2.73	2.77	2.82	2.86	2.90	38
53	2.55	2.58	2.62	2.66	2.69	2.73	2.77	2.81	2.85	2.90	2.94	37
54	2.58	2.62	2.65	2.69	2.73	2.77	2.81	2.85	2.89	2.94	2.98	36
55	2.62	2.65	2.69	2.72	2.76	2.80	2.84	2.88	2.93	2.97	3.02	35
56	2.65	2.68	2.72	2.76	2.80	2.84	2.88	2 92	2.96	3.01	3.05	34
57	2.68	2.71	2.75	2.79	2.83	2.87	2.91	2 95	3.00	3.04	3.09	33
58	2.71	2.74	2.78	2.82	2.86	2.90	2.94	2.99	3.03	3.08	3.12	32
59	2.74	2.77	2.81	2.85	2.89	2.93	2.97	3.02	3.06	3.11	3.16	31
60	2.76	2.80	2 84	2.88	2.92	2.96	3.01	3.05	3.09	3.14	3.19	**30**
61	2.79	2.83	2.87	2.91	2.95	2.99	3.04	3.08	3.13	3.17	3.22	29
62	2.82	2.86	2.90	2.94	2.98	3.02	3.06	3.11	3.16	3.20	3.25	28
63	2.84	2.88	2.92	2.96	3.00	3.05	3.09	3.14	3.18	3 23	3.28	27
64	2.87	2.91	2.95	2.99	3.03	3.07	3.12	3.16	3 21	3.26	3.31	26
65	2.89	2.93	2.97	3.01	3.06	3.10	3.14	3.19	3.24	3.29	3.34	25
66	2.92	2.96	3.00	3.04	3.08	3.13	3.17	3.22	3.27	3.31	3.37	24
67	2.94	2.98	3.02	3.06	3.10	3.15	3.20	3.24	3.29	3.34	3.39	23
68	2.96	3.00	3.04	3.08	3.13	3.17	3.22	3.26	3.31	3.36	3.42	22
69	2.98	3.02	3.06	3.10	3.15	3.19	3.24	3.29	3.34	3.39	3.44	21
70	3.00	3.04	3.08	3.12	3.17	3.21	3.26	3.31	3.36	3.41	3.46	**20**
71	3.02	3.06	3.10	3.14	3.19	3.24	3.28	3.33	3.38	3.43	3.48	19
72	3.04	3.08	3.12	3.16	3.21	3.25	3.30	3.35	3.40	3.45	3.50	18
73	3.05	3.09	3.14	3.18	3.22	3.27	3.32	3.37	3.42	3.47	3.52	17
74	3 07	3.11	3.15	3.20	3.24	3.29	3.33	3.38	3.44	3 49	3 54	16
75	3.08	3.13	3.17	3.21	3.26	3 30	3.35	3.40	3.45	3.50	3.56	15
76	3.10	3.15	3.18	3.23	3.28	3.32	3.37	3.42	3.47	3.53	3.58	14
77	3.11	3.15	3.19	3.24	3.29	3.33	3.38	3.43	3.48	3.54	3.59	13
78	3.12	3.16	3.21	3.25	3.30	3.34	3.39	3.44	3.49	3.55	3.60	12
79	3.13	3.18	3.22	3.26	3.31	3.36	3.41	3 46	3.51	3.56	3.62	11
80	3.14	3.19	3.23	3.27	3.32	3.37	3.42	3.47	3.52	3.57	3.63	**10**
81	3.15	3.20	3.24	3.28	3.33	3.38	3.43	3.48	3.53	3.58	3.64	9
82	3.16	3.20	3.25	3.29	3.34	3.39	3.44	3.49	3.54	3.59	3.65	8
83	3.17	3.21	3.26	3.30	3.35	3.40	3.45	3.49	3.55	3.60	3.66	7
84	3.18	3.22	3.26	3.31	3.35	3.40	3.45	3.50	3.55	3.61	3.66	6
85	3.18	3.22	3.27	3.31	3.36	3.41	3.46	3.51	3.56	3.61	3.67	5
86	3.19	3.23	3.27	3.32	3.36	3.41	3.46	3.51	3.57	3.62	3.68	4
87	3.19	3.23	3.28	3.32	3.37	3.42	3 47	3.52	3.57	3.62	3.68	3
88	3.19	3.23	3.28	3.32	3.37	3.42	3 47	3.52	3.57	3.62	3.68	2
89	3.19	3.24	3.28	3.33	3.37	3.42	3.47	3.52	3.57	3.63	3.68	1
90	3.19	3.24	3.28	3.33	3.37	3.42	3 47	3 52	3 57	3.63	3.68	**0**

TABLE LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	74½°	74¾°	75°	75¼°	75½°	75¾°	76°	76¼°	76½°	76¾°	77°	77¼°	ζ
1°	.06	.07	.07	.07	.07	.07	.07	.07	.07	.08	.08	.08	89
2	.13	.13	.13	.14	.14	.14	.14	.15	.15	.15	.16	.16	88
3	.20	.20	.20	.21	.21	.21	.22	.22	.22	.23	.23	.24	87
4	.26	.27	.27	.27	.28	.28	.29	.29	.30	.30	.31	.32	86
5	.33	.33	.34	.34	.35	.35	.36	.37	.37	.38	.39	.40	85
6	.39	.40	.40	.41	.42	.42	.43	.44	.45	.46	.46	.47	84
7	.46	.46	.47	.48	.49	.50	.50	.51	.52	.53	.54	.55	83
8	.52	.53	.54	.55	.56	.57	.58	.59	.60	.61	.62	.63	82
9	.58	.59	.60	.61	.62	.64	.65	.66	.67	.68	.70	.71	81
10	.65	.66	.67	.68	.69	.71	.72	.73	.74	.76	.77	.79	**80**
11	.71	.73	.74	.75	.76	.77	.79	.80	.82	.83	.85	.86	79
12	.78	.79	.80	.82	.83	.85	.86	.88	.89	.91	.92	.94	78
13	.84	.86	.87	.88	.90	.91	.93	.95	.96	.98	1.00	1.02	77
14	.91	.92	.94	.95	.97	.98	1.00	1.02	1.04	1.06	1.08	1.10	76
15	.97	.98	1.00	1.02	1.03	1.05	1.07	1.09	1.11	1.13	1.15	1.17	75
16	1.03	1.05	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.20	1.23	1.25	74
17	1.09	1.11	1.13	1.15	1.17	1.19	1.21	1.23	1.25	1.28	1.30	1.32	73
18	1.16	1.17	1.19	1.21	1.23	1.25	1.28	1.30	1.32	1.35	1.37	1.40	72
19	1.22	1.24	1.26	1.28	1.30	1.32	1.35	1.37	1.39	1.42	1.45	1.47	71
20	1.28	1.30	1.32	1.34	1.37	1.39	1.41	1.44	1.46	1.49	1.50	1.55	**70**
21	1.34	1.36	1.38	1.41	1.43	1.46	1.48	1.51	1.54	1.56	1.59	1.62	69
22	1.40	1.42	1.45	1.47	1.50	1.52	1.55	1.58	1.60	1.63	1.66	1.70	68
23	1.46	1.49	1.51	1.54	1.56	1.59	1.62	1.64	1.67	1.70	1.74	1.77	67
24	1.52	1.55	1.57	1.60	1.63	1.65	1.68	1.71	1.74	1.77	1.81	1.84	66
25	1.58	1.61	1.63	1.66	1.69	1.72	1.75	1.78	1.81	1.84	1.88	1.91	65
26	1.64	1.67	1.69	1.72	1.75	1.78	1.81	1.84	1.88	1.91	1.95	1.99	64
27	1.70	1.73	1.75	1.78	1.81	1.85	1.88	1.91	1.95	1.98	2.02	2.06	63
28	1.76	1.78	1.81	1.84	1.87	1.91	1.94	1.97	2.01	2.05	2.09	2.13	62
29	1.81	1.84	1.87	1.90	1.94	1.97	2.00	2.04	2.08	2.11	2.15	2.20	61
30	1.87	1.90	1.93	1.96	2.00	2.03	2.07	2.10	2.14	2.18	2.22	2.27	**60**
31	1.93	1.96	1.99	2.02	2.06	2.09	2.13	2.17	2.21	2.25	2.29	2.33	59
32	1.98	2.01	2.05	2.08	2.12	2.15	2.19	2.23	2.27	2.31	2.36	2.40	58
33	2.04	2.07	2.10	2.14	2.18	2.21	2.25	2.29	2.33	2.38	2.42	2.47	57
34	2.09	2.13	2.16	2.20	2.23	2.27	2.31	2.35	2.40	2.44	2.49	2.53	56
35	2.15	2.18	2.22	2.25	2.29	2.33	2.37	2.41	2.46	2.50	2.55	2.60	55
36	2.20	2.24	2.27	2.31	2.35	2.39	2.43	2.47	2.52	2.56	2.61	2.66	54
37	2.25	2.29	2.33	2.36	2.40	2.44	2.49	2.53	2.58	2.63	2.67	2.73	53
38	2.30	2.34	2.38	2.42	2.46	2.50	2.55	2.59	2.64	2.69	2.74	2.79	52
39	2.35	2.39	2.43	2.47	2.51	2.56	2.60	2.65	2.70	2.75	2.80	2.85	51
40	2.40	2.44	2.48	2.52	2.57	2.61	2.66	2.70	2.75	2.80	2.86	2.91	**50**
41	2.45	2.49	2.53	2.58	2.62	2.66	2.71	2.76	2.81	2.86	2.92	2.97	49
42	2.50	2.54	2.58	2.63	2.67	2.72	2.77	2.81	2.87	2.92	2.97	3.03	48
43	2.55	2.59	2.63	2.68	2.72	2.77	2.82	2.87	2.92	2.98	3.03	3.09	47
44	2.60	2.64	2.68	2.73	2.77	2.82	2.87	2.92	2.98	3.03	3.09	3.15	46
45	2.65	2.69	2.73	2.78	2.82	2.87	2.92	2.97	3.03	3.08	3.14	3.20	45

Table LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	74½°	74¾°	75°	75¼°	75½°	75¾°	76°	76¼°	76½°	76¾°	77°	77¼°	ζ
46°	2.69	2.73	2.78	2.82	2.87	2.92	2.97	3.03	3.08	3.14	3.20	3.26	44°
47	2.74	2.78	2.83	2.87	2.92	2.97	3.02	3.08	3.13	3.19	3.25	3.31	43
48	2.78	2.82	2.87	2.92	2.97	3.02	3.07	3.13	3.18	3.24	3.30	3.37	42
49	2.82	2.87	2.92	2.96	3.01	3.07	3.12	3.18	3.23	3.29	3.35	3.42	41
50	2.87	2.91	2.96	3.01	3.06	3.11	3.17	3.22	3.28	3.34	3.41	3.47	**40**
51	2.91	2.95	3.00	3.05	3.10	3.16	3.21	3.27	3.33	3.39	3.45	3.52	39
52	2.95	3.00	3.04	3.09	3.15	3.20	3.26	3.31	3.38	3.44	3.50	3.57	38
53	2.99	3.04	3.09	3.14	3.19	3.24	3.30	3.36	3.42	3.48	3.55	3.62	37
54	3.03	3.08	3.13	3.18	3.23	3.29	3.34	3.40	3.47	3.53	3.60	3.67	36
55	3.07	3.11	3.16	3.22	3.27	3.33	3.39	3.45	3.51	3.57	3.64	3.71	35
56	3.10	3.15	3.20	3.26	3.31	3.37	3.43	3.49	3.55	3.62	3.68	3.76	34
57	3.14	3.19	3.24	3.29	3.35	3.41	3.47	3.53	3.59	3.66	3.73	3.80	33
58	3.17	3.22	3.28	3.33	3.39	3.45	3.51	3.57	3.63	3.70	3.77	3.84	32
59	3.21	3.26	3.31	3.37	3.42	3.48	3.54	3.61	3.67	3.74	3.81	3.88	31
60	3.24	3.29	3.35	3.40	3.46	3.52	3.58	3.64	3.71	3.78	3.85	3.92	**30**
61	3.27	3.33	3.38	3.44	3.49	3.55	3.62	3.68	3.75	3.82	3.89	3.96	29
62	3.30	3.36	3.41	3.47	3.53	3.59	3.65	3.72	3.78	3.85	3.92	4.00	28
63	3.33	3.39	3.44	3.50	3.56	3.62	3.68	3.75	3.82	3.89	3.96	4.04	27
64	3.36	3.42	3.47	3.53	3.59	3.65	3.72	3.78	3.85	3.92	4.00	4.07	26
65	3.39	3.45	3.50	3.56	3.62	3.68	3.75	3.81	3.88	3.95	4.03	4.11	25
66	3.42	3.47	3.53	3.59	3.65	3.71	3.78	3.84	3.91	3.99	4.06	4.14	24
67	3.44	3.50	3.56	3.62	3.68	3.74	3.81	3.87	3.94	4.02	4.09	4.17	23
68	3.47	3.53	3.58	3.64	3.70	3.77	3.83	3.90	3.97	4.05	4.12	4.20	22
69	3.49	3.55	3.61	3.67	3.73	3.79	3.86	3.93	4.00	4.07	4.15	4.23	21
70	3.52	3.57	3.63	3.69	3.75	3.82	3.89	3.95	4.03	4.10	4.18	4.25	**20**
71	3.54	3.60	3.65	3.71	3.78	3.84	3.91	3.98	4.05	4.13	4.20	4.28	19
72	3.56	3.62	3.67	3.74	3.80	3.86	3.93	4.00	4.07	4.15	4.23	4.31	18
73	3.58	3.64	3.69	3.76	3.82	3.89	3.95	4.02	4.10	4.17	4.25	4.33	17
74	3.60	3.65	3.71	3.78	3.84	3.91	3.97	4.04	4.12	4.19	4.27	4.36	16
75	3.61	3.67	3.73	3.79	3.86	3.92	3.99	4.06	4.14	4.21	4.29	4.38	15
76	3.64	3.69	3.75	3.82	3.88	3.94	4.01	4.08	4.16	4.23	4.31	4.40	14
77	3.65	3.70	3.76	3.83	3.89	3.96	4.03	4.10	4.17	4.25	4.33	4.41	13
78	3.66	3.72	3.78	3.84	3.91	3.97	4.04	4.11	4.19	4.27	4.35	4.43	12
79	3.67	3.73	3.79	3.86	3.92	3.99	4.06	4.13	4.21	4.28	4.36	4.45	11
80	3.68	3.74	3.81	3.87	3.93	4.00	4.07	4.14	4.22	4.30	4.38	4.46	**10**
81	3.70	3.75	3.82	3.88	3.94	4.01	4.08	4.16	4.23	4.31	4.39	4.48	9
82	3.71	3.76	3.83	3.89	3.96	4.02	4.09	4.17	4.24	4.32	4.40	4.49	8
83	3.72	3.77	3.84	3.90	3.96	4.03	4.10	4.18	4.25	4.33	4.41	4.50	7
84	3.72	3.78	3.84	3.91	3.97	4.04	4.11	4.18	4.26	4.34	4.42	4.51	6
85	3.73	3.79	3.85	3.91	3.98	4.05	4.12	4.19	4.27	4.35	4.43	4.51	5
86	3.73	3.79	3.85	3.92	3.98	4.05	4.12	4.20	4.27	4.35	4.43	4.52	4
87	3.74	3.79	3.86	3.92	3.99	4.06	4.13	4.20	4.28	4.36	4.44	4.53	3
88	3.74	3.80	3.86	3.92	3.99	4.06	4.13	4.20	4.28	4.36	4.44	4.53	2
89	3.74	3.80	3.86	3.93	3.99	4.06	4.13	4.21	4.28	4.36	4.44	4.53	1
90	3.74	3.80	3.86	3.93	3.99	4.06	4.13	4.21	4.28	4.36	4.44	4.53	**0**

TABLE LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	77½°	77¾°	78°	78¼°	78½°	78¾°	79°	79¼°	79½°	79¾°	80°	ζ
1°	.08	.08	.08	.09	.09	.09	.09	.09	.10	.10	.10	89
2	.16	.16	.17	.17	.18	.18	.18	.19	.19	.20	.20	88
3	.24	.25	.25	.26	.26	.27	.27	.28	.29	.29	.30	87
4	.32	.33	.34	.34	.35	.36	.37	.37	.38	.39	.40	86
5	.40	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50	85
6	.49	.49	.51	.51	.52	.54	.55	.56	.57	.59	.60	84
7	.56	.57	.59	.60	.61	.62	.64	.65	.67	.69	.70	83
8	.64	.66	.67	.68	.70	.71	.73	.75	.76	.78	.80	82
9	.72	.74	.75	.77	.78	.80	.82	.84	.86	.88	.90	81
10	.80	.82	.84	.85	.87	.89	91	.93	.95	.98	1.00	**80**
11	.88	.90	.92	.94	.96	.98	1.00	1.02	1.05	1.07	1.10	79
12	.96	.98	1.00	1.02	1.04	1.07	1.09	1.11	1.14	1.17	1.20	78
13	1.04	1.06	1.08	1.10	1.13	1.15	1.18	1.21	1.23	1.26	1.30	77
14	1.12	1.14	1.16	1.19	1.21	1.24	1.27	1.30	1.33	1.36	1.39	76
15	1.20	1.22	1.25	1.27	1.30	1.33	1.36	1.39	1.42	1.46	1.49	75
16	1.28	1.30	1.33	1.35	1.38	1.41	1.44	1.48	1.51	1.55	1.59	74
17	1.35	1.38	1.40	1.44	1.47	1.50	1.53	1.57	1.60	1.64	1.68	73
18	1.43	1.46	1.49	1.52	1.55	1.58	1.62	1.66	1.70	1.74	1.78	72
19	1.51	1.53	1.57	1.60	1.63	1.67	1.71	1.75	1.79	1.83	1.87	71
20	1.58	1.61	1.65	1.68	1.72	1.75	1.79	1.83	1.88	1.92	1.97	**70**
21	1.65	1.69	1.72	1.76	1.80	1.84	1.88	1.92	1.97	2.01	2.06	69
22	1.73	1.77	1.80	1.84	1.88	1.92	1.96	2.01	2.06	2.11	2.16	68
23	1.81	1.84	1.88	1.92	1.96	2.00	2.05	2.09	2.14	2.20	2.25	67
24	1.88	1.92	1.96	2.00	2.04	2.08	2.13	2.18	2.23	2.29	2.34	66
25	1.95	1.99	2.03	2.07	2.12	2.17	2.22	2.27	2.32	2.38	2.43	65
26	2.02	2.07	2.11	2.15	2.20	2.25	2.30	2.35	2.41	2.46	2.52	64
27	2.10	2.14	2.18	2.23	2.28	2.33	2.38	2.43	2.49	2.55	2.61	63
28	2.17	2.21	2.26	2.31	2.36	2.41	2.46	2.52	2.58	2.64	2.70	62
29	2.24	2.28	2.33	2.38	2.43	2.48	2.54	2.60	2.66	2.73	2.79	61
30	2.31	2.36	2.40	2.46	2.51	2.56	2.62	2.68	2.74	2.81	2.88	**60**
31	2.38	2.43	2.48	2.53	2.58	2.64	2.70	2.76	2.83	2.89	2.97	59
32	2.45	2.50	2.55	2.60	2.66	2.72	2.78	2.84	2.91	2.98	3.05	58
33	2.52	2.57	2.62	2.67	2.73	2.79	2.85	2.92	2.99	3.06	3.14	57
34	2.58	2.64	2.69	2.75	2.80	2.87	2.93	3.00	3.07	3.14	3.22	56
35	2.65	2.70	2.76	2.82	2.88	2.94	3.01	3.08	3.15	3.23	3.30	55
36	2.72	2.77	2.83	2.89	2.95	3.01	3.08	3.15	3.23	3.30	3.38	54
37	2.78	2.84	2.90	2.95	3.02	3.08	3.15	3.23	3.30	3.38	3.47	53
38	2.85	2.90	2.96	3.02	3.09	3.16	3.23	3.30	3.38	3.46	3.55	52
39	2.91	2.97	3.03	3.09	3.16	3.23	3.30	3.37	3.45	3.53	3.62	51
40	2.97	3.03	3.09	3.16	3.22	3.29	3.37	3.45	3.53	3.61	3.70	**50**
41	3.03	3.09	3.16	3.22	3.29	3.36	3.44	3.52	3.60	3.69	3.78	49
42	3.09	3.15	3.22	3.29	3.36	3.43	3.51	3.59	3.67	3.76	3.85	48
43	3.15	3.21	3.28	3.35	3.42	3.50	3.57	3.66	3.74	3.83	3.93	47
44	3.21	3.27	3.34	3.41	3.48	3.56	3.64	3.72	3.81	3.91	4.00	46
45	3.27	3.33	3.40	3.47	3.55	3.62	3.71	3.79	3.88	3.97	4.07	45

Table LXX.

FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

ζ	77½°	77¾°	78°	78¼°	78½°	78¾°	79°	79¼°	79½°	79¾°	80°	ζ
46°	3.32	3.39	3.46	3.53	3.61	3.69	3.77	3.86	3.95	4.04	4.14	44°
47	3.38	3.45	3.52	3.59	3.67	3.75	3.83	3.92	4.01	4.11	4.21	43
48	3.43	3.50	3.57	3.65	3.73	3.81	3.89	3.98	4.08	4.18	4.28	42
49	3.49	3.56	3.63	3.71	3.79	3.87	3.96	4.05	4.14	4.24	4.35	41
50	3.54	3.61	3.68	3.76	3.84	3.93	4.02	4.11	4.20	4.30	4.41	**40**
51	3.59	3.66	3.74	3.82	3.90	3.98	4.07	4.17	4.26	4.37	4.48	39
52	3.64	3.71	3.79	3.87	3.95	4.04	4.13	4.22	4.32	4.43	4.54	38
53	3.69	3.77	3.84	3.92	4.01	4.09	4.19	4.28	4.38	4.49	4.60	37
54	3.74	3.81	3.89	3.97	4.06	4.15	4.24	4.34	4.44	4.55	4.66	36
55	3.78	3.86	3.94	4.02	4.11	4.20	4.29	4.39	4.50	4.60	4.72	35
56	3.83	3.91	3.99	4.07	4.16	4.25	4.34	4.44	4.55	4.66	4.77	34
57	3.88	3.95	4.04	4.12	4.21	4.30	4.39	4.50	4.60	4.72	4.83	33
58	3.92	4.00	4.08	4.16	4.25	4.35	4.44	4.55	4.65	4.77	4.88	32
59	3.96	4.04	4.12	4.21	4.30	4.39	4.49	4.60	4.70	4.82	4.94	31
60	4.00	4.08	4.17	4.25	4.34	4.44	4.54	4.64	4.75	4.87	4.99	**30**
61	4.04	4.12	4.21	4.29	4.39	4.48	4.58	4.69	4.80	4.92	5.04	29
62	4.08	4.16	4.25	4.34	4.43	4.53	4.63	4.73	4.85	4.96	5.08	28
63	4.12	4.20	4.29	4.38	4.47	4.57	4.67	4.78	4.89	5.01	5.13	27
64	4.15	4.24	4.32	4.41	4.51	4.61	4.71	4.82	4.93	5.05	5.18	26
65	4.19	4.27	4.36	4.45	4.55	4.65	4.75	4.86	4.97	5.09	5.22	25
66	4.22	4.31	4.40	4.49	4.58	4.68	4.79	4.90	5.01	5.14	5.26	24
67	4.26	4.34	4.43	4.52	4.62	4.72	4.82	4.94	5.05	5.18	5.30	23
68	4.28	4.37	4.46	4.55	4.65	4.75	4.86	4.97	5.09	5.21	5.34	22
69	4.32	4.40	4.49	4.58	4.68	4.79	4.89	5.00	5.12	5.25	5.38	21
70	4.34	4.43	4.52	4.61	4.71	4.82	4.93	5.04	5.16	5.28	5.41	**20**
71	4.37	4.46	4.55	4.64	4.74	4.85	4.96	5.07	5.19	5.32	5.45	19
72	4.39	4.48	4.57	4.67	4.77	4.88	4.98	5.10	5.22	5.34	5.48	18
73	4.42	4.51	4.60	4.70	4.80	4.90	5.01	5.13	5.25	5.37	5.51	17
74	4.44	4.53	4.62	4.72	4.82	4.93	5.04	5.15	5.27	5.40	5.53	16
75	4.46	4.55	4.65	4.74	4.84	4.95	5.06	5.18	5.30	5.43	5.56	15
76	4.48	4.57	4.67	4.76	4.87	4.97	5.09	5.20	5.32	5.45	5.59	14
77	4.50	4.59	4.68	4.78	4.89	4.99	5.11	5.22	5.35	5.47	5.61	13
78	4.52	4.61	4.70	4.80	4.91	5.01	5.13	5.24	5.37	5.50	5.63	12
79	4.54	4.63	4.72	4.82	4.92	5.03	5.14	5.26	5.39	5.52	5.65	11
80	4.55	4.64	4.74	4.84	4.94	5.05	5.16	5.28	5.40	5.54	5.67	**10**
81	4.56	4.65	4.75	4.85	4.95	5.06	5.18	5.30	5.42	5.55	5.69	9
82	4.57	4.67	4.76	4.86	4.97	5.08	5.19	5.31	5.43	5.56	5.70	8
83	4.59	4.68	4.78	4.87	4.98	5.09	5.20	5.32	5.45	5.58	5.72	7
84	4.60	4.69	4.79	4.88	4.99	5.10	5.21	5.33	5.46	5.59	5.73	6
85	4.60	4.69	4.79	4.89	5.00	5.11	5.22	5.34	5.47	5.60	5.74	5
86	4.61	4.70	4.80	4.90	5.00	5.11	5.23	5.35	5.47	5.61	5.74	4
87	4.62	4.71	4.81	4.90	5.01	5.12	5.23	5.35	5.48	5.61	5.75	3
88	4.62	4.71	4.81	4.91	5.01	5.12	5.24	5.36	5.48	5.61	5.75	2
89	4.62	4.71	4.81	4.91	5.01	5.12	5.24	5.36	5.49	5.62	5.76	1
90	4.62	4.71	4.81	4.91	5.02	5.13	5.24	5.36	5.49	5.62	5.76	**0**

335. Comparison of Time.—After time has been thus observed the chronometers at the two stations should be compared by telegraph. This constitutes the *automatic exchange of signals.* The chronometer at one station being in circuit with the chronograph and recording upon it, that at the other station is switched into the telegraphic circuit, by which it is brought to the first station and switched into the local circuit there, so that the two chronometers register upon the same chronograph, their beats being marked side by side by the same pen. After this has gone on for a minute or more the operation is reversed, the chronometer at the first station is switched into the telegraphic circuit and made to record upon the chronograph with the chronometer at the second station. Of course the observers are informed of the hour and minute at which the joint record upon the several chronographs begins.

The arbitrary exchange of signals is made as follows: Each chronometer recording on its own chronograph as usual, and each local circuit being connected with the main-line circuit through a relay, the observer at one station breaks the circuit by means of the main-line talking-key, which break is recorded on the chronograph sheets at both stations. The breaks are repeated at every two seconds for at least one full minute. The operation is then reversed by the observer at the second station, making the breaks which are recorded at both stations as before. The *differences of time between the chronometers* at the two stations are read from the chronograph sheets at each station and corrected for error of the chronometers. The results from the two chronograph sheets will differ by an amount equal to twice the time occupied in transmission of signals. The mean of the two is therefore the approximate difference of longitude.

This result is yet to be *corrected for personal equation*, or the difference between the errors of observing of the two observers. Every observer has the habit of recording a transit a little too early or too late, the difference between two observers not infrequently being as great as a fourth of a second. To measure this difference, the observers usually meet, preferably at the known station, both before and after the campaign, and observe for time each with his own instrument, or with one similar in all respects to that used in the campaign. A comparison of the time determinations made by the two observers gives an approximation to the personal equation.

A better method, but one not always practicable, is for the observers, having completed half of the observations for time and longitude, to exchange stations for the remainder of the work. The mean of the results before and after exchange of stations will practically eliminate personal equation.

There is one error incident to this work which cannot be eliminated. This is the unequal attraction of gravity, or *local attraction*, or, as it is sometimes called, *station error*. The neighborhood of a mountain mass will attract the plumb-line and deflect the spirit-level to such an extent as to cause serious errors in astronomical determinations of latitude and time. The same result is frequently produced by a difference in density of the underlying strata of rock, so that station errors of magnitude often appear where they are not expected. Indeed, the station error cannot be predicted with any certainty either as to amount or even direction.

The only practical method of even partially eliminating this error is to select a number of stations for astronomic location, under conditions as widely diverse as possible, connect them by triangulation, and by this means reduce all these astronomical determinations to one point, thus obtaining for this point a number of astronomic determinations each having a different station error. The mean of these gives for this point a position from which—in part, at least—station error has been eliminated, and this mean position can be transferred back by means of the triangulation to the several astronomic stations, thus giving each of them a position similarly comparatively free from station error, a position so determined is referred to as a *geodetic position.*

CHAPTER XXXVI.

SEXTANT AND SOLAR ATTACHMENT.

336. **Sextant.**—This is a hand instrument for measuring the angle subtended by any two objects. The *principle of the measurement* is dependent on the fact that the angle subtended by the eye by lines passing to it from two distant objects may be measured by so arranging two glasses that one object is looked at directly, while the image of the other is seen as reflected from the silvered or mirrored surface of one glass to that of the other, and from the second to the eye. The mirror of the first glass is then moved so that the double reflected image of the second object is made to coincide with the object as seen directly.

The sextant is *especially useful on exploratory surveys and at sea* because of its lightness and portability and because it requires no fixed support. With it can be obtained results of sufficient accuracy for all the purposes of navigation and of exploratory determination of astronomic position. The sextant is also extensively used in measuring the heights of objects from the sea or from land, and in measuring horizontal angles between two objects, especially in hydrographic surveying for the location of soundings.

Sextants are of various forms, which differ according to the maker. They are sometimes made of wood mounted with ivory, but such materials are liable to warp. The most satisfactory sextant for all-round surveying is made of brass with a silvered arc of sufficient extent to permit of measuring angles up to 80°.

The *principal parts* of the sextant (Fig. 183) are:

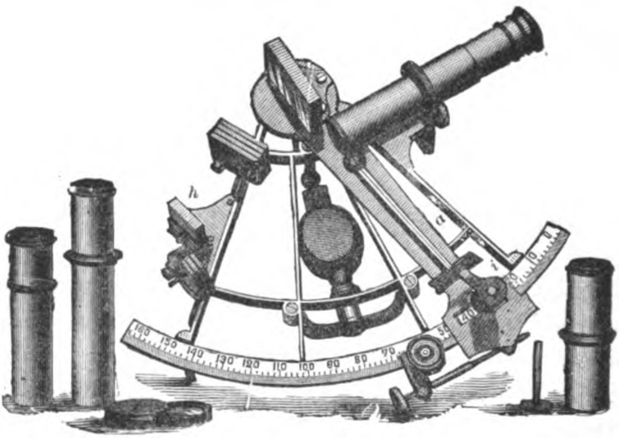


FIG. 183.—SEXTANT.

1. A mirror *i*, called the *index glass*, which is rigidly attached to the movable arm *a*, called the index arm; also

2. A mirror *h*, called the *horizon glass*, rigidly attached to the frame of the instrument; and

3. The *arc* on which the angles are read by means of the vernier at the end of the index arm.

The planes of the two mirrors are so fixed as to be parallel, one to the other, when the vernier points to zero degrees.

337. **Adjustment of Sextant.**—Among the more important adjustments of the sextant are those of—

1. The index glass;
2. The horizon glass;
3. The telescope;
4. Correction for index error.

The reflecting surface of the *index glass* must be perpendicular to the plane of graduated arc of the instrument. To test it, set the index near the middle of the arc, then place the eye very near the index glass and plane of the instruments, and observe whether the reflected image of the arc forms a continuous or broken line with the arc as seen direct. If continuous, the glass is perpendicular to the plane of the instrument. If the reflected image drops, the glass is leaning backward; if it rises, forward. The adjustment is made by means of a key on the back, the latter being turned to the left if the image is dropping, and to the right if rising.

The reflecting surface of the *horizon glass* should be perpendicular to the plane of the instrument. To test this, put in the telescope and point it towards a star, holding the instrument vertical, then move the instrument until the reflected image is in a horizontal line with the direct image. If it is exactly in coincidence with the direct image, the horizon glass and index glass must be parallel in that position, and as the index glass has been adjusted perpendicularly to that plane, in any position. If they do not coincide, put the adjusting key on the screw at the back and turn to the right to move the reflected image to the right, and to the left to move it to the left.

To make the *line of collimation* of the telescope *parallel* to the *plane of the instrument*, the sextant should be rested on a plane surface with the telescope directed at a well-defined point about 25 feet distant. Two objects of equal height are then placed on the extremities of the arc, and these serve to establish a plane of sight parallel to the arc. They may be two small sticks of sufficient height to make the plane of sight of the same height above the arc as is the line of collimation of the telescope. If the line of collimation now intersects the line defined by the two pointers, the instrument is in adjustment. If not, the error is corrected by the screws on the holder of the telescope.

Another mode of performing this adjustment is the following: Place a telescope which has two wires in the field of view that are parallel to each other and equidistant from the center of the field, in the telescope ring of the sextant, and turn the eyepiece until the wires are parallel to the plane of the instrument. Measure the angular distance between the two objects which are apart as far, say, as 60 degrees or more; when the reflected and direct images are in contact on one wire, clamp the index firmly and make a precise contact by using the tangent screw; now move the instrument so as to bring the object on the other wire; if they remain in exact contact, the telescope is parallel to the plane of the instrument. If not, it may be adjusted by altering the screws in the ring so as to change the angle of the collar which holds the telescope.

To correct the index error sight at some well-defined object, as a star, and move the index arm until the direct and reflected images coincide, when the vernier should read zero. If not, the difference may be recorded as an index error or be corrected by adjustment.

338. **Using the Sextant.**—In measuring any angle with the sextant it is held in one hand in the plane of the two objects. The telescope is then directed towards the fainter object by looking through the unsilvered portion of the horizon glass. With the other hand the index arm is then moved until the second object as seen by double reflection is brought in exact coincidence with that seen directly.

If it is desired to read the horizontal angle between two objects which are at different elevations, some object, as a tree, building, or a plumb hung in line with one of them, must be found which is directly above or below it. The angle is then measured from this to the other object by holding the sextant horizontally in its plane. If no suitable object can be seen, some point about 60 degrees from one of the objects may be selected and angles be read between each object and that point. The difference between these two angles will be approximately the horizontal angle.

If it is desired to measure an angle between two objects which are very near together, the angle between each and a third object may be measured and the difference taken.

Should the angle to be read be too large to come within the range of one measurement of the arc, the sum of the angles between each object and an intermediate object may be measured.

To measure vertical angles for determination of altitudes or for ascertaining heights of celestial bodies, the horizon is used as a reference point. At sea this is done by sighting the true horizon while on land an artificial horizon must be employed. The latter consists of a small bath of mercury protected from the wind by a glass cover. The observer stands or kneels near this reflecting surface of mercury and looks directly on the object the height of which is to be measured, and also at the reflection of this object in the mercury bath, and the contact is made between these two. The angle measured from the reflecting surface is necessarily twice the angular elevation of the object observed above the true horizon.

339. Solar Attachment.—The object of this instrument, which may be attached either to a compass or a transit, is the *determining of the meridian, latitude, and time* by observation on the sun. It is extensively used in the subdivision of the public lands in the West. In the past the solar compass was mostly employed, but now little work is done with a compass, all meridians being run with the engineer's transit by projecting from Polaris observations for azimuth, or with the solar attachment.

This instrument was originally invented by Wm. A. Burt of Michigan, but at present there are several modifications of the original Burt attachment made by various manufacturers. There are, however, but two forms in popular use by surveyors; these are the Burt Solar Attachment, as modified by Messrs. W. and L. E. Gurley, and the Smith Meridian Attachment, made by Messrs. Young and Sons. The adjustment and use of these is described in the following Articles.

340. Burt Solar Attachment.—This consists essentially of an axis which is parallel to the earth's axis, and a line of sight or pointer which is set at an angle to the instrumental polar axis equal to the declination of the sun for the time of observation. The polar axis is placed at right angles to the tube of the telescope by attaching it to the telescope tube by adjusting-screws. In a plane right-angled to the polar axis is a small circle, graduated on its outer edge into fractional 24 hours and called the equatorial or hour circle. Attached to the polar axis, and swinging about it with its lower side parallel to the plane of the hour axis, is an arm carrying a small arc with vernier attachment, called the *declination arc*.

The *polar axis* is attached to the telescope by a small circular disk of an inch and a half diameter, and on this as a pivot rests the enlarged base of the axis surrounded by the *hour circle*, the disk being attached to the base of the pivot of the polar axis by four capstan-headed screws which serve to adjust the polar axis. The hour circle can be fastened at any point desired by two flat-headed screws on its upper side, and the hours marked upon it are divided into 5 minutes of time, which are read by a small index fixed to the declination circle and moving with it. The *declination arc* is of about 5 inches radius, divided into 30 degrees, reading by the vernier to single minutes. It is attached to the polar axis by a hollow cone or socket moving snugly upon it by a milled-head screw on top, and to this is securely fastened the declination arc by two large screws. The declination arc has two lenses and two silver plates on which equatorial and hour angles are ruled by parallel lines at right angles on two opposite ends of the radial arms of the sector; it has also a clamp and tangent movement, and the declination arc may be turned on its axis and one or the other of the solar lenses used, according as the sun is north or south of the equator.

341. Adjustment of Burt Solar Attachment.—The adjustments of a solar are simple. These are first to make the lines of collimation parallel to each other and at right angles to the polar axis, when the declination arc reads zero; and

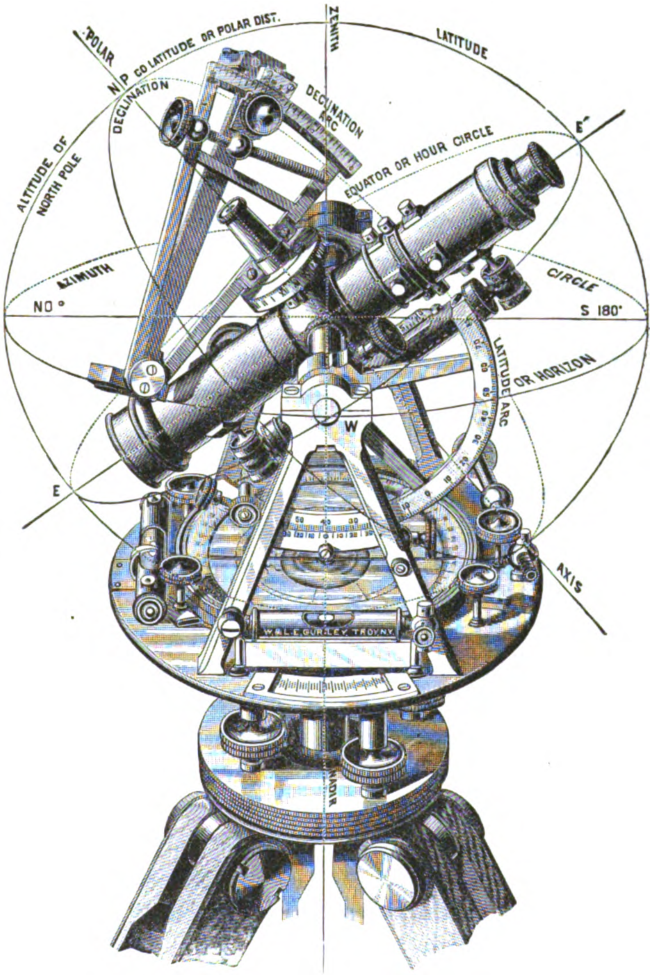


FIG. 184.—GRAPHIC ILLUSTRATION OF THE SOLAR ATTACHMENT.

second, to make the polar axis perpendicular to the telescope. In addition, the ordinary adjustments of the telescopic alidade must be made.

The *lines of collimation are made parallel* by making each line parallel to the edges of the blocks containing them. This is done by removing the declination bar or bar carrying the lines of collimation, which is done by removing the clamp and tangent screws and the conical center with the small screws by which the arm is attached to the arc. Then a bar which is furnished with each instrument, called the adjusting-bar, is substituted for the declination arm; and the conical center bar screwed into its place at one end and the clamp-screw into the other are inserted in the hole left by the removal of the tangent screw. The arm is then turned so as to bring the sun into one line of collimation, and then the bar is quickly revolved or turned over, but not end for end. If the image still falls in the square, the line of collimation is parallel to the two edges of the blocks. If not, the silver disk must be moved through half the apparent error of the sun's image, and the same operation repeated. Then the bar must be reversed end for end by the opposite faces of the blocks upon it, and the other line of collimation adjusted until the image will remain in the center of the equatorial lines.

To *adjust the polar axis* the instrument is first carefully leveled, the tangent movement of the vertical arc of the telescope being used in connection with the leveling-screws of the striding-level of the alidade. Then the equatorial centers on top of the blocks are placed as closely together as practicable with obtaining a distinct view of a distant object. Having previously set the declination arm at zero, sight through the interval of the equatorial centers and blocks at some distant object, the declination arm being placed over either pair of capstan-headed screws on the under side of the disk; now the instrument is turned on its axis and the same object sighted, while the declination arm is at the same time kept with one hand upon the object originally sighted. If the sight line strikes either above or below, the instrument must be releveled by the two capstan-headed screws under the arm by such an amount as will eliminate half the error, and the operation again repeated until the sight strikes both objects in the same position of the instrument. The instrument may now be turned at right angles, keeping the sights still upon the same object as before, and if it does not strike the same point when sighted, the axis is not truly vertical in the second position of the instrument, and the correction must be made by the capstan-headed screws under the declination arc by means of reversing it as before.

To *adjust the hour arc*, which should read apparent time when the instrumen is set in the meridian, loosen the two flat-headed screws on top of the hour circle and with the hand turn the circle around until the index of the hour arc reads apparent time, when the screws may be fastened.

342. **Smith Meridian Attachment.**—As this is a telescopic solar and thus permits of clearer definition of the sun and hence better work, it is preferred by many surveyors. This attachment is placed on the left side of the transit and is attached to the standard with a light plate by small butting-screws. A counterpoise is placed on the corresponding right side, and both can be easily removed when not in use. The solar telescope, *C* (Fig. 185), revolves in collars, and its line of collimation and axis of revolution coincide with the polar axis, *PP*. These collars are attached to the *latitude arc*, *l*, which has a horizontal axis, the whole being mounted on a frame which is attached to the transit standards, *f*, *f'*. On the side of the telescope is fixed the *declination arc*, *d*, the vernier of which is attached to an arm, *e*, which turns on its axis a reflector, *c*, placed before the object-glass of the telescope. Both the latitude and declination arcs have tangent screws to impart slow motion. The arm holding the declination vernier when placed at zero, is so arranged that the plane

of the reflector makes an angle of 45° with the axis of the telescope. If the telescope is revolved on its polar axis, the reflected line of collimation will describe the celestial equator, thus by setting off any given declination north or south the

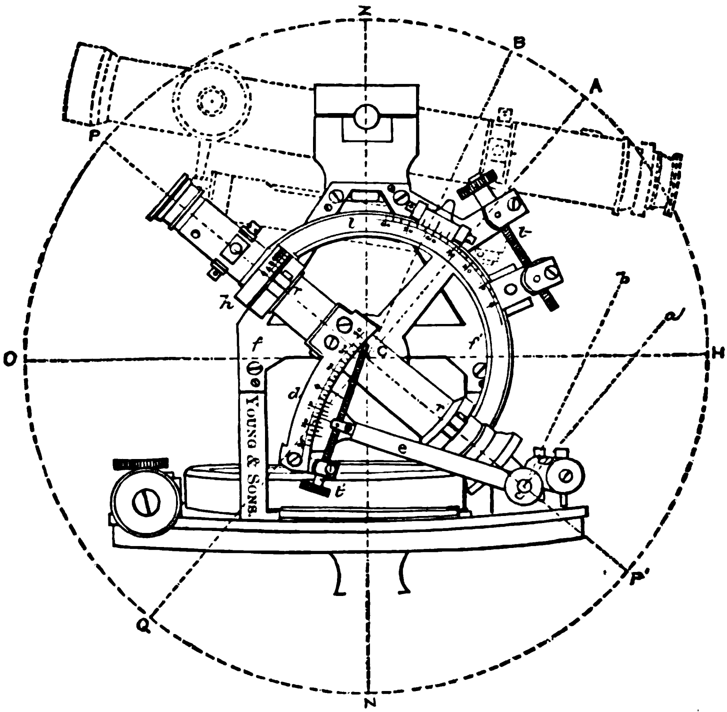


FIG. 185.—SMITH MERIDIAN ATTACHMENT.

image of the sun may be kept in the field of view from its rising to its setting by revolving the telescope. The *hour arc*, *h*, is attached to the telescope and revolves at right angles with the polar axis.

343. **Adjustment of Smith Meridian Attachment.**—These adjustments are made in the following manner and order, as arranged by Mr. Hargreaves Kippax:

1. The *adjustment of* the *line of collimation* of the solar telescope is made first. It differs in no respect from the like

adjustment in a Y level, and consists in rotating the telescope in its collars until the intersection of the center cross-hairs remains on a selected object or point throughout the possible extent of the rotation. In preparing for this adjustment, the apparatus may be revolved bodily on the axis of the latitude arc until the solar telescope becomes nearly level or assumes such other position as will conveniently observe the selected object or point. That sufficient light may be had, the reflector should be placed edgewise; this can be conveniently done by removing the lug by which the vernier tangent-block is attached to the declination-arm. The tangent-block and the attached vernier-arm can then be swung so that the reflector shall be edgewise, and it may be retained in that position by a rubber band or other device. Should this expedient not give sufficient light, it will be necessary to remove the reflector and its attachments, not by removing the ring to which the bearings of the reflector are attached, but by carefully removing the four screws which secure the two caps over the journals of the reflector-block. When replacing these caps, be careful not to screw them down so tightly as to cause too much friction on the journals, else there may be danger of disturbing the position of the vernier-arm which is attached to the block by a small plug-screw, thus creating an index error unawares. It is well to have the object or point used in the adjustment of collimation somewhat remote, to avoid subsequent error through focusing on the sun.

2. Having adjusted the collimation, it is convenient to see if the *solar telescope travels in vertical plane.* This can be best done while the reflector is removed. Carefully adjust the plate-levels of the instrument before attempting this adjustment. The angle of a building known to be vertical may be used, or, better still, an elevated object and its reflection from an artificial horizon of mercury or other suitable fluid. It will be found convenient to remove the latitude clamp and tangent during this adjustment which is made by

the four pairs of screws which attach the frame-plate to the standard. See test and adjustment 4, below.

3. The next adjustment is to determine whether the *line of collimation of* the *solar* telescope *is perpendicular to* the *axis of* the *latitude arc* upon which the solar telescope revolves in altitude. This adjustment is supposed to be made permanently by the maker; the surveyor should not disturb it unless certain that a change is needed. Once perfected, the adjustment will seldom need attention.

4. *To test the parallelism of the two telescopes*, carefully adjust the transit telescope for collimation and elevate or depress it before making this test. A target may be used upon which two points are placed at a distance equal to the eccentricity of the solar telescope. By training the transit telescope on one point and the solar telescope on the other, the parallelism of the two lines of sight can be assured. A center mark may be placed on the first target, and the use of the second target be thus avoided. The adjustment is made by the four pairs of binding and butting screws by which the frame carrying the solar apparatus is attached to the standard of the transit. This adjustment may disarrange the second adjustment which has to be made by the same screws, and it may be found necessary to carry on both at the same time. After being satisfied of the preceding adjustments, the portions detached can then be replaced and attention given to the index errors of the latitude and declination arcs.

5. *To adjust index error of latitude arc*, set at zero, clamp and place striding-level on telescope. Level with tangent screw. Reverse the level, and if the bubble returns to first position, the axis may be considered horizontal. If any deviation is noticed, move the bubble half the distance by the tangent screw. Reverse the level, and if the bubble takes a like position in the opposite direction, the adjustment is accomplished.

6. *To adjust index error of declination arc*, set off the latitude and observe the sun on the meridian by bringing its image exactly between the horizontal lines or equatorial wires. If any difference is noted between the observed and calculated declinations after correcting for refraction, move the arc by loosening the three screws on top of it until the difference is eliminated. Never attempt to remove the declination index error by manipulating the small plug-screw which attaches the vernier-arm to the reflector.

7. After adjusting as above, and making a careful observation, it will usually happen that the *transit telescope will* still *deviate* one or two minutes *from* the *meridian*. If, on test, this appears to be a constant error, and you are satisfied with your former adjustments, the small deviation may be considered as the resultant of several residual undiscovered errors, and may be removed in the following manner. Observe with the solar at 9 A.M. Turn transit telescope south, and note the error east or west of the meridian. Place transit telescope on the meridian with tangent screw. By means of the small butting-screws attaching the plate to the standard move the south end of the plate east or west, as was the error, until the image is precisely between the wires. Verify the adjustment by an observation at 3 P.M.

8. *To adjust equatorial wires*, rotate the diaphram carrying the cross-wires by loosening the screws until the image follows the equatorial wires precisely.

344. Determination of Azimuth and Latitude with Solar Attachment.—The declination of the sun is given in the American Ephemeris or Nautical Almanac and is calculated for apparent noon at Greenwich. It can also be determined from tables sold by makers of solar attachments. To determine the declination for any other hour at a place in the United States, reference must be had to differences of time arising from longitude and the change of declination from day to day. The longitude of a place and therefore its difference in time may be obtained merely from platting on a good map, or from a watch which is kept adjusted within a few minutes. The best time at which to use the solar attachment for the determining of a meridian is not at noon, when the sun is passing the meridian, nor early or late in the day, when refraction is greatest, but between 8 and 11 o'clock in the morning and 1.30 and 5 o'clock in the afternoon.

The use of the solar attachment can best be explained by reference to an example. The following were prepared by Mr. A. F. Dunnington of the U. S. Geological Survey, from his field-notes:

EXAMPLE FOR MERIDIAN.—Set the instrument over the corner of sections 7, 8, 17, and 18, T. 2 N., R. 5 E., of the Black Hills meridian, South Dakota. Level the transit and point the telescope approximately north with the aid of the magnetic needle. Knowing the latitude of the place, set the same off on the latitude arc. Having computed the declination for the day and hour corrected for refraction, taken from the pocket Ephemeris, set it off on the declination arc. Place index at approximate local mean time on hour circle. Look into the solar telescope and the sun's image should be seen in the field of view, but not between the equatorial wires. Now move the telescope of the transit into the meridian, and if the horizontal plates have been set at zero, any angle can be set off from the meridian and the course run.

Record.—Aug. 4, 1898. Long. 103° 45′. At 7 h. 00 m. A.M., l. m. t., I set off 17° 11′ N. on the decl. arc; 44° 08½′ on the lat. arc, and determined a true meridian with the solar at the cor. of secs. 7, 8, 17, and 18, T. 2 N., R. 5 E., of the Black Hills meridian, South Dakota.

EXAMPLE FOR LATITUDE.—Some minutes before noon place instrument in position and level as before. Loosen clamp screw to the horizontal plate of the transit. Set off the computed declination for 12 M. corrected for refraction, and revolve the solar telescope in its collars until the index coincides with XII hours, making sure that this last setting is not disturbed. With the azimuth tangent screw of the transit bring the image in the field of view, and with the slow-motion screw of the latitude arc bring the image between the equatorial lines. As the image leaves the wires repeat this operation until the image appears to remain stationary for a few moments before leaving the wires in an opposite direction. At this moment the sun has reached its highest point, and the latitude of the place is read direct from its arc with the Smith meridian attachment, and the colatitude with the Burt attachment.

Record.—Aug. 31, 1897. Long. 103° 45′. At the cor. of secs. 13, 14, 23, and 24, T. 2 N., R. 3 E., of the Black Hills meridian, South Dakota, I set off 8° 22′ N. on the decl. arc; and at 0 h. 0.10 m. P.M., l. m. t., observe the sun on the meridian; the resulting latitude is 44° 02½′ N., which is about 0′.21 less than the proper latitude.

345. Solar Attachment to Telescopic Alidade.—As an instrument for use in topographic surveys the solar attachment has some advantages, especially in heavily timbered country, or where the magnetic declination is variable, as an aid to the rapid location of points in connection with the plane-table. Such locations are of necessity not of sufficient accuracy to permit of their being used in further extension of triangulation, but they are of sufficient accuracy ordinarily to permit of their being employed as tertiary control, either for the adjustment of traverses or the sketching-in of topographic details.

The mode of using the solar attachment to the telescopic alidade with this object is chiefly as a means of *orienting* the *plane-table* or placing it in true meridian when but one or two located points are visible; in other words, without the solar attachment a station to which sights have not yet been taken can be located only by means of the three-point problem (Art. 75) or reduction from three known stations.

With the aid of the solar attachment a *resection location* can be *accurately made* under most circumstances when two points only are in view. The method of procedure is as follows, and is rather similar to that employed in traversing with a plane-table when a magnetic needle is used for orientation: Having set up the plane-table at the point the position of which is desired, the telescopic alidade is placed on the board with its fiducial edge parallel to a true north and south line which is ruled somewhere on the paper, and after the solar observation has been made the board is swung into true meridian and clamped. Now swing the far end of the alidade about the positions of first one and then the other of the two known points, and drawing lines along the edge of the ruler, the intersection of these lines is the position of the point occupied, and this position is made more than an approximation by the third check secured by an intersection with a true meridian line obtained by the aid of the solar attachment.

The unknown factor is the *direction of the meridian*. The latitude of the point may be determined by observation, as with the solar transit, but in the case of plane-table triangulation conducted on small scales and based on primary triangulation the latitude can be platted from the plane-table sheet with sufficient accuracy. This is then set off on the larger vertical arc of the telescopic alidade as a colatitude, so that the polar axis when in meridian may point to the pole. On the declination arc is set off the declination for the time of the observation. Now, with the plane-table leveled so that revolutions about its vertical axis may be in azimuth only, the board is revolved horizontally and with the line of sight about the polar axis until the image of the sun is brought between the equatorial lines. Then the polar axis and the telescope will lie in meridian, and the instrument may be clamped and the meridian line ruled upon the board.

CHAPTER XXXVII.

PHOTOGRAPHIC LONGITUDES.

346. Field-work of Observing Photographic Longitude.—A photographic camera of particularly stable and rigid form is set up, so that the image of the moon is about in the center of the plate, and a *series of instantaneous exposures are made*, allowing such an interval between the exposures that the moon's images on the plate will not overlap; i.e., from 1½ to 2½ minutes, according to the moon's age. After a set of, say, seven moon exposures the camera is left untouched until bright stars of approximately the same declination as the moon have arrived at the same point in the heavens. The camera is then opened for periods of 15 to 30 seconds, and the stars allowed to impress their trails on the plate. These star exposures should be repeated four or five times.

It is obvious that if the local time of each moon and star exposure be known, such plate will give *all data necessary to compute* the moon's position either in right ascension, declination, azimuth, or lunar distance.

In *placing* the *sensitive plate in* the *slide*, care must be taken that the pointed screws against which it rests are only allowed to puncture and not to scratch the gelatine film, otherwise the exact position of the plate will be uncertain. The camera should be set up so that the moon will, as nearly as can be judged, cross the center of the field in about seven or eight minutes, so that the resulting photograph will show its seven images distributed on each side of the center. As it is difficult to insure this, it is a good plan to make a larger number of moon exposures, that the measurer may select those most centrally situated.

If the star exposures have been made first, it may be found that the moon does not cross the field near the center. In this case it will be necessary, after all the exposures have been completed, to move the camera, so that the moon is in the center, and take two or three additional exposures. These are only for the purpose of measuring the radius of the moon's image on the plate.

The *moon exposures should be instantaneous.* In making *star exposures* it is desirable to take two sets of stars in order to get trails both north and south of the moon. The *essential conditions* are that the local time of each star and moon exposure should be known and that there should be a series of trails of at least two stars on the plate. To prevent possible confusion it is advisable to make some small difference in the two sets of exposures by putting in one or two indicating exposures of twice the length of the others.

The proper time for the commencement of the star exposures should be determined by calculation, not by looking through the sighting arrangement. After the camera has been clamped ready for the first exposure it should not be approached to a nearer distance than 3 or 4 feet until all the exposures are complete. The minimum magnitude of star that can be used with a clear sky is about a third-magnitude star.

The following details of the process and the appended example have been worked out by Mr. Wm. J. Peters of the U. S. Geological Survey in connection with Capt. E. H. Hills's, (R. E.) published description of his experiments.

347. The Camera and its Adjustments.—The camera should be moderately heavy for good work, but may be lighter for approximate or exploratory work. Other things being equal, the longer the focal length the larger the scale of the photograph, and hence the more accurate the measurements. The means of transport available will probably be the guiding factor. The camera must be capable of being readily turned to any portion of the sky and of being firmly clamped in position. It is therefore best to use a photo-surveying camera or theodolite (Art. 125) or to mount the body between a pair of wyes with clamping arrangement in altitude, the wyes being on a base plate which can be rotated so that the whole instrument can be rotated.

Provided the mounting be strong and stable, it can be of the roughest character, as it is not necessary to know anything whatever about the position of the camera at the moment of exposure. The one essential is that the instrument shall not move during the whole time of exposure, often of several hours' duration. The *stand should be low*, a height of 20 inches to the base plate being ample. A tripod is probably best, provided it be firmly braced. The ends of the legs which rest on the ground should be flat, not pointed. The legs should be of well-seasoned wood, this being more constant under changes of temperature than metal.

In designing the *body of the camera* two points must be borne in mind: first, that the focus, when once found, shall not require any alteration to compensate for change of temperature; and secondly, that the center of the plate, i.e., the point where the axis of lens and camera cuts it, shall not shift. These conditions are perfectly fulfilled by making the body of stout brass tubing, which will expand or contract symmetrically, and the plate will therefore maintain the same position with reference to the axis of the instrument.

It may be observed that it is of no importance to know the actual focal length of the *lens*, the quantity not being required in the formulas of reduction. Some form of *finder*, as a pin-hole and sight-vane, must be provided to enable the observer to direct the camera so that the center of the field falls at any desired point in the sky. The exposing arrangement that seems preferable is a simple *flap shutter*, actuated by a pneumatic ball, with sufficient length of tubing to allow the observer to keep at a distance of about four feet from the camera, and thus to enable it to maintain the high degree of stability essential to success. The slight shock of the opening of the light flap or blind does not move a heavy camera to any measurable extent.

The *plate-holder* should be of metal, and must be provided with some means by which the position of the sensitive plate in the slide can be readily determined, and with an adjustment which will cause the plate to take up a position truly perpendicular to the optical axis. This is completely effected by making the plate rest against three sharp-pointed screws, which puncture the gelatine film and give three points from which the position of the plate can be exactly determined; the necessary adjustment being made by moving the screws in or out.

The *lens* employed must be one giving an absence of optical distortion over a large field, and must, therefore, be of the doublet form. A good lens of this class would show no appreciable distortion up to a distance of 7° from the center of the plate, which field is amply large for the purpose.

The following definitions will aid in an understanding of the adjustments of the camera:

1. The optical center of the plate is the point where the axis of the lens cuts it.

2. The geometrical center of the plate is the foot of the perpendicular from the center of the lens to the plate.

The *adjustments* required are:

1. Focus;

2. Finding the geometrical center of the plate; and

3. Bringing the geometrical and optical centers into coincidence.

Of these 1 and 2 are accomplished in one operation, as follows: The camera is placed in a vertical position, lens downwards, over a mercury bath. The plate-holder is inserted and a piece of plate glass is placed, resting on three pivoted screws, in the position that a sensitive plate would occupy. The camera is then moved until the top surface, and therefore the bottom surface, of this glass plate is truly level. A mark is made at the middle of the lower surface of the plate, and by examining with an eyepiece the reflected image of this mark can be seen somewhere about the same position as the mark itself. The lens is moved in or out until the image is brought to a focus in the same plane as the object, and the glass plate or mark on it is moved until image and object coincide. The mark is then at the geometrical center of the plate and at the true focus of the lens. The center thus formed must coincide with the optical center, i.e., must lie over the axis of the lens. Should this be found not to be the case, the screws in the plate-holder must be altered until geometrical and optical centers coincide. A small error in this point will produce a quite appreciable error in the results, but should it be desired to test it there are well-known methods available which it is not necessary to describe here. It will, therefore, suffice if the geometrical center of the plate is brought to the axis of the camera. The points where the glass plate rests on the three screws, when adjustment is complete, are marked on the glass, and the latter then forms a gauge from which the center of any plate can be marked on it after exposure and development. It is obvious that this adjustment is not one that is likely to be disturbed, and need, therefore, only be repeated at rare intervals.

348. Measurement of the Plate.—The measurements must be made with a micrometer, but it is not necessary to enter into a discussion of all the details of making such measurement of a photographic plate. The *quantities to be measured* are the coordinates of each moon image and star trail from any two axes on the plate, expressed in any scale. The axes need not be mutually perpendicular, but they must be straight. A réseau may be used, this being a plate coated with an opaque substance and ruled with two sets of fine transparent lines at intervals of 2 or 5 millimeters. It is placed in contact with the photographic plate previous to development, and, in that position, exposed to the light. The lines are, therefore, impressed on the plate and the réseau is developed together with the stars. The method is undoubtedly a very accurate one, but is not perhaps advisable, as adding another operation to be performed in the field. An alternative method is to use a positive réseau, i.e., black lines on a transparent ground, and to clamp the star plate and réseau plate, film to film, for measurement. A simple method, and one susceptible of quite sufficient accuracy, is to rule two axes on the gelatine film with a fine needle. This has the advantage, in common with the first method, that the plate can be readily remeasured at any future time.

The *measurement of the coordinates* of the star trails and moon's bright limb call for no special remark; the only difficulty met with is when attempt is made to measure the moon's radius in order to deduce the coordinate of the center. This is a point which has given a considerable amount of trouble, and calls for a somewhat detailed notice. The moon's image on the plate is, in general, not circular, but is *subject to two distortions*, the first due to the difference of refraction on the upper and lower limbs, and the second due to the photographic projection, the cone of rays from the moon through the center of the lens being cut obliquely by the plate. The moon image on the plate is therefore elliptical or very nearly so.

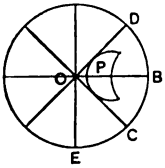
The first distortion will vanish when the moon is at a sufficient altitude, and the second when the image is near the center of the plate. In this work a negligible error may be provisionally defined as a quantity less than one second of an arc. In this case the quantity sought is the radius of the image of the moon, and therefore a distortion up to $2''$ in the diameter may occur, which means that the moon's altitude atthe time of exposure must be at least 30°, and the distance of the image from the center of the plate must not exceed 2°.

It is no doubt theoretically possible to devise a method for determining the position of the center of the moon's image from measurements to the limb on the supposition that the latter is an ellipse. Practically this has not been found a success, nor is it necessary, providing one or two of the images are within the requisite 2° of the center, when they can be treated as circular and their radii easily measured. In the case of the other moon, more remote from the center of the plate, the distance from limb to center of image can be determined by a simple calculation. It is therefore required to have on the plate at least one moon image within 2° of the center. The observer will find no difficulty in fulfilling this condition, which, while greatly facilitating the measurement of the plate, is not absolutely essential to it.

The problem of determining the radius of a central image is a comparatively simple one, and reduces itself to the question of finding the radius of a circle when a portion of the arc is given. It may at first sight be thought that, if the moon be more than half full, the diameter might be measured directly; but this is by no means the case, as will be apparent upon consideration. Except the moon be absolutely full the bright line will cover only a semicircle, and therefore, unless the measurement be made exactly on the line joining the ends of the terminator, it will be erroneous. This line is quite impossible to select with any precision on the plate, especially if the moon be nearly full.

One successful method is to rule a line across the image and measure the sine and versine. This is not very accurate for this reason: The cord cuts the limb obliquely, and consequently, in measuring its length in the micrometer, the cross-wire *cuts* the limb, while in measuring the versine the cross-wire is brought up, *touching* the limb. The edge of the image is not absolutely sharp, and the two measurements are not strictly comparable.

A much superior method, in which all the contacts are symmetrical, is as follows: The microscope is fitted with two



pairs of cross-wires inclined to each other at 45°. The line of motion of the micrometer being along the wire OB, the moon image is made to touch OD and OC; the screw is then turned until EF touches the limb, and the length OP is thus measured. The radius $= \frac{OP}{\sqrt{2} - 1}$.

It is to be noticed that the measurement of the radius is not necessarily an absolute one. Thus the observer may habitually make the wire encroach too much on the limb, and so measure the radius too small. This error, however, completely disappears, since he will measure the distance from the coordinate axis to the limb with the same bias. Hence the coordinate of the limb will be increased exactly the same amount as the apparent radius is decreased, and the deduced coordinate of the center will not be affected. It is therefore important not only that the same observer should make both sets of measurements, which may be regarded as absolutely essential, but also that he should make them at the same time and under the same conditions of lighting.

The two coordinate axes on the plate should be ruled approximately parallel and perpendicular to the meridian through the center. If, for the final result, the moon's right ascension is to be computed, which seems to be the preferable method, it will be readily seen that much greater weight attaches to the coordinates perpendicular to the meridian than to those parallel to it. It is consequently desirable to make at least twice as many readings on the one set as on the other. The actual number of readings taken must be left to the taste of the individual observer; but for those who have no particular bias it is suggested that four readings of one coordinate, eight of the other, and twenty-four for the radius of the image form a good working set. A plate with twelve star trails and seven moons can be measured thus in about four hours.

349. **Computation of the Plate.**—The time of each exposure and the measured coordinates of moon and star images and of the center of the plate are all the data necessary for determining the position of the moon. The result can be expressed in any form, as right ascension, azimuth, or lunar distance, and the strictly correct course would therefore be to select whichever of these be changing most rapidly at the time, and reduce the result to that form.

It may, however, be remarked: First, with regard to declination, that the rate of change of this quantity is so variable, and will so often be too small to be of any value, that the extra labor spent in computing it will rarely be repaid. Secondly, with regard to lunar distances, that this would only be applicable to certain stars; and in the case of a star near the same parallel of declination as the moon, the rate of change of the lunar distance is practically identical with the rate of change of the moon's right ascension. If, therefore, right ascension be computed, then, to all intents and purposes, the lunar distance is also computed, and to give separate attention to the latter would be a waste of time. The best method to adopt as the standard one is the computation of the moon's right ascension, which quantity is always changing at a practically uniform rate, thus securing the same measure of precision in the results, whatever be the moon's position.

There is, in certain cases, some advantage in taking the computation of the moon's azimuth, in that it disseminates one source of error, as will be seen later when the errors of the method are dealt with. The computation is, however, more laborious, and it seems doubtful whether it be in any case worth the additional labor. The only occasion where it would be at all desirable is when the moon is at a considerable distance from the meridian, and the observer is not near the equator.

A_1, P_1 = apparent right ascension (R. A.) and north-polar distance (N. P. D.) of center of plate;

a_0, p_0 = apparent right ascension and north-polar distance of known star;

a', p' = apparent right ascension and north-polar distance of unknown star or moon;

a, p = true apparent right ascension and north-polar distance of unknown star or moon;

x, y = measured coordinates of star or moon;

ξ, η = standard coordinates of star or moon (i.e., rectangular coordinates on a plane tangent to celestial spheres at AP perpendicular and parallel to the meridian through the center, expressed in parts of the radius);

a, b, c, d, e, f = plate constants;

θ = sidereal time (arc);

t = hour-angle;

δ = moon's declination;

π = moon's horizontal parallax;

ρ = earth's radius;

ϕ' = reduced latitude;

q, q_0 = auxiliary angles.

To compute the moon's right ascension from the plate the following is the procedure:

(1) Assume any epoch for the plate, conveniently the sidereal time T of the first moon exposure.

(2) The center of each star trail exposed at a sidereal time S is to be regarded as an imaginary star whose north-polar distance equals that of the star, and whose right ascension = R. A. of star $-$ $(T - S)$.

(3) Compute true L. S. T. of epoch (in arc).

(4) Estimate by any approximate method A and P, the R. A. and N. P. D. of the center of the plate.

(5) Select three star trails, of which not more than two can be of the same star, and compute their ξ, η by formulas:

$$\xi = \frac{\tan (a_0 - A) \sin q_0}{\cos (P - q_0)}; \quad . \quad . \quad . \quad . \quad (168)$$

$$\eta = \tan (P - q_0); \quad . \quad . \quad . \quad . \quad . \quad . \quad (169)$$

where $$\tan q_0 = \tan p_0 \cos (a_0 - A). \quad . \quad . \quad . \quad (170)$$

(6) Calculate approximately the six plate constants, a, b, c, d, e, f, from the six equations thus furnished of the form

$$\xi = ax - by - c;$$
$$\eta = dx - ey - f;$$

viz., one pair for each star.

(7) With approximate values of the constants thus found, and the measured coordinates of the center of the plate, calculate the ξ, η of the center, and hence its corrected R. A. and N. P. D. form the approximate formulas.

(8) With the new AP calculate ξ, η of all the stars.

(9) The plate constants can now be accurately determined from the comparison of the measured x, y, and the computed ξ, η for all the stars, either by least squares or by suitably grouping the stars in threes and taking the arithmetical mean of all the values thus found. This latter procedure is practically as accurate and is much less laborious.

(10) Calculate ξ, η of all the moons and their a' from the formulas

$$q = P - \tan^{-1}\eta, \quad . \quad . \quad . \quad . \quad . \quad . \quad (171)$$

$$\tan (a' - A) = \frac{\xi \cos (P - q)}{\sin q}. \quad . \quad . \quad . \quad (172)$$

(11) With an assumed approximate longitude calculate the moon's declination and horizontal parallax at time of each exposure, and deduce parallax in right ascension by ordinary formulas.

(12) There is thus obtained true right ascension of each image, and by adding the interval from assumed epoch we get moon's true right ascension at each exposure, and hence Greenwich M. T. and longitude.

350. **Sources of Error.**—The degree of accuracy of the longitude, as obtained from the photographic plate, is exhibited in Article 351 by examples taken at a place whose longitude is known. For their better comprehension it will be interesting to briefly discuss the errors that the method is liable to, their possible elimination, and their probable amount.

The possible sources of error may be classified as follows:

(*a*) Differential refraction;
(*b*) Aberration;
(*c*) Flexures of camera;
(*d*) Want of stability of camera;
(*e*) Optical distortion of lens;
(*f*) Lag in photographic action of a faint star;
(*g*) Error in estimating position of center of plate;
(*h*) Errors of measurement in the micrometer;
(*i*) Clock errors (i.e., local time and clock rate);
(*j*) Personal equation in making exposures;
(*k*) Movement of moon during exposure;
(*l*) Change of refraction between moon and star exposures.

(*a*), (*b*), (*c*) will entirely disappear owing to differential nature of measurements between moon and stars.

(*d*) The necessity of a high degree of stability in the instrument has already been insisted upon, and nothing more need be said on the point.

(*e*) With a suitable lens this source of error, which has been mentioned above, is quite negligible.

(*f*) A faint star will not act on the plate as rapidly as a bright one, but there it no theoretical reason why this should cause any errors, as the measurements are taken to both ends of the star trails.

(g) This is unlikely to have an appreciable effect on the result. Should the estimated position of the center differ by 2′ from the true one, the position of the unknown star or moon would be wrong by a maximum of about 1″. The resulting error of position varies directly as the error of the center, and as the square of the distance of the star from the center. There should be no difficulty in finding the position of the center within 2′.

(h) The error in the micrometer may be due to:

(1) Imperfections in the screws or scale;

(2) Coordinate axes not being straight;

(3) Errors of bisection on the image;

(4) Errors due to the moon's radius not being accurately measured;

(5) Distortion of the photographic film.

Of these (1) can be eliminated by well-known methods, and need not be more than mentioned here; (2) should not amount to a measurable quantity; (3) will totally disappear, in so far as systematic errors of bisection are concerned, if the plate be reversed during the measurement; (4) has already been discussed; (5) has been proved negligible in the case of the Astrographic Chart plates.

As an illustration of the degree of concordance that may be expected in a series of micrometer measurements of a moon or star image, the following set, which has been selected quite at haphazard, will be of interest. They are the measurements of one coordinate of the moon's limb expressed in millimeters. As the length of the lens was 19.5 inches, a unit in the third decimal place represents very nearly 0″.5.

Moon.									Mean.
1	53.024	31	36	28	32	31	33	21	53.030
2	49.067	70	68	71	72	71	76	61	49.070
3	45.096	92	01	95	00	94	03	92	45.097
4	41.130	35	41	35	38	31	41	28	41.135
5	37.168	62	78	68	72	65	73	61	37.168

An inspection of the above set will show that a mean of a series is not likely to be in error more than .002 mm. or 1″ arc, as far as the actual measurements are concerned.

As another example, a set of measurements for the radius of the moon may be given. For the quantity *OP* (diagram on p. 800) the following were the actual readings of the measurement:

.927	23	25	24	19	14	13	13
.924	23	27	23	19	14	13	15
.933	30	30	21	20	19	12	14

It is at once obvious that the readings in the second half of each line are consistently smaller than those in the first half. The reason of this is found in the fact that the cross-wires of the micrometer were not truly at right angles, and consequently measurements taken in adjacent quadrants were not identical. The diaphragm carrying the cross-wires was rotated through 90° between each set of four measurements.

To get a fair idea of the accuracy of these measurements we must, therefore, combine together the first and fourth in each line, second and fifth, etc. We then get the following values for *OP*:

.923	19	19	18
.924	19	20	19
.926	25	21	17

Dividing by $\sqrt{2} - 1$ to get radius, we have radius:

2.228	18	18	16	
2.231	18	21	18	2.223
2.236	33	23	14	mean

(*i*) An error in the local time will cause the same error in the resulting longitude, but an error in estimating the clock rate may have a somewhat more serious effect, inasmuch as it will alter the estimated interval between moon and star exposures, and thus tend to produce an error in the moon's right ascension and hence a large error in the longitude.

(*j*) This source of error will be practically negligible, as it will only affect the longitude by the same amount.

Suppose the observer systematically marked the exposures 0.1 second late. As this will apply equally to moon and star images, it makes the Greenwich time, and hence the longitude, 0.1 second wrong.

(*k*) As the exposure on the moon is not instantaneous, the image will move to a slight, but quite appreciable, extent during the time the shutter is open. If the duration of the exposure be 0.2 second, the moon's movement will be $3''$, and we should, therefore, tend to get a difference of this amount in the right ascension according as the image corresponds to the beginning or end of the exposures, i.e., according as the bright limb be following or leading. If the middle of the exposure corresponds to the recorded clock time, the moon's right ascension will be in error by $1''.5$. Account must be taken, therefore, of the fact that the effective exposure is somewhat less than the total time the shutter is open, and it is probably not far wrong to reduce the amount by one-third; hence the error caused by the moon's movement is not likely to exceed $1''$. This error changes its sign according as the bright limb be leading or following, and can, therefore, be completely eliminated by combining plates in pairs before and after full moon.

(*l*) If a considerable time elapses between moon and star exposures, the change in refraction may become a serious source of error. To take the most unfavorable case: If the observer be near the equator and the photograph be taken at an altitude of 30°, a variation in temperature of 10° Fahr. will cause a change of about $2''.5$ in refraction, and therefore about that amount of error in the right ascension. Such unfavorable conditions as these would be very rare, and in general the possible error would be only a small fraction of this amount. This error disappears if the photograph be taken on or near the meridian, as in that case a change in apparent altitude will not affect the right ascension. It will also entirely disappear if the moon's azimuth be used for computing the longitude, but this would only be of limited application, as, if the observer be on the equator, the azimuth is changing too slowly to be of any value.

351. Precision of Resulting Longitude.—As a general conclusion it seems not unfair to state that there is no one source of error which should in any case exceed 1″, unless it be the measurement of the moon's radius. A limit of errors in this case is somewhat difficult to fix, but we shall probably be not far wrong if we assume a limit of double the above amount, and hence conclude that the right ascension of the moon can be determined within 4 seconds of time. As will be seen immediately, a higher degree of accuracy has been realized with actual plates.

It is obvious that almost all the errors could be materially diminished if the method could be made a differential one, that is to say, if a duplicate photograph were taken with a similar instrument at about the same time at a fixed point.

It now remains to give the results of plates exposed at a place of known longitude, which is as follows:

Place, Chatham, England. True longitude, $2^m\ 08^s.13$ *E.*

Plate.	Date.	No. Moon Images.	Reference Stars.	Long. from Plate.	
				m.	*s.*
1	1894, Oct. 16	7	α Tauri; Jupiter	2	07.1
2	" " "	7	" "	2	07.6
3	1895, May 2	5	α Tauri; γ Leonis	2	09.4
4	" " "	3	" "	2	06.0
5	1895, May 4	5	γ Tauri; δ Virginis	2	07.0

All the above plates were exposed with camera resting on a solid masonry foundation, and it is not probable that quite such accurate results would be obtained in the field.

REFERENCE WORKS ON GEODESY.

No attempt has been made in the following list of reference works bearing on the subjects of geodesy and astronomy to include all those published which relate to the subject. The endeavor has been, however, to include those which have been consulted by the author in the preparation of this volume, and a few others which have a particular bearing upon the subject. They are printed here that the reader may know where to look for more detailed information on the various branches touched upon in the text.

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PART VII.

CAMPING, EMERGENCY SURGERY, PHOTOGRAPHY.

CHAPTER XXXVIII.

CAMP EQUIPMENT AND PROPERTY.

352. **Attributes of a Skillful Topographer.**—The skill of a topographer is necessarily judged not only by his ability to perform his technical duties in the most efficient manner, but also by the time and cost of making the survey. The ultimate *test of* the *ability of two men* on the same class of work, providing each be equally skillful, is the relative cost of their work. If two topographers are able to make an equally good survey of the same territory, he is the more useful who performs the work the more rapidly and cheaply. In the U. S. Geological Survey extreme instances of this have been noted where two men have been engaged in the survey of the same region and have produced maps of equal quality on the same scale. One required, however, as much as three times as long to obtain this result as the other, and the work of one cost $7 per square mile, and that of the other $20 per square mile. This great difference was due in large measure to their relative skill in managing the parties, in the planning and control of the work of the assistants, and in subsisting and transporting the members of the field force.

Where the topographer *subsists on the country* by living at hotels or farmhouses and hiring transportation, he need possess but few accomplishments beyond those of a technical knowledge of his business and sufficient executive ability to enable him to properly direct the work of the members of his party. Where, however, he *subsists in camp*, additional knowledge must be his, of that common order called "horse-sense" which comes to those who are brought up in the ways of the country and the woods. Finally, where the work takes him into inaccessible and unexplored regions he must possess, in addition to the qualifications already cited, a general knowledge of many things non-technical if he is to attain the relative success which would be looked for in similar work in other regions.

Under these conditions he must have a *general knowledge of all handicrafts* with which he has to deal, for he will frequently find himself unable to rely upon the members of his party for the little matters of care and repair of outfit where the outside aid of skilled artisans is not procurable. He will have to know not only how to harness a team and to adjust a saddle or pack (Fig. 201), but also how to repair a broken wagon or to shoe a mule, repair harness or make a riding- or pack-saddle. That he may make necessary repairs to the camp outfit and instruments, his equipment should include such tools as will enable him to do various kinds of rough work in wood, leather, and metal. Many of these are enumerated in Article 367, but those which he may have to use in the repair of instruments he must select according to his own judgment and skill.

An examination of the illustrations of the every-day life of the topographer in the far West (Figs. 186, 201 and 203) will give a clearer idea than words can convey of the nature of the travel and work which he will have to perform. Under such circumstances nearly all of the topographers of the United States Geological Survey have at one time or another had to repair or replace a broken tripod leg, a split plane-table board, an injured alidade or level. To replace cross-hairs or make a stadia rod are as common occurrences as the repair of a pair of old shoes, a torn coat, or a broken saddle-girth.

FIG. 186.—WHERE A PACK-MULE CAN GO.

353. Subsistence and Transportation of Party in Field. One of the most difficult problems connected with the execution of topographic field-work is the subsistence and transportation of the working force. Lack of judgment or experience in this figures largely in the output of the field force and the cost of obtaining a given result. Where the party can *live on the country cheaply*, that is, subsist in hotels or farmhouses and be transported by the people of the country, the work can be thus most economically managed. There are two modes of arranging *payment* for the services of the individuals of the party under such circumstances. One is to give them a per diem rate and to allow them to pay their own living expenses; the other, and that far more satisfactory where parties remain in the field a long time, is to pay the men by the week or month and to subsist and transport them. By this means the chief of the party has larger control of the movements and time of his working force. Subsistence may be had in various portions of the United States under such circumstances at from $1.00 to $2.50 per day per man. Single conveyances may be hired at from $1.00 to $2.00 per day, including the feed of the animals, and from $2.00 to $4.00 per day for a team with heavy wagon.

The other mode of subsisting a party in the field is by *camping*, when tents, cooking outfit, animals and conveyances for transportation must be procured or the latter be hired for a period of time. This plan must necessarily be resorted to in many regions where habitations are widely scattered. To aid the party chief in selecting his outfit the following memoranda have been prepared from a wide and varied experience, not only of the author in camping in various portions of the United States, Mexico, and India, but also from the experience of his associates on engineering work on railroad and government surveys.

354. **Selecting and Preparing the Camp Ground.** — When tents are to be pitched for a night or two only, it matters little where they are placed beyond choosing level and well-drained ground. When, however, the party is to remain in the same camp for several days, much care should be exercised in selecting the best camp ground. There is much more in this than the mere choice of level holding-ground for the tents, and considerations which apply in one region are entirely reversed in others.

In general the *ground should be nearly level*, having just slope enough to drain well. The soil should be preferably an open, earthy gravel, as this gives best underdrainage, holds tent-pins well, and is cleanly. Sand does not hold well, and loamy or clay soil is wet and damp after a rain, and when soaked will not hold the tent-pins. Moreover, it soon be-

FIG. 187.—A PRETTY CAMP GROUND, NORTH CAROLINA.

comes filthy and tramped into mud-puddles about the camp animals which have to lie on it. The camp site should not be near a town because of the lack of privacy and the annoyance from visitors. Where cattle or hogs roam at large it should be in a fenced field. It should be especially selected for convenience to an abundant supply of good water (Art. 378) and where fire-wood can be easily obtained (Art. 365).

In the dense, damp woods of the North it should be in a clearing, and in the burning sunlight of the South and West it should be in the shade, preferably of a grove of trees (Fig. 187). It should always be on slightly rising or high ground to assure good drainage, and as between camping in the bottom of a ravine or canyon or on top of a ridge, the latter should invariably be selected. There it is less damp and cold in early morning, and there the sunlight shines last at night. Care must always be exercised in selecting a camp site to choose one easily reached by wagons, pack-animals, etc., and which is convenient to forage or to pasture-land.

355. **Tents.**—The most satisfactory and comfortable tent for all general purposes is a *9 × 9 tent with 4-foot wall* and extension fly (see tent with flag, Fig. 187). The best pattern of tents is that made by contract for the United States Army. Tents of fair quality, and nearly as good as those of the Army, are to be had of numerous makers in various parts of the United States. They should be of full 12-oz. army duck, warranted free from sizing and mildew-proof. All seams should be lapped at least one inch and double-sewed. The opening should be in the middle of one end and should be protected by flap of at least eight inches width which can be tied both inside and outside. Near the top of each end should be a small opening for ventilation and for inserting the pole (Fig. 188), and this should be protected in stormy weather by a canvas flap which can be tied down over it from the inside. Inside the tent, on a level with the top of

the wall all around, at distances of two feet, should be tied strings, so that the wall can be raised in warm weather and tied up so as to allow the air to circulate freely.

At the bottom of the wall should be a sod-flap of lighter duck and about eight inches in width. This is to keep out the wind and rain, so that when the tent is pegged down from the outside this flap is turned inside near the floor or ground, and kept in place by laying upon it dirt, short strips of wood or stone, or anything else which will weight it down. The use of dirt is not recommended as it is liable to rot and

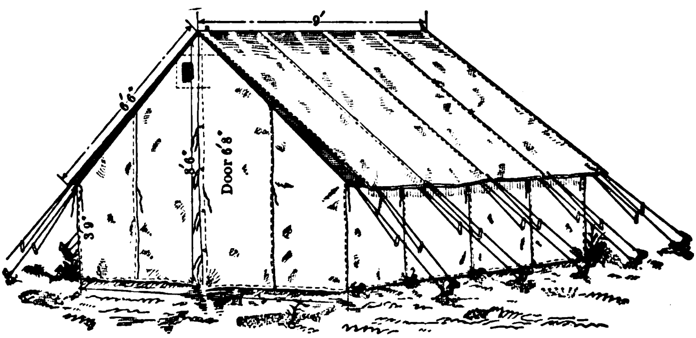


FIG. 188.—WALL TENT WITH FLY.

destroy the cloth. The ridge-pole should project about five or six feet in front of the tent, and be supported there by a third pole, and the fly should be this amount longer than the tent—that is, 14 to 15 feet in length, so as to extend like a porch as a shelter from sun and rain. An excellent modification of this extension fly is to so cut it that it will droop downward in a curved or turtleback form; then it will not be supported by an extension ridge, but will be merely guyed out by ropes.

Frequently for convenience of transportation or for lightness tents of other sizes must be used. In a large party it is often more convenient to have a 12 × 14 tent, in which a number of men may live or which may be used as a dining-tent. If for the latter purpose, such tent need not be provided with a fly. In very hot weather the most comfortable dining-tent is a simple fly which forms an awning as protection from sun and rain. Larger tents than 9 × 9 require at least two men to properly erect them.

Where tents must be transported on the backs of men or animals, they must be of especially light design. It is then impracticable to carry poles unless these be jointed. Where wood can be procured, poles can be quickly hewn, or a rope may be used for the ridge and be tied to two trees, and the tent hung from these and guyed out as though supported by ridge and poles. Where timber is not convenient, light jointed poles may be carried, but the ridge can be dispensed with, a rope being used as a ridge by guying it out to some distance in front and rear. Under such circumstances small 7 × 7 A tents of 8-oz. duck and without flies may be carried. These can be had of weights as light as 6 to 8 lbs. to the tent, and will furnish shelter to a party of three or four men in the most inclement weather, providing they are properly ditched and have steep enough inclination to their sides to quickly shed water.

In the *tropics* and where the heat is great, only the heaviest tents will furnish comfortable protection. A light tent or one of moderate-weight canvas will not keep out the burning rays of a tropic sun. At least two or even three tents of brown duck, each six inches to a foot smaller than the next, should be erected one inside the other so as to have an air-space of several inches about each; then if the wall at the bottom be raised to allow circulation of the air, the interior will be sufficiently cool for comfort. In India and Persia tents of light woolen cloth are used, and over the outside of these are placed tents of brown duck. These keep out the heat, retain the cool air of the early morning until late in the day, and protect from the rays of the sun.

356. Specifications for Army Wall Tents.—These and the following specifications are those issued by the Quartermaster's Department of the U. S. Army to bidders on contracts.

Dimensions.—Height, eight (8) feet six (6) inches; length of ridge, nine (9) feet; width, eight (8) feet eleven and one-half (11½) inches; height of wall, three (3) feet nine (9) inches; wall eaves, two (2) inches wide; height of door, six (6) feet eight (8) inches; width of door, twelve (12) inches at bottom, four (4) inches at top; from top of ridge to wall, six (6) feet six (6) inches.

Material.—To be made of cotton duck twenty-eight and one-half (28½) inches wide, clear of all imperfections, and weighing twelve (12) ounces to the linear yard.

Work.—To be made in a workmanlike manner, with not less than two and one-half (2½) stitches of equal length to the inch, made with double thread of five-fold cotton twine well waxed. The seams to be not less than one (1) inch in width, and no slack taken in them.

Grommets.—Grommets made with malleable-iron rings, galvanized, must be worked in all the holes, and be well made with four-thread five-fold cotton twine well waxed. Sizes of grommets: For eaves, one-half (½) inch rings; for foot-stops, three-quarter (¾) inch rings; and for ridge, three-quarter (¾) inch rings; the latter to be worked so that the center will measure one and three-eighths (1⅜) inches from edge of roof, so as to be in correct position to receive spindle of upright poles.

Door and Stay Pieces.—Door and stay pieces to be of the same material as the tent. Stay pieces on ends and ridge of tent to be six and a half (6½) inches square; those at corners of tent, at angle of roof and wall, to be eight (8) inches wide, let into the tabling at the eaves, and extending eight (8) inches up the roof and eight (8) inches down the wall; those on the sides to be four (4) inches wide and extending six (6) inches along the angles beneath the roof and six (6) inches along the walls.

Sod Cloth.—The sod cloth to be of eight (8) ounce cotton duck, eight and three-quarters (8¾) inches wide in the clear from the tabling, and to extend from door to door around both sides and ends of the tent.

Tabling.—The tabling on the foot of the tent, when finished, to be two and one-half (2½) inches in width.

Ventilator.—An aperture four (4) inches wide and eight (8) inches long, one (1) in the front and one (1) in the back end of the tent, placed six (6) inches from the top and two (2) inches from the center, on the right side of each end. The aperture to be reinforced with eight (8) ounce cotton duck, and to have the edges turned in and stitched all around. A flap or curtain on the inside eight (8) inches wide and fourteen (14) inches long, finished, to be made of two (2) ply eight (8) ounce cotton duck, stitched around the edges; to have one (1) "No. 1" sheet-brass grommet placed at the top for the purpose of tying it up to close the opening; strings made of "No. 2" gilling line to be used for tying the curtain in place.

Door Lines.—The door lines to be of six-thread manila line (large), three (3) feet long in the clear.

Wall Lines.—Eighteen (18) in number, to be two (2) feet long, to be made of "No. 3" gilling line, whipped at both ends and placed under the eaves on the seams, for tying the wall up.

Door Fastening.—Door fastening, as shown in sample tent, to consist of four (4) double door strings of one-fourth (¼) inch cotton rope one (1) foot long, on each side, passing through the door seam and secured by a "Mathew Walker" knot. Brass grommets, "No. 4," to be in corresponding position on edge of door piece, in which to tie the door cords. A one and one-half (1½) inch tabling to be made on the edge of door.

Foot-stops.—Foot-stops, seventeen (17) in number, to be loops four (4) inches long in the clear, of nine-thread manila line, both ends passing through a single grommet, worked in the tabling at seam, and to be held by what is known as the "Mathew Walker" knot.

Eave Lines.—Eave lines, ten (10) in number, to be of six-thread manila line (large), and to be eight (8) feet long in the clear, with an eye four (4) inches long, spliced on one end, and the other end properly whipped and furnished with "No. 3" metallic slip of Army standard.

The tabling at bottom, the sod cloth, and the foot-stops to be so arranged that the sod cloth falls inside and the foot-stops outside the tent.

All lines to be well whipped one (1) inch from the end with waxed twine, and properly knotted.

357. Specifications for Army Wall-tent Flies.

Dimensions.—Length, fifteen (15) feet and six (6) inches. Width, nine (9) feet when finished.

Material.—To be made of cotton duck, twenty-eight and one-half (28½) inches wide, clear of all imperfections, and weighing ten (10) ounces to the linear yard.

Tabling.—A two (2) inch tabling to be worked on ends, and a one and one-half (1½) inch tabling on sides.

Grommets.—Grommets made with malleable-iron rings, galvanized; to be worked in all the holes with four (4) thread five (5) fold cotton twine, well waxed. Size of grommets for eave lines, one-half (½) inch in diameter. and for upright spindle, three-fourths (¾) of an inch in diameter; the latter to be placed so as to measure one and three-eighths (1⅜) inches from their centers to edge of fly, so as to be in proper position to receive spindle.

Stay-pieces.—Stay-pieces on corners, triangular in shape, eleven (11) inches on base and perpendicular when finished, and on ridge six and one-half (6½) inches finished.

Work.—The fly is to be made in a workmanlike manner in every respect, with not less than two and a half (2½) stitches of equal length to the inch, made with double thread of five (5) fold cotton twine, well waxed.

Seams.—The seams not less than one (1) inch in width and no slack taken in them.

Eave Lines.—Eave lines, ten (10) in number, to be of six-thread manila line (large) and be seven (7) feet long in the clear, with an eye spliced on one end, four (4) inches long, the other end properly whipped, and furnished with a metallic slip No. 3, Army standard.

All lines to be well whipped one (1) inch from the end with waxed cotton twine and properly knotted.

358. Specifications for Army Wall-tent Poles.

A set of poles to consist of two (2) uprights and one (1) ridge, the former to be made of ash or white pine, and the latter of white pine, clear, strait grained, and free from knots or other imperfections.

Ridge.—Ridge nine (9) feet long, two and three-quarters (2¾) inches wide, two (2) inches thick; on each end a band, two and three-quarters (2¾) inches wide, of galvanized iron, secured by four (4) one and one-

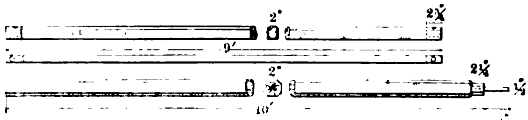


FIG. 189.—RIDGE AND POLE FOR WALL TENT.

quarter (1¼) inch copper nails. A five-eighths (⅝) of an inch hole bored through at a distance of one and one-quarter (1¼) inch from each end for the spindle of uprights.

Uprights.—Uprights octagonal, ten (10) feet long and two (2) inches thick; band of galvanized iron, two and one-quarter (2¼) inches wide, on upper ends, secured by two (2) one (1) inch screws. Spindle of one-half (½) inch round iron, galvanized, driven three (3) inches into upper ends and projecting four (4) inches.

At set of pins for wall tent to consist of ten (10) pins twenty-four (24) inches double-notched, and eighteen (18) pins sixteen (16) inches single-notched.

359. Specifications for Army Shelter Tents (Halves).

Material.—To be made of Army standard cotton duck, thirty-three (33) inches wide, weighing from seven and one-half (7½) to eight (8) ounces to the linear yard, and capable of sustaining a strain of seventy-two (72) pounds in the warp and thirty (30) pounds in the filling to the one-half (½) inch, counting not less than fifty-two (52) threads warp, and forty-eight (48) threads filling to the square inch.

Dimensions and Workmanship.—To be about sixty-five (65) inches long on the ridge, and about sixty-one (61) inches wide when finished. The center seam to overlap one (1) inch.

The four corners and center at the bottom of each half tent to be reinforced with pieces of the same material firmly sewed on; said pieces to be about four (4) inches square when finished.

The top or ridge to have a tabling, three and one-half (3½) inches wide, and the bottom edge to be turned in and hemmed, making a three-eighths (⅜) inch seam neatly and securely sewed.

To have two (2) grommet holes worked at each corner and at center of bottom. The two grommet holes at the top and front to be one and three-fourths (1¾) inches to the center from the top, and the first hole one-half (½) inch from the edge, and one and one-eighth (1⅛) inches from center to center apart. The two grommet holes at the top and rear to be one and three-fourths (1¾) inches from the top; the first hole one-half (½) inch to center from the edge, and one and one-eighth (1⅛) inches from center to center apart. Along the bottom all the grommet holes to be one (1) inch from the bottom to the center, the first hole at the front one (1) inch from the edge, and about one and one-half (1½) inches from center to center apart; the middle holes to be one and one-half (1½) inches from center to center apart, and the rear holes to be one (1) inch from edge to center, and about one and one-half (1½) inches from center to center apart.

The closed end to measure three (3) feet eleven (11) inches from ridge to base, and three (3) feet seven (7) inches along the base; the grommet holes in this end to be one (1) inch from the edge and one and one-half (1½) inches from center to center apart. All smaller-size grommet holes to be worked over a three-eighths (⅜) inch galvanized-iron ring,

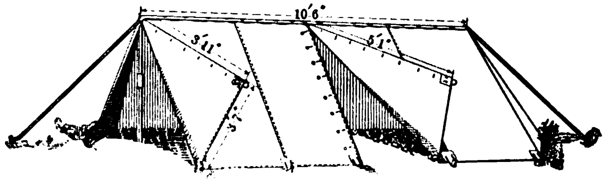


FIG. 190.—SHELTER TENTS.

with two (2) ply of five (5) fold cotton twine, well waxed; and the two (2) larger grommet holes made to receive the shelter-tent poles over a five-eighths (⅝) inch galvanized-iron ring, worked with two (2) ply of five (5) fold cotton twine, well waxed. To have nine (9) buttons and buttonholes along the top, the first buttonhole to be one-half (½) inch from the top, and three-fourths (¾) inch from the edge; the others about seven and seven-eighths (7⅞) inches apart, with a white-metal button below each buttonhole, three (3) inches from the edge.

The side of each half tent to have seven (7) buttons and buttonholes, the side buttonholes to be worked one-half (½) inch from the edge; the first buttonhole to be about eight (8) inches from the bottom, and the others spaced about seven and one-half (7½) inches apart. The buttons to be firmly sewed on about three and one-half (3½) inches from the edge. Along the closed end there shall be seven (7) buttons and buttonholes, the first buttonhole to be about five (5) inches from the top, and all to be about one-half (½) inch from the edge, and spaced about six (6) inches apart; and the buttons to be about two and one-half (2½) inches from the edge, and spaced about six (6) inches apart.

Each half tent to be furnished with a guy line, and four (4) foot-stops, made of 6-thread manila line, about one-fourth (¼) inch in diameter; the former about six (6) feet seven (7) inches long in the clear with an eye-splice of about two (2) inches at one end; each foot-stop to be about sixteen (16) inches long in the clear, all whipped at both ends. All sewing, including buttonholes, to be done with W. B. linen thread, good quality, No. 70.

360. Specifications for Army Shelter-tent Poles.

A set of Shelter Tent Poles shall consist of two (2) uprights, made round, about one (1) inch in diameter, when joined to make a pole forty-six (46) inches in length from lower end to shoulder at top, with a neatly turned spindle at top about one (1) inch long and one-half (½) inch in diameter, making a total length of forty-seven (47) inches.

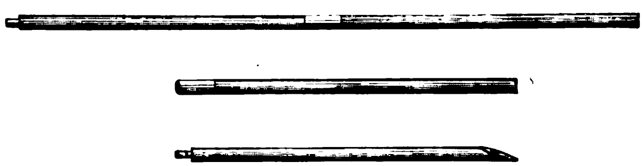


FIG. 191.—JOINTED SHELTER-TENT POLES.

Each upright to be in two parts of about equal length, about two and one-half (2½) inches bevel, and joined in a tin socket four (4) inches long, made of twenty-three (23) gauge tin (U. S. standard gauge), joined by a groove seam, neatly turned and soldered full length of seam, and secured to lower part of the pole by two (2) tacks, neatly and squarely driven.

The pole to be of poplar wood, free from knots, and smoothly finished.

361. Erecting the Tent.—To properly set up a tent it should be taken by the ridge and dragged away until laid out flat. The ridge-poles should be inserted through the ventilation-holes, the supporting poles inserted in the ridge-pole, and the whole raised and the corners at once guyed out. The corner ropes by which the tent is first stretched should be drawn in a diagonal direction so as to make an angle of about 45° with the walls. The door should be tied up so that the tent may be given its proper shape, and the wall-corner loops pegged down and door fastened to hold the whole in place. Then the side ropes should be guyed out and the tent stretched taut by tightening a little on each rope at a time.

The fly must be laid over the tent when on the ground, and be raised with it. Then it must be so stretched as to touch the tent at no point excepting at the ridge, while at the eaves it should be from 6 to 10 inches above the roof of the tent. (Fig. 187.) This result can be obtained by several methods. One is to use pegs with two notches, on the lower of which the tent-guys are fastened, and on the upper the fly-guys; or an additional row of pegs may be set a foot beyond the tent-pegs for the support of the fly-guys. Where much rain or heat is encountered, short crotched poles about 10 inches longer than the height of the wall should be cut and one of these be set under each of the corner fly-guys to raise the fly away from the tent roof. As a protection in high winds long guys should be stretched from each end of the ridge-pole in front and rear, otherwise storms blowing end on may carry the tent away.

362. Tent Ditching and Flooring.—Where the ground is moist or rains are to be provided against, the tent must be ditched in order that the water shall not run under it and wet the soil inside the tent; and where the camp is to remain in the same place for some time, the comfort of the party will be greatly increased by adding a floor to the tent. To *ditch a* *tent* a sharp spade or mattock should be used, and the soil be cut squarely or vertically just outside the foot of the wall. (Fig. 192.) The soil should be pitched away and an easy slope left on the outside of the spade-cut. Dirt should never be banked up against the outer wall of the tent, as it rapidly rots and destroys the canvas. The ditch should be cut sufficiently deep to assure its carrying off any ordinary rainfall, and should be made deep or shallow in various parts according to the slope of the ground, so that its bottom may have a uniform slope towards the lowest ground. At such point the ditch should be carried away from the tent a short distance in order to assure egress of the water from the ditch.

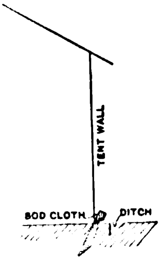
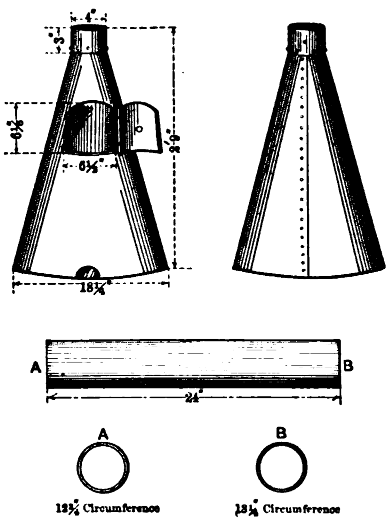


FIG. 192.—SOD-CLOTH AND DITCH.

The comfort of the occupants of the tent is increased by using a small strip of canvas or similar material as a floor on which to stand in dressing. Still better is a *canvas floor* of the full size of the interior of the tent, and this can rest upon the sod-cloth to keep out the wind. Where facilities for transportation permit, a *wooden floor* of tongue-and-grooved planks the length of the tent—say 9 feet—and fastened together by cleats in sections of 3 feet width may be provided. These 3 × 9-foot sectional floors can be easily handled in moving, and the whole tent can be floored with them or only one or more sections be placed in the space between two cots.

A still *more substantial floor* for a permanent *winter camp* consists in laying 2 × 4 scantling as floor-joists and planking these over so as to make a full floor which shall extend outside the canvas walls. The rain will run under this, and a little carpet or canvas on it will keep the wind out. At each corner a 2 × 4 joist should be erected the height of the wall, and these corner posts should be connected by smaller scantling so as to form a railing the height of the wall. Over this the tent will be stretched, the framing of scantling holding it out in shape. It is unnecessary except in very high winds to guy out tents stretched in this manner, the guys to the fly being sufficient protection.

363. Camp Stoves, Cots, and Tables.—The most comfortable camp stove is the *oil-heater*. With this it is unnecessary to cut any hole in the tent as an outlet for a smoke-pipe. It can be quickly lighted and extinguished, furnishes sufficient heat, and can be moved to any part of the tent with ease. Where oil cannot be carried for such a heater, a *Sibley stove*, made of sheet iron similar to that used in making stove-pipes, is one of the most simple and satisfactory heaters. This can be made by any tinner, is conical, the top being of the dimensions of ordinary small stove-pipe (Fig. 193), the



193.—TENT STOVE AND PIPE.

bottom 18 inches in diameter, and the height about 3 feet. A small hinged door must be cut and fitted in one side of this conical heater, the bottom being left open. In other words, it is an inverted funnel of stove-piping which rests on the bare earth. The sticks of wood are placed in it on end and rapidly ignite and produce a strong heat. The *fire* is *easily controlled* by banking up the outer edge of the stove at the bottom, so as to prevent the ingress of the air, thus at once dampening it, or by digging away the earth underneath it a little, so as to admit the air, when the fire quickly draws up.

The *stove-pipe* may be carried with a joint through one end of the tent. In this way the canvas will not be injured as when it is carried up straight through the roof, thus introducing danger of fire from sparks and admitting rain around the pipe. In order to protect the canvas from burning by the heat of the pipe, a rectangular hole should be cut in the canvas on three sides, the fourth side of the hole being left so that the canvas can be turned down, and when the pipe is removed it can be laid back again. On either side of the canvas surrounding this hole there should be fastened by rivets or wire thread a sheet of tin with a circular hole sufficiently large to permit the passage of the stove-pipe.

The most convenient camp bed is a *spring cot*, where such can be transported, or one of the various forms of folding cots. Where for convenience of transportation cots cannot be carried and where hay can be procured, this or straw makes an excellent couch on which to lay the blankets, first placing canvas beneath these to protect them from particles of hay. In the pine and fir forests of the North a *bed of boughs* can be made by breaking off the small twigs from spruce or balsam boughs and laying these on end with the butt in the ground, their length not exceeding 12 to 16 inches. When enough of these are laid in this manner, one close against the other, they make a substantial, warm, springy mattress. A less comfortable couch of spruce or balsam boughs may be made by cutting these in lengths of 2 feet and laying them with the leaf ends to the center and the butts out, crosswise of the bed, in such manner that the butts project to either side.

Various forms of folding *camp tables* may be purchased. These may, however, be made by a carpenter quite as conveniently and cheaply and even more satisfactorily. Of those varieties which may be purchased, as convenient a form as any is the folding sewing-table one yard in length. For large tables the simplest is a pair of trestles on which to lay planks, and a similar arrangement may be provided as a bench on either side of the table. Of portable tables, one of the most satisfactory forms is that shown in Fig. 194, which may be

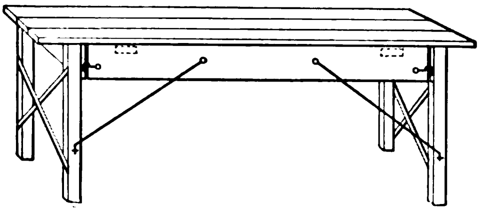


FIG. 194.—FOLDING CAMP TABLE.

readily constructed under the instructions of the topographer. This table is extensively used in the camps of the United States Geological Survey. The top consists of three 12-inch boards of suitable length screwed to two cleats. Each pair of legs is fastened at the table ends to wing boards 8 inches wide, and these are hinged to the top so as to fold inwards with the legs, thus lying flat to the table top. Lengthwise of the sides of the table top are hinged two other wing boards. When the legs are opened out these side boards are let down and hooked to the legs, thus keeping them in place. Long side hooks of iron add rigidity to the whole.

364. Specifications for Sibley Tent Stoves.

Stove.—The stove to be in the form of the frustrum of a cone, and to be made of No. 14 U. S. standard gauge common annealed plate-iron. To be in one piece (except the collar and door), and the seam at back to be fastened with twenty-four (24) rivets. The collar at top to be of the same material as the stove. To be two and a half ($2\frac{1}{2}$) inches deep, and be secured to the stove by six (6) rivets. Aperture for door to be about six (6) inches high by six (6) inches wide, the upper corners of which shall be rounded as in sample. The door to be sufficiently large to lap over the aperture; to be securely hinged to the stove, and to be properly molded to its form. An " A"-shaped vent at the bottom of stove directly under the door, about two (2) inches high by three (3) inches wide; the top to be rounded.

Dimension and Weight.—Height to top of collar, twenty-eight (28) inches. Circumference (outside) at bottom, fifty-eight (58) inches; at top, thirteen (13) inches. Distance from bottom of door aperture to base of stove, fourteen (14) inches. Weight about (19) pounds.

365. How to Build Camp-fires.—To *kindle a spark* into a flame the spark should be received in a loose nest of the most inflammable substance at hand, which ought to be prepared before the tinder is lighted. When by careful blowing or fanning the flame is once started, it should be fed with little sticks or pieces of bark until it has gained strength to grapple with thicker ones.

There is something of a knack in *finding fire-wood.* It should be looked for under bushes. The stump of a tree that is rooted nearly to the ground has often a magnificent root fit to blaze throughout the night. Damp or very sappy wood should be avoided. Dry manure of cattle is a fair fuel. Dry fuel gives out far more heat than damp fuel. Bones of animals also furnish a substitute for fire-wood. Wood should be cut into lengths of one foot and about two inches square. When nothing but brushwood is to be had, it should be burned in a trench. Where fuel is scarce, it is well when moving camp to gather and throw into the wagon all the dry wood which may be found along the road.

366. Cooking-fire for a Small Camp.—Lay down two green poles, 5 by 6 inches thick and 2 feet long, and spaced 2 or 3 feet apart, and with notches in the upper side about 10 to 12 inches apart. Lay two more green poles, 6 by 8 inches thick and 4 feet long, in the notches. Procure a good supply of dry wood, bark, brush, or chips, and start the fire on the ground between the poles. The air will circulate under and through the fire, and the poles are the right distance apart to support a camp-kettle, frying-pan, or coffee-pot.

If several meals are to be cooked in this place, it will pay to put up a *crane*. This is built as follows: Cut two green posts 2 inches thick and 3 feet long; drive these into the ground a foot from either end of the fire. If these poles are not forked, split the top end of each with the axe; then cut another green pole of same size and long enough to reach from one to the other of these posts; flatten the ends and insert them in the crotches or splits. The posts should be of such height that when this pole is passed through the bail of the camp-kettle or coffee-pot they will swing just clear of the fire. A less satisfactory crane is made by resting three poles together like a tripod and fastening them at the top by wire. Then a wire hook is hung from the center of these low enough to bring a kettle just over a fire built between the tripod legs.

367. Camp Equipment.—For a party of six and where transportation is by wagon, the following covers most of the essentials of the living equipment for the camp—that is, the equipment exclusive of that required for transportation:

Four 9 by 9 *tents*, with flies, poles, and pegs; one for party chief, one for three assistants, one for cook and kitchen, and one for dining and storage.

Canvas or sectional wooden *floors* for tents.

In winter, three heating-*stoves*, also one small (cast-iron) wood cooking-stove with pipes.

Two *mess-boxes*, one for cooking-utensils, the other for tableware and light provisions, of pine screwed together, with hinged tops and compartments; also an inside cover the full width of the top, which may be used as a bread-board. When the lids are opened out and the two mess-chests placed together, they form a table of the width of the mess-chests, and a length four times their thickness. These chests should be 20 inches deep, 20 inches wide, and 24 to 30 inches in length, so as just to fill a wagon-bed.

Mess-kit should consist of the following articles:

2 wash-basins
2 pepper-and-salt boxes
2 buckets
1 dipper
1 bread-pan
2 frying-pans
2 two-quart stew-pans
1 half-gallon coffee-pot
Table-cloths
Dish-towels
4 sheet-iron camp-kettles with covers, sizes ranging from 1 to 3 gallons so as to nest one within the other
2 carving-knives
1 spring-balance
6 pans one and one half inches deep, six inches in diameter, for soup, oatmeal, etc.
1 chopping-bowl and chopper
1 iron broiler
½ dozen cups and saucers
kerosene-oil can
1 dish-pan
2 four-quart stew-pans
10 plates
1 quart tea-pot
napkins
2 one-quart cups
1 coffee-mill
½ dozen plated or aluminum table-knives, forks, table-spoons, and tea-spoons
1 galvanized iron basting-fork and spoon
3 pans two inches deep and eight inches in diameter, as serving-dishes

All dishes, basins, etc., should be of *granite- or porcelain-lined ware.* The stew-, coffee- and tea-pots, etc., should also be of granite ware or have copper bottoms. To the above may be added numerous miscellaneous articles if transportation facilities will permit, as a wash-tub and board, rolling-pin, etc.

Where *transportation* is on the *backs of animals* tin and

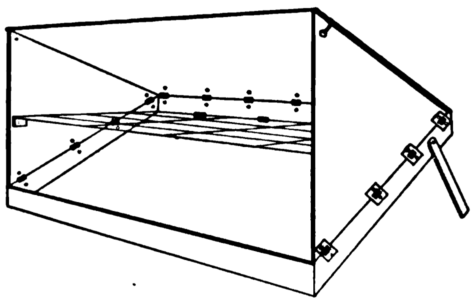


FIG. 195.—FOLDING TIN REFLECTING BAKER.

galvanized iron will have to be substituted for granite-ware to reduce weight, and many of the above articles must be dispensed with. The stove will be replaced for baking by a Dutch oven 12 inches in diameter, or by a tin reflector (Fig. 195).

For *transportation on men's backs* practically everything will be dispensed with but a few tin plates and cups, knives, forks, and spoons, a coffee-pot, frying-pan, and stew-pan. A tin reflector should also be carried for baking.

The miscellaneous *camp tools* may consist of some or all of the following:

- 1 or 2 axes and extra axe-helves
- 1 hatchet
- Bits and augers
- Screw-driver
- Assorted screws and nails
- Broom
- Quart canteens covered with cloth and canvas, or, in arid regions, a one-gallon canteen to each man
- Small files
- Rochester burners or other good lamps for drafting and reading
- Lanterns
- Assorted rope and string
- Whetstone
- Mattox
- Shovel
- Spade
- Saw

368. Provisions.—The best estimate of the amount of provisions required for a camping party can be obtained by consulting the following ration list, which has proved most satisfactory after long experience in the field-work of the United States Geological Survey.

A *ration* is the food estimated to be necessary to subsist one man one day. The amounts of the various articles in the ration are designed to be sufficiently liberal for all circumstances. They are maximum amounts, which should not be exceeded.

The Survey ration is made up of the articles and amounts given at the top of page 834.

On the basis of this list a *party of six* will consume six rations a day. One hundred rations will therefore subsist such a party seventeen days. The *cost of* the above *ration* will vary necessarily with the locality. Near large markets and convenient to railways the ration—that is, the food of one man for one day on the above basis—costs from 45 to

TABLE LXXI.

RATION LIST.

Article.	Unit.	100 Rations.
Fresh meat, including fish and poultry (*a*)	Pounds	100
Cured meat, canned meat, or cheese (*b*)	do.	50
Lard	do.	15
Flour, bread, or crackers	do.	80
Corn-meal, cereals, macaroni, sago, or corn-starch	do.	15
Baking-powder or yeast-cakes	do.	5
Sugar	do.	40
Molasses	Gallons	1
Coffee	Pounds	12
Tea, chocolate, or cocoa	do.	2
Milk, condensed (*c*)	Cans	10
Butter	Pounds	10
Dried fruit (*d*)	do.	20
Rice or beans	do.	20
Potatoes or other fresh vegetables (*e*)	do.	100
Canned vegetables or fruit	Cans	30
Spices	Ounces	4
Flavoring extracts	do.	4
Pepper or mustard	do.	8
Pickles	Quarts	3
Vinegar	do.	1
Salt	Pounds	4

(*a*) Eggs may be substituted for fresh meat in the ratio of 8 eggs for 1 pound of meat.

(*b*) Fresh meat and cured meat may be interchanged on the basis of 5 pounds of fresh for 2 pounds of cured.

(*c*) Fresh milk may be substituted for condensed milk in the ratio of 5 quarts of fresh for 1 can of condensed.

(*d*) Fresh fruit may be substituted for dried fruit in the ratio of 5 pounds of fresh for 1 pound of dried.

(*e*) Dried vegetables may be substituted for fresh vegetables in the ratio of 3 pounds of fresh for 1 pound of dried.

55 cents. It rarely exceeds 75 cents in the most inaccessible localities in the United States.

Where *transportation is difficult*, as by pack-animals, the above must be varied by omitting the heavier provisions, those containing the most moisture, such as all canned goods, and these must be replaced by additional amounts of flour, beans, and dried fruits. Where fresh meat cannot be obtained it must be replaced by additional bacon and corned beef.

Where *provisions* must be *carried on men's backs* a still further cut must be made in the heavier articles. Under the most unfavorable conditions an abundance of flour, bacon, rice, beans, oatmeal, cornmeal, tea, sugar, dried fruit, and salt must be provided.

To the above ration list are to be added such quantities of *matches* and *soap* as may appear necessary.

CHAPTER XXXIX.

TRANSPORTATION EQUIPMENT.

369. Camp Transportation; Wagons.—The manner of transporting the camp outfit must depend necessarily on the conveniences of the country in which the work is being executed. On the plains or where there are sufficient roads, and forage can be provided for animals, transportation in heavy wagons is necessarily the most convenient and satisfactory. Even in the roughest country a large camp wagon with four animals will transport the outfit, including tents, beds, and provisions, for a party of six or eight. One of the most convenient arrangements in hilly country is to have two smaller wagons, as they are more easily loaded and unloaded than a large one, and to have but one team to each wagon; then on the heavy hills the teams may be doubled up until the summits are reached.

There should be bows to the wagon, that a canvas cover may be hung over these to protect the load from rain. Covers should not be laid on the load, as the latter will soon wear holes in it and render it useless. The load should be well tied down with a long quarter-inch lash-rope passed back and forth over the whole, otherwise the various articles will jostle about and wear holes or injure each other. Care should be taken in loading to place the heaviest and most durable property in the wagon bed, and the tents, bedding, etc., on top, especial care being taken that nothing which will wear holes in the tents shall touch them.

There should be a tool-box in front of the dashboard to carry axle-grease, wrench, hatchet, wire, rope, nails, and similar articles which may be useful in event of a breakdown. The tailboard should be removed for easy loading, and in its place a long leather strap or chain be fastened about the mess-chests, which should occupy the rear of the wagon-bed. (Fig. 196.)

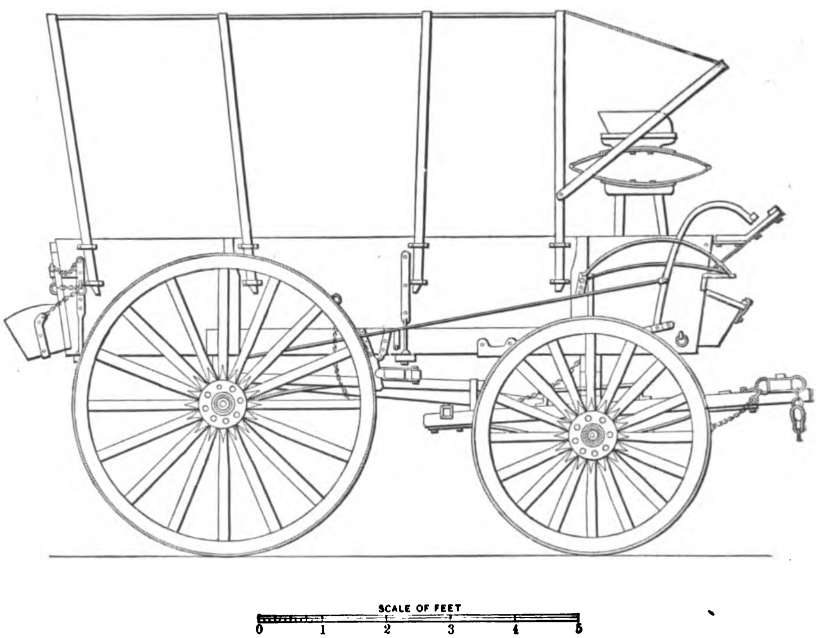


FIG. 196 —CAMP WAGON.

370. Pack Animals and Saddles.—The best pack-animals are short-coupled, short-legged, stocky mules of less than one thousand pounds weight. A heavy load when moving camp at a slow gait is about 30 per cent of the animal's weight or between 250 and 300 pounds. Where the animal is to move at a trot it should be loaded with from 150 to 200 pounds at the outside. A day's journey for laden animals is 20 to 25 miles, and the best way of making the move is not to stop for a noon rest, but to set out early in the morning, continuing to the end of the journey without unpacking. Where longer trips have to be made, however, the packs and saddles should be removed and a full hour given for rest, otherwise the animals may be galled.

The pack-saddle is fitted on the animal in the same manner as is a riding-saddle with a heavy *saddle-blanket* or pad underneath it. It is so constructed that it can be placed a little further forward than the riding-saddle. In addition to the ordinary saddle-cincha the saddle is sometimes provided with a crupper, but this is not as satisfactory for heavy work as is a *breeching*. The latter is made by screwing stay-straps to the rear end of each of the wooden pads of the saddle and carrying these back to a ring over the animal's rump. Thence the stay-straps fall off to and support the forward ends of the breeching-straps as with ordinary harness. To the front ends of the breeching-straps are attached snap-hooks to catch into the cincha-rings on either side.

In the Southwest the *aparejo* is generally employed as pack-saddle by the trained packer. This consists of two leather bags stuffed with hay which are connected at one end by a leather apron so that they may be hung over the sides of the mule. They are about 3 feet in length and 2 feet in width, and when fully stuffed about 4 to 6 inches in thickness. They are fastened to the animals by a wide girth thrown over the aparejo and under the animal's stomach, and are firmly cinched on. Upon them is placed the load, divided into two parts of equal weight, one resting on each side of the aparejo and fastened in position by means of a long rope tied with a diamond hitch. This apparatus can, however, only be used by skillful packers, and is then rarely as satisfactory as is the Moore army saddle or the ordinary crosstree pack-saddle where the animals are compelled for any reason to move at a brisk gait.

The *crosstree pack-saddle* can be purchased of dealers in

St. Louis, Denver, and similar supply centers. It can also be readily constructed. It consists of two pads of wood curved and shaped somewhat like the tree of an ordinary riding-saddle so as to fit the back of the animal, and these are joined together by two strips of oak or other stout wood screwed to the outsides of the wooden pads. They are fastened to each other at their junction, at which point they cross, one at front and the other at rear of the saddle.

Panniers or *alforjes* of canvas, about 2 feet in length, 14 inches deep, and 6 to 8 inches through, are hung on the saddle-forks by means of leather straps. To the outer sides of these are fastened at one end a long leather thong, and at the other a loop or metal ring. The thongs are thrown across the back of the animal, passed through the loop on the opposite alforje, and tied up so as to raise the load on to the tree of the saddle and away from the sides of the animal. The center of the load may be filled with loose and light articles, as blankets or a tent, to give the whole shape and body. Over this should be thrown a canvas cover, and the pack be tied on by means of the diamond hitch. The *lash-rope* which fastens the load on the aparejo or pack-saddle should be of $\frac{5}{8}$ inch manila rope 42 feet in length. This should be fastened to a wide girth of canvas which comes under the belly of the animal, and on the other end of the girth should be an iron hook or a hook made from the crotch or forked branch of some hard wood.

371. **Moore Pack-saddle.**—The United States Army uses an improved saddle (Fig. 197) for packing which is a modification of the Mexican aparejo, over which it has several advantages, chiefly in that it is more easily handled by inexperienced packers and is more readily kept in good condition. The Moore saddle, as it is called after its inventor, consists, like the crosstree saddle, of a number of parts, including the saddle proper, two pads similar to those of the aparejo, a crupper instead of the breeching used with the

crosstree saddle, a corona or pad placed next to the animal's back under the pad, and a large canvas pack cover; also

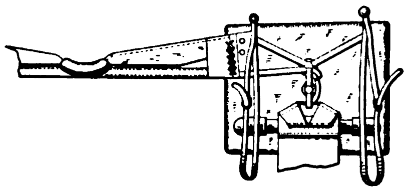


FIG. 197.—FULL-RIGGED MOORE ARMY PACK-SADDLE.

a canvas cincha ten inches in width, of varying length according to the animal (Fig. 198); half-inch manila rope

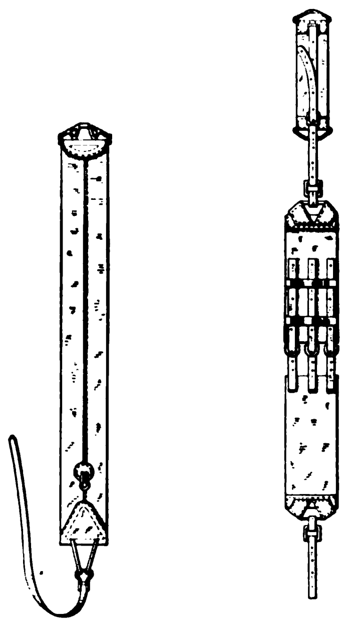


FIG. 198.—PACK-SADDLE CINCHES.

twenty-two feet long for sling-rope, and a lash-rope similar to that used with the crosstree saddle.

The army saddle is adjusted to the animal somewhat differently from the crosstree saddle. The cincha goes entirely over the saddle, coming under the animal's belly and over his back, thus completely encircling or girdling him and the saddle. The pack is loaded in a manner similar to that described for the crosstree saddle.

372. **Throwing the Diamond Hitch.**—It requires two men to lash a pack with the diamond hitch unless the packer possess unusual skill. Calling the two packers, respectively, thrower and cincher, the latter stands on the off or right side of the animal, and the former on the left or near side. The thrower first casts the girth under the animal's body to the cincher, who grasps the hook, point to front, in his left hand. The thrower immediately casts the end of the rope backward over the left shoulder of the animal across his right hip, then taking the short or girth end in his right hand and the long or loose end in his left hand—that is, the end toward the head of the animal—he casts a short loop of the rope over the back of the animal (Fig. 199, *A*) to the cincher, who passes this through the hook of the girth and draws it slightly taut. The cincher at the same time throws the remainder of the slack of the rope over the back and towards the head of the animal and on the side of the thrower (Fig. 199, *B*).

The thrower next passes the long end of the loop backward over the rope first cast, thus making a bight in it, and he also carries it in front of the forward corner of the pack on his side, leaving the short end hanging backward—that is, to his right (Fig. 199, *C*).

The cincher now takes his slack loop in his right hand, and reaching beyond the bight just made by the thrower in the cinch rope, he passes his loop backward under and forward over the cinch or first rope, thus making a second bight in it. Immediately he takes the loose end, which is that to his left, and passes it forward and across the pack under the rope which he has just looped (Fig. 199, *D*).

The cincher now takes the end of the rope which is caught in the hook and, pressing his foot against the side of the animal, draws this as taut as he can, while the thrower, turning his back to the pack, takes in the slack by holding it tautly over his shoulder. This slack he passes over the front corner

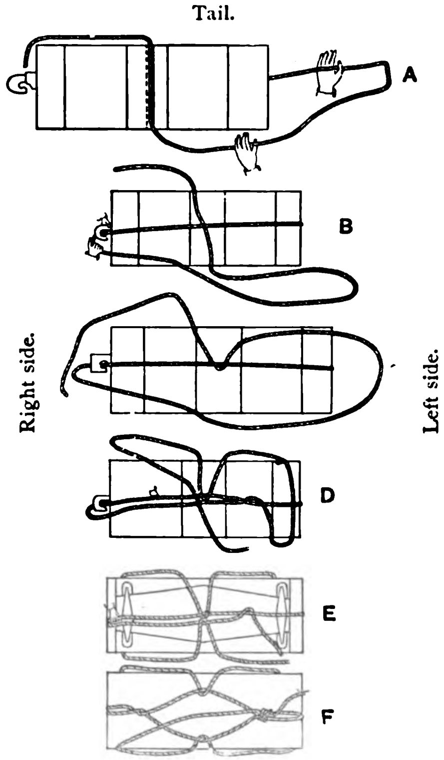


FIG. 199.—LASHING PACK WITH DIAMOND HITCH.

of the pack and, still holding it firmly, passes it under the same and backward, pressing his foot against the rear corner of the pack to draw it as taut as possible (Fig. 199, *E*). Then the cincher, standing to the rear of his side of the pack, takes in the slack given him by the thrower and, pressing his foot against the rear of his pack, draws the rope as taut as possible.

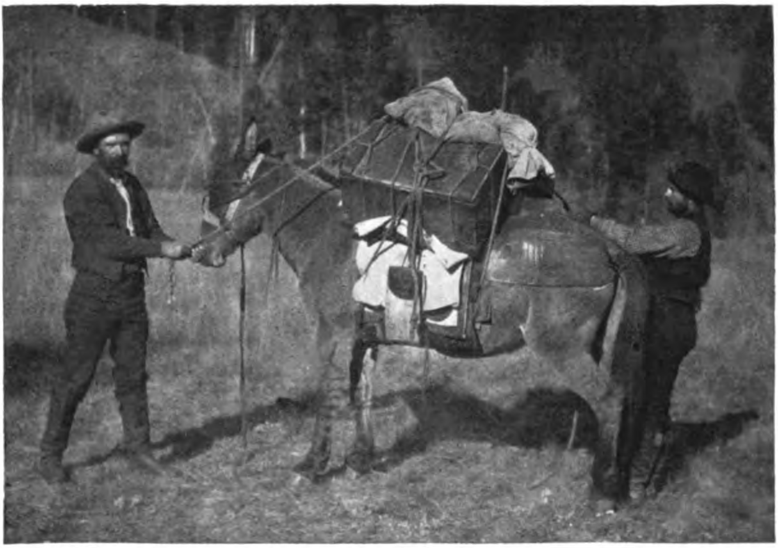


FIG. 200.—LOADING PACK-MULE WITH MESS-BOXES.

The slack he passes around the rear end of the pack on his side, under it and up the forward side, pressing his foot against the pack from the front, while the thrower, using his foot against the front side of the pack on his side, takes in the slack given him by the cincher (Fig. 200).

Having the entire pack now fastened, it will be noted that the two bights open the loops in the form of the diamond hitch. The thrower then takes the slack end, which he now holds, and ties it on his side across the front and outer side of the pack in such manner as to firmly bind the whole together (Fig. 199, *F*).

373. **Packmen.**—Where camp equipment must be transported on men's backs, as in some portions of the Adirondacks (Fig. 201), in the Northwest, and in Alaska, the loads may be arranged thus: Blankets, clothing, etc., may be rolled inside of rubber or canvas into bundles of about 24 inches in length, 18 to 20 inches width, and 15 inches thickness. These should be strapped and slung over the shoulders by wide leather straps fitted in a manner described below for pack-baskets. For heavy provisions and miscellaneous small articles *baskets* of the type used in the Adirondacks, or canvas panniers, furnish the most satisfactory mode of carrying packs.

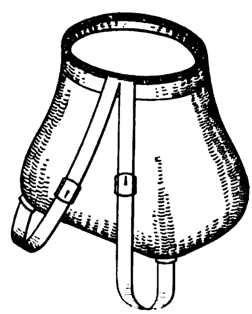


Fig. 202.—Pack-basket.

These baskets are shaped as shown in Fig. 202, averaging about 18 inches in depth, 17 inches in width at the bottom, and 15 inches in width at the top, with the thickness at bottom and top 12 inches. A heavy leather strap is run around the top under the rim, and to this are attached two carrying-straps which come close together and pass through the same loop at the top. These straps pass down the body side of the basket close to the latter, and are caught up at the bottom of the basket at their outer extrem ity, so as to form the letter "A" as viewed against the basket. Thence they run up and buckle to the ends which come from the upper portion of the basket, leaving wide loops through which the arms can be passed, while the buckles give necessary freedom for adjustment. This pack should be carried as high as possible on the shoulders, and the closeness of the straps at the top keeps it well on to the shoulders without a confining breast-strap.

A heavy *load for a packman* over good trails and for tramps of 15 to 20 miles is 60 to 75 pounds. A light load for heavy traveling and mountain work is 35 to 50 pounds.

374. **Transportation Repairs.**—In addition to camp wagon and harness or pack-saddles, as the case may be, the following should be carried for repair and use in connection with the animals and outfit.

A *farrier's kit* for shoeing where blacksmiths cannot be had: this should include one clinch-cutter, one clinching-iron, one shoeing-hammer, one pair shoeing-pincers, one shoeing-rasp, assorted horse or mule shoes already fitted and corked, and assorted nails.

A *saddler's kit*, for use with pack and saddle outfits: this should include sewing-palm, bradawls, sail-needles, twine, wax and sewing-thread, assorted buckles, assorted copper rivets, rivet and iron set, riveting-pincers, rivet-nippers and cold-chisels, assorted rings, leather whangs, lace leather, also some heavy harness leather and copper wire.

In addition to the above the following *miscellaneous utensils* should be provided:

Axle-grease, nose- or feed-bags, horse-brush, currycomb, halters, whips, riding saddles and bridles, saddle-blankets, wagon-jack, monkey-wrenches, and canvas pack-covers.

To the above may be added, under certain conditions: hopples, bells, tethering-ropes and long pivot picket-pins with rings at top, water kegs or barrels, heavy canvas wagon-covers with bows for supporting same, lash-ropes for tying packs on wagons.

The amount of *forage* required by animals doing heavy work may be estimated roughly from the following:

14 lbs. hay or fodder, 12 qts. oats, or 8 qts. corn	per horse per day	Hay, when pressed, 11 lbs. to cubic foot, 32 lbs. to bushel, 25.71 to cubic foot. Grain 56 lbs. to bushel, 45.02 to cubic foot.

375. Veterinary Surgery.—Some general remedies should be carried for the use of the animals. These may consist chiefly of the following:

Liniments of ammonia or strychnine for external application, as for sprains, by reducing heat without blistering. *Soap liniments* and iodine compounds for external application to swellings.

Cleansing agents for decomposing sores, consisting of sulphate of copper or bluestone or of carbolic acid in weak solutions. All excellent for curing "scratches."

Astringents to diminish the discharge of wounds, as alum or sulphate of zinc.

Healing agents for wounds, as collodion and arnica.

Emollients to soften and relax muscles, as olive oil and poultices.

Cathartics, as Epsom salts, castor-oil, aloes.

Stimulants for the stomach, as ginger, gentian, and caraway seeds.

For *cramps* salicylic acid, oil of turpentine.

Diuretics for bladder and kidneys, as turpentine, sweet spirits of niter.

CHAPTER XL.

CARE OF HEALTH.

376. Blankets and Clothing.—The personal property to be carried by each individual of the party will depend necessarily, as do the other articles of the camp outfit, upon the mode of transportation and the region in which the party is to work. Where wagon transportation is provided and the party may carry all essentials, each individual may take a small steamer-trunk for his clothing and should roll and strap his blankets in the form of a cylindrical bundle in a piece of No. 6 canvas or 16-oz. duck. The *canvas cover* should be 7 feet long by 6 feet wide, so that when the bed is laid down it may rest on half of the canvas to keep out moisture if on the ground, and air if on a cot, and the other half of the width of canvas should be passed over the bed to protect it from air and moisture.

An excellent *mattress* consists of a good large comforter folded three times endways, the width being about as long as a man's body. Additional wool blankets and a comforter should be taken for covering, the number depending upon the climate. *Sleeping-bags*, such as are now sold by dealers in sporting goods, furnish the warmest and most comfortable bed for almost any condition of camping.

Where rain is to be encountered a mackintosh or rubber coat is of little value. A heavy *oil-slicker* is the most serviceable garment; and for horseback, leggings and pea-jacket of oil-slicker. The most serviceable hat is a heavy, wide-brimmed soft felt sombrero or army campaign hat for all climates and conditions. The more intense the heat of the sun's rays and the more penetrating, as in the tropics, the heavier should be the head-covering. Under such circumstances a heavy pith helmet may be used; but, be helmet or heavy felt sombrero worn, a band made of light linen or India silk, folded to about three inches in width and of a length of about two yards, should be wrapped around the hat close to the brim so as to make a thick pad over the temples to keep out the penetrating rays of a tropic sun.

Rubber boots should never be used even in snow or water. In deep snow or intense cold, arctics or a wrapping of gunny-sack over the leather shoe may be employed. For climbing a *shoe* is much more comfortable and supple than a boot. For riding leather leggings may be added, or else water-proof leather boots may be used. In any event, in cold or wet and for heavy climbing water-proof leather shoes with thick extension soles should be worn (Fig. 203). In the tropics the foot-covering should be light and supple, but the soles should be heavy both to protect the feet from moisture and to keep out the heat of the soil. Light canvas shoes should be carried to rest the feet, which easily blister in tropic lands.

FIG. 203.—PLANE-TABLE STATION ON MOUNTAIN IN ALASKA.

Where much foot-work is done, very heavy, coarse cotton *socks* should be worn. In cold weather heavy woolen socks, and in intense cold and deep snow German felt socks, must be worn. As a substitute for leather boots and arctics, felt boots may be worn over the German socks.

In high altitudes the *underwear* should always be of heavy wool regardless of how high the temperature may be in the daytime. Medium-weight wool may be carried, and two suits be worn, one over the other, in very cold weather. The sudden changes at evening and night render heavy underwear an essential to health. In the tropics light silk gauze or a mixture of silk and wool underwear should be worn next the skin to absorb the moisture of the body.

For *sleeping-clothes*, pajamas only should be used, and in the tropics especially these should be of light flannel. Also in the tropics flannel *cholera-bands* should be invariably worn over the abdomen, and never removed except to change.

For work in the brush or woods the most satisfactory *outer garments* are made of brown duck, or light overalls may be pulled on over woolen trousers. In cold and windy weather, such as is experienced at high altitudes, flannel-lined hunting-coats and trousers of duck should be worn, the duck keeping out the wind. The best coat for wind protection is the blanket-lined leather hunting-coat. A canvas or leather hunting-coat, lined or unlined, is a most convenient garment for the surveyor because of its numerous pockets. In the cold and at high altitudes a woolen *sweater* should be carried or worn in preference to an overcoat (Fig. 203).

In addition to the above the novice in camping should not neglect to take towels, soap, and miscellaneous *toilet articles*. Where the party is to sleep in the open air, or when the weather is very cold, the head should be covered with a knit nightcap of worsted, the most satisfactory being the conical toboggan cap, which can be pulled down well over the ears and head.

The lack of a tooth-brush, even in the Arctics, has been known to produce sore mouth and gums in one accustomed to its use. Toilet-paper is essential, especially in extremely hot or cold climates, or piles may result. Camping is at best uncleanly, and every effort should be made to keep the person and the camp as clean as possible. Even then much dirt will have of necessity to be encountered.

377. General Hints on Care of Health.—In camping or working in high altitudes the topographer is liable to contract a disease known as *mountain fever*, which is allied to typhoid. It is caused chiefly by carelessness in becoming overheated under the hot rays of the midday sun and then suddenly chilling off in the night air. The precaution already described of covering the head at night and of wearing heavy woolen underwear, despite the intensity of the heat at midday, will generally suffice to protect the camper from any sickness in the healthful climate usually found at high altitudes.

In the *tropics* the traveler is liable to sickness from malarial fevers, dysentery, and cholera. With proper attention to food and clothing, if living a healthful outdoor life, one is hardly more liable in the tropics than elsewhere to contract other diseases than malaria if great care is exercised in carrying out the following suggestions:

Wear thick-soled *shoes* of soft leather, and change or dry these, going barefooted meanwhile if necessary, as soon after they become wet as practicable. In other words, do not keep wet shoes on the feet, and do not wear rubber to protect against moisture. If the body become wet from rain or fording streams, the clothes should be taken off and dried as soon as possible thereafter. The body should never be allowed to steam while covered with drying shoes or clothes.

Flannel *cholera-bands* should be worn at all times. Clothing worn in the daytime should invariably be changed at night for flannel pajamas. The *head-covering* must be of the heaviest, and the protection over the temples should be especially heavy. The topographer should not expose himself to the direct rays of the sun more than absolutely necessary, and where practicable should be shaded by an umbrella. The back of the neck should be shaded from the level rays of the early morning and late afternoon sun by a *cloth veil* hung from the back of the hat.

The camper should sleep in a *hammock* or on a *cot*. He should, if possible, never go to sleep wet or on wet ground, and when this is unavoidable he should endeavor to sleep in dry woolen blankets, or, if he must sleep in wet blankets, these should be of light wool and should be next his body. Above all, the head should always be protected from the *night dews* either by some temporarily improvised shelter, by covering with a sheet, or the canvas bed-cover, or mosquito-netting fine enough to keep out the moisture. He should avoid rising before the sun has dispelled the night dew. *Early rising* is very *dangerous* in malarious regions.

Where possible, *drinking-water* should always be boiled and allowed to cool. (Art. 378.) At work it is best to carry in the canteen boiled water or thin coffee or tea. Lime-juice should be freely used in water which is not boiled. Weak ginger tea made of a thin effusion of Jamaica ginger with a little sugar is a palatable and safe beverage, especially where the water is alkaline. Unless absolutely unavoidable, water which is standing in the sun, especially running water in shallow streams, should never be drunk without previously boiling or adding whiskey or lime-juice to it. Water should be kept shaded from the sun as far as practicable, and only water which has stood overnight to cool should be used if possible.

Water may be kept fairly cool in *canteens* throughout the day if they are heavily covered with one-half inch of woolen blanketing shielded outside by heavy canvas. This covering should be soaked in the morning, and as it evaporates it keeps the canteen water cool. When it dries off it should if possible be again soaked, perhaps several times during the course of the day. The covering should be omitted on the edges under the carrying-strap.

Heavy foods and *flesh foods* should be used sparingly. Fresh meats once a day and in moderate quantities should be eaten to keep up the system, but not more than one such meal a day should be consumed. Jerked or sun-dried meat, chipped beef, or the "carne seca" of Spanish America will not putrify under the most unfavorable circumstances, and make a palatable dish when stewed with canned corned beef, potatoes, onions, or other vegetables. Bacon may, in spite of the fact that it contains fat, be used once a day. Eggs should not be indulged in too freely. Cereal foods, as rice, cornmeal, and good bread, beans and peas, should be used freely.

Fresh fruit should be used most carefully and sparingly. It may be safely eaten in the morning providing it has been picked overnight and allowed to cool in the night air. It should never be eaten after ten o'clock in the morning, not only because of the heat of the body, but also because the fruit itself is hot. It is most dangerous when picked ripe from the tree in the hot sun. Not only over-indulgence but any indulgence in fresh fruit after the heat of the day has come on is most dangerous. Fruit which has been kept on ice or otherwise cooled may be eaten sparingly after sundown.

Excess in drinking or eating should be scrupulously avoided in all climates. Alcoholic liquors should never be indulged in, especially in the tropics, excepting for medicinal purposes. Prolonged immersion in bathing should be avoided in all climates, especially if the water be cold. A quick plunge or sponge-bath may be indulged in daily in early morning or late evening.

378. **Drinking-water.**—Nothing is more certain to secure endurance and capability of long-continued effort than the avoidance of everything as a drink except cold water, and at breakfast a little coffee. The less drunk of these on a long tramp the better, since one suffers less in the end by controlling the thirst, however urgent.

Poisonous matter of many descriptions may be taken into the stomach in *drinking bad water.* Dysentery and malarial diseases ensue from its use. With *muddy water* the remedy is to filter; with *putrid water*, to boil, to mix with charcoal, or expose to the sun and air, or, what is best, to use all three methods at the same time. With *salt water* nothing avails but distillation.

Sand, charcoal, sponge, and wool are the substances most commonly used in filtering muddy water. A small piece of *alum* or, better, *powdered alum* is very efficacious in purifying water from organic matter, which is precipitated by the alum, and left as a deposit in the bottom of the vessel. Above all, whenever there is the least uncertainty as to the quality of the water, *boil it*. Nothing is so sure a preventive of sickness in camping in warm climates as the exclusive use of boiled water for drinking.

379. **Medical Hints.**—As the topographer and the explorer are frequently so circumstanced as to be unable to promptly procure proper medical attention, and as the nature of their duties is such as to render them liable to certain classes of sickness and to violent injury, the following suggestions have been prepared for the emergency treatment of the sick and injured. In all cases of serious illness or of fractured limbs the best medical advice procurable must be sought at once, however far it may be necessary to seek it or to move the patient. In Article 384 is given a list of the most useful emergency medicines with their uses and size of dose.

Malarial Fevers.—These are, of all diseases, the most likely to be contracted in camping in semic-tropic and tropic regions. They should be treated by administering 15 to 20 grains of quinine before the expected attack. This should be preceded invariably at first by one to two compound cathartic pills. If the dose be given twelve hours previous to the renewal of attack, it will have better results. In malarial localities a tablespoonful of whiskey with 4 to 6 grains of quinine should be taken daily as a tonic. In severe cases of malaria there should be given, excepting in the hot stage, quinine in doses of 15 grains at intervals of eight hours. A Dover's tablet should be given every three hours with quinine in obstinate cases of malarial fevers. Wherever possible, even at the expense of suffering to the patient, he should be removed to a higher and dryer situation, if such be accessible.

Colic is treated by giving 1½ ounces of castor-oil with 20 drops of tincture of opium. Also it may be treated with 10 drops of essence of peppermint or a teaspoonful of Jamaica ginger in hot water. Hot turpentine fomentations should be applied to the abdomen, and 3 grains of calomel and soda may be given instead of the castor-oil.

For *Constipation* give compound cathartic pills, a saline purgative, as Epsom salts, or two tablespoonfuls of castor-oil. In obstinate cases, enemas of warm water with olive or castor oil or castile soap should be given, the patient meantime lying down.

For *Frostbite* moderate friction should be opplied to the parts affected. They should not be warmed until recovery is well advanced. Where snow is procurable, friction should be produced at first with this or with sponges dipped in ice-water. As the parts become warmer and less congested they should be encased in dry flannel or cotton wool.

In all cases of *Poisoning* vomiting should be at once encouraged. The simplest ways in which to induce it are by large draughts of lukewarm mustard-water, ipecacuanha, soapy water, or by tickling the throat from the inside. After this soothing liquids should be administered, as beaten raw egg, flour and water, or milk in large quantities. If the sufferer be much depressed and have cold hands or feet, and blue lips, some stimulant may be administered, preferably strong hot tea or coffee.

380. Diarrhea and Dysentery.—Errors in diet resulting in simple *Diarrhea* may be treated with a mild laxative of castor-oil or cathartic pills. A change of diet should be made to milk and well-boiled arrowroot. A glass of port wine and brandy with plenty of sugar and nutmeg may also be administered occasionally, and the patient be kept as quiet as possible. If diarrhea refuses to yield to the above, take 3 grains of calomel and soda at a dose. Should it occur after a chill or in localities where dysentery is prevalent, 20 to 30 drops of chlorodyne should be given, followed at bedtime by five Dover's powders.

Though one of the most feared of all tropical diseases, *Dysentery* yields quite readily to timely treatment. It is most commonly caused by sudden or prolonged chills, or results from bad drinking-water or food. Symptoms are diarrhea followed by irregular and shooting, griping pains, straining and discharge of mucus from the bowels. As the disease advances the pains are more distressing and the actions more frequent, the discharge being tinged with blood and of most offensive odor.

This is the first stage of the disease, the treatment of which consists of immediate rest in bed and turpentine fomentations on the abdomen followed by a large linseed poultice. A mustard-leaf should be placed on the pit of the stomach and 20 drops of tincture of opium in water be administered, followed by 20 to 30 grains of ipecacuanha powder mixed in water. Fluids should be abstained from to avoid vomiting. Repeat ipecacuanha powders twice at intervals of six hours, and give five to ten grains of Dover's powder at bedtime. The food during and for some time after the disease should consist of boiled milk, weak meat broths, and well-boiled arrowroot. Beef tea should be avoided as too heavy.

Where malaria is present 15 grains of quinine may be given in addition to the above. In advanced cases Dover's powders should be given instead of ipecacuanha. Diet is all-important in this dread disease, as the smallest particle of solid food may set up an irritation which will prove fatal.

381. **Drowning and Suffocation.**—Drowning may sometimes be of such duration as to cause natural breathing to cease. *Treatment* consists in the re-establishment of the action of breathing by means of artificial respiration. The body must be at once freed from clothing which binds about the neck, chest, and waist and be *turned on* the *face*, a finger being

thrust into the mouth and swept around to remove anything which may have accumulated there. Respiration may then be restored by Sylvester's method, which is as follows:

The body is *laid out flat* on the *back*, with a folded blanket, shawl, coat, or stick of wood under the shoulders, so as to cause the neck to be stretched out and the chin to be carried far away from the chest. The tongue is drawn carefully forward out of the mouth by holding it with a cloth.

Some one now places himself on his knees behind the head, *seizes both arms* near the elbows, and sweeps them round horizontally, away from the body and over the head until they meet above it; when a good, strong pull is made upon them and kept up for a few seconds. This effects an *inspiration*—fills the lungs with air—by drawing the chest-wall up and so enlarging the cavity of the chest.

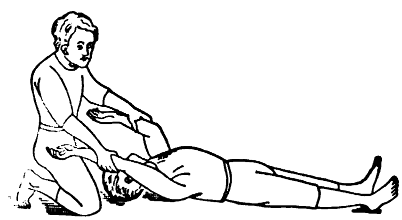


FIG. 204.—INDUCING ARTIFICIAL RESPIRATION.

The arms are now swung back to their former position alongside the chest, making strong pressure against the lower ribs, so as to drive the air out of the chest and to effect an act of *expiration*. This need occupy but a second of time.

If this plan is regularly carried out, it will make about sixteen complete acts of respiration in a minute. It should be kept up for a long time, until there is no doubt that the heart has ceased to beat or until natural respiration is re-established. The cessation of the pulse at the wrist amounts to nothing as a sign of death. Often life is present when only a most acute and practiced ear can detect the sound of the heart.

Respiration having been re-established, *stimulants* should be given as soon as they can be swallowed. A teaspoonful of whiskey in a tablespoonful of hot water may be given every few minutes until the danger point is passed. *Warmth* must be secured immediately by any means available, as hot bottles, plates or bricks, warm blankets and wraps. The body must be constantly and effectively rubbed, the direction of the rubbing being towards the heart to help the labored circulation of the blood. Meantime every effort should be continued to restore respiration. No attempt should be made to remove the patient, unless he be in danger from cold, until the restoration has been thoroughly accomplished.

382. **Serpent- and Insect-bites.**—The bites of *Poisonous Snakes* demand instant *cauterization or excision* of the injured part. A handkerchief should be fastened above the wound and a stick be passed through it and twisted to prevent the poisoned blood from moving towards the trunk and heart. It may be well at first to *scarify the wound* to enable it to bleed freely. Some one should then suck it. If practicable, the injured part may be soaked in hot water and squeezed to draw the blood out after incision. Immediate application of ammonia may be of advantage. The safest procedure of all is immediate excision of the part, or cauterization with a needle heated to redness.

Among *Insect-bites*, the most annoying are those of the *chigre*. The treatment must be applied immediately and before the insect lays its eggs. This consists in anointing the bites with a 10% solution of iodoform in collodion. Where the pest abounds, each individual should wear close-fitting leggings or top boots, and each day on returning to camp should bathe the whole body with salt water. Lime-juice. lemon-juice, kerosene oil, or salt pork rubbed over the infected parts of the body prevent the chigres from entering the skin by removing them.

383. **Surgical Advice.**—In cases of *Burns* or *Scalds* remove immediately with scissors all clothing about the injured

part. Then dress with sweet-oil, castor-oil, or sweet lard, but no oil containing salt should be used. Caron-oil, which is a mixture of linseed-oil and lime-water, gives the greatest relief.

In *Sprains* of all sorts, as those of the wrists or joints, the immediate effort should be to rest the tendons by covering the parts with cotton wool followed by a soft, firm bandage. Next, the inflammation should be allayed by the application of hot water; finally, the absorption of inflammatory products should be promoted by friction, kneading of the joint, careful motion of it, and alternate hot and cold douching.

Wounds or Clean Cuts should be treated by bringing the edges together after washing with antiseptic solution, and then supporting them in that position by long strips of adhesive plaster. These should not be applied to the wound, but first to one side of it and, drawing the flesh together, to the other side so as to bring the cut parts in contact.

Hemorrhage of Vein-blood should be treated by the elevation of the part and the application of cold water, ice, snow, salt, or vinegar. In addition to a severe application of cold, firm intense pressure should be applied *below* the wound, and this generally suffices to stop it.

Arterial Hemorrhage, known by the bright color of the blood and its spouting in jets, must be controlled from *above*, i.e., on the side towards the heart, and in the same manner as for venous hemorrhage, but by the application of firm pressure over the artery, if it can be located—which it frequently can by noticing its pulsations. Stimulants should not be given at all, or with the greatest caution, in case of hemorrhage, as they excite the circulation of the blood.

384. **Medicine-chest.**—No. 1. *Tincture of opium:* sedative. Dose, 10 to 30 drops in water, not to be repeated for six hours. In diarrhea, dysentery, pleurisy, colic, sleeplessness, etc.

No. 2. *Paregoric:* sedative. Dose, 15 to 60 drops in water. In colds, coughs, bronchitis.

No. 3. *Chlorodyne.* Dose, 5 to 25 drops in water. In seasickness, diarrhea, colic, cramps, spasms, neuralgia.

No. 4. *Turpentine.* For fomentations; to be sprinkled on flannels wrung out of boiling water and at once applied to the skin. In colic, dysentery, pleurisy, pneumonia.

No. 5. *Carbolic acid.* Used in solution and externally only: 1 part to 100 parts water to remove foul odors or to wash wounds; 1 part to 20 parts olive- or linseed-oil as an application to ulcers, to prevent attacks from insects, to destroy ticks, etc.

No. 6. *Olive-oil.* For use with above; also as a local application to burns, etc.

No. 7. *Opium pills,* one grain each. Dose, one pill. In diarrhea, rupture, spasms, colic, etc.

No. 8. *Dover's powder,* in capsules or tablets of five grains. Dose, one to two capsules. In bronchitis, coughs, colds, pleurisy, dysentery, fevers, etc.

No. 9. *Calomel and soda,* in one-grain compressed tablets. Dose, 1 to 5 tablets. In torpid liver, disordered stomach, liver congestion, pleurisy, diarrhea, etc.

No. 10. *Quinine,* in five-grain capsules. Dose, one to five capsules. In malarial fevers, etc.

No. 11. *Ipecacuanha powder,* in five-grain capsules. Dose, one to six capsules. In dysentery, especially in the premonitory or acute stages; also as an emetic after poisons.

No. 12. *Salicylate of soda* (purest procurable), in ten-grain capsules. Dose, one to eight capsules a day. For rheumatism of all kinds.

No. 13. *Vaseline.* For use as simple ointment.

No. 14. *Permanganate of potash,* in two-grain pills. For snake-bite, internally; as surgical wash or as a gargle for sore throat, one dissolved in a cup of water; also for snake-bite injected hypodermically close to the wound.

No. 15. *Adhesive plaster,* tape rolled in tin.

No. 16. *Mustard*, in tin, and *mustard-leaves*. For counter-irritation and as an emetic.

No. 17. Two clinical *thermometers* in cases. For certain detection of fevers when temperature is noted above 99° F. This invaluable but fragile instrument should be carried in duplicate in case of accident.

No. 18. Several long *cotton roller bandages*, various widths; a rubber bandage and two pairs of triangular bandages for fractures.

No. 19. *Borated lint and absorbent cotton*. For dressing wounds and sores.

No. 20. *Arrowroot*. As a food after fevers and dysentery and after violent vomiting.

No. 21. *Persulphite of iron*. Applied to wounds to stop violent hemorrhage.

No. 22. *Sun cholera tablets*. For use in cases of diarrhea, cholera, etc. Dose, one every two hours until three or four have been taken.

No. 23. *Extract of beef* (Liebig). For beef tea and broth.

No. 24. *Collodion* with 2½% salicylic acid. For insect-stings, skin eruptions, and corns, to be used as a paint.

No. 25. *Collodion* with 10% iodoform. To be painted on wounds as a dressing.

No. 26. *Carbolized vaseline*. For dressing wounds.

No. 27. *One hypodermic syringe*.

No. 28. One dozen *assorted surgical needles* and silk.

No. 29. *Styptic cotton*. For nose-bleed at high altitudes.

No. 30. *Iodoform*. For dressing wounds and sores.

No. 31. *Vegetable compound cathartic pills*. For torpid liver and constipation. Dose, one to three pills.

No. 32. *Linseed*. For poulticing boils, the abdomen, etc.

No. 33. *Castor-oil* in capsules. As a mild laxative or purgative. Dose, one-half to one fluid ounce.

No. 34. *Bichloride of mercury tablets*. For antiseptic wash for wounds and sores.

CHAPTER XLI.

PHOTOGRAPHY.

385. Uses of Photography in Surveying.—As a map record alone is insufficient to completely illustrate the results of an exploratory survey, requiring for the fuller understanding of the discoveries made a written report as an accompaniment, so also is such a report incomplete unless accompanied by illustrations (Chap. IV). A military reconnaissance must likewise be accompanied by a report, and this is made more comprehensive, and is often more rapidly and lucidly prepared, when illustrated by sketches or photographs (Chap. V).

The present stage reached in the development of the science of photography is such that any one possessing the qualifications necessary for the execution of an exploratory, geographic, or military reconnaissance could easily acquire the skill necessary to make photographs for the proper illustration of the accompanying report. The varieties of work to be executed under such circumstances are many. They include chiefly outdoor or landscape photography, but in addition must frequently be accompanied by illustrations of the inhabitants, the fauna and flora, as well as details of the geology of the regions traversed. Finally, photography is now employed quite extensively in the making of topographic surveys (Chap. XIV), and may be used in determining longitudes (Chap. XXXVII).

386. Cameras.—There are two general types of photographic cameras:

1. The hand camera;

2. The tripod camera.

The former are of three general kinds, namely:

a. Those with lenses having universal focus.

b. Those which require to be focused by means of a scale attached to the camera; and

c. Those which, in addition to being used as hand cameras, may be mounted on tripods and focused as are stand cameras.

The *stand camera* is mounted upon a tripod by which it must always be leveled when used. It is provided with a ground glass on which the image sighted can be focused by means of a ratchet motion and the bellows attachment of the lens. For general exploratory uses the extension bellows should be of red Russia leather, as red ants will not eat this. According as the rear or front end of the camera is moved by the rack motion the camera is said to have a front or back focus. In addition it should be possible to move the lens vertically through a short space in order to take in images which cannot be reached without tilting the camera out of level, which should never be done. The lens and the plate-holder should both tilt a little to aid in seeing objects without the range of view.

Hand cameras are made in all sizes, from those carried in the pocket, which take pictures 2½ by 3½ inches, up to those which take pictures 5 by 7 inches and are provided with all of the refinements which permit of doing the best tripod work. Hand cameras, however, are rarely provided with a sufficiently fine lens for the highest grade of photographic landscape work. For this reason and because the larger hand cameras have to be focused and are therefore not so handy nor so satisfactory either in manipulation or results as the smaller hand cameras of universal focus, the best camera outfit for the explorer would be a small fixed-focus hand camera and a first-class tripod camera. Such a tripod camera should be capable of taking pictures either of 5 by 7 or of $6\frac{1}{2}$ by $8\frac{1}{2}$ inches, and should be provided with the best combination wide and narrow angle lenses procurable.

For *photographic surveying* the relative positions of plate and lens in the camera must be invariable, and when adjusted the plate must be exactly vertical. Accordingly but few of the makers are able to supply suitable cameras where the work is to be accurately executed. Nearly any camera may be readily adapted to the work of reconnaissance surveying by photographic methods when there is oriented upon it a compass for directions and cross-hairs or needle pointers to fix initial directions.

A scale showing the position of the lens for focusing on objects at various distances should be applied to all stand cameras in the same manner as it is applied to high-grade hand cameras. All good tripod cameras should in addition have the position of the universal focus and the corresponding stop number, as F 16, etc., marked on the focusing-rack. Where the camera is provided with ground glass and opportunity permits, this should always be used in focusing.

The tripod camera requires a *focusing-cloth* to be thrown over the head of the operator when focusing on the ground glass, thus producing for him a small dark room in which to observe the glass. This focusing-cloth should be of rubber or of stout black baize or similar material, having a quantity of leaden shot sewn in a hem around the edges to prevent its being blown about in the wind. An additional ground glass should always be carried in the field as a precaution against breakage. Celluloid covered with ground glass substitute is useful in case of breakage. If a little powdered emery be carried, a glass plate may be cleaned off and the same quickly turned into ground glass by rubbing with emery and cloth.

387. **Lenses and their Accessories.**—For the ordinary purposes of photography there are three classes of lenses, namely:

1. Portrait lenses;
2. Landscape lenses;
3. Copying lenses.

Portrait lenses require a large aperture compared with the focal length, so as to admit a large volume of light in the subdued atmosphere of a room. They are aplanatic; that is, they can be used without a stop. They have little depth of focus, narrow field, and great rapidity.

Copying lenses must be achromatic and anastigmatic. They should be rapid and rectilinear.

The type of lens best suited to the purposes of the explorer or surveyor is that which may be used for general view-work, where great flatness of field is unnecessary and distortion must be a minimum.

There are two general classes of *landscape lenses*, namely:

1. Single achromatic lenses;
2. Combination lenses.

The *single lens* is that most used in instantaneous hand cameras, and is of the achromatic converging-meniscus type. The flatter the lens the more rapid. The defects of such lenses are, distortion of the image, moderate angular view and slowness, but when used with small stops they have depth of focus and produce crisp negatives. They are nonaplanatic and must be used with stops. The defects of the single landscape lens are largely corrected by the *combination* lens, which possesses many of the best qualities of the copying lens. *Combination wide and narrow angle lenses*, now made for the best landscape work, will take wide-angle views with the two glasses, and narrow-angle views with one glass removed.

Distortion as applied to lenses is due to the greater refraction of the rays from the margin of the lens towards the axis. This defect is most pronounced in single lenses without diaphragms. *Aberration* is due to the impossibility of obtaining a good definition in attempting to focus on the ground glass. The image appears as a circular patch of light, decreasing in intensity from the center to the edges. It increases as the square of the aperture of the lenses, and inversely as its focal length. It is of two kinds: 1. Lateral; and 2. Longitudinal. As aberration increases so rapidly with the aperture, *stops* or *diaphragms* are employed to reduce it. Their effect is to cut off the marginal rays. In use the camera is focused with a large diaphragm, and a smaller one is employed in making the exposure. In a single lens diaphragms are placed in front. In combination lenses they are placed between the lenses. As the brightness of the image depends upon the quantity of light admitted by the diaphragm, it is proportioned to its *aperture* or the square of its diameter. The larger the aperture the more light admitted. Brightness further varies inversely as the square of the focal length. Thus by doubling the focal length the dimensions of the image are doubled and the light admitted is distributed over four times the area. The brightness of the image is reduced in proportion.

For the highest type of *photographic surveying* the *lens* must be (1) rectilinear; (2) free from distortion; (3) it should cover an angular field of about sixty degrees, and (4) the definition should be uniform over the entire plate. Slowness is preferable to rapidity in order to get strength of shades and definition. For these purposes what are known as wide-angle lenses must be employed. They are doublets of two lenses between which is placed the diaphragm. For photographic surveying on the Canadian Government a Zeiss anastigmatic of F 18 aperture and 14-millimeter focus is preferred. Such doublets consist of an achromatic interior in which the flint has the higher refractive index. Among rapid rectilinear anastigmatic combinations are those made from Jena glass.

When attempting panoramic view-work from the summit of a mountain it becomes occasionally desirable to take a more detailed view of some object in the field. This cannot be done without approaching more nearly to the object with the camera. There is, however, an attachment called a *tele-photo combination* which consists in the addition of a negative or magnifying element in the rear of the combination proper. This produces larger images of distant objects, but it must be remembered that by reducing the light it necessarily reduces the rapidity of the combination lens.

The lenses supplied with cameras have, as a part of their construction, some kind of *shutter* for making both time and instantaneous exposures. These are operated by cylinders in which a piston is actuated by compressed air from a hand-bulb. The pressure on this bulb opens and closes the shutter. The best shutters are so automatic in their construction that by setting a pointer to the required speed in seconds or fractions of a second, as marked upon it, the proper time is given by the shutter upon applying pressure to the bulb, or it sets the shutter in the time exposures.

388. Dry Plates and Films.—The effort in all good photographic work is to obtain results full of detail and clearly defined. The most difficult operations in photography are the procuring of clearly defined landscape views because of the character of the light diffused by the atmosphere. As a result aerial perspective is much exaggerated as produced by photography, because of the strong actinic effect of the blue haze through which distance is seen and which speedily blurs out the details of the image. The presence of smoke or dust in the air contributes to the same result.

This effect can be eliminated to a certain extent by selection of dry plates especially suited to such work. Ordinary plates are sensitive to the blue and violet rays only. *Ortho-chromatic* or *isochromatic* plates are manufactured which are acted upon by the colors at the other end of the spectrum, although the maximum sensitiveness is still to blue and violet. By using a *screen or ray filter* of orange or reddish-orange glass it is possible to exclude the greater volume of the light rays other than the green, yellow, and red, and such a screen in connection with the orthochromatic plates partly solves the difficulties of photographing through haze. All that is required in such a plate is that it be especially sensitive to other rays than blue and violet, because these are largely cut off by the screen.

The proportion between direct sunlight and skylight varies with the altitude of the sun and with the absorption of the atmosphere. Shadows look more intense when the sun is high than when it is low. Accordingly, on mountains and in general landscape view-work at high elevations the contrast is greatest because the atmosphere is very light and the coefficient of absorption proportionately small. For this reason good photographs of mountain scenery are scarce, and also because of the wide contrast ranging from snow and sunlight to dark woods and shade. In such work satisfactory results can only be expected when orthochromatic plates or color screens are used.

When a subject presenting strong contrast is given long exposure to the action of the light, the image appears to spread upon the plate. The edges of the high lights merge into the shadows by a gradually decreasing tint, according to the intensity of the light and the length of the exposure. This is called *halation* and is due to the light which has passed through the film, striking the rear surface of the glass plate and being reflected by it on the back of the film, causing there a halo. The remedy of halation is to stop the light when it reaches the back surface of the plate by coating the latter with some non-actinic material which will absorb light. Any kind of opaque material will not do. The coating must be in optical contact with the glass, and the refractive index of the coating must be the same as that of the glass. Such a coating is produced by painting the back of the glass with a solution of fine lampblack mixed in sandarac dissolved in alcohol. Nonhalation produced by such a coating is rarely necessary in ordinary photography, providing one uses dry plates which are *orthochromatic*. Where, however, it is necessary to photograph towards light, as viewing in the direction of the sun or towards electric lights at night or their reflections, even the best grades of orthochromatic nonhalation plate may be reinforced by the aid of the nonhalation backing. Before developing the plate so backed, the lampblack must be washed off with alcohol.

The *isochromatic nonhalation* plates furnished by the dealers have been made nonhalation by coating them with two or three layers of the silver emulsion. Thus the Seed nonhalation plates are given a coat of emulsion of 23 sensitometer test followed by a second coat of emulsion of 26 sensitometer test. Wuerstner triple-coated isochromatic nonhalation plates have three coats numbered 1, 2, and 3. The principle on which these work is that the light coming from the blue and the violet rays makes its way more quickly through the first coating, and is impressed upon the second or third coating at about the same time that the light coming from the less rapid rays reaches the first or second coating.

Where the best work is attempted in photographing distant views, dry plates prepared as above must be used. For the general purposes of the photographer, explorer, or military photographer, however, where the effort is merely to procure a good record of objects seen, the most satisfactory plates are the simple single emulsion plates without isochromatic or nonhalation character. These may be of glass or on *cut celluloid films*. The latter give excellent results for all practical purposes of the amateur photographer. They will not break in transportation, are less heavy and bulky than glass, and are in every way more satisfactory where compactness of outfit is an item. For use in the instantaneous hand camera *roll films* should be used. With the larger sizes of hand cameras it is not possible to stretch the film sufficiently, and parts of it are thus out of focus, due to wrinkles in it. The best cut films are now prepared with isochromatic nonhalation coatings, and give results almost equal to the best glass dry plates.

389. **Exposures.**—The best results in exposing plates in photographic work are to be procured by using slow plates. This gives sufficient time to bring out the deep shadows, and there is less likelihood of error in properly timing with slow than with fast plates. Where the hand camera is used instantaneous exposures should be made whenever the light will permit, the best results only being had with such a camera by rapid work. Where landscape work or panoramic work from an eminence is to be done, or where detail of a strongly marked object is to be brought out, the tripod camera and slow exposure should be used.

For a plate having a speed which, under ordinary conditions, requires one-fifth of a second for exposure, a small fraction of a second overtiming or undertiming affects the plate more seriously than in the timing of a five-second plate. In the latter case a fraction of a second under timing or overtiming is of small moment. In exposing a slow plate it is best always to err in the direction of overtiming, a little more time doing less harm than undertiming.

The exposure to be given a plate is inversely proportional to the intensity of the light illuminating the object. A subject requiring an exposure of ten seconds with the intensity of light taken as one, will require an exposure of five seconds with an intensity of two. The light received by a landscape in direct sunshine consists of: 1, Direct rays of the sun; 2, The light diffused by the sky. As a result there is considerable change in the exposures required at the same time of day at sea-level and at great altitudes. It has been found that there is little change in the exposures required at great elevations until the sun approaches the horizon. According to Mr. E. Deville, taking the exposure with the sun at the zenith as one second at sea-level, the exposure at 10,000 feet altitude will be a trifle under one second. With the sun at 40 degrees altitude at sea-level, one and one-fourth seconds will be re-

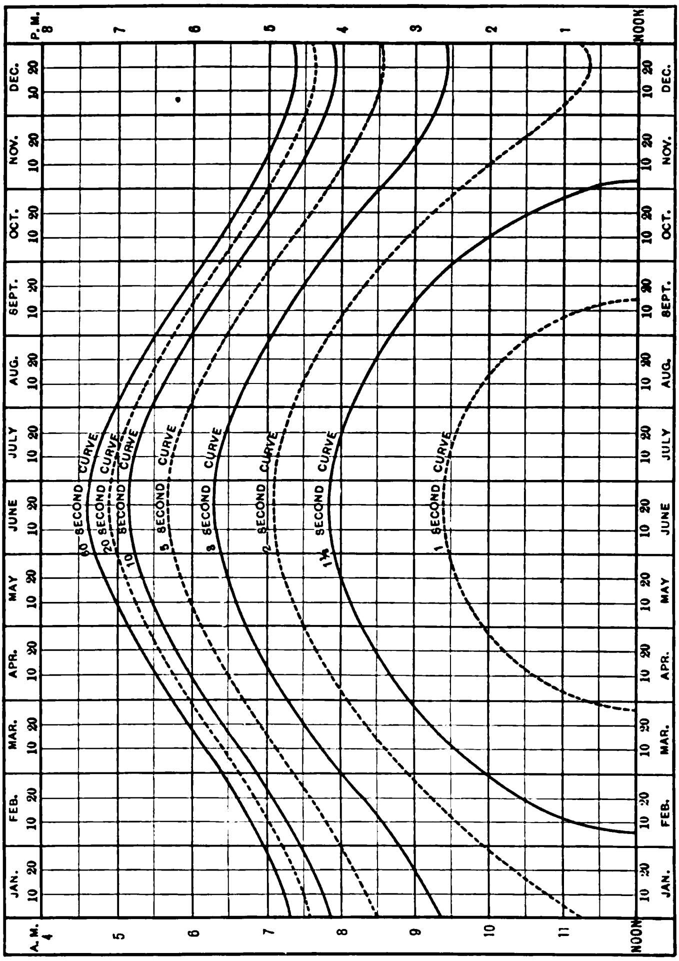


FIG. 205.—DIAGRAM SHOWING RELATIVE EXPOSURES AT DIFFERENT TIMES OF DAY AND YEAR.

quired, whereas at 10,000 feet altitude one second will still be sufficient. As the altitude of the sun decreases the difference is rapidly accentuated. At 25 degrees altitude one second is required at 10,000 feet, two seconds at sea-level;

at 15 degrees altitude one and one-fourth seconds at 10,000 feet, three and one-half seconds at sea-level.

Fig. 205, from Lieut. Reber's "Manual of Photography," shows how the photographic intensity of daylight varies with the time of day and of year. Table LXXII, also taken

TABLE LXXII.

RELATIVE TIMES OF EXPOSURE FOR DIFFERENT STOPS AND SUBJECTS.

Stop.	Open Landscape.	Landscape with Heavy Foliage in Foreground.	Under Trees.	Fairly Lighted Interiors.	Badly Lighted Interiors.
F 8	0.25	1.50	150	150	1440
F 11	0.50	3.00	300	300	2880
F 16	1.00	6.00	600	600	5760
F 32	4.00	24.00	2400	2400	23040
F 64	16.00	96.00	9600	9600	92160

from Reber, shows the times of exposure required with various sizes of stop in different subjects. This is arranged on the basis of the Carbutt B 16 plate with F 16 stop and normal exposure on open landscape on a day when three seconds of time is sufficient. Such an exposure is taken as unity. Accordingly, opposite F 8 in the table, under badly lighted interiors, the exposure 1440 should be multiplied by 3 seconds, the time given the unit Carbutt B 16 plate. As a result it is found that the time required under these changed conditions will be 1 hour 28½ minutes. If, in experimenting with another and faster plate, as Carbutt Eclipse 27, one-fourth of a second is required, the proper exposure with the F 64 stop on the landscape with foliage would be 96 multiplied by ¼ second, or 24 seconds. Thus by reference to this table and by making one experimental exposure the operator may know what exposure to give under different conditions. He must also keep in mind, however, the time of day and season of the year as related to that at which the experimental exposure was made. Then by reference to the diagram he will have a fair idea of the time required.

Referring now to the diagram (Fig. 205), if the three-second exposure taken above as unity were made at 5 P.M. in March or August, the same subject and plates would require but two seconds at 5 P.M. in June. It would require 60 seconds at the same hour in November or at 7 A.M. in the months of March, June, or August. Again, an exposure made at 7 P.M. in June would require only as much time as one made at 5 P.M. in November.

Much depends upon the *coloring and brilliancy of* the *object*, especially whether there is much green, yellow, brown, or red in it. Distinct views require less time than less clearly defined ones. Exposures are often made only for distance, others for foreground only. For deep shadows long exposure is required.

390. **Developing.**—Before developing a plate it should be cleaned with a camel's-hair brush to remove dust from the surface which would otherwise produce pin-holes in the negative. The plate should also have been dusted before placing it in the plate-holder before exposure. Where an effort is made to obtain the best results, only filtered or boiled *water* or water from melted ice should be used. The developing solution should be at a temperature of from 65 to 70 degrees, and must never be warmer than this. When water is warm enough to cause the emulsion to frill after development, the plates will be greatly helped by first flowing over the surface, previously wetted with water, a strong solution of *alum water*. The negative should be handled only by the edges, great care being taken not to touch the film side.

Before placing in the developer the plate should have *water flowed* over it to thoroughly wet it, and should be placed *emulsion side up* in the developing tray. The developer should then be flowed quickly back and forth over the surface of the film by a sweeping movement, so that no air-bubbles shall collect on the surface. The wet surface assists the developer in spreading evenly. The *rocking motion* given the tray should be continued throughout the process of development. In developing *roll films* these should be laid in the tray and then held under the faucet and the surface of the film be swabbed with a moist camel's-hair brush or a bunch of absorbent cotton. The tray should then be drained and the developer be applied. This is not necessary, however, with cut films.

The high lights in the negative should begin to show in 20 seconds. If they appear too quickly, the plate has been *over-exposed* and the action of the developer should at once be checked. For this purpose, pour off the developer, and if necessary wash the plate with a little water. Then begin again with a developer to which has been added 10 drops of a 10 per cent solution of a *potassium bromide*. If, on the other hand, the image does not appear in 30 seconds, the plate is probably *under-exposed*. It is more difficult to bring out the detail in an under-exposed plate than to check the development of an over-exposed plate. The details in the under-exposed plate may be helped by the use of a little more alkali, and the density of the negative may be helped by more reducing agent. *Long soaking in weak or old developer* is the best treatment. When the high lights develop properly and then thicken before the details come out in the remainder of the plate, the developer should be diluted with two or three times its volume in water, or some bromide solution be added, thus permitting the development to proceed slowly. It may also be necessary to leave the plate in a weakened developer for a half-hour in order to properly bring out the shadows.

Developing solutions can be purchased of all dealers. For easy and safe transportation they are now put up in tablet form by Wyeth & Co. of Philadelphia. If one desires to prepare his own developer, the following are recommended:

The more common developer, and perhaps the most valuable, is the *pyrogallic acid solution.* This will not keep well, however, excepting in well-corked bottles, and when old stains the negative yellow. For the best work fresh developer should be mixed for every two or three plates, according to the following formula:

	Crystallized sulphite of soda........	120 grains
	Crystallized carbonate of soda.......	60 "
(Or	Dry granular carbonate of soda......	30 ")
	Carbonate of potassium............	30 "

Then add 10 grains of Schering's or Merck's pyrogallic acid and 10 minims of a 10 per cent solution of bromide of potassium.

A *pyro solution which will keep* for a long time fairly well is the following:

PYRO DEVELOPER.

	Distilled or ice water............	10 ounces
	Oxalic acid......................	15 grains
(Or,	Sulphuric acid..................	15 minims)
	Bromide of potassium............	30 grains

Then add one ounce of Schering's or Merck's pyrogallic acid and enough water to make 16 fluid ounces.

An *alkali solution for* helping out the details of *under-exposed negatives* is the following:

ALKALI SOLUTION.

Distilled water..................	16 ounces
Sulphite of sodium, crystals........	4 "
Carbonate of sodium, crystals......	2 "
Carbonate of potassium...........	1 "

A *developer which keeps better* than the pyro and is less liable to stain when old is the following:

EIKO CUM HYDRO DEVELOPER.

SOLUTION NO. 1.

Distilled water	32 ounces
Sulphite of sodium, crystals	4 "
Eiconogen	330 grains
Hydrochinon	160 "

SOLUTION NO. 2.

Distilled water	32 ounces
Carbonate of soda, crystals	2 "
Carbonate of potassium	2 "

For instantaneous exposures take 4 ounces of water, 1 ounce of No. 1 and 1 ounce of No. 2; for normal exposures on rapid plates, 3 ounces of water, 1 ounce of No. 1 and ½ ounce of No. 2; for normal exposures on slow plates, 4 ounces of water, 1 ounce of No. 1 and ¾ ounce of No. 2.

391. Fixing.—After the negative has been developed it must be washed, preferably under the faucet, until all the developer has been removed. It is then placed immediately in a *clearing and fixing bath*, which may be prepared as follows:

SOLUTION A.

Warm distilled water	48 ounces
Hyposulphite of soda	16 "

SOLUTION B.

Crystallized sulphite of soda	2 ounces
Warm distilled water	6 "

Add 1 dram of sulphuric acid to 2 ounces of water and pour into B solution. This latter mixture should then be added to the hyposulphite or A solution. Before using the fixing-bath dissolve 1 ounce of chrome alum to 8 ounces of water and add this to it. This fixer will last a long while and may be used over and over. It both clears and hardens the negative. An inferior fixing-bath consists simply of hyposulphite of soda dissolved in 4 parts of water.

The *negative should be left in* the *fixer* about five minutes after the white milky bromide of silver has entirely disappeared from the film. Then it should be washed for three-quarters of an hour in running water and placed in a rack to dry. If for any reason rapid drying is necessary, this may be accomplished by flowing methyl alcohol or wood alcohol over the negative two or three times, which will take up the water. In *hot weather* a bath consisting of a *solution of alum* in water should be used both before and after fixing. If the negative is not sufficiently dense for printing, it should, after thoroughly washing the last traces of hyposulphite, be placed in an intensifying solution.

A *good intensifier* is a weak solution of equal parts of mercuric chloride (corrosive sublimate) and chloride of ammonium, and this should be flowed over the plate until its surface is slightly chalky; the longer the solution is used the denser will the plate become. Afterwards it should be washed with water, then with a weak solution of chloride of ammonium and, after being thoroughly washed, immersed in a bath of 10 minims of strong ammonia to each ounce of water until the plate blackens throughout, when it should be washed and dried in a rack. To *diminish* the *density* of an overdeveloped negative, treat the plate with one part of saturated solution of potassium ferricyanide mixed with ten parts of 10 per cent solution of hyposulphite of soda.

The following tabular arrangement from Reber is an index to the various causes of defects in negatives:

Defects in Negatives.	Cause.
1. Fog.	Over-exposure; white light entering camera or dark room; unsafe developing light; old and decomposed developer; silver nitrate or hyposulphite of soda in developer; developer too warm; too much alkali and not enough bromide in developer.

Defects in Negatives.	Cause.
2. Weak negatives with clear shadows.	Under-development.
3. Strong with clear shadows.	Under-exposure.
4. Weak negative with details well out in shadows.	Over-exposure and incorrect development.
5. Too much density.	Developer too strong or too warm, or too long applied.
6. Fine transparent lines.	Using too stiff a brush in dusting plates, or slide of plate-holder rubs against the surface of the plates or films.
7. Round transparent spots.	Air-bubbles on plate during development, or defects in emulsion.
8. Pin-holes.	Dust or muddy water.
9. Yellow stains.	Old developer or washing insufficient to eliminate hypo.
10. Mottled negatives.	Precipitation from old hyposulphite bath containing alum.
11. Crystallization on negative.	Imperfect elimination of hypo.
12. Halation.	Reflection into emulsion by the glass back of the light transmitted through emulsion. May be prevented by coating the back of negative with a black wash, or by using an emulsion of such thickness as to absorb all light falling on it.

392. **Printing and Toning.**—Various *papers* for printing can be procured of photographic-supply dealers. Special papers, as bromide paper for dead-black prints or platinum paper for black or sepia prints, are accompanied by full descriptions for printing. The common photographic print is made on silver or albumen paper, and also can be obtained more cheaply and satisfactorily from the manufacturers than it can be made, and it also is accompanied by descriptions of the method of printing.

It is better to *varnish the negative* before printing in order to prevent scratching or otherwise injuring the film. The following is a good, tough, hard and durable varnish:

NEGATIVE VARNISH.

Shellac........................	1½	ounces
Mastic.........................	½	"
Oil of turpentine................	½	"
Sandarac.......................	2½	"
Venice turpentine...............	½	"
Camphor.......................	20	grains
Alcohol........................	20	ounce

To *print*, the negative is placed in the printing-frame, film side up, and back of this is placed a piece of sensitized paper of the same size as the negative, with the silver side down or facing the negative. The whole is backed by blotting-paper to fill the frame, which is at once closed and stood on edge in such manner as to expose it to the direct rays of the sun. The printing should continue until the shadows bronze out well, the operator keeping in mind that the print will be less strong when toned and fixed. In examining the print this should be done only in subdued light, great care being taken to raise only one-half of the back at a time and not to let the negative or paper slip. Where the best class of work is attempted the printing should not be done under the direct rays of the sun, but under ground glass on a cloudy day, or in subdued light, thus procuring softer results.

After removing the prints from the frames they should be kept in a dark box until toned, which should be done within 24 hours. Prints are not fixed immediately after printing because of the disagreeable reddish color produced. To obviate this, where pleasing effects are desired, the print is first *toned* by placing it in a solution of chloride of gold, the salt of which comes in very small sealed bottles and is best kept by dissolving it in water in proportion of one grain to the ounce.

To get good purple and black colors immerse in a toning-bath of 9 ounces of water to 1 ounce of gold solution neutralized by a little carbonate of sodium until the bath is alkaline, as shown by the testing-paper. A rich warm tone recommended by Reber is 1 ounce of gold solution to 30 grains of acetate of sodium in 8 ounces of water. The longer the prints remain in this bath the browner the tone. The most satisfactory results are procured by preparing the bath at least a day before using.

Before toning prints they should be *washed* face downward by laying them in a basin of water until there is no trace of cloudiness in the water, each print floating separately by itself. The prints should then be laid face up in the *toning-bath* separately, the separation being produced by dropping the prints one at a time so that there shall be a layer of liquid between them. The tray should then be given a gentle rocking motion until the toning has progressed far enough, when the prints should be removed and placed face downward in the water to stop the action of the toning solution. A little salt added to the water stops the action more quickly and prevents the tendency to blister.

After toning and washing, the prints should be put in the *fixing-bath* of 1 part of hyposulphite of soda to 4 parts of water, in which they should be left for 15 minutes. When fixed the whites will appear colorless and the shadows be free from red spots. The fixing-bath will be improved by a dram of ammonia added to each 10 ounces of water, as the ammonia increases the speed of fixing and prevents blistering. *After fixing*, the prints should be *washed* in running water for several hours to remove every trace of hyposulphite, which would otherwise cause the prints to lose brilliancy and fade. There can now be procured of all dealers a *self-toning paper* which has only to be put in water and hypo. This paper is especially convenient in getting proofs in the field.

The following table is given by Reber as indicating the defects in prints and their causes:

Defects in Prints.	Causes.
1. Small white spots with black center.	Dust on paper.
2. Gray starlike spots.	Inorganic matter in paper.
3. Bronze lines, if straight.	Line of stoppage during floating of paper.
4. Bronze lines, curved.	Scum on sensitizing-bath.
5. Marbled appearance of print.	Baths too weak or not floated enough.

NOTE.—3, 4, and 5 refer especially to albumen paper.

Defects in Prints.	Causes.
6. Red spots on prints, especially in shadows.	Marks caused by moist fingers coming in contact with paper.
7. Weak prints.	Weak negatives.
8. Harsh prints.	Harsh negatives.
9. Too red a tone.	Undertoning.
10. Cold blue tone.	Overtoning.
11. Streaky prints.	Acid toning-bath.
12. Whites appear yellow.	Imperfect washing; imperfect toning; not long enough fixing.
13. Yellow spots when dry.	Imperfect elimination of hypo.
14. Prints refuse to tone.	Gold exhausted from toning-baths, or there is hypo in separate toning-baths.
15. Dark, mottled appearance in body of paper.	Improper fixing in too strong a light.
16. Blisters.	Saline solution between emulsion and paper. Can be prevented by salting the first wash-water.

393. **Blue-prints and Black-prints.**—Both blue- and black-print paper can be purchased of dealers in draughting and photographic supplies. Blue-print paper can be made readily, however, by floating close-grained drawing-paper in a bath of 1 part of ammoniocitrate of iron and 2 parts of water, to which is added 1 part of ferricyanide of potassium dissolved in 4 parts of water. This bath must be kept in the dark and used immediately. The paper may then be removed and thoroughly saturated and hung up to dry by spring clothes-pins. Blue-prints are printed by placing the tracing over the blue-print paper and exposing it to direct sunlight. Printing should continue until the surface is well bronzed or blue,

according to the paper used, when it is at once placed in an abundance of flowing water and washed until free from all blue or yellow. The blue-prints are then either hung up to dry or, better still, placed between blotting-paper.

Black-print paper, or that which produces black lines on white paper, may be prepared by immersing drawing-paper in the following solution:

Water	9 ounces
Gelatin	3 drams
Solution of perchloride of iron (U. S. Ph.)	6 drams
Tartaric acid	3 drams
Ferric sulphate	3 drams

After printing develop in the following solution:

Gallic acid	6 drams
Alcohol	6½ ounces
Water	32 ounces

Then wash and thoroughly dry.

Another formula for the same is that given by Mr. B. Howarth Thwaite, and is as follows:

1.	Gum arabic	12 drams
	Water	17 ounces
2.	Tartaric acid	13 drams
	Water	6 ounces
3.	Persulphite of iron	8 drams
	Water	6 ounces 6 drams

The paper to be prepared by immersion, separately in 1 and 2, and to be developed in 3.

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